**International Journal of Engineering Research and Development**
*e-ISSN: 2278-067X, p-ISSN: 2278-800X, www.ijerd.com*
*Volume 10, Issue 10 (October 2014), PP.30-38*

# A Study on Single Server Retrial Queue with Steady State Partial Generating Function

M.Renisagayaraj[1], Moganraj.D[2], R.Sathishkumar[3]
[1]*Sacred Heart College, Tirupattur, Tamilnadu, India.*
[2]*Arunai Engineering College, Tiruvannamalai, Tamilnadu India.*
[3]*Department of Computer Science, Sacred Heart College, Tirupattur, Tamilnadu, India.*

**Abstract:-** In this paper we are investigate the steady state and partial generating function of single server retrial queue. We concentrate the case of exponential service time and explicit expression for the stationary distribution and factorial moments for the single server retrial queue.

**Keywords:-** Queuing model, retrials queue, steady state, partial generating function

## I. INTRODUCTION

Queueing systems with repeated request have many useful applications in communication and computer system modelling. The main characteristic of the single server queue with repeated requests is that a customer finds the server busy upon arrival returns to the system after a random amount of time.

Most queuing system with repeated order are characterized by the following feature, if the server is free then the arriving customer enter service, if the server is occupied the customer must leave the service area and enter a pool of unsatisfied customers. An arbitrary customer in the pool generates a steam of repeat requests that is independent of the rest of customer in the pool. This type of situation arises in telephony.

M.MARTIN and J.R.ARTALEJO [2] considered a service system in which the processes must serve two types of impatient units. In the case of blocking the first type units leave the system whereas the second type units enter a pool and wait to be processed latter.

Fayolly [7] deals with simple telephone exchange with delayed feedback. In earlier Farahmand [9] derived single line queue with repeated demands quests. Falin [5] developed an exhaustive analysis of the system including embedded Markov chain, fundamental period and various classified stationary probability distributions, more specific performance measures, such as the number of least cost and other quantities are also developed. Now we considered an example for retrial queue.

A computer network in client – server host, there is an always-on host, called the server, which services requests from many other hosts, called clients. The client hosts can be either sometimes-on or always-on. A host computer wishes to send a message to another computer. If the transition medium is available, the host computer immediately sends the message; otherwise the message will be stored in a buffer and the host has to try again sometime later.

## II. THE MATHEMATICAL MODEL

We consider queuing system in which arrive according to Poisson stream with rate $\lambda$>0. Any customer who finds the server busy upon arrival leaves the service area and join a pool of unsatisfied customers.

The control policy to access from this pool to the server is governed y an exponential law with linear intensity $v_n = \alpha(1-\delta_{n_0}) + nv$, when the pool size n$\epsilon N$. The service times are general with probability distribution function B(x) and mean $\beta_1 < \infty$. The input stream of arrivals service times and intervals between successive repeat requests are assumed to be mutually independent. Also, let $\beta(s)$ be the corresponding Laplace-Stieltjes transform.

The state of the system can be described by the process X(t)={C(t),N(t), $\xi(t)$} where C(t) denote the state of the server 0 and 1 according to whether the server is free or busy, N(t) the number of unsatisfied customers at time t and if C(t)=1 the, $\xi(t)$ represents the elapsed time of the customer being served. We neglect $\xi(t)$ and consider only the pair {C(t), N(t)} whose state space is S={0,1}*N.

Now we study the necessary and sufficient condition for the system to be stable. To see this, we investigate the ergodicity of the embedded Markov chain at the departure epochs. Let $N_n$ be the number of the customer in the pool at the time of the $n^{th}$ departure.
We have the fundamental equation,

$$N_n=N_{n-1}-B_n+V_n$$

Where $V_n$ is the number of customers arriving the $n^{th}$ service time, and

$B_n=1$ if the $n^{th}$serve customer from the pool

=0 otherwise.

