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A Multi-Level Multi-Objective Quadratic Programming Problem with Fuzzy Parameters in Constraints

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***Abstract*:- This paper proposes an algorithm to solve multi-level multi-objective quadratic programming (MLMOQP) problem, involving trapezoidal fuzzy numbers in the right hand side of the constraint, the suggested algorithm uses the linear Ranking Methods to convert the mentioned problem to its equivalent crisp form then uses the interactive approach to obtain the satisfactory solution (preferred solution) in view of the satisfactoriness concept andε -constraint method with considerations of overall satisfactory balance among all of the three levels, an illustrative example is included.**

Keywords:- Multi-Level Programming, Trapezoidal Fuzzy Numbers, linear Ranking Methods, Interactive Approach,ε -constraint method.

I. INTRODUCTION

Multi-level programming (MLP) is indicated as mathematical programming which solves the coordinating problem of the decision-making process in decentralized systems by enhancing the objective of a hierarchical organization, while dealing with the tendency of the lower level of the hierarchy to enhance their own objectives. Three level programming (TLP) is a class of Multi-level programming problem in which there are three independent decision-makers. Multi-level programming (MLP) has been applied in many different kinds of the problems ([1], [2], [3], [5], [6], [8]and [9]).

When taking into account some cooperation among the decision makers in different levels with more than one objective function in every level, it is appropriate to use an approach or algorithm for obtaining a satisfactory solution to such model, many researches utilized the Interactive approach as Theoretical and methodological base for solving such kind of the problems ([5], [7], [9] and [10]). In [5] Emam proposed a bi-level multi-objective integer non-linear programming fractional problem with linear or non-linear constraints, at the first phase the convex hull of its original set of constraints was found then the equivalent problem was simplified by transforming it into a separate multi-objective decision-making problem and finally solving the resulted problem by using the e –constraint method. In [10] Mishra et al proposed an interactive fuzzy programming method for obtaining a satisfactory solution to a bi-level quadratic fractional programming problem.

The main idea behind the fuzzy number is that of gradual membership to a set without sharp boundary in concord with fuzziness of human judgment ([3], [6] and [8]). In [6] Youness et al, suggested algorithm to solve a bi-level multi-objective fractional integer .programming problem involving fuzzy numbers in the right-hand side of the constraints. In [8] Sakawa et al. Presented interactive fuzzy programming for multi-level linear programming problems with fuzzy parameters.

II. PROBLEM FORMULATION AND SOLUTION CONCEPT[2]

Let $x_i \in \mathrm{R}^n$, (i = 1, 2, 3), be a vector of variables which indicates the first decision level's choice, the second decision level's choice and the third decision level's choice and F_i : $\mathrm{R}^n \to \mathrm{R}^{N_t}$(i = 1, 2, 3), be the first level objective function, the second level objective function and the third level objective function, respectively. Assume that the first level decision maker is (FLDM), the second level decision maker is (SLDM) and the third level decision maker is (TLDM).N_1,$\mathrm{N}_2$and $\mathrm{N}_3 \geq 2$,the FLDM, SLDM, and TLDM haveN_1,N_2 and N_3objective functions, respectively. Let G be the set of feasible choices $\{(x_1, x_{2,} x_3)\}$. Therefor the multi-level multi-objective quadratic programming problem (MLMOQPP) with fuzzy numbers in the right hand side of constrains may be formulated as follows:

$[\mathbf{1^{st}\ level}]$

$$\max_{x_1} F_1(\bar{x}) = \max_{x_1} \left[f_{11}(\bar{x}), f_{12}(\bar{x}) \dots, f_{1N_1}(\bar{x})\right] \tag{1}$$

where x_2 , x_3 *solve*

[$\boldsymbol{2^{nd}}$ ***level***]

$$\max_{x_2} F_2(\bar{x}) = \max_{x_2} \left[f_{21}(\bar{x}), f_{22}(\bar{x}) \ldots, f_{2N_2}(\bar{x})\right] \tag{2}$$

where x_3 *solves*

[$\boldsymbol{3^{rd}}$***level***]

$$\max_{x_3} F_3(\bar{x}) = \max_{x_3} \left[f_{31}(\bar{x}), f_{32}(\bar{x}) \ldots, f_{3N_3}(\bar{x})\right] \tag{3}$$

