International Journal of Engineering Research and Development
e-ISSN: 2278-067X, p-ISSN: 2278-800X, www.ijerd.com
Volume 10, Issue 5 (May 2014), PP.41-48

Modelling and Surface Analysis of Deployable Mesh Reflector

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Abstract:- Focal length of an antenna is a critical parameter when it comes to setting the feed or the receiver. In this paper, a technique for least-squares fitting of the elliptic paraboloid in 3D data set is presented. The objective is to estimate the invariants and the focal length of the deployable mesh reflector using the linear least-squares based fitting mechanism. The 3D data set will be acquired from the flight model hardware using close range photogrammetry (CRP) and the points will be used as input to the least square based fitting mechanism. The technique also describes how to estimate the Euler angles and orientation of the antenna. Estimating the variations of the focal length in each deployment tests will help in deciding the placement of the feed or the receiver.

Keywords:- Parameter estimation, paraboloid, fitting mechanism, mesh reflector, focal length

I. INTRODUCTION

Future space based communication; remote sensing and space exploration missions will require large antenna reflectors capable of communicating with ground systems. They are also needed for missions beyond Earth's orbit where information must be transmitted over long distances. In order to support such large amounts of data, at long distance an antenna that can receive and transmit a large amount of radiation must be employed. If the distance between the mission and Earth grows to the order of astronomical units, there will be additional burdens on the antenna to radiate enough power to compensate for a large loss of the signal over such a large distance. The power that radiates from the antenna increases with the size of the reflector. Therefore, the need for such large data transfer rates leads to the necessity for aperture antennas which have large parabolic reflectors. Reflector antennas are particularly effective for the purpose of concentrating a signal over long distances. Unfortunately, the limited volume of a launch vehicle or spacecraft places restriction on the size of a rigid antenna reflector [2].

Since the late 1960's, deployable mesh reflectors have been favoured for their potential to fill large apertures with extremely lightweight hardware. To allow larger reflectors to serve these missions, a deployable antenna is desirable. Such an antenna has a reflector which is made of a light–weight material, can be collapsed or folded for compact stowing and is deployable at an appropriate time.

Measure of performance of these parabolic reflectors is the stability of reflecting surface after deployment on orbit. Reflecting surface is characterised by its invariants (major axis, minor axis, focal length) by measurement of coordinates and estimation. Test will be repeated to show that these parameters do not change in each deployment test on ground. In the following sections a methodology to estimate parameters of an elliptical paraboloid reflecting surface is described.

II. PROBLEM DESCRIPTION

The antenna is a parabolic mesh reflector of 6m aperture diameter. It consists of a mesh reflector made of gold plated molybdenum wire. The truss is made up of carbon fibre reinforced plastic (CFRP) and the vertical tie support is made up of Kevlar. When the antenna is deployed, it is essential to achieve a parabolic surface with a surface accuracy of less than 2mm. But due to error accumulations occurring in the manufacturing and assembly process, designed parabolic surface will have some errors. Variation in the focal length of the reflector will affect the location of the feed in the assembly. This study attempts to show that the invariants and errors are stable in different trials of deployment tests and estimate the magnitude of the same.

A least square based fitting technique to recover the parameters of the surface from its 3D data, of co-ordinates measured at the nodes, is discussed in the reminder of this paper. The antenna specification is shown below:

- Surface accuracy : < 2 mm
- Antenna aperture size (d) : Ø 6000 mm
- Focal length (f) : 4380 mm

- f/d ratio : 0.73
- Antenna type : Offset type (550mm offset)

The objectives of the methodology are as follows:

- To estimate the unknown parameters of quadratic surface given by photogrammetric coordinates of unfurlable antenna.
- To prove that the obtained quadratic surface is an elliptical paraboloid.
- To determine the invariants and focal length of the elliptical paraboloid.