Now $\{N_n, n\varepsilon N\}$ is irreducible and aperiodic,

To prove ergodicity we shall use Foster's Condition,

## III. ERGODICITY OF EMBEDDED MARKOV CHAIN

**3.1 Foster's Condition:**

An irreducible and aperiodic Markov chain is ergodic if there exists a non-negative function f(s), s$\epsilon$ N and $\epsilon$>0 such that the mean drift

$$\varphi_s = E[f(N_{n+1}) - f(N_n)/N_n = s] \quad (3.1)$$

Is finite for all s $\epsilon$N and$\varphi_s \leq -\epsilon.$ $\forall s\epsilon N$Expect a finite number,

We consider the function f(s) =s. We then obtain.

$$\varphi_j = \begin{Bmatrix} \gamma, j=0 \\ \gamma - \frac{\alpha+jv\alpha}{\lambda+\alpha+jv}, j=1,2\ldots\ldots \end{Bmatrix}$$

Where $\gamma = \alpha\beta_1$

Clearly if $\gamma$ satisfies the inequality

$$\gamma < 1 - \frac{\lambda}{\lambda+\alpha}\delta_{0v} \quad (3.2)$$

Then we have $\lim_{j\to\infty}\varphi_j < 0.$

Therefore the embedded Markov chain $\{N_n, N\epsilon N\}$ is ergodic .The same condition is also necessary for ergodicity .The proof follows by Sennot et al., which states that we can guarantee non-ergodicity if $\{N_n, n\epsilon N\}$ satisfies Kaplan's condition, $\varphi_j < +\infty (j \geq 0)$ and there is a $j_o$ such that $\varphi_j \geq 0 (j \geq j_0)$ .

$P_{ij} = 0 (j < i - k, i > 0)$Where $P=P_{ij}$ (i, j=0, 1,…….) is the transition matrix associated to $\{N_n, n\epsilon N\}$.

Since the arrival stream is Poisson process, We use Burke's theorem given by Cooper [4].

**3.2 Burke's Theorem:**

The limiting probabilities $P_{ij} = lim_{t\to\infty} P_r\{C(t), N(t) = (i,j)\}$, (i, j)$\epsilon S$

Exist and positive iff the inequality (3.2) holds.

**3.3 Analysis of the steady state probabilities**

We now study steady state distribution of our queuing system .Let $P_i(z)$ be the generating function of the sequence $P\{_{ij}, j\epsilon N\}$ for i $\epsilon\{0,1\}$. We also consider in the steady state and

$$k_j=P_r[Y=j]$$
$$k_0=P_{00}$$
$$k_1=P_{01}+P_{10}$$
$$k_2=P_{02}+P_{ii}$$
$$\ldots\ldots$$
$$K_j=P_{ij}+P_{ij-1\ldots\ldots}(j>=1)$$

The generating function of the customers in system in steady state is given by

$$k(z)=\sum_{j=1}^{\infty}(Poj + P_{1j-1})z^j$$

$$= P_0(z)+zP_1(z), \text{for } |z| \leq 1 \quad (3.3)$$

Finally we denote by Q(z) the generating function of the number of customers present in the system in the standard M/G/1 queue. This is given by

$$Q(z) = (1-\gamma)\frac{(1-z)\beta(\lambda-\lambda z)}{\beta(\lambda-\lambda z)-z}, for\ |z| \leq 1 \quad (3.3)$$

Where$\beta(s)$ is the Laplace –Stieltjes transform defined by

$$\beta(s) = \int_0^{\infty} e^{-st} d\beta(t), s > 0.$$

**Theorem3.3 (a)**

If v>0,$\alpha > 0\ and\ \gamma < 1,$ then

$$P_0(z)=z^{-\alpha/v}H(z)(1-(\delta_{0\alpha})\frac{\int_z^1 x^{\alpha/v-1}H^{-}(x)dx}{\int_0^1 x^{\alpha/v-1}H^{-}(x)dx} \quad (3.5)$$

$$P_1(z)=\frac{1-\beta(\lambda-\lambda z)}{\beta(\lambda-\lambda z)-z}P_{0(Z)} \quad (3.4)$$

Where

$$H(z)=(1-\gamma)\exp\{\frac{\lambda}{v}\int_z^1\frac{\beta(\lambda-\lambda x)-1}{\beta(\lambda-\lambda x)-x}dx \quad (3.5)$$

Is the partial generating function of $\{P_{0j}, j\epsilon N\}$. In the classical retrial queue associated with the case v>0 and $\alpha=0$.

