

Analysis of Extreme Value at Risk to Amazon Stocks

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ABSTRACT: The VaR of the negative log returns of E-commerce stock Amazon stock (AMZN) (during the period of October 31, 1997 to October 30, 2017) are studied, and based on the statistical exam and analysis, it turns out that the returns is a stationary, uncorrelated time series, and not normally distributed or t-distributed. Therefore, one cannot use the traditional way to compute the value at risk (VaR) of the returns based on either normal distribution or t-student distribution. How to evaluate the VaR of the returns becomes a very tricky problem. To avoid the assumption of distribution of the returns, in this paper, we use the Extreme Value Theory to approach VaR and conclude that the calendar anomalies which are the lower risk seasons are Q2 and Q3 and higher risk seasons are Q1 and Q4. But the investors should realize that the resulting principle of AMZN stock may not apply to other portfolios or stocks. Especially, comparing with empirical results of traditional portfolio SPDR S&P 500 ETF SPY (SPY), we conclude that the seasonal effect of E-commerce and traditional commerce are quite different.

KEYWORDS: AMZN stocks, SPY stocks, Value-at-Risk, Extreme Value Theory, seasonal effect.

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I. MOTIVATION AND INTRODUCTION

Throughout the 1990s, the spread of the Internet and World Wide Web has swept the world. With the increasing use of the Internet in society, companies start to look the web as a new way of doing business. Selling products through the Internet offers a variety of options and opportunities. Amazon.com Inc. is a well-known American E-commerce company and primarily a retail site, it sells almost everything including electronic products, clothes, food, games, groceries, health and personal-care items, etc. At the same time it has its own manufactures which also produce their own electronics for consumers, including the Kindle, which is an electronic reading device etc. As internet and its carriers (such as iphone, ipad, laptop and desktop) have the convenient shopping functions, in the past ten years, more and more physical stores have replaced by E-commerce companies. Nothing should be surprised to this phenomena, shopping online has greatly saved people's time and energy, and costumers could face to a wider shopping world, have more choices for everything. As the E-commerce becomes very popular in recently, Amazon.com is the biggest E-commerce company in the years, to study the risk of investment of E-commerce companies also becomes very important to researchers. Especial world, to study the value at risk of AMZN has typical meaning.

It is well known that the SPY is the most liquid and heavily traded security in the world, and it is the most popular ETF used to access this exposure with more than \$243.3 billion in assets[1]. SPY has large institutional ownership, a robust secondary trading market has allowed SPY to establish and maintain itself as an attractive vehicle for accessing the S&P 500 Index.

We introduce AMZN and SPY here, because we not only study the VaR of AMZN, but also compare the seasonal effect of these two stocks in this paper by using the method of value at risk (VaR)[2][3][4], which is a measure of the risk of investments. A VaR statistic has three components: a time period, a confidence level and a loss amount (or loss percentage)[5]. To study the VaR of AMZN stocks, the first of all, we have to test if the log returns follow the normal distribution, if so, we can use the Variance-Covariance Method to evaluate VaRs. Unfortunately, in this paper we found the log returns of AMZN does not follow the normal distribution or t-student distribution significantly. Therefore, we have to find another way to evaluate the VaRs of AMZN log returns.

Extreme value theory (EVT) or extreme value analysis (EVA) is a branch of statistics dealing with the extreme deviations from the median of probability distributions. It tries to assess, from a given ordered sample of a given random variable, the probability of events that are more extreme than any previously observed[6]. It is possible that there is no appropriate distribution for extremes, but if there is one, it must be from the Generalized Extreme Value (GEV) family (block maxima) or the Generalized Pareto (GP) family (excesses over a high

threshold). The two families are related. The Generalized Pareto Distribution (GPD) was introduced by Pikands III[7] and studied by E. Castillo[8][9] and others [10] [11]. Thereafter,

it is suitable to use the extreme value analysis on the tested AMZN returns to study the seasonal effect [12][13][14] in stock market. But all of these studies focused on a calendar anomalies in stock returns and volatility, they did not study the seasonal effect of E-commerce and traditional commerce. However, in this paper, we use EVT to approach the VaR of AMZN, furthermore, we also compare the VaR of seasonal effect of E-commerce AMZN and traditional commerce SPY, and discover that the seasonal effect of these two stocks are quite different. Finally, we analyze the possible factors which caused the seasonal anomalies of these two stocks.