III. METHODOLOGY

Surface recovery is commonly requisite in automatic object recognition and visual inspection, particularly in surface model-based systems. In such procedures the surface recovery is typically a key processing activity. Processing schema that enable recovery of a surface-based model from a 3D data typically include tasks of local surface property estimation and reconstruction which involves finding the parameters of the best fitting surface [6]. The least squares approach is a popular method in statistical estimation and its application is found common in computer vision. In order to determine the focal length of the paraboloid, the unknown parameter is to determined first, followed by estimation of the shape from the 3D data and then reconstruction based on the estimated parameters. Reconstruction helps to determine the invariants in order to find the focal length of the antenna.

A. Determining the unknown parameters

The Gauss-Markoff model is used to estimate the unknown parameters. The Gauss-Markoff model is represented as follows:

$$X\beta = Y + e \tag{1}$$

Where, 'X' is an *n* x *u* matrix of given coefficients, 'β' a *u* x *1* matrix of unknown fixed parameters and 'Y' an *n* x *1* random vector of observations. The matrix 'e' represents the error matrix. The above equation is also known as observation equation [5].

The polynomial model often serves the purpose to fit a planar curve or a surface in three-dimensional space E^3 to measured data. If for instance a curve is given in E^2, where a rectangular (x, y) coordinate system is defined, and if on the curve points P_i with coordinates (x_i, y_i) are selected such that for given abscissae x_i the ordinates y_i are measured, then the curve may be represented by the polynomial '$\beta_1 + x_i\beta_2 + x_i^2\beta_3 +\ldots+ x_i^{u-1}\beta_{u-1}$ equal to $E(y_i)$ with $i \varepsilon \{1,2\ldots,n\}$, where $\beta_1, \beta_2,\ldots, \beta_{u-1}$ denote the unknown parameters of the polynomial model.

The equation for a general quadric in Cartesian space is,

$$\beta_1x^2 + \beta_2y^2 + \beta_3z^2 + \beta_4xy + \beta_5yz + \beta_6xz + \beta_7x + \beta_8y + \beta_9z + \beta_{10} = 0 \tag{2}$$

It is possible to re-write Eq. (2) in a form with nine coefficients since only nine of its ten coefficients are independent. For example, for the cases that $\beta_{10} \neq 0$, Eq. (2) can be rewritten as

$$\beta_1x^2 + \beta_2y^2 + \beta_3z^2 + \beta_4xy + \beta_5yz + \beta_6xz + \beta_7x + \beta_8y + \beta_9z = 1 \tag{3}$$

Reconstructing a general quadric requires determining the nine independent coefficients β_i (i = 1, 2, …, 9).

Next, is a description of how the nine quadric coefficients of Eq. (3) can be recovered using a linear least-squares approach. It is assumed the least-squares fitting is performed on an input of *m* 3D points (x_i, y_i, z_i). If there are m sample points, there are m(x, y, z) values. Theoretically, the nine unknown parameters can be solved from a group of nine linear equations of the form of Eq. (3), each of which has one data point (x_i, y_i, z_i) assigned to its corresponding variables x, y and z. In this way, the values of coefficients can be acquired with only nine input sample points. However, due to sampling error, such results are not robust [8].

To enable robust fitting, an over-constrained system of m linear equations (i.e., m > 9) Eq. (1) is considered where,

$$X_{mxn} = \begin{bmatrix} x_1^2 & y_1^2 & z_1^2 & x_1y_1 & y_1z_1 & x_1z_1 & x_1 & y_1 & z_1 \\ x_2^2 & y_2^2 & z_2^2 & x_2y_2 & y_2z_2 & x_2z_2 & x_2 & y_2 & z_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ x_m^2 & y_m^2 & z_m^2 & x_my_m & y_mz_m & x_mz_m & x_m & y_m & z_m \end{bmatrix} \tag{4}$$

Eq. (4) is the coefficient matrix of the linear equation group and n = 9.

