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Three loop Lateral Missile Autopilot Design in Pitch Plane using State Feedback & Reduced Order Observer (DGO)

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Abstract—*A flight path rate demand modified three-loop lateral missile autopilot design methodology for a class of guided missile, based on state feedback, output feedback, reduced order Das & Ghosal observer (DGO) is proposed. The open loop undamped model of three-loop autopilot has been stabilized by using pole placement and state feedback. The non-minimum phase feature of rear controlled missile airframes is analyzed. The overall response of the three-loop autopilot has been significantly improved over the classical two-loop design. It has been established through this paper that the initial negative peak occurring in the time response due to the non-minimum phase zeros, the reduction of which posed a major challenge so far in autopilot design, is reduced to some extent as compared to that of two-loop design. Steady state value of flight path rate has also improved over the classical two-loop design. Body rate demand is met exactly due to integrator applied in the forward path thus making the steady state body rate error zero. Reduced order Das & Ghosal observer is implemented successfully in this design to estimate two states elevator deflection and its rate while the other two states are measured by accelerometer & rate gyro. It has also been established that addition of an observer (an auxiliary dynamic system) to the system does not impair the system stability; it only appends its own poles (Eigen values) with the original system poles. Finally a numerical example has been considered and the simulated results are discussed in details.*

Keywords—Three Loop Pitch Missile Autopilot, Angle of attack, Flight path rate demand loop, Rate Gyro, Accelerometers, Aerodynamic control, Luenberger Observer, Das & Ghosal Observer, Generalized Matrix Inverse, LQR, and Ackermann.

I. INTRODUCTION

This paper deals with the modified three-loop lateral missile autopilot design methodology in pitch plane based on its state space model (fig. 3.3). In literature **[1] & [2],** detailed design of classical two loop flight path rate demand autopilot (fig. 3.1) is given. Here the accelerometer provides the main output (flight path rate) feedback and the rate gyro enables the body rate feedback (inner loop) thus resulting in two loops. The authors presented three different design situations of two loop lateral autopilot for a class of guided missile. Frequency domain approach had been taken in those papers. In conventional two loop autopilot system there is no provision for direct control over the missile body rate. However, Tactical homing missiles require **explicit control on body rate**. Three such specific requirements are **a)** body rate generation not to exceed predetermined maximum; **b)** body acceleration limit; and **c)** it could produce moderate actuator rates without rate saturation for sudden pitch rate demands. In literature **[3],** authors modified the design of two loop lateral autopilot and proposed an additional rate gyro feedback to be applied at the input of an integrating amplifier block (fig. 3.2) which integrates the body rate (i.e. pitch rate here) error to obtain direct control over the missile body rate. This enhanced model is referred to as Three Loop Lateral Autopilot. The three loop autopilot has a larger dc gain and a relatively small high frequency gain compared to a two-loop autopilot. This feature effectively improves the steady state performance and loop stiffness as well as reduces the initial negative peak of the time response. The three-loop autopilot attempts to reduce the adverse effect of non-minimum phase zeros. In reference **[6],** Prof. G. Das and T. K. Ghosal have derived a new method of reduced order observer construction based on Generalized Matrix Inverse theory **[7]** which possesses some certain advantages over the well known and well-established Luenberger observer **[9] & [10].** In paper **[4],** Lin Defu, Fan Junfang, Qi Zaikang and Mou Yu have proposed a modification of classical three-loop lateral autopilot design using frequency domain approach. Also they have done performance and robustness comparisons between the two-loop and classical three-loop topologies. Tayfun Cimen has discussed in paper **[5]** the concept of *extended linearization* (also known as *state-dependent coefficient parameterization*) for *state-dependent nonlinear formulation* of the vehicle dynamics in a very general form for the development of a generic and practical autopilot design approach for missile flight control systems. Any *extended linearization control methods,* such as State-Dependent Riccati Equation (SDRE) methods, can then be applied to this state-dependent formulation for missile flight control system design. He has also discussed about different aspects of LQR based state feedback design since it generally gives good performance characteristics and stability margins, with the availability of the states required for implementation.