II. DATA DESCRIPTION AND SOME STATISTICAL RESULTS

A. Data Description

In this paper, we use daily adjusted closing prices of the AMZN and SPY (from Yahoo Finance¹) from October 31, 1997 to October 30, 2017. The adjusted closing price has formed an accurate tracking record of stock performance, furthermore, log returns of adjusted closing price makes samples stable, which is used to study the VaR of a stock. Let P_t represents the adjusted closing price of the day t of a stock, and then the logarithmic rates of returns is determined by

$$r_t = 100 \left(\log \frac{P_t}{P_{t-1}} \right) = 100 (\log P_t - \log P_{t-1})$$

Fig. 1 shows a time series chart of the adjusted closing price and daily log returns of AMZN stock from 31 October 1997 to 30 October 2017. And it shows that AMZN's stock price has soared more than 200 times since 2007, and the volatility of AMZN's daily log returns is relatively stable even though we observe many peaks.

Table 1 summarizes the basic statistical features of the AMZN stock daily log return series during the test period from October 31, 1997 to October 30, 2017. The test results notice us that on one hand, there may be a possibility of time-varying variance and non-normal behavior; on the other hand, in order to calculate the VaR properly, we need to examine stationarity and the normality of the daily log return series.

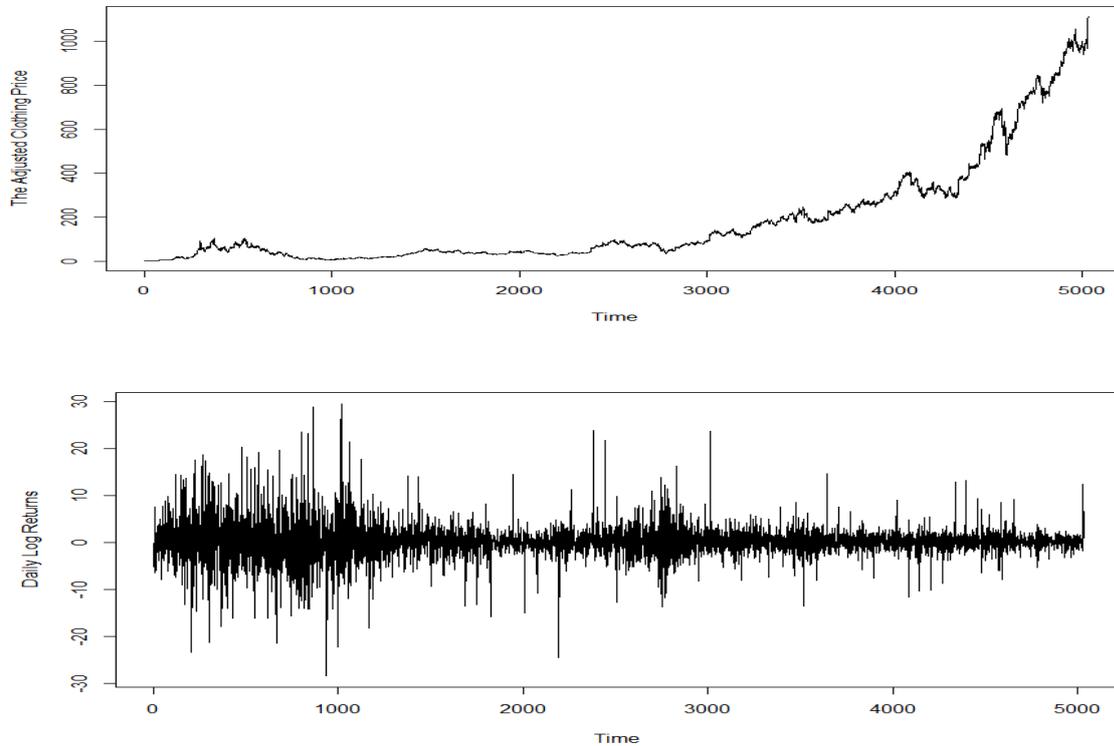


Fig. 1: Time plots of AMZN stock. The upper panel is for adjusted closing price, and the lower panel is for daily log returns