Subject to:

$$G(\bar{x}, \tilde{b}) = \left\{\bar{x} \in R^n \middle| A\bar{x} \leq \tilde{b}, \bar{x} \geq 0\right\} \tag{4}$$

where $F_i\ (\bar{x})$, (i = 1, 2,.., n)can be suggested in the following form $\left(\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}\, x_i x_j + \sum_{j=1}^{n} C_j x_j\right)$,$\bar{x} = (x_1, x_{2,} x_3) \in R^{\,n_1+n_2+n_3}$,$G$ is a linear constraint functions set, and $\tilde{b}$is a trapezoidal fuzzy number denoted by *(A, B, α, β)* where *A, B, α, β* are real numbers and its membership function is given blow:

$$\mu_{\tilde{b}}(x) = \begin{cases} 1 - \frac{A-x}{A-\alpha} & if \quad A - \alpha \leq x < A \\ 1 & if \quad A \leq x < B \\ 1 - \dfrac{x - B}{B + \beta} & if \quad b < x \leq b + \beta \\ 0 \ otherwise & \end{cases} \tag{5}$$

III. RANKING METHOD

To solve (MLMOQPP) with fuzzy parameters at the right hand side of the constraint the linear ranking method technique is used to convert fuzzy number form into equivalent deterministic crisp form.

Definition 1 [11]:
If ($\tilde{E}$) = (*A*, *B*, *α*, *β*) ∈ $F(R)$, then the linear ranking function is definedas
$\Re(\tilde{E})= \frac{1}{2}\left[A + B - \left(\frac{4}{5}\alpha + \frac{2}{3}\beta\right)\right]$which is known by Yager Ranking Method. (6)

Definition 2 [11]:
$\tilde{E}(A_1, B_1, \alpha_1, \beta_1)$,$\tilde{H}(A_2, B_2, \alpha_2, \beta_2)$are two Trapezoidal fuzzy numbers and $x \in R$. *a* Ranking function is a convenient method for comparing the fuzzy numbers which is a map from *F(R)* into the real line. So, the orders on *F(R)* asfollow:

1. $\tilde{E} \geq \tilde{H}\ \ if$ and only if $\Re(\tilde{E}) \geq \Re(\tilde{H})$.
2. $\tilde{E} > \tilde{H}\ \ if$ and only if $\Re(\tilde{E}) > R(\tilde{H}\ )$.
3. $\tilde{E} = \tilde{H}\ \ if$ and only if $\Re(\tilde{E}) = \Re(\tilde{H}\ )$.

Where$\tilde{E}$ *and* $\tilde{H}$*are in F(R).*
Now after applying linear ranking method the problem will be formulated as follow:

[$\boldsymbol{1^{st}}$ ***level***]

$$\max_{x_1} F_1(\bar{x}) = \max_{x_1} \left[f_{11}(\bar{x}), f_{12}(\bar{x}) \ldots, f_{1N_1}(\bar{x})\right] \tag{1}$$

where x_2 , x_3*solve*

[$\boldsymbol{2^{nd}}$ ***level***]

$$\max_{x_2} F_2(\bar{x}) = \max_{x_2} \left[f_{21}(\bar{x}), f_{22}(\bar{x}) \ldots, f_{2N_2}(\bar{x})\right] \tag{2}$$

where x_3 *solves*

[$\boldsymbol{3^{rd}}$***level***]

$$\max_{x_3} F_3(\bar{x}) = \max_{x_3} \left[f_{31}(\bar{x}), f_{32}(\bar{x}) \dots, f_{3N_3}(\bar{x})\right] \quad (3)$$

Subject to:

$$\bar{G}(\bar{x}, b) = \{\bar{x} \in R^n | A\bar{x} \leq b, \bar{x} \geq 0\} \quad (7)$$

,
Where $\bar{G}$ is a linear constrain functions set and $b$is a deterministicnumber.

