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Chance Constrained Multi-Objective Linear Plus Linear Fractional Programming Problem Based on Taylor's Series Approximation

Surapati Pramanik[1], Durga Banerjee[2]
[1]*Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, P.O.- Narayanpur, District – North 24 Parganas, Pin Code-743126, West Bengal, India*
[2]*Ranaghat Yusuf Institution,Rathtala,P.O.-Ranaghat,District-Nadia,Pin Code-741201,West Bengal, India*

***Abstract*—**In this paper, we present fuzzy goal programming approach to solve chance constrained multi-objective linear plus linear fractional programming problem based on first order Taylor's series approximation. The chance constraints with right hand parameters as random variables of known distribution are converted into equivalent deterministic constraints by using the given confidence levels. In the model formulation, we first determine the individual best solution for each objective function subject to the deterministic system constraints. The objective functions are then transformed into linear objective function at their individual best solution point by using first order Taylor polynomial series. We then construct the membership function for each linearized objective function by considering the individual best solution of linearized objective function as fuzzy goal and using pay-off matrix. We formulate three fuzzy goal models for solving the problem by minimizing the negative deviational variables. To determine the best compromise solution obtained from the three fuzzy goal programming models, we use Euclidean distance function. To demonstrate the proposed approach a numerical example is solved.

***Keywords*—**Chance-Constraints, Euclidean Distance Function, Fractional Programming, Fuzzy Goal Programming, Linear Plus Linear Fractional Programming, Multi-objective Linear Fractional Programming Problem, Taylor Series.

I. INTRODUCTION

In the Management Science, there are numerous decision making problems where the objective functions are the sum of linear and linear fractional functions. This type of functions can be found in game theory, portfolio selection, inventory problems, agriculture based management systems.

Multi-objective linear fractional programming problem (MOLFPP) with system constraints is a prominent tool for solving many practical decision making problems. MOLFPP can be extended in the field where the objective functions are the sum of linear and linear fractional functions and the problem is then called Multi-objective linear plus linear fractional programming problem (MOLPLFPP). MOLPLFPP can be converted into multi-objective linear programming problem (MOLPP) by using variable transformation method or Taylor's series approximation method. Charnes and Cooper [1] proposed variable transformation method to solve MOLFPP. MOLFPP can be solved by Bitran and Noveas [2] by adopting the updating objective functions method. Kornbluth and Steuer [3] studied goal programming approach for solving MOLFPP. In the goal programming approach the target goals are stated precisely. But in many real life decision making situations, it is impossible to describe the target levels accurately due to imprecise nature of human judgement, availability of resources. To overcome these difficulties, Luhandjula [4] proposed linguistic variable approach based on fuzzy sets to solve MOLFPP. Luhandjula's linguistic technique was modified by Dutta et al. [5]. Dutta's approach was further modified by Minasian and Pop [6]. Interactive approach for solving MOLFPPs with block angular structure involving fuzzy numbers was studied by Sakawa and Kato [7]. With the help of variable transformation method, Chakraborty and Gupta [8] converted the original MOLFPP to MOLPP using fuzzy set theoretic approach. Taylor's series approximation method is another approach to linearize MOLFPP to MOLPP. Guzel and Sivri [9] used Taylor's series approximation method for MOLFPP. Toksarı [10] developed Taylor series approach for dealing with MOFLPP in fuzzy environment. In the recent past Pramanik and Dey [11] studied MOLFPP in fuzzy environment. They [12] also studied priority based FGP for MOLFPP.

Teterav [13] introduced linear plus linear fractional programming problem (LPLFPP) and also introduced optimality criteria for it. In 1977, Schaible [14] made a note on the sum of a linear and linear fractional function. In 1993, Chadha [15] developed a dual of the sum of linear and linear fractional programming. Chadha's approach was modified by Hirche [16]. Jain and Lachhwani [17] studied LPLFPP under fuzzy rules constraints. In the recent past, Jain et al. [18] introduced a solution method for MOLPLFPP containing non differential coefficients. Jain and Lachhwani [19] developed MOLPLFPP with homogeneous constraints using fuzzy approach. Recently, Singh et al. [20] developed an algorithm for solving MOLPLFPP with the help of Taylor's series. Singh et al. [21] also studied FGP approach to solve MOLPLFPP. Pramanik et al. [22] developed FGP model for solving MOLPLFPP based on Taylor's series approximation.