**Proof:**

Our queuing theory system can be viewed as a vacation model verifying the assumption given in Fuhrmann and Cooper [8]. Thus the number of customer present in the system at a random point can be decomposed into the sum of two independent random variables, where one of the these is the number of customers in the corresponding standard M/G/1 system and the other is the number of customer in the model under study given that the server is on vacation. This property is known as stochastic decomposition law. Stochastic decomposition for retrial queue was observed by and is applications were discussed by Artalejo,J.R and Falin [3]. By this property we have,

$$K(z)=Q(z)\frac{P_0(z)}{P_0} \quad (3.6)$$

Where

$P_0=P_0(1)$.

On the other hand, we also note that the number of transition in a cycle at which the number in the pool increase from j-1 to j equals the number of transitions at which the number of customers in the pool decreases from j to j-1.Then we get ,

$$\alpha+(j+1)v)P_{0j+1}=\lambda P_{1j}\, j\,\epsilon N \quad (3.7)$$

On taking generating functions we have

$$\sum_{j=0}^{\infty}\alpha+(j+1)v)P_{0j+1}z^j=\sum_{j=0}^{\infty}\lambda P_{1j}z^j$$

$$\alpha\sum_{j=0}^{\infty}P_{0j+1}\,z^j+v\sum_{j=0}^{\infty}(j+1)\,P_{0j+1}z^j=\lambda\sum_{j=0}^{\infty}P_{1j}z^j$$

$$\alpha(P_{01}z^0+P_{02}z^1+P_{03}z^2+P_{04}z^3\ldots\ldots+v(P_{01}+2P_{02}z\ \ +3P_{03}z^2+\cdots..)=\lambda P_1(z)$$

$$\frac{\alpha}{z}(P_{01}z^1+P_{02}z^2+P_{03}z^3+P_{04}z^4\ldots\ldots+v(P_{01}z+2P_{02}z^2+3P_{03}z^3+\cdots..)=\lambda P_1(z)$$

$$\frac{\alpha}{z}(P_0(z)-P_{00})+v(P_0{}'(z)=\lambda P_1(z)$$

$$vz(P_0'(z)+\alpha(P_0(z)-P_{00})=\lambda P_1(z)$$

$$=(K(z)-P_0(z))\lambda$$

$$=(Q(z)P_0^{-1}P_0(z)-P_0(z))\lambda$$

$$vz(P_0'(z)+P_0(z)(\alpha+\lambda(1-Q(z)P_0{}')=\alpha P_{00} \quad (3.8)$$

$$\left(P_0'(z)+\frac{P_0(z)(\alpha+\lambda(1-Q(z)P_0{}'}{vz}\right)=\frac{\alpha}{vz}P_{00} \quad (3.9)$$

Solving we obtain

$$P_0(z)e^{\int_{1/v}q(u)du}=\int\frac{\alpha}{vz}P_{00}\,e^{\int_{1/v}q(u)du}dz+c$$

Where

$$q(u)=\left(\frac{(\alpha+\lambda(1-Q(z)P_0{}'}{vz}\right)$$

$$\therefore P_0(z) = \exp\{\tfrac{1}{v}\int_z^1 q(u)du\}(P_0 - \alpha v^{-1}P_{00}\int_z^1 x^{-1}\exp\{\tfrac{-1}{v}q(u)du\})dx \qquad (3.10)$$

To determine $P_0$ and $P_{00}$ we set z=0 in (3.10) and we have

$$P_0 = Q(0) = 1-\gamma \qquad (3.11)$$

Replacing $P_0$ in (3.10) and setting z=0 we obtain after algebraic manipulation.

$$P_{00} = \delta_{0\alpha}H(0) + (1-\delta_{0\alpha})\frac{v}{\alpha}\left(\int_0^1 x^{\alpha/v-1}H^{-1}(x)dx\right)^{-1} \qquad (3.12)$$