II. AUTOPILOT

Autopilot is an automatic control mechanism for keeping the spacecraft in desired flight path. An Autopilot in a missile is a closed loop system and it is a minor loop inside the main guidance loop. If the missile carries accelerometers and/or rate gyros to provide additional feedback into the missile servos to modify the missile's course of motion then the flight control system i.e. the missile control system is usually called an Autopilot. When the autopilot controls the motion in the pitch or yaw plane, they are called Lateral Autopilot. For a symmetrical cruciform missile, pitch and yaw autopilots are identical. The guidance system detects whether the missile's position is too high or too low, or too much right or left. It measures the deviation or errors and sends signals to the control system to minimize the acceleration (latex) according to the demand from the guidance computer. For aerodynamically controlled skid to run missile, the autopilot activates to move the control-surfaces i.e. wings and fins suitably for orienting the missile body with respect to the desired flight path. This control action generates angle of attack and consequently the latex demand for steering the missile following the desired path. In this paper, such a lateral autopilot (Three Loop) has been designed in pitch plane using reduced order observer (DGO) based state feedback control.

III. DEVELOPMENT OF MODIFIED THREE-LOOP AUTOPILOT FROM THE CONVENTIONAL ONE

The following block diagrams (fig. 3.1 & 3.2) represents the transfer function model of flight path rate demand two loop and three loop autopilot respectively in pitch plane **[1], [2] & [3].**

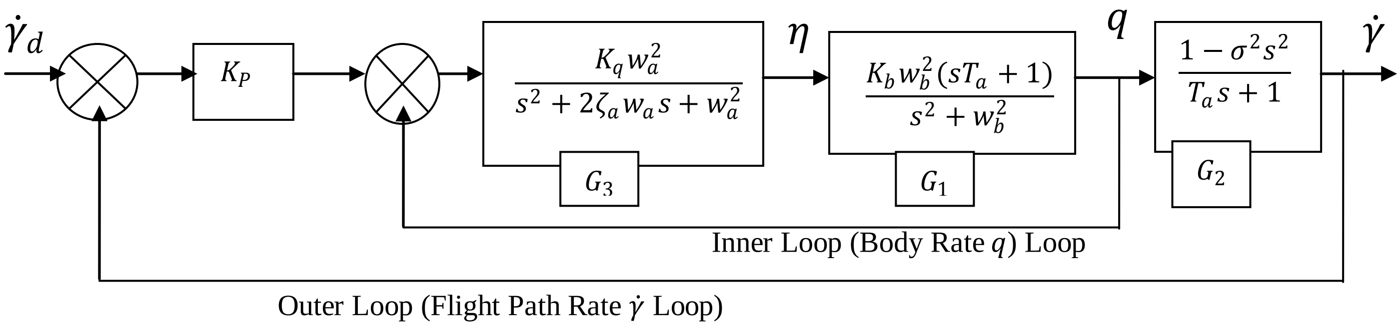


Fig - 3.1: Conventional Flight Path Rate Demand Two Loop Autopilot Configuration (Transfer Function Model)

Where G_1 & G_2 $is\ the\ Aerodynamic\ transfer\ function\ and\ G_3 is\ the\ Actuator\ transfer\ function$