IV. INTERACTIVE APPROACH FOR (MLMOQP) PROBLEM

To obtain the satisfactory solution (preferred solution) for (MLMOQP) problem, the interactive approach through the satisfactoriness concept,ε -constraint method and The Kth best method is used. The FLDM gives the preferred or satisfactory solutions that are acceptable for the FLDM in rank order to the SLDM, Then SLDM will seek for the preferred solution which is close for the FLDM and then SLDM gives the preferred solution in rank order to the TLDM then the TLDM do exactly like the SLDM. The FLDM decide the satisfactory solution (preferred solution) of the (MLMOQP) problem according to the minimal satisfactoriness constant and solution test [5].

Definition 1[4]:
To define the range of the objective function, the best and the worst solutions of the function is defined, as follow:

$$b^*_{lj} = \underset{x \in \bar{G}}{Max}\, f_{lj}(\bar{x}),\ w'_{lj} = \underset{x \in \bar{G}}{Min}\, f_{lj}(\bar{x}),\ j = 1,2,..N_n, l = 1, 2 \dots L \quad (8)$$

Definition 2[4]:
Let s=$s_0$at the beginning, and let s=s_1 , s_2 ... s_n. respectively, where $s_0 \in [0,1]$ is the satisfactoriness of the decision maker and ${}^{\delta}{}_{lj}$ is calculated as follow:

$$^{\delta}{}_{lj} = (b^*_{lj} - w'_{lj})s + w'_{lj} \; .l = 1, 2 \dots L, j = 1, 2, \dots n \quad (9)$$

Definition 3[4]:
Themain concept of the ε -constraint method is to optimize one of the objective functions using the other objective functions as constrains,theε -constraint problemincluding satisfactoriness is defined as follow:

Max $f_{l1}(\bar{x})$ (10)

Subject to:

$\bar{x} \in \bar{G}$

$f_{lj}(\bar{x}) \geq {}^{\delta}{}_{lj},\ l = (1, 2, \dots L) j = (2, \dots. N_1)$

The ε -constraint problem has no solution if $f^l_{l1}(\bar{x}) < {}^{\delta}{}_{l1}$, l=1, 2 ...L. so the satisfactoriness should be adjusted to s=$s_{lt+1} < s_{lt}$, l $= 1, 2 \dots$ L, $t = 1, 2, \dots n$ till the solution of the problem is obtained.

Definition4 [8]:
The Kth best method starts from a point which is a non-inferior solution to the problem of the upper level and checks whether it is also non-inferior solution to the problem of the lower level or not. If the first point is not non-inferior solution, the Kth best continues to examine the second best solution to the problem of the upper level and so on.

The solution of the (MLMOQP) problem will be obtained by solving FLDM, SLDM, TLDM problems each one separately.

A. The problem of FLDM:

The FLDM problem may be formulated as follow:

$[\boldsymbol{1^{st}\ level}]$

$$\max_{x_1} F_1(x_1, x_2, x_3) = \max_{x_1} \left[f_{11}(x_1, x_2, x_3), f_{12}(x_1, x_2, x_3)\dots, f_{1N_1}(x_1, x_2, x_3)\right] \quad (1)$$

Subject to:

$(x_1, x_2, x_3) \in \bar{G}$ (11)

First, the best and the worst of the objective function is calculated (8), Then first level multi-objective decision making problem is transformed into the single- objective decision making problem using the concept of the ε-constraint method including satisfactoriness as follows:

$max\, f_{11}(x_1, x_2, x_3)$, (12)

Subject to:

$(x_1, x_2, x_3) \in \bar{G}$ (13)

$f_{1j}(x_1, x_2, x_3) \geq {}^{\delta}{}_{1j}, j = (2, \dots . N_1),$

${}^{\delta}{}_{1j} = \left(b^{*}_{1j} - w'_{1j}\right)s_{1t} + w'_{1j}, j = (1,2 \dots . n)$.where$s_{1t}, t = 1, 2, \dots n$ is given by the FLDM indicted to FLDM satisfaction.

