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Spatiotemporal Dynamics on Interaction of Species with Mutualism

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Abstract:-In this article, a two dimensional continuous time mutualistic model is considered. This model is characterized by a pair of non-linear coupled partial differential equations. The role of diffusivity on stability about interior equilibrium point and persistence criteria of the system is studied. Finally we have inspected the steadiness and dynamical behaviour of the system with computer simulations.

Keywords:-Mutualism, Positive Equilibrium, stability, diffusion

I. INTRODUCTION

Mathematical demonstration is an influential device in discovering innumerable features of populace dynamics and illuminating the existing complexity in real ecosystems. The growth, developments and extensions takes place in environmental and evolutionary representations have a widespread diversity of solicitations across the life sciences and are appropriate to many other divisions of science. Mathematical ideal is shortened description of the world that is used to learning the crucial physiognomies and dynamics of actual world phenomenon. The theoretical methodologies in demonstrating ecosystems are started in the early 19th century [1,2] and established faster with various scientific frame works [3,4]. Mathematical model is a set of interrelated estimates about the ecosphere and the foremost use of modelling is to shorten the complexity of factual environments and interrelationship between two species such as predation, commensalism, ammensalism, mutualism and so on.

Mathematical prototypical are so significant because they provide deep understanding of patterns of plenty of species in a complex ecosystem without any workshop experiments which are extremely expensive and long-time developments. Many investigators cushing [4], Paul [5]Freedman[6], Kapur[7, 8] and etc., are contributed to the enlargement of qualitative theory for execution of system analysis with nonlinear prototypes in dynamics of inhabitants.

The purpose of the current work is to study the dynamics of two mutually interacting species in the presence of diffusion. Mutualism is a relationship between two species that benefits both the species. Mutualistic understandings are most likely to enlarge between organisms with widely differing living requirements. Mutualism is a type of interaction between two species where one species provides some benefit that it has in abundance in exchange for some benefit that the other species has in relative abundance [9,12]. The benefit exchanged might be good or a service. Some examples of mutual interactions are following:

Insect-mediated pollination, plants exchange carbon energy in the form of nectar or pollen which it has in relative abundance due to its mobility. In this case a resource is traded for a service. This idea of trading goods with differing availability to different species is very similar to the economic concept of comparative advantage [9-12].The consequence of transmission on the spatiotemporal mutualistic model has been explored by many scientists to their insightful work [13-16]. Recently, the consequence of self as well as cross-diffusion in diffusion systems has established much attention by both environmentalists and theoreticians also. B. Dubey et.al.,[17], motivated us to do this diffusion analysis of the proposed model.

II. MATHEMATICAL FORMULATION AND DIFFUSION ANALYSIS

The present work emphasizes the mathematical modelling of mutualistic interaction between two species in the presence of diffusion. We considered an ecological system two species are living together. We here consider the movement of species in the vertical direction, then $m_1 = m_1\left(x,t\right)$, $m_2 = m_2\left(x,t\right)$ where x a space variable and keeping, these in view and following, the dynamics of the diffusive system may be governed by the following non- linear partial differential equations.

$$\left(m_1\right)_t = i_1 m_1 - \frac{i_1 m_1^{\,2}}{C_1 + p m_2} + T_1\left(m_1\right)_{xx} \quad (2.1)$$

$$\left(m_2\right)_t = i_1 m_2 - \frac{i_1 m_2^{\,2}}{C_2 + q m_1} + T_2\left(m_2\right)_{xx} \quad (2.2)$$

where m_1, m_2 represents the sizes of the populations of first and second species at time ' t '. C_1, C_2 represent the carrying capacities of first and second species, i_1 represents the birth rate of first and second species and p & q are arbitrary constants. T_1, T_2 representthe diffusive coefficients for both the species of populations. We are assuming the following conditions on the populations of species $m_1\left(x,t\right),\ m_2\left(x,t\right)$ in $0 \le x \le K$, $K > 0$ as follows

$$\frac{\partial m_1(0,t)}{\partial t} = \frac{\partial m_1(K,\mathrm{t})}{\partial t} = \frac{\partial m_2(0,t)}{\partial t} = \frac{\partial m_2(K,\mathrm{t})}{\partial t} = 0 \qquad (2.3)$$

