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Formalization of an Approach for Improvement of Maintenance Policy on Multi-State Systems

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Abstract:- This work includes part of the results of I.W.Soro on performance evaluation of Multi-State Systems (MSS) about the preventive maintenance policy. It was to assess the availability and the rate of production of a multi-state system based on a rate of transitions in the level of β degradation. The formalism of calculation based on Markov chains used and Chapman-Kolmogorov equations induce as many calculations as possible cases of β transition rates to deduce the one that brings the best drift of the availability curves and production rates. Moreover, the representation of multi-state system by a Markov graph quickly becomes dense and difficult to use. In this paper, it will first be presented formalization of the transition process of multi-state system (MSS) by Bayesian Networks (especially compact) and the rules governing promotion from the Markov graph. In a second step, it will be exhibited, the cost function of preventive maintenance and the best method for identifying the β transition rates and thus the best preventive maintenance policy to adopt. The optimization has done by reinforcement learning.

Keywords:- Multi-State Systems, Graph Markov, Dynamic Bayesian Networks, Preventive Maintenance , Modeling

I. INTRODUCTION

Today, the complexity of industrial systems and production requirements prompt maintenance services to make a management even more rigorous in their task and a continuous search for improvement of maintenance strategies. That will require the availability of tools for decision support on the choice of these strategies according to indicators, notably the cost and the productivity.

Commonly, to satisfy production requirements such as productivity improvements, the production machines are forced to operate continuously under several levels of performance with the least possible downtime. This operating mode called "multi-state" will cause multiple damage (fatigue, wear, physico-chemical alterations, etc..) without the input of a process of national policy maintenance (preventive and corrective).

It is in this context that is currently oriented researches on the reliability, the modeling and optimization of the preventive maintenance of Multi-State Systems (MMS).

Our objective of this paper is, firstly, to present our work on the formalization of various system states by the Dynamic Bayesian Network from a degradation model by Markov chain and on the other hand, the formulation evaluation of the availability, the cost function and the preventive maintenance method for identifying the best combination of β transition rates and thus the best preventive maintenance policy to adopt continuously. The remain of the paper is organized as follows. Section 2 is devoted to a review of the MSS. Section 3 compares the formalism of Dynamic Bayesian Network (DBN) to the Markov chain. Section 4 presents the proposed approach. Section 5 is the application of the approach with the results. Section 6 provides a conclusion of the work.

II. STATE OF THE ART

2.1 Concept of Multi-State

In the classical concept, in binary mode systems worked either in perfect condition or completely failed state. The theory of this binary system in [1] paved the way to the mathematical theory and statistical reliability. In practice with production issues, we realized the need of another mode of operation that may confer to production services and maintenance flexibility of operating their facilities therefore to have the desired availability and maintenance costs reasonable. The system will integrate several operating states corresponding to levels of system performance, hence the name of multi-state systems.

In reality, the system components can operate at different levels of degradation. This degradation varies between states of operation and the total failure of the element [2] . For example: sheller's status can be 0, 1, 2, 3, 4 corresponding to 0%, 25%, 50%, 75%, 100% of its total capacity.

The theory of multi-state systems emerged with the work in [3] which defines the system state as the state of the worst component at best minimal link, or the state of the best component at minimal cut. The performance of any system depends on the state of its components and there are different configurations: the systems in series, parallel, series-parallel and parallel-series, k-among-n, k-consecutive among-n. K-consecutive among-n systems are also the subject of interesting studies considering their better reliability compared with series systems, cheaper than parallel systems and their large application [4].

2.2 Methods of assessment and review of Multi-State Systems

Many studies have been done on analyzing the availability or reliability of multistate systems. We identified four main approaches in the literature to assess the availability and reliability of these Multi-State Systems (MSS):

- Stochastic [5],
- Monte Carlo[6],
- Functional [7],
- UMGF [8].