The FLDM ε-constraint problem has no solution if $f_{11}^{F} \leq {}^{\delta}{}_{11}$, so the satisfactoriness should be adjusted to $s_{1t+1} < s_{1t}$, $t = 1, 2, \dots n$ till the satisfactory solution of the problem $\left(x_1^F, x_2^F, x_3^F\right)$is obtained.

Secondly, according to the interactive mechanism of the MLMOQP problem the FLDM variables x_1^F should be given to the SLDM to seek the SLDM satisfactory solution, so the FLDM puts the preferred solutions in order in the format as follow:

Preferred solution$\left({x_1^F}^{k_1}, {x_2^F}^{k_1}, {x_3^F}^{k_1}\right)\dots.. \left({x_1^F}^{k_1+p}, {x_2^F}^{k_1+p}, {x_3^F}^{k_1+p}\right)$, preferred ranking (14)

$\left({x_1^F}^{k_1}, {x_2^F}^{k_1}, {x_3^F}^{k_1}\right) \succ {x_1^F}^{k_1+1}, {x_2^F}^{k_1+1}, {x_3^F}^{k_1+1}) \succ \dots \succ \left({x_1^F}^{k_1+p}, {x_2^F}^{k_1+p}, {x_3^F}^{k_1+p}\right)$

Where k_1=0, P= (1, 2,…..n).

B. The problem of SLDM:

The SLDM problem can be formulated as follows:

[$\boldsymbol{2^{nd}\ level}$]

$$\max_{x_2} F_2\left(x_1^F, x_2, x_3\right) = \max_{x_2} \left[f_{21}\left(x_1^F, x_2, x_3\right), f_{22}\left(x_1^F, x_2, x_3\right) \dots, f_{2N_2}(x_1^F, x_2, x_3)\right] \tag{15}$$

Subject to:

$\left(x_1^F, x_2, x_3\right) \in \bar{G}$ (16)

The best and the worst solution of SLDM problem is calculated (8) then the ε-constraint problem of the SLDM including satisfactoriness formulated as follows:

$$max\, f_{21}\left(x_1^F, x_2, x_3\right) \tag{17}$$

Subject to: (18)

$\left(x_1^F, x_2, x_3\right) \in \bar{G}$

$f_{2j}(x_1^F, x_2, x_3) \geq {}^{\delta}{}_{2j}, j = (2, \dots . N_2)$

${}^{\delta}{}_{2j} = \left(b^{*}_{2j} - w'_{2j}\right)s_{2t} + w'_{2j}$,where$s_{2t}$,$t = 1, 2, \dots n$ is given by the SLDM indicted to SLDM satisfaction.

The ε-constraint problem of the SLDM including satisfactoriness has no solution; if $f_{21}^{S} \leq {}^{\delta}{}_{21}$, so the satisfactoriness should be adjusted to $s_{2t+1} < s_{2t}$, $t = 1, 2, \dots n$ till the satisfactory solution of the SLDM problem $\left(x_1^F , x_2^S , x_3^S \right)$is obtained.