The zero isoclines of model equations (2.1) – (2.2) also give the steady state which are same as we have obtained for homogeneous system. To study the dynamics of the above diffusive model (2.1) – (2.2), it is required to linearize the system (2.1) – (2.2) about the interior steady state and we obtain

$$\left(M_1\right)_t = -i_1 M_1 m_1^{*} + T_1\left(M_1\right)_{xx} \quad (2.4)$$

$$\left(M_2\right)_t = -i_1 M_2 m_2^{*} + T_2\left(M_2\right)_{xx} \quad (2.5)$$

by assuming $m_1 = m_1^{*} + M_1$ and $m_2 = m_2^{*} + M_2$.

Let the solution of the system (2.4)-(2.5) be the form

$$M_1(x,\mathrm{t}) = \alpha e^{\mu t} e^{irx} \quad (2.6)$$

$$M_2(x,t) = \beta e^{\mu t} e^{irx} \quad (2.7)$$

where α , β are amplitudes and r is the wave numeral of the solution. M_1 , M_2 are propagations of populations. The characteristic equation corresponding to the diffusive system (2.4)-(2.5) is observed as

$$\mu^2 - r_1\mu + r_2 \qquad (2.8)$$

where $r_1 = i_1\left(m_1^{*} + m_2^{*}\right) - r^2(T_1 + T_2)$; $r_2 = \left(i_1 m_1^{*} + T_1 r^2\right)\left(i_1 m_2^{*} + T_2 r^2\right)$ or r_2 can be written as $i_1^{\,2} m_1^{*} m_2^{*} + i_1 m_1^{*} T_2 r^2 + i_1 m_2^{*} T_1 r^2 + T_1 T_2 r^4$ and r_2 can be written as $N\left(r^2\right)$

where $N\left(r^2\right) = T_1 T_2\left(r^2\right)^2 + \left(i_1 m_1^{*} T_2 + i_1 m_2^{*} T_1\right) r^2 + i_1^{\,2} m_1^{*} m_2^{*}$

Now, our main objective is to derive the criteria for diffusive instability of model system (2.1)-(2.2).The system (2.1)-(2.2) is unstable if one of the above roots of the equation (2.8) is positive. A necessary and sufficient condition for a root to be positive is that $r^2(T_1 + T_2) + i_1\left(m_1^{*} + m_2^{*}\right) > 0$,

which implies that $r^2 > \dfrac{-i_1\left(m_1^{*} + m_2^{*}\right)}{T_1 + T_2}$ (2.9)

Since the wave numeral r is real, then the above statement is feasible, if $m_1^{*} + m_2^{*} < 0$. Thus the necessary condition for diffusive instability of the system is $m_1^{*} + m_2^{*} < 0$. The sufficient condition for positivity of one of the roots of the equation (2.8) is $N(r^2) < 0$. Since $N(r^2)$ is an expression in r^2 where r the wave numeral, non-zero positive quantity, the minimum of $N(r^2)$ occurs. Let $\left(r^2\right)_{\min}$ be the corresponding value of r^2 for minimum value of $N(r^2)$.

Then $\quad \left(r^2\right)_{\min} = \dfrac{-i_1\left[m_2^{*} T_1 + m_1^{*} T_2\right]}{T_1 T_2} > 0$ provided $m_2^{*} T_1 + m_1^{*} T_2 < 0$

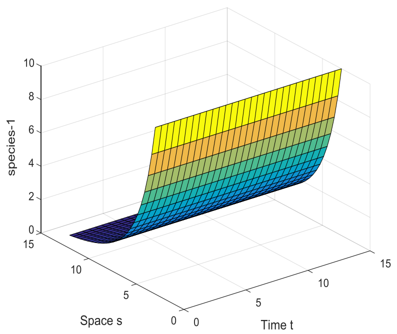
The corresponding minimum value of $N(r^2)$ is $N\left(r^2{}_{\min}\right)=\dfrac{-\left(i_1 m_1{}^* T_2+i_1 m_2{}^* T_1\right)^2}{4T_1T_2}+i_1{}^2 m_1{}^* m_2{}^*>0$

So, the sufficient condition reduces to $\left(m_1{}^* T_2-m_2{}^* T_1\right)^2>0$ which is saying that $\dfrac{m_1{}^*}{m_2{}^*}>\Gamma$ (2.10)

where $\Gamma=\dfrac{T_1}{T_2}$. Thus the diffusion of the prey predator populations derives the ecological system into unstable oscillation when (2.9) & (2.10) are satisfied.