As for estimating the availability of larger systems, the UMGF method is the best applied among other methods (Stochastic, Monte Carlo). A literature review relatively exhaustive on the reliability of MSS can be found for example in [9]. Many researchers have focused on the study of MSS and their application in various fields such as industry, medicine etc., each with different approaches or formalisms more or less varied as follows:

Reference [10] has shown properties for deterministic and probabilistic system performance.

Reference [11] used the approach of a Markovian system in three states. He led a study, of the availability status, frequency of failure and mean time to failure.

Reference [12] developed a simulation algorithm to calculate the probability distribution of system state and also used the theory of Markov chain to give the component reliability and the system.

Reference [13] developed a model to assess the availability, the production rate and the reliability function of degraded multi state systems subjected to minimal repairs and imperfect preventive maintenance . They associated to each state of its system Markov model a performance rate. The aim is that the rate of system performance at time t exceeds the customer's request. The transition from one state to another is made according to the exponential law. The analytical model is established by the Chapman-Kolmogorov equations. However we find that this customer demand (production rate) is constant this is not the case in practice.

Reference [14] proposed a study and a construction of a general model for representing generic term models that can adapt to multi-state systems, with the laws of any stay time and possibly a contextual dependency. To do this, they propose a particular Dynamic Bayesian Network appointed Model Graphical Time (MGD).

Reference [15] established an integrated planning of preventive maintenance and production of multi-state systems, the work provides planning templates to generate an optimal production plan at the tactical level and the moments when response intervals for preventive maintenance actions (acyclic or cyclic). To obtain optimal solutions, they developed methods of assessing time and cost of maintenance, capabilities relating to systems and some algorithms of resolution. This work provides an economic impact by integrating the planning of preventive maintenance and production.

Reference [16] based on the dynamic Bayesian network, on the one hand they offered a cost function to evaluate maintenance policies and on the other hand an optimization algorithm type genetics in order to retain the optimal preventive maintenance policy. Their approach is applied to a distribution system for three valves.

III. MARKOV CHAIN FORMALISM AND DYNAMIC BAYESIAN NETWORK

We present here Dynamic Bayesian Networks and Markov Chain while exposing the strengths and benefits of each.

3.1 Dynamic Bayesian Networks

Dynamic Bayesian Networks (DBN) are really an extension of Bayesian networks in which the temporal evolution of the variables is represented in ([17], [18], [19]). Dynamic Bayesian Networks are also shown as an extension of Markov Chains [20]. In many works on the representation of complex systems, probabilistic graphical models such as DBN hold a prominent place in the modeling of dynamic systems with discrete and finite states [21].

It aims to model the probability distribution of a series of variables $\left(X_t\right)_{1<t<T} = \left(X_{1,t}, X_{2,t}, ..., X_{n,t}\right)_{1<t<T}$ on a sequence of length T∈ℕ. The process is represented by a node X_t^i at time step t with a finite number of

possible states $S_{X^i} : \left\{ S_i^{X^i}, ..., S_N^{X^i} \right\}$ and arcs represent dependencies between time points. A state space Ω is the cross product of the values of states for individual state variables: $\Omega = \prod_{i=1}^{N} S_{X^i}$. $p\left(X_t^i\right)$ being the probability distribution on variable states at step time t. The nodes correspond to state variables that can be partitioned into two sets: one corresponding to the state variables at time step t and the other corresponding to the system state at next time step (t +1). The variable is then represented at successive times in this case.
The following figure shows a dynamic Bayesian network with two time steps t and (t +1), the network is called dynamic Bayesian network with two slices named DBN-2.

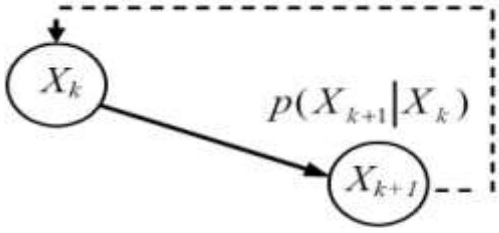

Fig. 1: Modeling of a 2-DBN

Many studies speak of the relationship or the link of Markov chains to Dynamic Bayesian Network.
A correspondence between Markov chains and Dynamic Bayesian Network is presented case by case, an advantage of the Bayesian network of Markov chains is highlighted in [22].
Indeed the Dynamic Bayesian Network is most suitable and appropriate in the reliability analysis of large complex systems.