The satisfactory solution of the SLDM problem $\left(x_1^F , x_2^S , x_3^S \right)$is tested to examine the closeness to the FLDM satisfactory solution or it may be changed by the following test rule:

$$\frac{\| F_1(x_1^F , x_2^F , x_3^F) - F_1(x_1^F , x_2^S , x_3^S) \|_2}{\| F_1(x_1^F , x_2^F , x_3^F) \|_2} \prec \delta^F \tag{19}$$

x_1^F and x_2^S should be given to the TLDM to seek the TLDM non-inferior solution, so the SLDM puts the preferred solution in order in the format as follow:

Preferred solution(${x_1^F}^{k_2}$, ${x_2^S}^{k_2}$, ${x_3^S}^{k_2}$)….. (${x_1^F}^{k_2+p}$, ${x_2^S}^{k_2+p}$, ${x_3^S}^{k_2+p}$) , preferred ranking (20)

$$({x_1^F}^{k_2} , {x_2^S}^{k_2} , {x_3^S}^{k_2}) \succ ({x_1^F}^{k_2+1} , {x_2^S}^{k_2+1} , {x_3^S}^{k_2+1}) \succ \dots \succ ({x_1^F}^{k_2+p} , {x_2^S}^{k_2+p} , {x_3^S}^{k_2+p})$$

Where k_2=0, P= (1, 2,…..n).

C. The problem of the TLDM:

Thirdly, variables x_1^F and x_2^S should be given to the TLDM; hence, the TLDM do exactly like the SLDM till the satisfactory solution of the TLDM problem $\left(x_1^F , x_2^S , x_3^T \right)$ is obtained.

The satisfactory solution of the TLDM problem$\left(x_1^F , x_2^S , x_3^T \right)$is tested to examine the closeness to the SLDM preferred solution or it may be changed by the following test rule:

$$\frac{\| F_1(x_1^F , x_2^S , x_3^S) - F_1(x_1^F , x_2^S , x_3^T) \|_2}{\| F_1(x_1^F , x_2^S , x_3^S) \|_2} \prec \delta^S \tag{21}$$

where δ^S is a fairly small positive number given also by FLDM.

So $\left(x_1^F , x_2^S , x_3^T \right)$ is the satisfactory solution (preferred solution) for (MLMOQP) problem.

V. AN ALGORITHM

In this section an algorithm is presented to solve (MLMOQPP) with fuzzy parameters in the right hand side of constrains, the algorithm is illustrated in the following series steps:

Step 1: use Yager ranking method and Compute $\Re(\widetilde{A})$for all the coefficients of the (MLMOQPP) with fuzzy number in the right hand side of constrains, where $\widetilde{A}$ a trapezoidal fuzzy number.

Step 2: Convert the (MLMOQPP) with fuzzy number in the right hand side of constrains from the fuzzy form to the crisp form.

Step 3: Formulate the (MLMOQPP)

Step 4: Use interactive approach to solve the (MLMOQPP).

Step 5: solve the problem of the FLDM.

Step 6: Calculate the best and the worst solution of the FLDM problem.

Step 7: FLDM sets $s_{1t}, t = 1, 2, \dots n$

Step 8: formulate and solve the ε-constraint problem of the FLDM including satisfactoriness.

Step 9: If $f_{11}^{F} \leq {}^{\delta}{}_{11}$, adjust satisfactoriness $s_{1t+1} < s_{1t}$, $t = 1, 2, \dots n$ and go to step 8 otherwise go to step 10.

Step 10: obtain the satisfactory solution (preferred solution)of the FLDM $\left(x_1^F , x_2^F , x_3^F \right)$

Step 11: Set$k_1 = 0$.

Step 12: The FLDM orders the satisfactory solutions$\left(x_1^F , x_2^F , x_3^F \right)$

Step 13: Given $x_1 = x_1^F$ to the SLDM problem,

Step 14: solve the problem of the SLDM.

Step 15: Calculate the best and the worst solution of the SLDM problem.

Step 16: SLDM sets $s_{2t}, t = 1, 2, \dots n$.

Step 17: formulate and solve the ε-constraint problem of the SLDM.

Step 18: IF $f_{21}^{S} \leq {}^{\delta}{}_{21}$, so the satisfactoriness should be adjusted to $s_{2t+1} < s_{2t}$, $t = 1, 2, \ldots n$ and go to step 17 otherwise go to step 19.