We have

$$\exp\left\{\frac{1}{u}\int_z^1 q(u)du\right\} = \frac{Z^{-\alpha/v}}{(1-\gamma)}H(z)$$

Substituting (3.11) and (3.10) in (3.12) and rearranging leads to

$$P_0(z) = \frac{Z^{-\alpha/v}}{(1-\gamma)}H(z)\Bigg[(1-\gamma) - \alpha v^{-1}\left\{\delta_{0\alpha}H(0) + (1-\delta_{0\alpha})x\frac{v}{\alpha}\left[\int_0^1 x^{\alpha/v-1}H^{-1}(x)dx\right]^{-1} x\int_z^1 x^{-1}x^{\alpha/v}\frac{(1-\gamma)}{H(x)}dx\right\}\Bigg]$$

$$= \frac{z^{-\alpha/v-1}H(z)}{(1-\gamma)}[(1-\gamma) - \alpha v^{-1}(\delta_{0\alpha}H(0) + (1-\delta_{0\alpha})\frac{v(1-\gamma)\int_z^1 x^{\alpha/v-1}H^{-1}(x)\,dx}{\alpha\int_0^1 x^{\alpha/v-1}H^{-1}(x)\,dx}]$$

$$P_0(z) = Z^{-\alpha/v}H(z)[1-(1-\delta_{0\alpha})\frac{v\int_z^1 x^{\alpha/v-1}H^{-1}(x)\,dx}{\alpha\int_0^1 x^{\alpha/v-1}H^{-1}(x)\,dx}]$$

We have K (z) = $P_0$ (z) + $zP_1$ (z)

$zP_1$ (z) = K (z)-$P_0$ (z)

$zP_1$ (z) = $\frac{Q(z)P_0(z)-P_0(z)}{P_0}$

$zP_1$ (z) = $(\frac{Q(z)}{P_0} - 1)P_0(z)$

$zP_1$ (z) = $((1-\gamma)\frac{(1-z)\beta(\lambda-\lambda z)}{\beta(\lambda-\lambda z)-z} - 1)P_0(z)$

$zP_1$ (z) = $((1-\gamma)\frac{(1-z)\beta(\lambda-\lambda z)-\beta(\lambda-\lambda z)+z}{\beta(\lambda-\lambda z)-z})P_0(z)$

$zP_1$ (z) = $\frac{z(1-\beta(\lambda-\lambda z)}{\beta(\lambda-\lambda z)-z})P_0(z)$

$P_1$ (z)=$\frac{(1-\beta(\lambda-\lambda z)}{\beta(\lambda-\lambda z)-z})P_0(z)$

Note that in the special case v=0,(3.10) reduces to a simple expression that is consistent with the result given in Farahmand[9].\

## IV. EXPLICIT FORMULA FOR THE CASE OF EXPONENTIAL SERVICE TIMES

We assume that the service times are exponential distributed with rate $\mu$ .Let F be the hyper geometric series given by

$$F(a,b;c,z) = \sum_{n=0}^{\infty}\frac{(a)_n(b)_n z^n}{(c)_n\, n!} \qquad (4.1)$$

Where $(x)_n$ is the pochhammersymbol, which is given by

$$(x)_n = \begin{Bmatrix} 1, for\ n=0 \\ x(x+1)\ldots\ldots\ldots(x+n-), for\ n\geq 1 \end{Bmatrix}$$

We shall derive the following result.

**Theorem (4.1) (a)**

If v>0, $\alpha > 0\ and\ \gamma < 1$ then

**(i)** **The partial generating functions $P_i$ (z),$i\epsilon(0,1)$ are given by**

$$P_0(z) = (1-\gamma z)P_{00}(z)F(\frac{\gamma+\alpha}{v}+1,1;\frac{\alpha}{v}+1;\gamma z) \quad (4.2)$$

$$P_1(z) = (\gamma)P_{00}(z)F\left(\frac{\lambda+\alpha}{v}+1,1;\frac{\alpha}{v}+1;\gamma z\right) \quad (4.3)$$