3.2 Markov Chain

The sequence of random variables X_1, X_2,..., X_n forms a Markov chain with discrete state space if for all $n \in \mathbb{N}$ and all possible values of X_n random variables , we have :

$i_1 < i_2 < ... < i_n$.

$$P\left(X_n = j \mid X_{n-1} = i_{n-1}, X_{n-2} = i_{n-2}, ..., X_1 = i_1\right) = P\left(X_n = j \mid X_{n-1} = i_{n-1}\right) \quad (1)$$

This conditional probability $P\left(X_n = j \mid X_{n-1} = i_{n-1}\right)$ is called the transition probability.

Indeed the transition probability allows for a transition from E_i state at step (n-1) to the E_i state at step nth.
The Markov chain is said to be homogeneous when this probability does not depend on n, that is to say

$$P_{ij} = P\left(X_n = j \mid X_{n-1} = i_{n-1}\right) \quad (2)$$

The following figure shows an example of a Markov chain with two states, the model represents the transition probabilities that are associated with each arc.

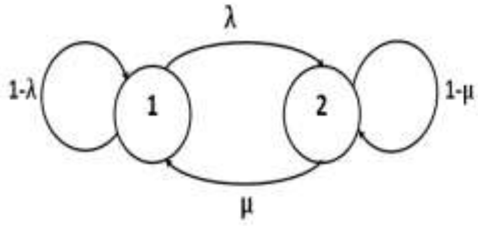

Fig. 2: Example of Markov chain

In the case of a Markov process with time independent the failure rate is considered constant while in the Markov process at this time dependent this rate is not constant so variable. This is explained by the fact that degradation of the component of a system within an industrial environment is constantly changing over time due to its use or age.
However the use of Markov models have limitations in particular the combinatorial explosion in the number of states likely to be occupied by the system which is desired to model the behaviour [23] (Innal, F., et al., 2006).

IV. APPROACH

Our goal is to provide an approach and a tool for decision support that enable searching the optimal preventive maintenance policy based on a simulation of an entire operating horizon of material with a learning gradually(history) decisions and performance (failure rate and availability) obtained each time. In addition, the tool should allow to add expert knowledge of a cognitive nature. As a dysfunctional representation of the equipment, we start from a state representation via a graph and Markov chain:

- **i.** We produce the structure of a Dynamic Bayesian Network (DBN) and define the conditional probability tables (modeling the transition parameters of multi-state systems). The nodes correspond to the transition parameters;
- **ii.** We define the rules for the passage of the Markov graph to a Dynamic Bayesian Network (setting rule of conditional probability tables CPT, ...). These are generic rules of passage and not specific to application case treated;
- **iii.** We integrate the indicators performance for the assessment (availability, maintenance cost, ...) in DBN;
- **iv.** We simulate the behavior of the equipment (multi-state) on a service life where the parameters are stochastic and not constant. The target node is the variable state of degradation. The stochastic evolution of PM levels will be associated with each iteration, the availability and cost of operational maintenance of equipment;
- **v.** We use a reinforcement learning algorithm to obtain the optimal level of preventive maintenance in view of the simulation.

V. BAYESIAN MODELLING OF MULTI-STATE SYSTEM

We start from the representation of the Markov graph that shows the different states that could have a production system.

The parameters of transitions between states are:

λ_i : Failure rate from i state to i+1 state

α_i : Degradation rate from i state to i+1 state

β_i : Rate of passage from degraded state to next degraded state

μ_i : Repair rate from failure state to degraded state

To follow the evolution of a given system, we consider a decision variable of preventive maintenance policy called x in the graph of Markov chain (Fig.9).

5.1 Bayesian model of MSS

From the Markov graph, we establish the structure of our Dynamic Bayesian Network as shown in Fig.3.