Step 19: obtain the satisfactory solution of the SLDM problem $\left(x_1^F , x_2^S , x_3^S \right)$

Step 20: test the satisfactory solution of the SLDM problem $\left(x_1^F , x_2^S , x_3^S \right)$

Step 21: If $\left(x_1^F , x_2^S , x_3^S \right)$is close solution to FLDM satisfactory solution, go to step 22 otherwise, set $k_1 = k_1 + 1$ then goes to step 12.

Step 22: Set$k_2 = 0$.

Step 23: the SLDM orders the satisfactory solutions $\left(x_1^F , x_2^S , x_3^S \right)$

Step 24: Given $x_1 = x_1^F$and $x_2 = x_2^s$ to the TLDM problem,

Step 25: solve the problem of the TLDM.

Step 26: Calculate the best and the worst solution of the TLDM problem.

Step 27: TLDM sets s_{3t} , $t = 1, 2, \ldots$ n.

Step 28: formulate and solve the ε-constraint problem of the TLDM.

Step 29: If $f_{31}^{T} \leq {}^{\delta}{}_{31}$, so the satisfactoriness should be adjusted to $s_{3t+1} < s_{3t}$, $t = 1, 2, \ldots n$ and go to step 28 otherwise go to step 30.

Step 30: obtain the satisfactory solution of the TLDM problem $\left(x_1^F , x_2^S , x_3^T \right)$

Step 31: test the non-inferior solution of the TLDM problem $\left(x_1^F , x_2^S , x_3^T \right)$

Step32: If $\left(x_1^F , x_2^S , x_3^T \right)$is a close solution to SLDM satisfactory solution, go to step 33 otherwise, set $k_2 = k_2 + 1$ then goes to step 23.

Step 33:$\left(x_1^F , x_2^S , x_3^T \right)$ is the satisfactory solution (preferred solution) to the (MLMOQP) problem.

VI. EXAMPLE

To demonstrate the solution for MLMOQP problem with fuzzy parameter at the right hand side, let us consider the following example:

$[\mathbf{1^{st}\ level}]$

$$\max_{x_1} F_1(x_1, x_2, x_3) = \max_{x_1} [3x_1^2 + 2x_2^2 + x_3 \quad , \quad 6x_1^2 + 3x_2^2 + x_3]$$

Where x_2 solves

$[\mathbf{2^{nd} level}]$

$$\max_{x_2} F_2(x_1, x_2, x_3) = \max_{x_2} [5x_1^2 + 8x_2^2 - 2x_3 \quad , \quad x_1^2 + 8x_2^2 - x_3]$$

Where x_3 solves

$[\mathbf{3^{rd} level}]$

$$\max_{x_3} F_3(x_1, x_2, x_3) = \max_{x_3} [x_1 + x_2 + 4x_3^2 \quad , \quad x_1 + x_2 + 6x_3^2]$$

Subject to:

$$\begin{gathered} (x_1, x_2, x_3) \in G = \{(x_1, x_2, x_3) \\ 4x_1 + x_2 + x_3 \leq (24,16,5,12) \\ x_1 + 3x_2 + x_3 \leq (23,7,10,18) \\ 2x_1 + x_2 + 2x_3 \leq (20,16,15,12) \\ x_1 , x_2 , x_3 \geq 0\} \end{gathered}$$

By using equation (5) the given problem is converted from the fuzzy form to the crisp form then the problem is formulated as follow:

$[\mathbf{1^{st}\ level}]$

$$\max_{x_1} F_1(x_1, x_2, x_3) = \max_{x_1} [3x_1^2 + 2x_2^2 + x_3 \quad , \quad 6x_1^2 + 3x_2^2 + x_3]$$

Where x_2 solves

$[\mathbf{2^{nd} level}]$

$$\max_{x_2} F_2(x_1, x_2, x_3) = \max_{x_2} [5x_1^2 + 8\,x_2^2 - 2x_3 \;,\; x_1^2 + 8\,x_2^2 - x_3]$$