Additionally,

$$\beta = [\beta_1\ \ \beta_2\ \ \beta_3\ \ \beta_4\ \ \beta_5\ \ \beta_6\ \ \beta_7\ \ \beta_8\ \ \beta_9]^T \tag{5}$$

is the vector of unknown variables. The right-side term is

$$Y_{m\times 1} = [1\ 1\ .\ .\ .\ 1]^T \tag{6}$$

Normally, the least squares solution does not satisfy all equations in the group, but it minimizes the value of the residual error. The least-squares solution 'X' to this system of equations is the solution that minimizes the residual error

$$e = \|X\beta - y\|_2 \qquad (7)$$

and this solution is optimal in the least-squares sense. The solution 'β' can be computed using the normal equation

$$\beta = (X^T X)^{-1} X^T Y \qquad (8)$$

If the sample data points are not concentrated at a single point, line or plane (which are the reduced dimension cases), then the rows and columns in matrix X will not be linearly co-related, and therefore the matrix X^TX will be non-singular. When X^TX is non-singular, there is guaranteed to exist a least-squares solution vector β [8].

Thus far, the coefficients of Eq. (3) have been determined. However, the coefficients of the equation do not have clear geometric significance, such as the symmetric centre, if any, of the quadratic surface, its translation from the origin, and the orientation of the surface. Therefore the next step is to estimate the surface type or shape.

B. Estimating the shape of the quadratic surface

Surface type can be determined using the surface's invariants Δ and D, which are invariant with respect to translation and rotation [7]. For an arbitrary general quadric defined in the form of Eq. (2), the invariant Δ is

$$\Delta = \begin{vmatrix} \beta_1 & \beta_4/2 & \beta_6/2 & \beta_7/2 \\ \beta_4/2 & \beta_2 & \beta_5/2 & \beta_8/2 \\ \beta_6/2 & \beta_5/2 & \beta_3 & \beta_9/2 \\ \beta_7/2 & \beta_8/2 & \beta_9/2 & \beta_{10} \end{vmatrix} \qquad (9)$$

The invariant D is

$$D = \begin{vmatrix} \beta_1 & \beta_4/2 & \beta_6/2 \\ \beta_4/2 & \beta_2 & \beta_5/2 \\ \beta_6/2 & \beta_5/2 & \beta_3 \end{vmatrix} \qquad (10)$$

The relationship between quadric surface type and the invariants is summarized in Table I.

Table I: Quadric surface classification via invariant values

	D ≠ 0 Central quadric surface	**D = 0 Non-central quadric surface**
Δ > 0	**Single-sheet hyperboloid**	**Hyperbolic paraboloid**
Δ < 0	**Ellipsoid or dual-sheet hyperboloid**	**Elliptical paraboloid**
Δ = 0	**Cone**	**Cylinder or plane**

Comparing the values of D and Δ with Table I will help in determining the shape of the quadratic surface [7]. After determining the quadratic surface, the standard equation of the type of surface is considered and the invariants are determined.

C. Paraboloid reconstruction

The standard equation of an elliptic paraboloid is

$$f(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - z = 0 \qquad (11)$$

Eq. (11) is formed for the assumed parameter c = 1. In other words, a and b are the shape parameters of the paraboloid, i.e., the half lengths of its cross-sectional ellipse at z=1.

According to principle axes theorem, the Eq. (2) can be written in matrix form as follows:

$$x^T A x + Gx + H = 0 \qquad (12)$$

where, $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $A = \begin{bmatrix} \beta_1 & \beta_4 & \beta_6 \\ \beta_4 & \beta_2 & \beta_5 \\ \beta_6 & \beta_5 & \beta_3 \end{bmatrix}$, $G = [\beta_7 \quad \beta_8 \quad \beta_9]$ and $H = \beta_{10}$. If the coefficients of cross-products are zero then no rotation is necessary. Also if it's zero, then A is symmetric and also there exist an orthogonal matrix R such that R^TAR is equal to a diagonal matrix. The matrix R must be orthogonal and its determinant will be ± 1. This matrix can be thought of a sequence of three rotations, one about each principle axis, first about x-axis, then the y-axis and finally z-axis [1]. The matrix will be of the form

$$R = R(\theta_x)R(\theta_y)R(\theta_z) \qquad (13)$$

The standard equation of an elliptic paraboloid can be re-written in terms of matrices and vectors, to yield a form similar to Eq. (12), as