In this paper, we consider MOLPLFPP with chance constraints. Many real situations may arise when constraints cannot be deterministically stated but can be stochastically described. Dantzig [23, 24] introduced stochastic programming using probability theory. Chance constrained programming (CCP) [25] is one of the most important one in stochastic programming. In CCP, we transform the chance constraints into deterministic constraints by using known distribution function.

In this article, we use Taylor's series approximation to transform the MOLPLFPP into MOLPP. FGP models are used to solve MOLPP after converting chance constraints into deterministic constraints. Lastly, the optimal solution with minimum Euclidean distance [26] is considered as the best compromise optimal solution.

Rest of the paper is organized in the following way: Section II presents formulation of chance constrained MOLPLFPP. Section III describes the process of transforming chance constraints into equivalent deterministic constraints. Section IV presents linearization technique of MOLPLFPP by using Taylor polynomial series approximation. Section V is devoted to present proposed FGP formulation for MOLPLFPP. Section VI provides Euclidean distance function for identifying the best compromise optimal solution. Numerical example has been solved in Section VII to show the efficiency of the proposed FGP models. Finally, Section VIII presents the concluding remarks.

II. FORMULATION OF CHANCE CONSTRAINED MOLPLFPP

The objective functions of linear plus linear fractional programming problem can be formulated as:

$$\text{Max } Z_k(\bar{x}) = (\bar{p}_k^T \bar{x} + \gamma_k) + \frac{\bar{c}_k^T \bar{x} + \alpha_k}{\bar{d}_k^T \bar{x} + \beta_k}, \quad k = 1, 2, \ldots, K \tag{1}$$

$$\bar{x} \in S` = \left\{ \bar{x} \in \overline{\Re}^n : \Pr(\overline{A}\bar{x} \begin{pmatrix} \le \\ \ge \end{pmatrix} \bar{b}) > \bar{I} - \bar{\varepsilon},\ \bar{x} \ge \bar{0},\ \bar{b} \in \overline{\Re}^m \right\} \tag{2}$$

$\overline{A}$ is $m \times n$ matrix, $\bar{b}$, $\bar{\varepsilon}$ are $m \times 1$ matrices, $\bar{I}$ is a matrix of order $m \times 1$ having all components equal to 1 (unity), $\bar{x}$, $\bar{p}_k^T$, $\bar{c}_k^T$, $\bar{d}_k^T \in \overline{\Re}^n$ and α_k, β_k, γ_k are constants, T denotes transposition, $\bar{\varepsilon}$ denotes the level of confidence and $\bar{d}_k^T \bar{x} + \beta_k > 0$ for all $\bar{x} \in S`$, $S`$ is non empty, convex and compact in $\overline{\Re}^n$.

III. CONSTRUCTION OF EQUIVALENT DETERMINISTIC CONSTRAINTS

First, we consider the chance constraints of the form:

$$\Pr\left(\sum_{j=1}^{n} a_{ij} x_j \le b_i\right) \ge 1 - \varepsilon_i, \ i = 1, 2, \ldots, m_1.$$

$$\Rightarrow \Pr\left(\frac{\sum_{j=1}^{n} a_{ij} x_j - E(b_i)}{\sqrt{var(b_i)}} \le \frac{b_i - E(b_i)}{\sqrt{var(b_i)}}\right) \ge 1 - \varepsilon_i, \ i = 1,2,\ldots, m_1$$

$$\Rightarrow \varepsilon_i \ge 1 - \Pr\left(\frac{\sum_{j=1}^{n} a_{ij} x_j - E(b_i)}{\sqrt{var(b_i)}} \le \frac{b_i - E(b_i)}{\sqrt{var(b_i)}}\right)$$

$$\Rightarrow \varepsilon_i \ge \Pr\left(\frac{\sum_{j=1}^{n} a_{ij} x_j - E(b_i)}{\sqrt{var(b_i)}} > \frac{b_i - E(b_i)}{\sqrt{var(b_i)}}\right)$$

$$\Rightarrow \Psi^{-1}(\varepsilon_i) \ge \frac{\sum_{j=1}^{n} a_{ij} x_j - E(b_i)}{\sqrt{var(b_i)}}$$

$$\Rightarrow \Psi^{-1}(\varepsilon_i)\sqrt{var(b_i)} \ge \sum_{j=1}^{n} a_{ij} x_j - E(b_i)$$

$$\Rightarrow \sum_{j=1}^{n} a_{ij} x_j \le E(b_i) + \Psi^{-1}(\varepsilon_i)\sqrt{var(b_i)}, \ i = 1, 2, \ldots, m_1 \tag{3}$$