**(ii)** **The limiting probabilities $\{P_{ij},(i,j)\epsilon S\}$ are given by**

$$P_{0j}(z) = F\left(\frac{\lambda+\alpha}{v}+1,1;\frac{\alpha}{v}+1;\gamma z\right)^{-1}$$

$$P_{0j}(z) = P_{00}(z)\gamma^j(\frac{\lambda^{\frac{\lambda+\alpha}{v}}}{\lambda+\alpha\left(\left({}^{\alpha}/_{v}\right)+I\right)_j}, j \geq 1$$

$$P_{1j}(z) = P_{00}(z)\gamma^{j+1}(\frac{(\frac{\lambda+\alpha}{v}+1)}{\left(\left({}^{\alpha}/_{v}\right)+I\right)_j}, j \geq 0$$

**(iii)** **The partial factorial moments** $M_k^i\, for\; i\epsilon\{0,1\}\;\; and\; k\epsilon N$ **are given by**

$$M_0^0 = (1-\gamma z)P_{00}(z)F)$$

$$M_k^0 = (1-\gamma z)P_{00}(z)(\lambda+\alpha)^{-1}k!\,\mu\gamma^{k+1}\frac{(\frac{\lambda+\alpha}{v}+1)_k}{\left(\left({}^{\alpha}/_{v}\right)+I\right)_k}F\left(\frac{\lambda+\alpha}{v}+k,k+1;\frac{\alpha}{v}+k+1;\gamma), k \geq 1\right)$$

$$M_0^1 = P_{00}(z)\gamma F\left(\frac{\gamma+\alpha}{v}+1,1;\frac{\alpha}{v}+1;\gamma\right) M_k^1 = P_{00}(z)k!\,\gamma^{k+1}\frac{(\frac{\lambda+\alpha}{v}+1)_k}{\left(\left({}^{\alpha}/_{v}\right)+I\right)_k}F \quad (4.4)$$

**Proof:**

Since B(x) = $1-e^{-\mu x}, (x \geq 0)$

$$H(x) = (1-\gamma)\exp\left\{\frac{\lambda}{v}\int\frac{\beta(\lambda-\lambda x)-1}{\beta(\lambda-\lambda x)-x}dx\right\}$$

$$= (1-\gamma)\exp\left\{\frac{\lambda}{v}\int_x^1\frac{\frac{\mu}{(\lambda-\lambda x)-\mu}}{\frac{\mu}{\lambda-\lambda x-\mu}-x}-1\,dx\right\}$$

$$= (1-\gamma)\exp\left\{\frac{\lambda}{v}\int_x^1\frac{(x-1)}{(x-1)(x-{}^{\mu}/_{\lambda})}dx\right\}$$

$$= (1-\gamma)\exp\left\{-\log\left({}^{((x-\mu)}/_{\lambda}\right)^{\lambda/v})\right\}$$

$$= (1-\gamma)\exp\left\{-\log\frac{(\gamma x-1)^{\lambda/v}}{(1-\gamma)}\right\}$$

Thus $H(x) = (1-\gamma)\quad\left\{\frac{(1-\gamma)}{(1-\gamma x)}^{\lambda/v}\right\}$

Substitution of (3.8) into (3.9) yields

$$P_{00} = \frac{v}{\alpha}(\int_0^1 x^{\alpha/v-1}{}^{H^{-1}(x)dx})^{-1}$$

$$P_{00} = \frac{v}{\alpha}\frac{1}{[(1-\gamma)^{(\lambda/v)+1}]^{-1}}(\int_0^1 x^{\alpha/v-1}{}^{(1-\gamma x)^{\lambda/v}dx})$$

Let t=$\gamma x$ the integral in (3.10) reduces to

$$\int_0^1 x^{\alpha/v-1}(1-\gamma x)^{\lambda/v}\,dx = \int_0^{\gamma} \frac{t^{\alpha/v-1}}{\gamma^{\alpha/v-1}}(1-t)^{\lambda/v}\frac{dt}{\gamma}$$

$$=\gamma^{\alpha/v}\int_0^{\gamma} t^{\alpha/v-1}(1-t)^{\lambda/v}dt$$

Using hyper geometric series, formula given by Abramowitz and Stegun [1].