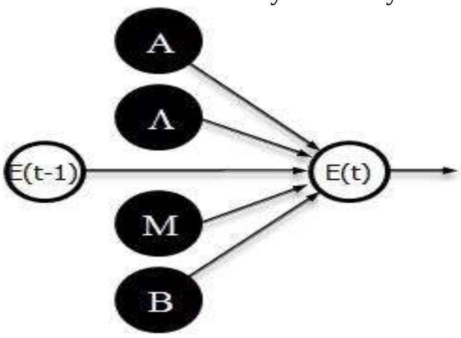

Fig. 3: **Bayesian model of MSS**

$E(t)$ et $E(t+1)$ respectively denote the state of system at time t, the state of system at time (t+1).

$A_1(t)$ et $B(t)$ respectively denote vectors consisted of α_i and β_i avec $i=1,2,...,n$.

$\Lambda(t)$ et $M(t)$ respectively denote the vectors formed of λ_i and μ_i with $i=1,2,...,n$

$$E(t) = \begin{pmatrix} E_1(t) \\ E_2(t) \\ \dots \\ E_n(t) \end{pmatrix}; E(t+1) = \begin{pmatrix} E_1(t+1) \\ E_2(t+1) \\ \dots \\ E_n(t+1) \end{pmatrix}$$

$$\mathrm{A1}(t) = \begin{pmatrix} \alpha_1(t) \\ \alpha_2(t) \\ \dots \\ \alpha_n(t) \end{pmatrix}; B(t) = \begin{pmatrix} \beta_1(t) \\ \beta_2(t) \\ \dots \\ \beta_n(t) \end{pmatrix}$$

$$\Lambda(t) = \begin{pmatrix} \lambda_1(t) \\ \lambda_2(t) \\ \dots \\ \lambda_n(t) \end{pmatrix}; \quad M(t) = \begin{pmatrix} \mu_1(t) \\ \mu_2(t) \\ \dots \\ \mu_n(t) \end{pmatrix}$$

In this article we will use the following structure of the DBN

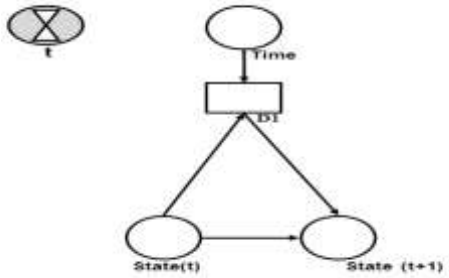

Fig. 4: Bayesian model used

To fill in the conditional probability tables of our DBN structure, we use the Chapman-Kolmogorov equations to determine the transitions probability of system states.

$$\begin{aligned}
&P_1^{(k+1)} = \left(1-\alpha_1^{(k)}-\lambda_1^{(k)}\right)P_1^{(k)} + \mu_1^{(k)}P_2^{(k)} + x_1^{(k)}\beta_1^{(k)}P_{2d-1}^{(k)} \\
&\forall j\in\{2,\dots,d-1\}: P_{2j-1}^{(k+1)} = \left(1-\alpha_j^{(k)}-\lambda_j^{(k)}\right)P_{2j-1}^{(k)} + \mu_1^{(k)}P_{2j}^{(k)} + \alpha_{j-1}^{(k)}P_{2j-3}^{(k)} + x_j^{(k)}\beta_j^{(k)}P_{2d-1}^{(k)} \\
&P_{2d-1}^{(k+1)} = \left(1-\alpha_d^{(k)}-\lambda_d^{(k)}-\sum_{j=1}^{d-1}x_j^{(k)}\beta_j^{(k)}\right)P_{2d-1}^{(k)} + \mu_d^{(k)}P_{2d}^{(k)} + \alpha_{d-1}^{(k)}P_{2d-3}^{(k)} \\
&P_{2d+1}^{(k+1)} = \left(1-\alpha_{d+1}^{(k)}\right)P_{2d+1}^{(k)} + \alpha_d^{(k)} + P_{2d-1}^{(k)} \\
&\forall j\in\{2,\dots,d-1\}: P_{2d+j}^{(k+1)} = \left(1-\alpha_{d+j}^{(k)}\right)P_{2d+j}^{(k)} + \alpha_{d+j-1}^{(k)}P_{2d+j-1}^{(k)} \\
&P_{2d+m}^{(k+1)} = P_{2d+m}^{(k)} + \alpha_{d+m-1}^{(k)}P_{2d+m-1}^{(k)} \\
&P_1^{(0)} = 1, P_j^{(0)} = 0; \forall j\in\{2,\dots,n\} \\
&\sum_{i=1}^{n} P_i^{(k)} = 1; 0\le t\le T
\end{aligned} \qquad (3)$$