$$x^T A x + G x = H \tag{14}$$

where,

$$x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{15}$$

$$A = \begin{bmatrix} \frac{1}{a^2} & & \\ & \frac{1}{b^2} & \\ & & 0 \end{bmatrix} \tag{16}$$

$$G = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \tag{17}$$

'H' is zero in Eq. (14). After certain translation and rotation Eq. (14) becomes

$$x`^T A_1 x` + G_1 x` = H_1 \tag{18}$$

where,

$$x` = \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix} \tag{19}$$

$$A_1 = R^T A R \tag{20}$$

$$R = R(\theta_x, \theta_y, \theta_z) = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \tag{21}$$

and, $G_1 = G^T R$. R is a 3x3 standard orthogonal matrix representing a rotational transform, and θ_x, θ_y and θ_z are three rotation angles that describe the quadratic's orientation with respect to the standard orientation. Since G_1 is the product of G^T and R, it is a vector denoted as follows:

$$G_1 = [w_1 \quad w_2 \quad w_3] \tag{22}$$

Because matrix A_1 is symmetric, there are only six independent elements in it. By expanding '$x`^T A x` = 0$' and comparing it to Eq. (2), the translation values x_0, y_0, z_0 and the 6 independent elements of A_1 can be computed. The process is as follows.

If it is assumed that,

$$A_1 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \tag{23}$$

Then according to Eq. (19) and Eq. (23), $x`^T A_1 x` = 0$ can be written as:

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{23}yz + 2a_{13}zx - (2a_{11}x_0 + 2a_{12}y_0 + 2a_{13}z_0)x - (2a_{12}x_0 + 2a_{22}y_0 + 2a_{23}z_0)y - (2a_{13}x_0 + 2a_{23}y_0 + 2a_{33}z_0)z + a_{11}x_0^2 + a_{22}y_0^2 + a_{33}z_0^2 + 2a_{12}x_0y_0 + 2a_{23}y_0z_0 + 2a_{13}z_0x_0 = 0 \tag{24}$$

After comparing Eq. (24) with Eq. (2), the coefficients of Eq. (2) can be represented as follows:

$$\beta_1 = a_{11}/u \tag{25}$$

$$\beta_2 = a_{22}/u \tag{26}$$

$$\beta_3 = a_{33}/u \tag{27}$$

$$\beta_4 = 2a_{12}/u \tag{28}$$

$$\beta_5 = 2a_{23}/u \tag{29}$$

$$\beta_6 = 2a_{13}/u \tag{30}$$

$$\beta_7 = -(2a_{11}x_0 + 2a_{12}y_0 + 2a_{13}z_0)/u \tag{31}$$

$$\beta_8 = -(2a_{12}x_0 + 2a_{22}y_0 + 2a_{23}z_0)/u \tag{32}$$

$$\beta_9 = -(2a_{13}x_0 + 2a_{23}y_0 + 2a_{33}z_0)/u \tag{33}$$

$$u = -(a_{11}x_0^2 + a_{22}y_0^2 + a_{33}z_0^2 + 2a_{12}x_0y_0 + 2a_{23}y_0z_0 + 2a_{13}z_0x_0) \tag{34}$$

For paraboloids, the values 'w_i' in the Eq. (22) are non-zero; $w_1=r_{31}$, $w_2=r_{32}$, and $w_3=r_{33}$. Since w_1, w_2, and w_3 are not zero, the values of x_0, y_0, z_0 cannot be solved directly using the linear least-squares formulation. Since these values cannot be found, the values which directly and indirectly depend on them also cannot be determined. Specifically, the value for u and the elements of A_1 cannot be found, which prevents determination of the shape and orientation parameters. Therefore, the next step is to find the location, orientation, and shape parameters for elliptic paraboloid. First, a variable v is introduced, where

$$v = 1/u \tag{35}$$

and perform the orthogonal decomposition (diagonalization) on a matrix,

$$A_2 = vA_1 = \begin{bmatrix} \beta_1 & \beta_4/2 & \beta_6/2 \\ \beta_4/2 & \beta_2 & \beta_5/2 \\ \beta_6/2 & \beta_5/2 & \beta_3 \end{bmatrix} \tag{36}$$