Here $\Psi(.)$ and $\Psi^{-1}(.)$ represent the distribution function and inverse of distribution function of standard normal variable respectively.

Considering the case when $\Pr(\sum_{j=1}^{n} a_{ij}x_j \geq b_i) \geq 1-\varepsilon_i$,

$i = m_1+1, m_1+2, \ldots, m.$

The constraints can be rewritten as:

$$\Rightarrow \Pr\left(\frac{\sum_{j=1}^{n} a_{ij}x_j - E(b_i)}{\sqrt{\mathrm{var}(b_i)}} \geq \frac{b_i - E(b_i)}{\sqrt{\mathrm{var}(b_i)}}\right) \geq 1-\varepsilon_i \text{ , } i = m_1+1, m_1+2, \ldots, m.$$

$$\Rightarrow \Psi\left(\frac{\sum_{j=1}^{n} a_{ij}x_j - E(b_i)}{\sqrt{\mathrm{var}(b_i)}}\right) \geq 1-\varepsilon_i$$

$$\Rightarrow 1-\Psi\left(-\frac{\sum_{j=1}^{n} a_{ij}x_j - E(b_i)}{\sqrt{\mathrm{var}(b_i)}}\right) \geq 1-\varepsilon_i$$

$$\Rightarrow \Psi^{-1}(\varepsilon_i) \geq -\frac{\sum_{j=1}^{n} a_{ij}x_j - E(b_i)}{\sqrt{\mathrm{var}(b_i)}})$$

$$\Rightarrow \sum_{j=1}^{n} a_{ij}x_j \geq E(b_i) - \Psi^{-1}(\varepsilon_i)\sqrt{\mathrm{var}(b_i)} \text{ , } i = m_1+1, m_1+2, \ldots, m \tag{4}$$

$$\bar{x} \geq \bar{0} \tag{5}$$

Let us denote the deterministic system constraints (3), (4) and (5) by S. Here, S` and S are equivalent constraints.

IV. LINEARIZATION OF MOLPLFPP BY FIRST ORDER TAYLOR'S SERIES APPROXIMATION

Here, we find optimal solution of each objective function separately subject to the deterministic constraints. After solving each objective function, the ideal solution point for each objective function is taken into account. Let $\bar{x}_k^* = \left(x_{k1}^*, x_{k2}^*, \ldots, x_{kn}^*\right)$ be the ideal solution for the k-th objective function. For the linearization, we use Taylor's series of first order expanding about the ideal solution point for each objective function. The series can be expressed as:

$$Z_k(\bar{x}) \approx Z_k(\bar{x}_k^*) + \sum_{j=1}^{n}(x_j - x_{kj}^*)\left(\frac{\partial}{\partial x_j} Z_k(\bar{x})\right)_{\text{at } \bar{x} = \bar{x}_k^*}$$

$$= \hat{Z}_k(\bar{x}) \quad k = 1, 2, \ldots, K. \tag{6}$$

Here we assume that $Z_k(\bar{x}_k^*) = \max_{x \in S} Z_k(\bar{x})$

V. FGP FORMULATION OF CHANCE CONSTRAINED MOLPLFPP

Using (6), we obtained the transformed linear objective functions as $\hat{Z}_1(\bar{x}), \ \hat{Z}_2(\bar{x}), \ldots, \ \hat{Z}_k(\bar{x})$.

Let $\hat{Z}_k(\bar{x}_1^o) = \max_{x \in S} \hat{Z}_k(\bar{x}) = \hat{Z}_k^B$ be the aspiration level for the function $\hat{Z}_k(\bar{x})$. $\hat{Z}_k^B$ is the individual best solution or ideal solution for the objective function $\hat{Z}_k(\bar{x})$.