5.2 Performance indicators

We consider in our approach the following performance indicators:

The availability of multi-state system is the probability of being in an acceptable state of operation at time t :

$$A(t) = \sum_{j=1}^{n} P_{2j-1} \qquad (4)$$

The states (2j-1) correspond to states of the system degraded.

The cost level of a component is:

$$C^v(t) = \sum_{a\in A}\left(c^{act}(a) + c^{pen}(a)g_t(a)\right) + \sum_{k\in x}\left(c^{state}(k)P(X_t = k)\right) \qquad (5)$$

$c^{state}(k)$: utility associated with the state k of the component v.

$c^{act}(a)$: utility (cost) associated with a maintenance action (repair or replacement) belonging to A, which denotes all maintenance actions.

$c^{pen}(a)$: utility associated with the penalty due to the sudden failure of the system. We assume that cpen (a) is always 0 for preventive maintenance. On the other hand, for a curative preventive maintenance policy 1 , this utility can quantify itself for example the loss in preparation time of the maintenance team (to get spare parts, to call logistics technicians ..) and this before effective repair of the system.

Traditional models assume that the component after the preventive maintenance tasks is "as good as new".

But in some cases, the system is not really refurbished, after preventive maintenance. This preventive action is called imperfect maintenance and unsatisfactory. In this work we model the effects of imperfect preventive maintenance by reducing the effective age of the component held, using an adjustment factor.

Suppose that the preventive maintenance is performed every $k\tau$ time, such as k = 0, 1, 2, 3, ... and designates the time step. The total replacement of the component is expected after an operating time greater than $N_{*\tau}$.
The probability that the component is in good working order after a preventive maintenance action is:
$P\left(X_{k\tau}\right)=(1-\alpha)P\left(X_1\right)$ where α is the adjustment factor.
The total cost of system maintenance is:

$$C\left(T\right)=\sum_{t=1}^{N}C^{V}\left(t\right)+\sum_{s\in S}C^{sys}\left(x\right)P\left(sys_t=s\right) \quad (6)$$

Where S and $C^{syst}\left(s\right)$ respectively denote the set of states and the utility of the s state of the Sys system, during a time unit.
We integrate our structure by RBD performance indicators such as the maintenance cost and availability (Fig. 5).

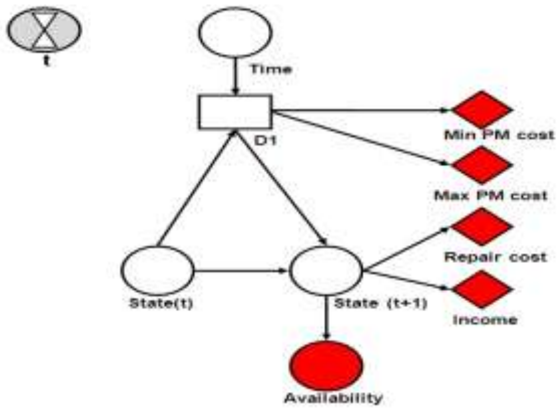

Fig. 5: Integration indicators

VI. SIMULATION

It is considered that the transition parameters are constant and their values are taken in the table.

Table I: The parameters of transition

α_1	α_2	α_3	λ_1	λ_2	λ_3	μ_1	μ_2	μ_3
0.03	0.05	0.07	0.005	0.008	0.01	0.01	0.02	0.04

Then given the Table I and Chapman-Kolmogorov equations, probability distributions of the various nodes of our RBD are calculated and put in their conditional probability tables (CPT).
The simulation is made over a period of two teams working 17600h or 8h on 20 working days in the month and during five years.