The result must be,

$$A_2 = R^T (vA_1)\, R \tag{37}$$

The R in Eq. (37) is same as the R in Eq. (21). Therefore the values of w_i can be found from eigen-analysis of Eq. (36). If Eqs. (25) – (30) and Eq. (35) are substituted in Eqs. (31) – (33), then,

$$2\beta_1 x_0 + \beta_4 y_0 + \beta_6 z_0 = -\beta_7 - r_{31} v \tag{38}$$

$$\beta_4 x_0 + 2\beta_2 y_0 + \beta_5 z_0 = -\beta_8 - r_{32} v \tag{39}$$

$$\beta_6 x_0 + \beta_5 y_0 + 2\beta_3 z_0 = -\beta_9 - r_{33} v \tag{40}$$

Here x_0, y_0, z_0 and v are unknown. The unknown translational variables x_0, y_0 and z_0 can be represented by expressions of v according to Eqs. (38) – (40),

$$x_0 = g_1(v) \tag{41}$$

$$y_0 = g_2(v) \tag{42}$$

$$z_0 = g_3(v) \tag{43}$$

After dividing both sides of Eq. (34) by u and then substituting Eqs. (25) – (30) and (41) – (43) into it, the result is an expression of the form:

$$pv^2 + qv + s = 0 \tag{44}$$

Where, q = 0 and, p and s are functions of the known coefficients β_1, β_2,.., β_9 and r_{31}, r_{32}, r_{33}. Furthermore, the value s is:

$$s = \begin{vmatrix} \beta_1 & \frac{\beta_4}{2} & \frac{\beta_6}{2} & \frac{\beta_7}{2} \\ \frac{\beta_4}{2} & \beta_2 & \frac{\beta_5}{2} & \frac{\beta_8}{2} \\ \frac{\beta_6}{2} & \frac{\beta_5}{2} & \beta_3 & \frac{\beta_9}{2} \\ \frac{\beta_7}{2} & \frac{\beta_8}{2} & \frac{\beta_9}{2} & -1 \end{vmatrix} \tag{45}$$

Eq. (45) is exactly the invariant Δ for a general quadratic [7].

In addition the value of p is

$$p = \begin{vmatrix} \beta_1 & \frac{\beta_4}{2} & \frac{\beta_6}{2} & \frac{r_{31}}{2} \\ \frac{\beta_4}{2} & \beta_2 & \frac{\beta_5}{2} & \frac{r_{32}}{2} \\ \frac{\beta_6}{2} & \frac{\beta_5}{2} & \beta_3 & \frac{r_{33}}{2} \\ \frac{r_{31}}{2} & \frac{r_{32}}{2} & \frac{r_{33}}{2} & 0 \end{vmatrix} \tag{46}$$

Thus p is,

$$p = \frac{-s}{v^2} \tag{47}$$

Since each of p and s can be expressed as a real multiple of the invariant Δ of a paraboloid surface, and since Δ must be negative for elliptic paraboloids (Table I), $s > 0$ and $p < 0$; a real solution for v is ensured. Therefore, the value of v can be found using

$$v = \pm\sqrt{-\frac{s}{p}} \tag{48}$$

It is necessary to determine the sign of v. Because of Eq. (37), vA is a diagonal matrix. Furthermore, according to the definition of matrix A in Eq. (16), its non-zero eigenvalues (diagonal entries) must be positive. Therefore, the sign of v can be determined by finding A_2's eigenvalues; if A_2's eigenvalues are negative, then v must be negative [1].

Once v has been determined, evaluation of Eqs. (16) and (37)–(40) allows determination of x_0, y_0, z_0, a, and b. Thus all of the location, orientation, and shape information can be extracted for elliptic paraboloid antenna.