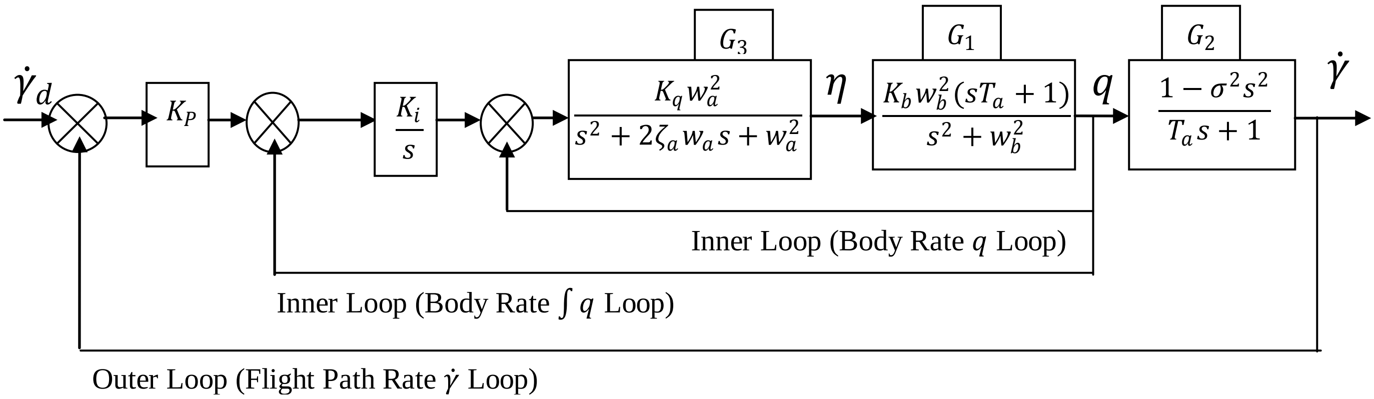


Fig – 3.2: Conventional Three Loop Autopilot Configuration (Transfer Function Model)

The open loop model i.e. the cascaded combination- $G_1 G_2 G_3$of fig. 3.2 can be converted to the corresponding state space model given by $\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}$ & $\boldsymbol{y} = \boldsymbol{Cx}$ (discussed in the next section) and the above configuration can be redrawn as :

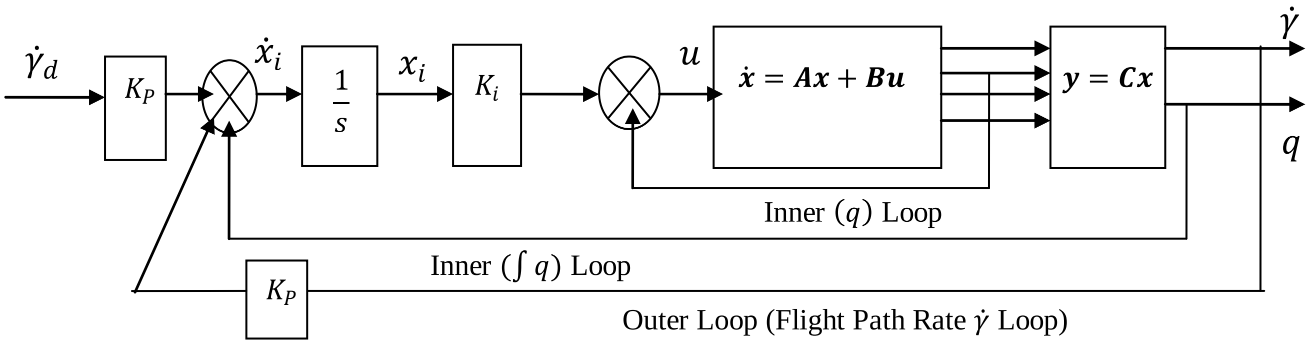


Fig – 3.3: Three Loop Missile Autopilot Equivalent State Space Configuration

Now the modified configuration of three-loop lateral autopilot using state feedback and DGO is presented below in fig. 3.4. The gain matrix $\boldsymbol{K} = [\boldsymbol{K_1} \quad \boldsymbol{K_2} \quad \boldsymbol{K_3} \quad \boldsymbol{K_4} \quad \boldsymbol{K_5}]$ is obtained by using **Ackermann** algorithm on the basis of some optimal pole locations giving satisfactory time domain performance.

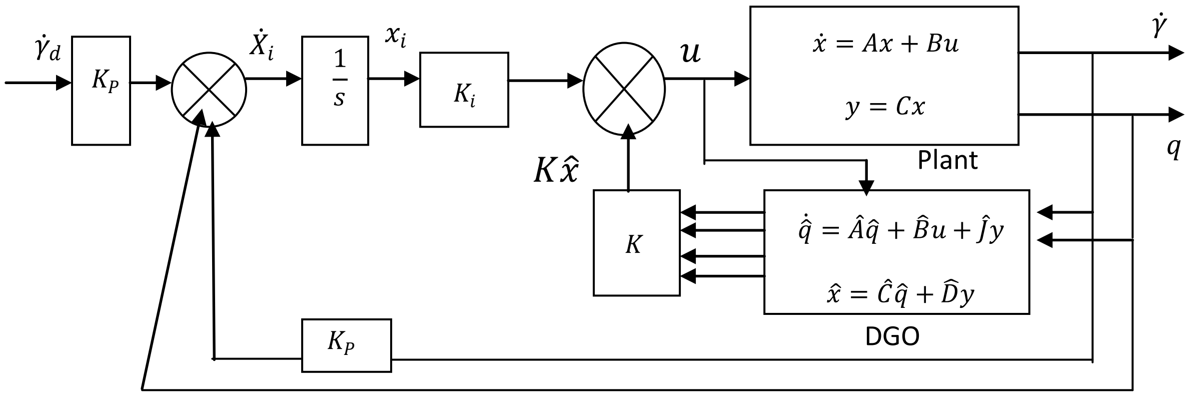
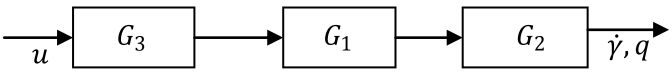