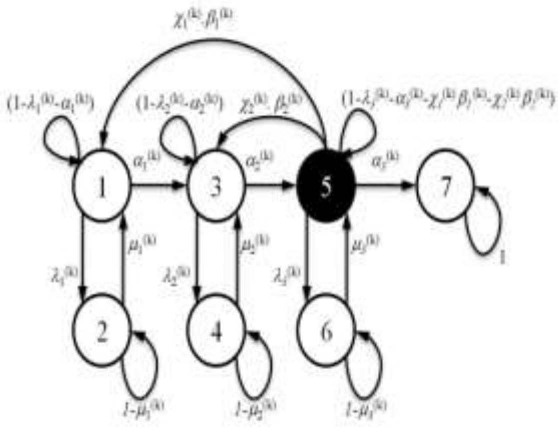

Fig.6: Markov graph with constant parameters

We consider a system with six states (Fig. 6) and simulation studies give us about 35% of availability with an average hourly income of 24% or about 2,457 euro (fig. 7).

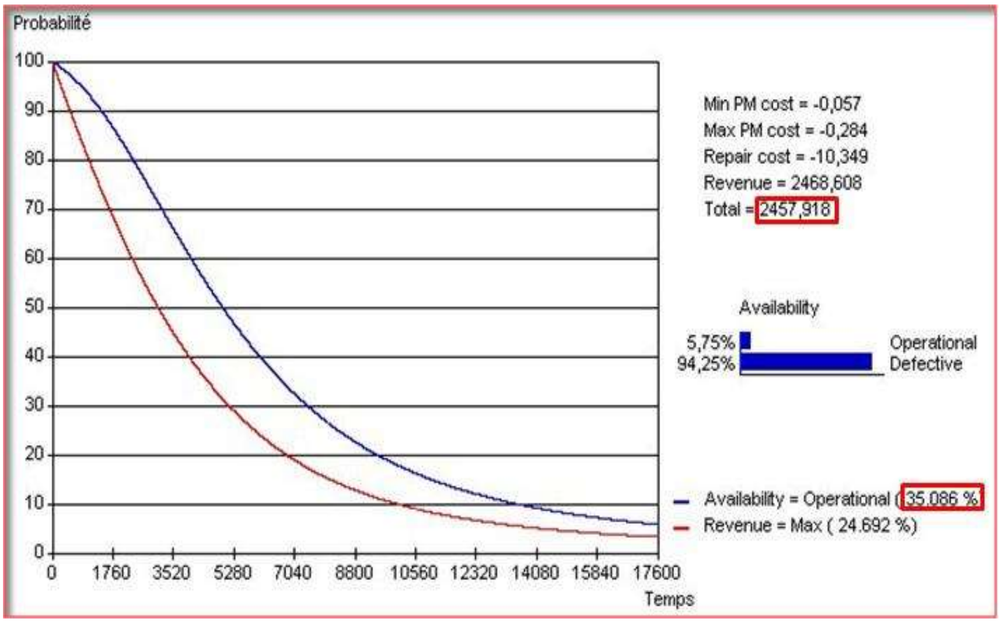

Fig. 7: Simulation curve

To better analyze and get the optimal preventive maintenance level, we set here three ways:

- no preventive maintenance
- minimal preventive maintenance
- maximal preventive maintenance

The D1 decision node imposes the one of maintenance levels cited above and a learning algorithm to make a good decision among the terms at each iteration and the occurrence of the state 5.

We note by learning the system studied has about an availability of 38% against 35% in the previous case without learning with average hourly income of 27% against 24% in the simulation (Fig. 8). So a 3% increase in availability and in income compared to simulation without learning.