D. Estimating the focal length

Now that the profile parameters 'a' and 'b' are known at an assumed value of 'c' the focal length can directly be calculated using the formula [7]:

$$F_l = \frac{(2b)^3}{16 \times 2a \times c}$$

IV. RESULT

Seven sets of three-dimensional data required for the analysis was obtained using photogrammetry and the antenna was plotted using MatLab software. The methodology discussed in the previous section was applied and the analysis was conducted using MatLab. The results are discussed as follows.

The diagram consists of all the coordinates of the mesh reflector as well as the coordinates of the truss. A total of 163 coordinates constitutes the mesh reflector and 18 coordinates belong to the deployable truss. Since the surface analysis was conducted only for the mesh reflector, the coordinates belonging to the truss was eliminated and the meshed diagram of the reflector was plotted.

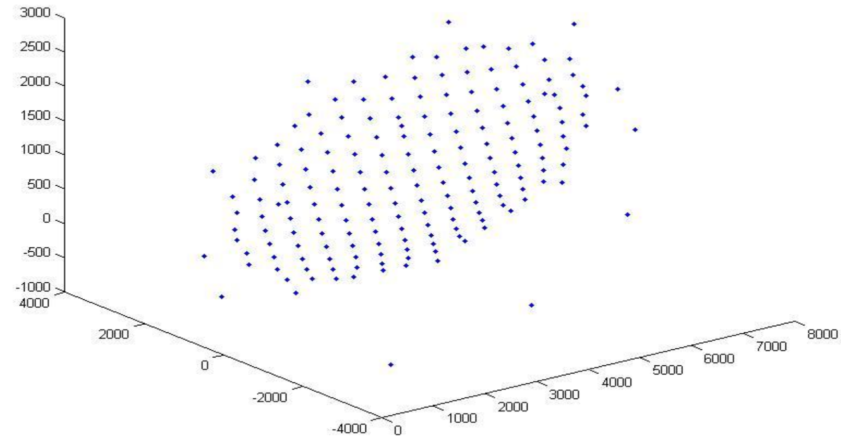

Fig. 1: Isometric view of the coordinates of the UFA

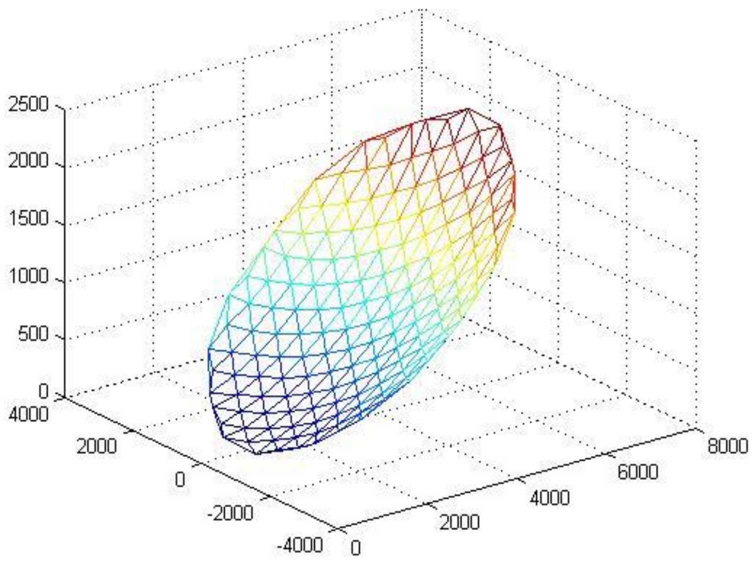

Fig. 2: Isometric view of the mesh reflector

A. Estimating unknown parameters

Eq. (3) is selected as the standard equation to estimate the unknown parameters. The coordinates of the mesh reflector are known. Substituting the coordinates into Eq. (5) the coefficient matrix 'X' is obtained. Matrix 'Y' is known as the unit vector matrix or the response matrix. Eq. (8) is solved to find the unknown parameters and therefore the 'β' matrix is obtained.

Since X, β and Y are now known matrices, Eq. (9) can now be solved to get the error plot.