Fig – 3.4: Modified Three Loop Autopilot Configuration using Das & Ghosal Observer (DGO)

Notations & Symbols used:
$\boldsymbol{\dot{\gamma}}$ *is flight path rate*; q *is pitch rate*;
$\boldsymbol{w_a}$ *is natural frequency of Actuator*; $\boldsymbol{\zeta_a}$ *is damping ratio of actuator*; $\boldsymbol{K_P}, \boldsymbol{K_q}, \boldsymbol{K_b}$ *are the control gains*;
$\boldsymbol{w_b}$ *is weather cock frequency*; $\boldsymbol{T_a}$ *is the incidence lag of the air frame*;
$\boldsymbol{\eta}$ *is Elevator deflection*;
$\boldsymbol{\sigma}$ *is aquantity whoose invese determines the locations of minimum phase zeros*;
$\boldsymbol{K_i}$ *integrator gain*;

IV. STATE VARIABLE MODELING OF THREE LOOP AUTOPILOT

The open loop model of three loop autopilot shown below



can be converted to state variable form based on the following four state variables:

$\boldsymbol{x_1} = \boldsymbol{\dot{\gamma}}$ (***Flight path rate demand***);
$\boldsymbol{x_2} = \boldsymbol{q}$ (***pitch rate***);
$\boldsymbol{x_3} = \boldsymbol{\eta}$ (***elevator deflection***);
$\boldsymbol{x_4} = \boldsymbol{\dot{\eta}}$ (***rate of chance of elevator deflection***

out of them x_1 *and* $\boldsymbol{x_2}$ have been considered to be as outputs.
Thus three loop autopilot model is a SIMO (single input – multiple output) system.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_a} & \frac{1+\sigma^2 w_b^2}{T_a} & -\frac{K_b \sigma^2 w_b^2}{T_a} & -K_b \sigma^2 w_b^2 \\ -\frac{1+w_b^2 T_a^2}{T_a(1+\sigma^2 w_b^2)} & \frac{1}{T_a} & K_b w_b^2 T_a \, {(1-\frac{\sigma^2}{T_a^2})}\Big/{(1+\sigma^2 w_b^2)} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -w_a^2 & -2\zeta_a w_a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ K_q w_a^2 \end{bmatrix} u \ldots\ldots (4.1a)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \ldots\ldots (4.1b)$$

Now the state space equivalent model (fig. 3.3) of conventional three loop autopilot (fig. 3.2) can be thought of as the open loop state space model (given by eqn. 4.1a & 4.1b) along with an additional integrator in the forward path with a gain (w_i) plus single state feedback and output feedback connection. Hence the order of the three-loop plant becomes 5 (4 states + 1 additional state).

Ultimately the modified three loop flight path rate demand autopilot (fig. 3.4) has been introduced by incorporating Das & Ghosal observer (DGO) along with the state space equivalent model, shown in fig. 3.3 and the Ackermann pole placement technique is adopted to ensure desired closed loop pole placement. The combined system (fig. 3.4) is governed by the following set of equations:

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}; \quad \boldsymbol{y} = \boldsymbol{Cx};$$
$$\dot{\boldsymbol{X}}_i = \boldsymbol{r} - \boldsymbol{K_f y} = \boldsymbol{r} - \boldsymbol{K_f Cx} = \boldsymbol{r} - \boldsymbol{C_1 x};$$
$$\boldsymbol{u} = \boldsymbol{K_i X_i} - \boldsymbol{K\hat{x}}$$
$$\dot{\hat{\boldsymbol{q}}} = \hat{\boldsymbol{A}}\hat{\boldsymbol{q}} + \hat{\boldsymbol{B}}\boldsymbol{u} + \hat{\boldsymbol{J}}\boldsymbol{y}; \quad \hat{\boldsymbol{x}} = \hat{\boldsymbol{C}}\hat{\boldsymbol{q}} + \hat{\boldsymbol{D}}\boldsymbol{y} \ldots\ldots (4.2)$$