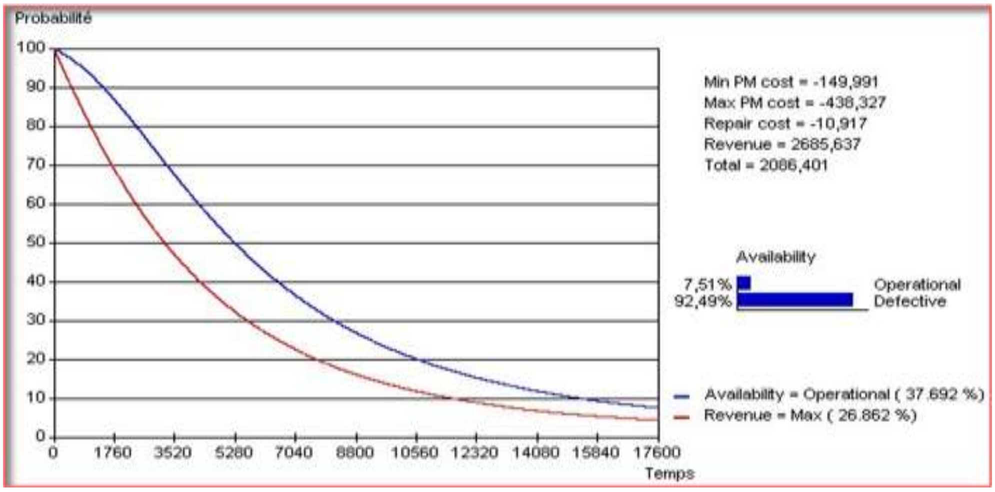

Fig. 8: Simulation curve - learning on preventive maintenance

VII. CONCLUSION

The study of multi-states is very interesting and complex in nature and objective assessment of actual ability. In this paper, we provide a review of multi-state systems, an assessment of availability and maintenance cost model and optimization approach provides a better choice of maintenance policy.

We consider in our study aspects of transitions where the parameters are constant or variable over time. Our formalism is based on stochastic processes including the graph associated with the Markov chain to model the dysfunctional behavior of production systems in time by the DBN.

A simulation study is conducted over a period of 5 years to find the best configurations of policy choices maintenance curves and deduce indicators of system performance.

The proposed model can be applied by the manufacturers subject to variability of the maintenance policy. \

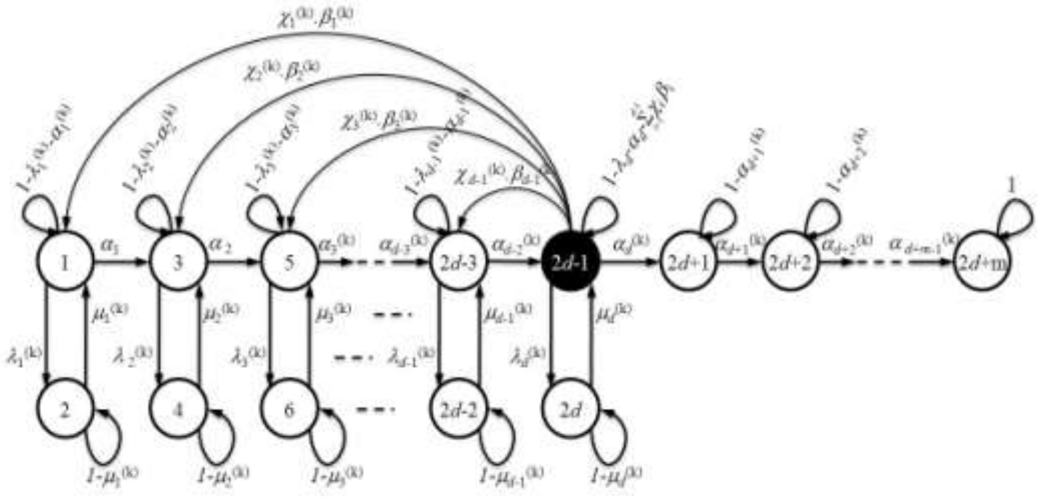

Fig. 9: Graph of Markov chain considered

REFERENCES

[1]. A. Birnbaum, Some latent trait models and their use in inferring an examinee’s ability, In F. M. Lord & M. R. Novick (Eds.), Statistical theories of mental test scores, 1968.