Then the fuzzy goals appear as $\hat{Z}_k(\bar{x}) \gtrsim \hat{Z}_k^B$, $k = 1, 2, \ldots, K$ (7)

Now, we construct pay-off matrix of order $K \times K$ by using individual best solution of the objective function.

$$\begin{array}{c|cccc} & \hat{Z}_1 & \hat{Z}_2 & \ldots & \hat{Z}_K \\ \hline \bar{x}_1^o & \hat{Z}_1(\bar{x}_1^o) & \hat{Z}_2(\bar{x}_1^o) & \ldots & \hat{Z}_K(\bar{x}_1^o) \\ \bar{x}_2^o & \hat{Z}_1(\bar{x}_2^o) & \hat{Z}_2(\bar{x}_2^o) & \ldots & \hat{Z}_K(\bar{x}_2^o) \\ . & . & \ldots & . & \\ . & . & \ldots & . & \\ . & . & \ldots & . & \\ \bar{x}_K^o & \hat{Z}_1(\bar{x}_K^o) & \hat{Z}_2(\bar{x}_K^o) & \ldots & \hat{Z}_K(\bar{x}_K^o) \end{array}$$

The maximum value of each column of the pay-off matrix provides upper tolerance limit for the objective function $\hat{Z}_k$, $\hat{Z}_k(\bar{x}_k^o) = \max_{\bar{x} \in S} \hat{Z}_k(\bar{x}) = \hat{Z}_k^B$, k = 1, 2, …, K. The minimum value of each column provides lower tolerance limit for the k-th objective function i.e.

$\hat{Z}_k^L$ = Minimum of $\{\hat{Z}_k(\bar{x}_1^o), \hat{Z}_k(\bar{x}_2^o), \ldots, \hat{Z}_k(\bar{x}_k^o)\}$, k = 1, 2, …, K.

VA. Construction of Membership Function

The membership function (see Fig. 1) for the fuzzy goal of the form $\hat{Z}_k(\bar{x}) \gtrsim \hat{Z}_k^B$ can be written as:

$$\mu_k(\bar{x}) = \begin{cases} 1, & \text{if } \hat{Z}_k(\bar{x}) \geq \hat{Z}_k^B, \\ \dfrac{\hat{Z}_k(\bar{x}) - \hat{Z}_k^L}{\hat{Z}_k^B - \hat{Z}_k^L}, & \text{if } \hat{Z}_k^L \leq \hat{Z}_k(\bar{x}) \leq \hat{Z}_k^B, \\ 0, & \text{if } \hat{Z}_k(\bar{x}) \leq \hat{Z}_k^L \end{cases} \tag{8}$$

k = 1, 2, …, K, where $\hat{Z}_k^L$ is the lower tolerance limit of the k-th fuzzy goal.

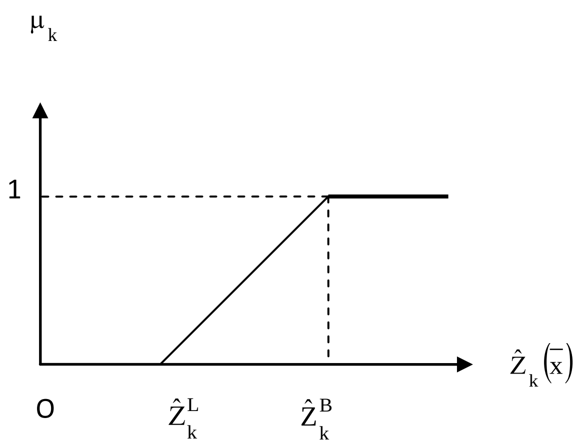

Fig. 1. Membership function of $\hat{Z}_k(\bar{x})$

Following Pramanik and Dey [27] and Pramanik and Roy [28] we can write

$$\mu_k(\bar{x}) + d_k^- = 1, \; k = 1, 2, \ldots, K \tag{9}$$

Here d_k^- (≥ 0) is the negative deviational variable for the k-th objective goal.