III. COMPUTER SIMULATIONS

In this division, we established the analytical findings through numerical simulations using MATLAB.



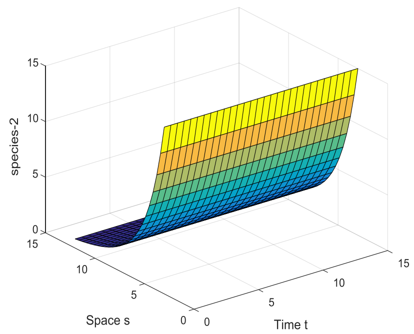


Figure-1 Figure-2

Figure 1 and Figure 2denotes the steady fluctuations of the mutualistic species against space and time with $i_1=0.132;\ C_1=0.33;\ C_2=0.144;\ p=0.112;\ q=0.3;\ T_1=0.1; T_2=0.2$

IV CONCLUDING REMARKS

In this, it is premeditated about a two species ideal with the interaction of mutualism and diffusion for both the species, which has a great role in shaping the dynamics of the model. Also,analysed instability condition for diffusive structure of the ideal structure (2.1)-(2.2). It is also verified the stable oscillations of the species populations against time and space in figures 1-2.

REFERENCES

[1]. Lotka, A.J.,1925. Elements of Physical Biology. Williams & Wilkins, Baltimore, New York.
[2]. Volterra, V., 1931. Lecons sun la theoriemathematique de la lutte pour la vie. Gauthier - Villars, Paris.
[3]. Carletti, M. (2007) ‘Numerical simulation of a Campbell-like stochastic delay model forbacteriophage infection’, Mathematical Medicine and Biology-A Journal of the IMA (Instituteof Mathematics and its applications), Vol. 23, No. 4, pp.297–310.
[4]. J.M. Cushing, Integro differential Equations and delay models in population dynamics, lecture notes in Bio-Mathematics-springer verlag, 20, 1997.
[5]. Paul Colinvaux, 1986. Ecology. John Wiley and Sons Inc., New York.
[6]. Freedman, H.I., 1980. Deterministic Mathematical Models in Population Ecology. Marces Decker, New York.
[7]. Kapur, J.N., 1985. Mathematical Models in Biology and Medicine. Affiliated East - West.
[8]. Kapur, J.N., 1985. Mathematical Modeling. Wiley, Easter.
[9]. J.L. Bronstein. Conditional outcomes in mutualistic interactions. Trends Ecol. Evol., 1994,9: 214 - 217.
[10]. R. Noe, P. Hammerstein. Biological markets, Tree 1995, 10: 336 - 339.
[11]. M.W. Schwartz, J.D. Hoeksema. Specialization and resource trade: biological markets as a model of mutualisms. Ecology, 1998, 79: 1029 - 1038.
[12]. J.N. Holland. Benefits and costs of mutualism: dem organic consequences in a pollinating seed-consumer interaction. Proceedings of the Royal Society of London Series B-Biological Sciences, 2002, 269: 1405 - 1412.

[13]. Hastings, A., 1978. "Global Stability in Lotka- Volterra Systems with Diffusion" J. Math. Biol., 6: 163-168.
[14]. Okubo, A., 1980. Diffusion and Ecological Problem: Mathematical models. Springer- Verlag, New York.
[15]. Rothe, F., 1984. Global solution of Reaction Diffusion systems. Lecture Notes in Math. 1072, Springer-Verlag, Berlin Heidelberg, New York, Tokyo.
[16]. Nisbet R.M., Gurney W.S.C. "The systematic formulation of population models for insects with dynamically varying instar duration" Theoret. Popul. Biol., 23 (1) 1982: 114-135.
[17]. B Dubey, NituKumari, R K Upadhyay, Spatiotemporal pattern formation in a diffusive predator-prey system: an analytical approach, J. Appl. Math. Comput., 31 (2009) 413-432.