V. REDUCED ORDER DAS & GHOSAL OBSERVER (DGO) APPLIED TO THREE-LOOP AUTOPILOT

Reduced order Das and Ghosal observer **[6]** is governed by the following equations and conditions.

$x = C^g y + L\,h \ldots\ldots\ldots (5.1)$ (eqn. 13 of [6])

$\dot{h(t)} = L^g AL\,h(t) + L^g AC^g\, y(t) + L^g\, B\, u(t) \ldots\ldots\ldots (5.2)$ (eqn. 15 of [6])

$\dot{y} = CALh + CAC_g\, y + CB\, u \ldots\ldots (5.3)$ (eqn. 18 of [6])

$\dot{\hat{h}} = (L^g AL - MCAL)\hat{h} + (L^g AC^g - MCAC^g)y + (L^g - MCB)u + M\dot{y} \ldots\ldots (5.4)$ (eqn. 19 of [6])

$\dot{\hat{q}} = (L^g AL - MCAL)\hat{q} + \{(L^g AC^g - MCAC^g) + (L^g AL - MCAL)M\}y + (L^g - MCB)u \ldots\ldots (5.5)$ (eqn. 20 of [6])

$where\ \hat{q} = \hat{h} - My \ldots\ldots (5.6)$ (Page-374 of [6])

Equation (5.5) can be expressed in short form: $\dot{\hat{q}} = \hat{A}\hat{q} + \hat{J}y + \hat{B}u \ldots\ldots (5.7)$

$And\ \hat{x} = L\hat{q} + (C^g + LM)y \ldots\ldots (5.8)$ (eqn. 21 of [6])

Equation (5.8) can also be expressed in short form: $\hat{x} = \hat{C}\hat{q} + \hat{D}y \ldots\ldots (5.9)$

VI. MATLAB SIMULATION RESULTS

The following numerical data for a class of missile have been taken for Matlab simulation:

$$T_a = 0.36\,sec;\ \sigma^2 = 0.00029\ \mathrm{sec}^2;$$
$$w_b = 11.77\frac{rad}{sec};\ \zeta_a = 0.6;\ K_b = -10.6272\,per\,sec;$$
$$v = 470\frac{m}{sec};\ K_p = 4.95;\ K_q = -0.12;$$
$$w_a = 180\frac{rad}{sec}; K_i = 22.02;$$

Using these values, state space model (4.1a) & (4.1b) becomes,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}-2.77 & 2.77 & 1.1860 & -0.4269\\ -50.6161 & 2.77 & 508.388 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -32400 & -216\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ -3888\end{bmatrix}u \ldots\ldots (6.1a) and$$

$$y = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \ldots\ldots (6.1b)$$

The above state model given by eqns. (6.1a) & (6.1b) actually describes two loop autopilot configuration. The three loop autopilot is the modified configuration of two loop autopilot where an integrator and a gain element are used additionally (fig. 3.2) in the forward path to make the body rate steady state error zero.