Now, the FGP models for chance constrained MOLPLFPP can be formulated as:

<u>Model-I:</u>

$$\text{Min } \Lambda = \sum_{k=1}^{K} w_k d_k^- \tag{10}$$

subject to

$\mu_k(\bar{x}) + d_k^- = 1$, k = 1, 2, …, K,

$\sum_{j=1}^{n} a_{ij} x_j \leq E(b_i) + \Psi^{-1}(\varepsilon_i)\sqrt{\text{var}(b_i)}$, i = 1, 2, …, m_1,

$\sum_{j=1}^{n} a_{ij}x_j \geq E(b_i) - \Psi^{-1}(\varepsilon_i)\sqrt{var(b_i)}$, i = $m_1 + 1, m_1 + 2, \ldots, m$,

$1 \geq d_k^- \geq 0$, k = 1, 2, …, K,

$\bar{x} \geq \bar{0}$

Here, w_k is the weight associated with the k- th membership function. The weight $w_k = \dfrac{1}{\hat{Z}_k^B - \hat{Z}_k^L}$ is determined based on Mohamed [29]. Alternatively, the decision making unit may suggest the weight according to the importance of the goals in the decision making situation.

<u>Model-II</u>

$$\text{Min}\, \zeta = \sum_{k=1}^{K} d_k^- \tag{11}$$

subject to the same constraints of FGP Model I.

<u>Model-III</u>

$$\text{Min}\ \lambda \tag{12}$$

subject to

$\mu_k(\bar{x}) + d_k^- = 1 \quad k = 1, 2, \ldots, K$

$\sum_{j=1}^{n} a_{ij}x_j \leq E(b_i) + \Psi^{-1}(\varepsilon_i)\sqrt{var(b_i)}$, i = 1, 2, …, m_1,

$\sum_{j=1}^{n} a_{ij}x_j \geq E(b_i) - \Psi^{-1}(\varepsilon_i)\sqrt{var(b_i)}$, i = $m_1 + 1, m_1 + 2, \ldots, m$,

$1 \geq d_k^- \geq 0$, k = 1, 2, …, K,

$\lambda \geq d_k^-$, k = 1, 2, …, K,

$\bar{x} \geq \bar{0}$.

VI. EUCLIDEAN DISTANCE FUNCTION

Euclidean distance function [26] can be defined as:

$$L_2\,(k) = \left(\sum_{k=1}^{K}\left(1-\mu_k(\bar{x})\right)^2\right)^{1/2} \tag{13}$$

Here $\mu_k(\bar{x})$ is the membership function for the k-th objective goal. This function is calculated for three proposed models and the solution with minimum distance L_2 would be considered as best compromise optimal solution.

VII. NUMERICAL EXAMPLE

We consider the following numerical example to illustrate the proposed approach.

Find $\bar{x}\,(x_1, x_2, x_3)$ so as to (14)

$\max Z_1 = (2x_1 + 3x_2 - x_3) + \dfrac{x_1 + x_2 + x_3 + 3}{x_1 + 2x_2 + 4x_3 + 5}$

$\max Z_2 = (-6x_1 + 2x_2 + 5x_3) + \dfrac{7x_1 - x_2 + 3x_3}{2x_1 + 9x_2 - 5x_3} + 10$

$\max Z_3 = (3x_1 - 4x_2 + 2x_3 + 15) + \dfrac{-5x_1 + 2x_2 - x_3 + 3}{9x_1 + x_2 + x_3 + 2}$

subject to

$\Pr(3x_1 - x_2 + x_3 \leq b_1) \geq 1 - \varepsilon_1$

$\Pr(-6x_1 + 4x_2 + 9x_3 \leq b\,) \geq 1 - \varepsilon_2$

$\Pr(11x_1 - 5x_2 + 7x_3 \geq b_3) \geq 1 - \varepsilon_3$

The means, variances and the confidence levels are given below:

$E(b_1) = 4$, $var(b_1) = 1$, $\varepsilon_1 = 0.05$

$E(b_2) = 9$, $var(b_2) = 4$, $\varepsilon_2 = 0.03$

$E(b_3) = 16$, $var(b_3) = 9$, $\varepsilon_3 = 0.01$

Using (4), (5) the chance constraints involved in problem (14) are transformed into equivalent deterministic constraints as:

$3x_1 - x_2 + x_3 \le 5.645$

$-6x_1 + 4x_2 + 9x_3 \le 12.77$

$11x_1 - 5x_2 + 7x_3 \ge 9.025$

Using Taylor's series approximation as described in (6), the transformed linear objective functions are obtained as:

$\hat{Z}_1(\bar{x}) = 2.013362x_1 + 2.994672x_2 - 1.042708x_3 + 0.585820,$

$\hat{Z}_2(\bar{x}) = -5.182459x_1 + 1.097886x_2 + 5x_3 + 10.599995,$

$\hat{Z}_3(\bar{x}) = 2.866745x_1 - 3.839293x_2 + 1.95467x_3 + 14.9127616,$

We obtained the individual best solutions for each linearized objective function subject to transformed deterministic system constraints as $\hat{Z}_1(\bar{x}_1^o) = \hat{Z}_1^B = 39.51842$, at (5.004688, 9.778438, 0.409375); $\hat{Z}_2(\bar{x}_2^o) = \hat{Z}_2^B = 17.69444$, at (0, 0, 1.418889); $\hat{Z}_3(\bar{x}_3^0) = \hat{Z}_3^B = 22.49230$, at (1.152576, 0, 2.187273).

The fuzzy goals appear as:

$\hat{Z}_1(\bar{x}) \gtrsim 39.51842$, $\hat{Z}_2(\bar{x}) \gtrsim 17.69444$, $\hat{Z}_3(\bar{x}) \gtrsim 22.49230$.

The obtained Pay-off matrix is represented as: $\begin{pmatrix} 39.51842 & -2.55411 & -7.48217 \\ -0.89367 & 17.69444 & 17.68622 \\ 0.62568 & 15.56318 & 22.49230 \end{pmatrix}$

The corresponding membership functions are constructed with the help of pay-off matrix as follows:

$$\mu_1(\bar{x}) = \begin{cases} 1, & \text{if } \hat{Z}_1(\bar{x}) \ge 39.51842 \\ \dfrac{\hat{Z}_1(\bar{x}) + 0.89367}{39.51842 + 0.89367}, & \text{if } -0.89367 \le \hat{Z}_1(\bar{x}) \le 39.51842 \\ 0, & \hat{Z}_1(\bar{x}) \le -0.89367 \end{cases}$$

$$\mu_2(\bar{x}) = \begin{cases} 1, & \text{if } \hat{Z}_2(\bar{x}) \ge 17.69444, \\ \dfrac{\hat{Z}_2(\bar{x}) + 2.55411}{17.69444 + 2.55411}, & \text{if } -2.55411 \le \hat{Z}_2(\bar{x}) \le 17.69444, \\ 0, & \text{if } \hat{Z}_2(\bar{x}) \le -2.55411 \end{cases}$$

$$\mu_3(\bar{x}) = \begin{cases} 1, & \text{if } \hat{Z}_3(\bar{x}) \ge 22.49234, \\ \dfrac{\hat{Z}_3(\bar{x}) + 7.48217}{22.49234 + 7.48217}, & \text{if } -7.48217 \le \hat{Z}_3(\bar{x}) \le 22.49234, \\ 0, & \text{if } \hat{Z}_3(\bar{x}) \le -7.48217 \end{cases}$$

Using FGP models (10), (11), and (12), the obtained results are shown in Table 1.
It is clear from the third column of the Table 1 that the FGP Model III gives the most compromise optimal solution.
Note1. Lingo Software version 11.0 is used to solve numerical example.

TABLE 1

Comparison of Optimal Solutions

Model	Membership values	L_2
FGP Model I	μ_1 =0.037596, μ_2 =0.894745, μ_3 = 1	0.968142
FGP Model II	μ_1 =0.037596, μ_2 =0.894745, μ_3 = 1	0.968142
FGP Model III	μ_1 = 0.489329, μ_2 = 0.489328, μ_3 = 0.489328	0.884509

VIII. CONCLUSIONS

In this paper, chance constrained multi-objective linear plus linear fractional programming problem with random variables is presented. To transform the fractional objectives into linear forms first order Taylor's series approximation is used. Three FGP models are presented to illustrate the proposed approach. For the future research, the proposed approach can be extended to multi-objective linear plus linear fractional programming problem with fuzzy parameters. The proposed approach can also be used for solving bi-level as well as multilevel linear plus linear fractional programming problem with crisp and fuzzy parameters.

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