Where x_3 solves

[3^{rd} level]

$$\max_{x_3} F_3(x_1, x_2, x_3) = \max_{x_3} [x_1 + x_2 + 4x_3^2 \;,\; x_1 + x_2 + 6x_3^2\,]$$

Subject to:

$(x_1,\ x_2,\ x_3) \in \bar{G} = \{(x_1,\ x_2,\ x_3)$

$4x_1 + x_2 + x_3 \le 12,$

$x_1 + 3x_2 + x_3 \le 10,$

$2x_1 + x_2 + 2x_3 \le 8,$

$x_1, x_2, x_3 \ge 0\}$

First, the FLDM solves his/her problem separately as follows:
Calculate the best and the worst solution of the FLDM problem by using equation (8) we get

$(b_{11}^*, b_{12}^*) = (29.7, 54)$, $(w'_{11}, w'_{12}) = (0, 0)$.

By using equations (12) and (13) the ε-constraint problem of the FLDM including satisfactoriness s_1=0.9, which is given by the FLDM is formulated as follow:

$$\max_{x_1} f_{11}(x_1, x_2, x_3) = 3x_1^2 + 2x_2^2 + x_3$$

Subject to:

$(x_1,\ x_2,\ x_3) \in \bar{G} = \{(x_1,\ x_2,\ x_3)$

$4x_1 + x_2 + x_3 \le 12,$

$x_1 + 3x_2 + x_3 \le 10,$

$2x_1 + x_2 + 2x_3 \le 8,$

$6x_1^2 + 3x_2^2 + x_3 \ge 48.6,$

$x_1, x_2, x_3 \ge 0\}$

The FLDM solution is $(x_1^F, x_2^F, x_3^F) = (2.36, 2.54, 0)$.

Second, $x_1 = x_1^F$ is given in order by FLDM to SLDM then, the SLDM solves his/her problem as follows:
Calculate the best and the worst solution of the SLDM problem by using equation (8) we get

$(b_{21}^*, b_{22}^*) = (88.8, 88.8)$, $(w'_{21}, w'_{22}) = (8, 4)$

By using equations (17) and (18) the ε-constraint problem of the SLDM including satisfactoriness s_2=0.6 which is given by the SLDM is formulated as follow:

Max $f_{21}(x_1^F, x_2, x_3) = 5x_1^2 + 8\,x_2^2 - 2x_3$

Subject to:

$(x_1^F\ \ x_2,\ x_3) \in \bar{G} = \{(x_1^F, x_2,\ x_3)$

$4x_1 + x_2 + x_3 \le 12,$

$x_1 + 3x_2 + x_3 \le 10,$

$2x_1 + x_2 + 2x_3 \le 8,$

$x_1^2 + 8x_2^2 - x_3 \ge 54.8,$

$x_1^F = 2.36,$

$x_2, x_3 \ge 0\}$

The SLDM solution is (x_1^F, x_2^S, x_3^S)= (2.36, 2.54,0)

By using (19) we will find that (x_1^F, x_2^S, x_3^S) is a satisfactory solution for the FLDM from the following test where δ_F= 0.2 is given by the FLDM.

$$\frac{\| F_1(2.36, 2.54, 0) - F_1(2.36, 2.54, 0) \|_2}{\| F_1(2.36, 2.54, 0) \|_2} \prec 0.2$$

Third, $x_1 = x_1^F$ and $x_2 = x_2^s$ is given in order by SLDM to TLDM then, the TLDM solves his/her problem as follows:
Calculate the best and the worst solution of the TLDM problem by using equation (8) we get

$(b_{31}^*, b_{32}^*) = (64, 96)$, $(w'_{31}, w'_{32}) = (-0.14, -0.2)$.