i.e,

$$\int_0^z t^{a-1}(1-t)^{b-1}dt = \frac{Z^a}{a}F(a,1-b;1+a;z) for\ 0 \leq z \leq 1, a, b > 0$$

Here Z= $\gamma$ , $a = \alpha/v$ , $b = (\lambda/v) + 1$

$$\therefore \int_0^1 x^{\alpha/v+1}(1-\gamma x)^{\lambda/v}dx = \{\left\{\frac{\gamma^{\lambda/v}}{\lambda/v}\right\}.F\left(\alpha/v, -\lambda/v; 1+\alpha/v; \gamma\right)\}\gamma^{-\alpha/v,}$$

Hence

$$P_{00} = \frac{v}{\alpha}(1-\gamma)^{(\lambda/v)+1}[\frac{v}{\alpha}F(\ \alpha/v, -\lambda/v; 1+\alpha/v\ ;\gamma)]^{-1}$$

$$P_{00} = (1-\gamma)^{(\lambda/v)+1}[F(\ \alpha/v, -\lambda/v; 1+\alpha/v\ ;\gamma)]^{-1} \tag{4.5}$$

From Bailey,

We have

$$F(c-a, c-b, c; z) = (1-z)^{a+b+c}F(a,b;c;z)$$

$$P_{00} = (1-\gamma)^{(\lambda/v)+1}[(1-\gamma)^{(\lambda/v)+1}F(\ \alpha/v, -\lambda/v; 1+q\,\alpha/v\ ;\gamma)]^{-1}$$

$$P_{00} = [F(\ \alpha/v, -\lambda/v; 1+\alpha/v\ ;\gamma)]^{-1}$$

Using this result from (3.5) we get

$$[\ P_0(z) = z^{-\alpha/v}H(z)(1-(\delta_{0\alpha})\frac{\int_z^1 x^{\alpha/v-1}H^-(x)dx}{\int_0^1 x^{\alpha/v-1}H^-(x)dx}]$$

$$P_0(z) = (1-\gamma z)P_{00}[F(\ \lambda+\alpha/v, +1,1; 1+\alpha/v\ ; z\gamma)]$$

Using this result from (3.6) we get

$$[P_1(z) = \frac{1-\beta(\lambda-\lambda z)}{\beta(\lambda-\lambda z)-z}P_0(z)]$$

$$P_1(z) = \gamma P_{00}F(\frac{\gamma+\alpha}{v}+1,1; 1+\frac{\alpha}{v}; \gamma z)$$

To prove (3.10) and (3.11) we have from (3.12)

$$P_0(z) = (1-\gamma z)P_{00}[F(\ \lambda+\alpha/v, +1,1; 1+\alpha/v\ ; z\gamma)]$$

$$\sum_{j=0}^{\infty} P_{0j}\quad (z) = (1-\gamma z)P_{00}\sum_{j=0}^{\infty}\frac{(\lambda+\alpha/v, +1)_j (I)_j z\gamma^j}{\frac{(1+\lambda)_j}{v}\, j!}$$

Solve this equation we get

$$P_{0j}(z) = P_{00} \quad \gamma^j \frac{\lambda}{\lambda+\alpha} \frac{(\frac{\lambda+\alpha}{v})_j}{(1+\frac{\alpha}{v})_j}, j \geq 1$$

We have from (3.12)

$$\sum_{j=0}^{\infty} P_{1j} z^j = P_{00} \quad \gamma \sum_{j=0}^{\infty} \frac{(\frac{\lambda+\alpha}{v})_j}{(1+\frac{\alpha}{v})_j} \frac{(\gamma z)^j (I)_j}{j!}$$

Equate we get,

$$P_{11} = P_{00} \ \gamma \ (\frac{(\frac{\lambda+\alpha}{v}+1)_0}{(\alpha+v+1)_0})$$

$$P_{12} = P_{00} \, \gamma^2 \ (\frac{(\frac{\lambda+\alpha}{v}+1)_1}{(\alpha+v+1)_1})$$