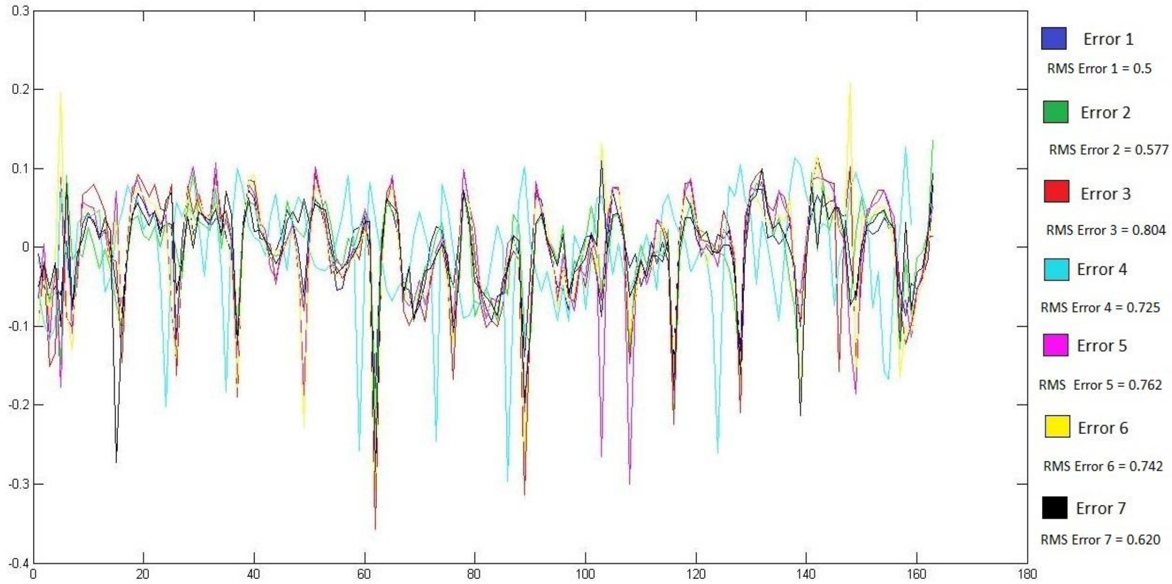

Fig. 3: Error plot

B. Estimating the shape of the quadratic surface

The values of matrix β are now known. Substituting the values in Eq. (9) and Eq. (10) values of 'Δ' and 'D' were found to be,

Δ = -2.1874e-10

D = 7.8144e-15 ≈ 0

The value of Δ is a negative value and the value of D is approximately equal to zero. Hence, from Table I, it can be concluded that the quadratic surface is an elliptic paraboloid.

C. Paraboloid reconstruction and focal length estimation

The invariants and focal length of all seven sets of data were found successfully, following the steps explained in sub-section C of section III. The ellipse was cut at an assumed value c = 44. The result is shown in Table II.

Table II: Final result

Deployment Test No.	Major	Minor	c	Focal Length	Ellipticity
1	**1960.5**	**1821.7**	**44**	**4380.23**	**1.076176**
2	**1962.0**	**1822.1**	**44**	**4379.822**	**1.076763**
3	**1961.9**	**1821.7**	**44**	**4377.067**	**1.076956**
4	**1962.2**	**1822.1**	**44**	**4379.346**	**1.076878**
5	**1961.8**	**1821.8**	**44**	**4377.7501**	**1.076859**
6	**1962.0**	**1821.9**	**44**	**4377.9034**	**1.076928**
7	**1961.9**	**1822.1**	**44**	**4379.929**	**1.076725**

V. CONCLUSION

An indirect technique of least-squares fitting of the elliptic paraboloid mesh reflector in 3D data has been presented. Using this approach the invariants and the focal length of the paraboloid reflector was estimated. The result matches with the design parameters. This shows that the technique was helpful in recovering the surface parameters. The limitation of this technique is that the inputs included must be from only one surface type. If the inputs include points from more than one surface type, fitting accuracy can be impacted.

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