TLDM do exactly like the SLDM, the ε-constraint problem of the TLDM including satisfactoriness s_3=0.3 which is given by the TLDM is formulated as follow:

Max $f_{31}\left(x_1^F, x_2^S, x_3\right) = x_1 + x_2 + 6x_3^2$

Subject to:

$\left(x_1^F, x_2^S, x_3\right) \in \bar{G} = \{\left(x_1^F, x_2^S, x_3\right)$

$4x_1 + x_2 + x_3 \leq 12,$

$x_1 + 3x_2 + x_3 \leq 10,$

$2x_1 + x_2 + 2x_3 \leq 8,$

$x_1^2 + 4x_3^2 \geq 19.1,$

$x_1^F = 2.36,$

$x_2^S = 2.54,$

$x_3 \geq 0\}$

So the $\left(x_1^F, x_2^S, x_3^T)\right)$ = (2.36, 2.54, 0.2) is the solution of the TLDM.

Finally by using (22) we will find that $\left(x_1^F, x_2^S, x_3^T\right)$is a satisfactory solution for the SLDM from the following test where δ_S= 0.1, which is given by the FLDM.

$$\frac{\| F_1((2.36, 2.54, 0)) - F_1(2.36, 2.54, 0.2) \|_2}{\| F_1((2.36, 2.54, 0)) \|_2} \prec 0.1$$

So $\left(x_1^F, x_2^S, x_3^T\right) = (2.36,2.54,0.2)$ is the satisfactory solution (preferred solution) to (MLMOQP) problem.

VII. CONCLUSION

This paper proposed an algorithm to solve multi-level multi-objective quadratic programming (MLMOQP) problem, involving trapezoidal fuzzy numbers in the right hand side of the constraint, the suggested algorithm used the linear Ranking Methods to convert the mentioned problem to its equivalent crisp form then used the interactive approach to obtain the preferred satisfactory solution (preferredsolution) in view of the satisfactoriness concept,ε -constraint method with considerations of overall satisfactory balance among all of the three levels. This algorithm can be applied to problems involving fuzzy number in the objective function or in both the objective function and the constraint.

REFERENCES

[1]. O. E. Emam, A fuzzy approach for bi-level integer nonlinear programming problem, Applied Mathematics and Computations, 172 (2006) 62–71.

[2]. M. S. Osman, M. A. Abo-Sinna, A. H. Amer and O. E. Emam, A multi-level non-linear multi-objective under fuzziness, Applied Mathematics and Computations, 153 (2004) 239-252.

[3]. E. A. Youness, O. E. Emam and M. S. Hafez, Simplex method for solving bi-level linear fractional integer programming problems with fuzzy numbers, International Journal of Mathematical Sciences and Engineering Applications, 7 (2013) 351-363.

[4]. O.E. Emam, Interactive Bi-Level Multi-Objective Integer Non-linear Programming Problem, Applied Mathematical Sciences, 5 (65)(2011) 3221 – 3232.

[5]. O.E. Emam, Interactive approach to bi-level integer multi-objective fractional programming problem, applied mathematics and computation 223 (2013)17-24.

[6]. E. A. Youness, O. E. Emamand M. S. Hafez, Fuzzy Bi-Level Multi-Objective Fractional Integer Programming, Applied Mathematics, 8(6) (2014)2857-2863.

[7]. Z. A. Kanaya, An Interactive Method for Fuzzy Multi-objective Nonlinear Programming Problems , JKAU, 22(1) (2010) 103-112 .

[8]. M. Sakawa, I. Nishizaki and Y. Uemura, Interactive fuzzy programming for multi-level linear programming problems with fuzzy parameters, Fuzzy Sets and Systems 109 (2000) 3-19.

[9]. M. Sakawa and Ichiro Nishizaki, Interactive fuzzy programming for decentralized two-level linear programming problems. Fuzzy Sets and Systems 125 (2002) 301–315.

[10]. S. Mishra and A. Ghosh, Interactive fuzzy programming approach to Bi-level Quadratic fractional programming problems, Ann Oper Res (143) (2006) 251–263

[11]. F .A. Adnan and I. H AIkanani, Ranking Function Methods For Solving Fuzzy Linear Programming Problems, Mathematical Theory and Modeling,4(4) (2014)65-72.