Proceeding like this

$$P_{1j} = P_{00} \, \gamma^{j+1} \ (\frac{(\frac{\lambda+\alpha}{v}+1)_j}{(\alpha+v+1)_j})$$

As $z \to -1 \ in \ P_i$(z) we obtain $M_0^i \ for \ i \epsilon \{0,1\}$

Since $P_i(1+z) = \sum_{k=0}^{\infty} \frac{M_k^i}{k!}, i\epsilon\{0,1\}$

$\sum_{j=0}^{\infty} P_{ij}(1+z)^j = M_0^i + M_1^i \frac{z^2}{2!} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$M_0^i = P_{i0} + P_{i1} + P_{i2}$, i=0 the above equation become

$M_0^0 = P_{00}(1)$)

$$\therefore M_0^0 = (1-\gamma) \quad M_0^0 = (1-\gamma) P_{00}(1) F(\frac{\lambda+\alpha}{v} + 1,1,1 + \frac{\alpha}{v}; \gamma)$$

Similarly we find other values of $M_0^0$

In the k[th] term is

$$M_k^1 = P_{00} \gamma^{k+1} k! \ (\frac{(\frac{\lambda+\alpha}{v}+1)_k}{(\frac{\alpha}{v}+1)_k}) F(\frac{\lambda+\alpha}{v} + k + 1, k+1, 1+k+\frac{\alpha}{v}; \gamma)$$

To find

$$P_0(z) = P_{00}(\lambda+\alpha)^{-1} \left[\alpha + \lambda F \frac{\lambda+\alpha}{v} + 1,1,1 + \frac{\alpha}{v}; \gamma z\right)$$

By the definition of hyper geometric function and (4.1), $M_k^0$ can be expressed as follows:

$$M_k^0 \quad = P_{00} \lambda (\lambda+\alpha)^{-1} \sum_{j=k}^{\infty} j(j-1) \ldots\ldots\ldots (j-k+1) \frac{(\frac{\lambda+\alpha}{v})_j}{(\frac{\alpha}{v}+1)_j} \gamma^j$$

By nothing that

$$j(j-1) \ldots\ldots\ldots (j-k+1) = \frac{(I)_j}{(I)_{j-k}}, j \geq k \geq 1 \ldots\ldots\ldots\ldots \qquad (4.6)$$

$$M_k^0 \quad = P_{00} \lambda (\lambda+\alpha)^{-1} \sum_{j=k}^{\infty} \frac{\left(\frac{\lambda+\alpha}{v}\right)_{j+k(I)_{j+k}}}{(\frac{\alpha}{v}+1)_{k+j(I)_j}} \gamma^{j+k} \qquad (4.7)$$

Using this ratio test we conclude that the series involved in $M_k^i$ converges when $\gamma < 1$ .In case $\gamma > 1$ the series diverges finally, if$\gamma = 1$ we cannot conclude from the ratio test anything at all about our series .We know from Raabes test that both series are divergent .Thus both series are convergent iff $\gamma < 1$ .These arguments guarantee that $M_k^i$ exists for all$k\epsilon N, i\epsilon\{0,1\}$.

For the case $\alpha \to 0$ and v->0, the results given in theorem 3.4(a) reduces to the expressions obtained by johin and Sedol [12]
Now for the case $\alpha \to 0$ and v->0the steady state probabilities.,

$$P_{0j}(z) = P_{00} \quad \gamma^j \frac{\lambda}{\lambda+\alpha} \frac{(\frac{\lambda+\alpha}{v})_j}{(1+\frac{\alpha}{v})_j}, j \geq 1$$

$$P_{ij} = P_{00}\,\gamma^{j+1} \left( \frac{\left(\frac{\lambda+\alpha}{v}+1\right)_j}{(\alpha+v+1)_j} \right), j \geq 0 \; becomes$$

$$P_{0j}(z) = \frac{\lambda}{\lambda+\alpha\left(1-\delta_{j0}\right)}(1-\rho)\rho^j, j \geq 0$$

$$P_{ij}(z) = \gamma(1-\rho)\rho^j, j \geq 0$$

Where

$$\rho = \lambda(\lambda+\alpha)\mu^{-1}\alpha^{-1}$$

The above result agrees with the inversion of the generating function given proposition 1 in Faylloe [10].