[2]. S. Bouri “Optimisation de la production et de la structure d'énergie électrique par les colonies de fourmis ”, Thèse, Université Jilali Liabès, 2007.

[3]. W. Barlow, and A. Wu, Coherent systems with multi-state components, Math. Oper. Res.volume(3), N°4, pp. 275-281, 1978.

[4]. S. Belaloui “Évaluation des systèmes de fiabilité a configuration complexe”, Thèse université, Mentouri Constantine, 2009.

[5]. J. Xue, K. Yang, Dynamic reliability analysis of coherent multi-state systems, IEEE Transactions on Reliability; 44: 683–8, 1995.

[6]. E. Zio, L. Podofillini, A Monte Carlo approach to the estimation of importance measures of multi-state components. In: Reliability and maintainability annual symposium (RAMS). p. 129–34, 2004.

[7]. S.M. Ross, Multivalued state component systems, Annals of Probability;7(2):379–83, 1979.

[8]. G. Levitin, Universal generating function, in: reliability analysis and optimization. Berlin (Heidelberg/New York): Springer-Verlag, 2003.

[9]. A. Lisnianski, G. Levitin, Multi-state system reliability: assessment, optimization and applications. World scientific Publishing Co. Pte Ltd, Singapore, 2003.

[10]. E. El-Neweihi, F. Proschan, and J. Sethuraman, Multi-state coherent systems, J. Appl.Prob., volume 15, pp. 675-688, 1978.

[11]. K.B. Misra , Reliability analysis and prediction: A methodology oriented treatment , Elsevier, 1992.

[12]. A.J. Li, W. Yu., K. Liu, Reliability estimation and prediction of M.S components and coherent system, Reliability Engineering System Safety, pp. 93-98, 2005

[13]. I.W Soro, M. Nourelfath, A.-K. Daouda, Performance evaluation of multi-state degraded systems with minimal repairs and imperfect preventive maintenance, Reliability Engineering and System Safety 95 (2), pages 65-69. 2009.

[14]. R. Donat, P. Leray, L. Bouillaut, P. Aknin, Réseaux bayésiens dynamiques pour la représentation de modèles de durée en temps discret, 2008.

[15]. C. Fitouhi, M. Nourelfath, “Optimisation de la planification intégrée de la maintenance préventive et de la production des systèmes multi-états” Thèse université de Laval, faculté des sciences et de génie, 2011.

[16]. I. Ayadi, L. Bouillaut, P. Aknin, Optimisation par algorithme génétique de la maintenance préventive dans un contexte de modélisation par modèles graphiques probabilistes, 17ème congrès de Maîtrise des Risques et de Sûreté de Fonctionnement, la Rochelle, 2011.

[17]. T. Dean, K. Kanazawa, A model for reasoning about persistence and causation, Computationnal Intelligence, 5(3), 1, 142–150, 1989.

[18]. D. Bellot, “Fusion de données avec des réseaux bayésiens pour la modélisation des systèmes dynamiques et son application en télémédecine ” Thèse, Université Henri Poincaré, Nancy 1, France, 2002.
[19]. J. Binder, K. Murphy and S. Russell, Space-efficient inference in dynamic probabilistic networks. in Proceedings of the 15th International Joint Conference on Artificial Intelligence(IJCAI’97), (pp. 1292–1296), Morgan Kaufmann Publishers, 1997.
[20]. P. Weber, L. Jouffe, Reliability modelling with Dynamic Bayesian Networks. 5th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes,Washington, D.C, USA, June 9-11, 2006.
[21]. H. Bouillaut, J.B. Dugan, Coherent system, Reliability Engineering System Safety, pp. 93-98, 2005.
[22]. A. Salem Ben, A. Müller, P. Weber, Dynamic Bayesian Networks in system reliability analysis, IFAC, 2006.
[23]. F. Innal, & Y. Dutuit, Evaluation de la performance d’un système de production et des contributions individuelles de ses unités constitutives, 2006.