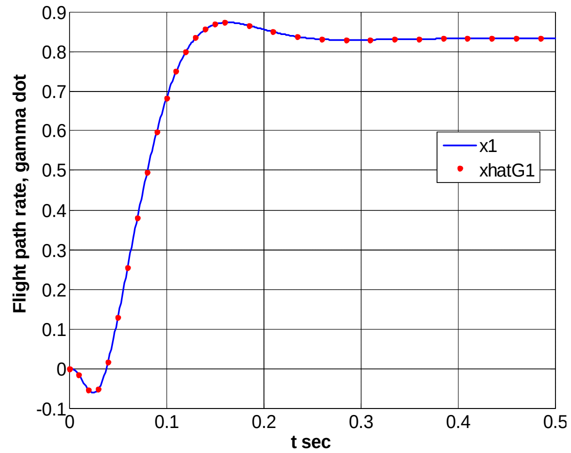


***Figure 6.1*:** x1 is plant state and xhatG1 is estimated state

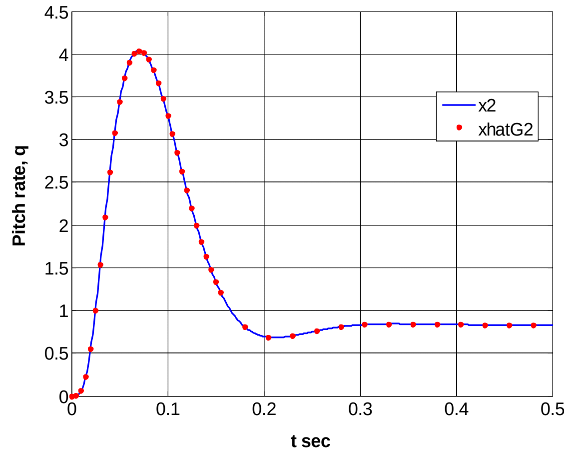


***Figure 6.2*:** x2 is plant state and xhatG2 is estimated state

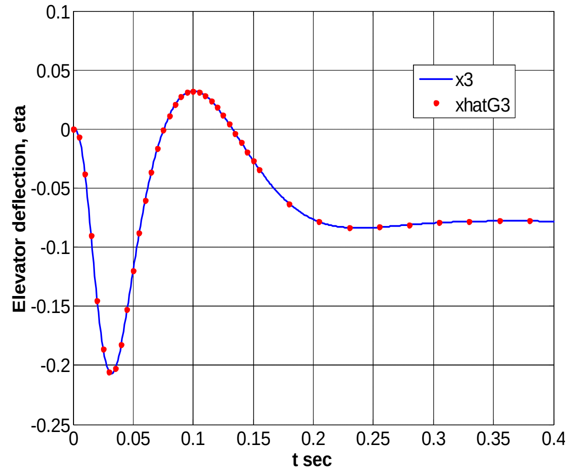


***Figure 6.3*:** x3 is plant state and xhatG3 is estimated state

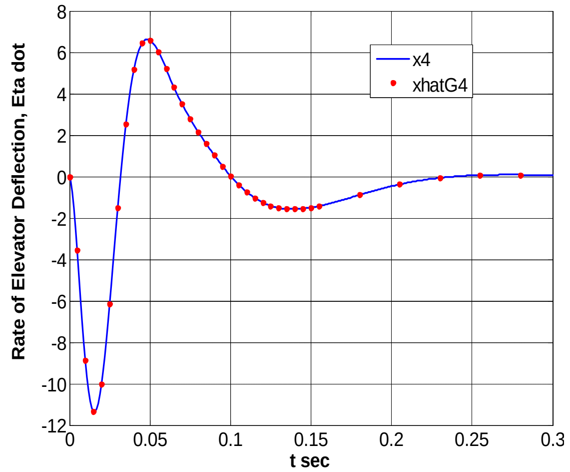


***Figure 6.4*:** x4 is plant state and xhatG4 is estimated state

VII. OBSERVATIONS AND DISCUSSIONS

In this paper, flight path rate demand three loop autopilot has been designed in state space model corresponding to the transfer function model given in literature **[1], [2] & [3].** Flight path rate $\dot{\gamma}$ and the pitch rate q have been used as outputs. In practical missiles these are generally measured by gyros and accelerometers. Reduced order Das & Ghosal observer is applied to measure the other two states i.e. elevator deflection η and rate of change of elevator deflection $\dot{\eta}$. Finally four states have been fedback to input to implement state feedback control. It is seen from the simulation graphs that the original states (blue continuous line) obtained from transfer function model and state space model overlap with each other indicating that both the modeling schemes are compatible. It has also been established through the simulation that Das & Ghosal observer **[6]** has successfully caught the system states within less than 0.02 seconds and without any steady state error or oscillations. Further the observation has also been carried out by using the very well known and well used Luenberger method **[9] & [10]** and it is seen that both of Luenberger and Das & Ghosal observer are giving exactly same dynamic performance (red dotted line indicates both of the observed states). So it can be inferred that Das & Ghosal observer is at par with reduced order Luenberger observer and in some cases it is superior **[13] & [14]** to the latter.

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