**Corr: 1**

**(i)** The probability of blocking is $P_b = P_1(I)M_0^1$.

**(ii)** The factorial moments of the pool size is in by $M_k = M_k^0 + M_k^1, for\ k\epsilon N.$

**(iii)** In the particular case$\alpha = 0\ and\ v > 0, the\ factorial\ moments\ become\ M_0^0 = 1-\gamma, M_0^1 = \gamma.$

**(iv)** In Particular for the case$\alpha > 0\ and\ v = 0, the\ factorial\ moments\ become\ M_0^0 = 1-\gamma, M_0^1 = \gamma.$

$$M_k = k!\left(\frac{\rho}{1-\rho}-\gamma\right)_k \left(\frac{\rho}{1-\rho}\right)^{k-1}, k \geq 1$$

Falin[3] and Hanske [12]

Independently obtained the generating functions of the stationary distribution of M/M/2.Characteristics of M/M/1 retrial queue discussed by Falin[2].

## V. CONCLUSIONS

In this paper detailed about the steady state distribution and partial generating functions of a single server queue. We concentrate on the case of exponential service times and finally we obtained explicit expression for the stationary distribution and the factorial moments for a single server queue.

## REFERENCES

[1]. Abramowitz, M and Stegun, I.A (1972) Hand book of Mathematical functions. Dover, New York.
[2]. Artalejo,J.R (1993) Explicit formulae for the characteristics of the M/$H_2$/I retrial queue. Operate Res.soc.44,(309-313).
[3]. Artalejo,J.R and Falin,G.I(1994),Stochastic decomposition for retrial queues top2, 329-342.
[4]. Cooper, R.B (1990) .Queuing theory. In Heyman,.D.P ., Sobel , M.J.eds stochastic Models Amsterdam: North-Holland (Elsevier),pp 469-518.
[5]. Falin,G.I.(1980) An M/M/1 queue with repeated calls in the presence of persistence function:Paper 1606-80.All-Union Institute for scientific and Technical information, Moscow.
[6]. Falin,G.I.(1984) An Double channel queuing system with repeated calls :Paper 4221-84.All-Union Institute for scientific and Technical information, Moscow.
[7]. Fayolly, G (1986) A simple telephone exchange with delayed feedbacks. In Teletraffic, Analysis and Computer performance evolutional .O.J.Boxma. J.W.Conen and H.C.Tijns, Elsevier, Amsterdam.

[8]. Fuhrmann, S.W .and Cooper, R.B (1985) stochastic decompositi0n in the M/G/1 queue with generalized vacations. Operate .Res 33,1117-1129.
[9]. Farahmand .K(1990) Single line queue with repeated demands QUESTSA6223-228.
[10]. Fayolle,G(1986) A simple telephone exchange with delayed feedbacks. In teletraffic analysis and computer performance evaluation et.O.J.Boxma,J.W.Conen and H.C.Tijns,Elsevier,Amsterdam.
[11]. Hanschke,T (1987) Explicit formulas for the characteristics of the M/M/2 queue with repeated attempts, J.Appl.Prob 24,486-494.
[12]. Jonin ,G.L an sedol,J.J(1970)Telephone systems with repeated calls.Proc $6^{th}$Int,Teletraffic congress 435/1-435/5.
[13]. Kendal,D.G (1957) Some problems in the theory of dams J.R statst Soc.B12,207-212.
[14]. Neal,S.R and Kuczura ,A (1973) A theory of traffic-measurement errors for loss systems with renewal input,BellSys.Tech J 52,967-990.
[15]. Ross,S.M (1993) .introduction to probability models $5^{th}$ed,San DiegoCa, Academic press.
[16]. Syski,R(1986) Introduction to congestion theory in telephone system $2^{nd}$ed.New York Elsevier.
[17]. James F.Kurose, Keith W.Ross, Computer Networking, Third Edition, Dorling Kindersley (India) Pvt. Ltd.