Table 1: Summary Statistics of the AMZN Daily Log Returns

Mean	Range	Std dev	Skewness	Kurtosis	Obervation
0.11	(-28.46,29.62)	3.75	0.43	8.68	5031

Table 2: KPSS Tests for the AMZN Daily Log Returns

	Null Hypothesis H_0	Stats	p-value	Test Result
KPSS test for stationary	The series is stationary around a straight line time trend	0.071	0.1	Accept H_0 ,The series is stationary.
	The series is stationary around a constant.	0.069	0.1	

B. Test for Stationary Property

For the stationarity hypothesis, the joint probability distribution of the log returns doesn't change when time goes. Using KPSS test [15] to examine stationarity. The hypothesis for the KPSS test is

$$H_0 : \sigma_\mu^2 = 0 \quad \text{vs} \quad H_1 : \sigma_\mu^2 \neq 0.$$

The rejection rule is that if the value of the KPSS statistic is more than the critical values estimated in [15], or the p-value is less than or equal to the significance level α , we reject H_0 , the series is non-stationary, otherwise, it is stationary.

From the above table of KPSS test in Table 2, we see that the daily log returns of AMZN is stationary during the test period.

C. Test for Normality

First of all, we draw a QQ-plot by the sample set of daily log returns of AMZN which against the normal distribution, and it has the fat tails, or is leptokurtic-see Fig.2.

We use the Shapiro-Wilk test [16] to verify the non-normal result. The Shapiro-Wilk test statistics is usually written as W, the value of W is between 0 and 1. If the values of W is small enough, it concludes the rejection of normality, whereas a larger value indicates the normality of data. The null hypothesis H_0 is W=1 which indicates the normal distribution. If the p-value of the test is less than the significance level, we reject the null hypothesis H_0 .

From Table 3, we know that the daily log returns of AMZN are not normal during the time period of October 31, 1997 to October 30, 2017.

Fig. 3 is the T QQ-plot of empirical distribution of the daily log returns(y-axis), it is obviously not the t-student distribution even if it fits better than the normal distribution compare to Fig. 2. The plots also show that empirical distribution of the daily log returns of AMZN has heavier tails than the normal distribution, this means that the prior assumption of normal distribution of the log returns is no sense.

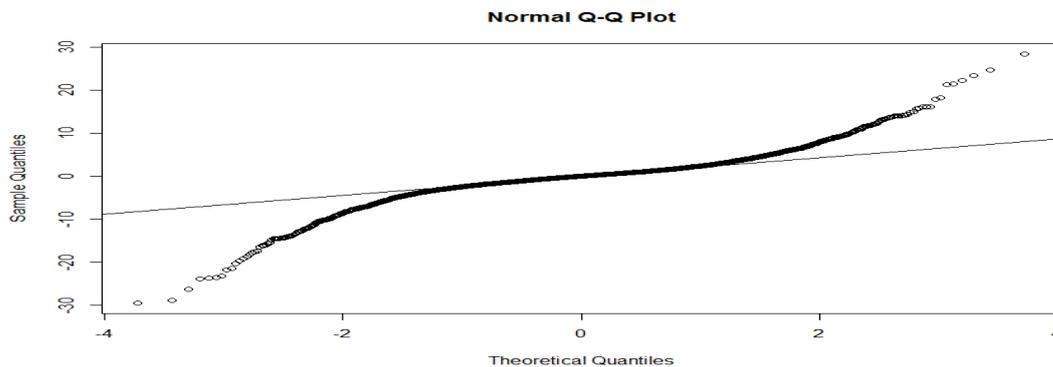


Fig. 2: QQ-plot of AMZN daily log returns against normal distribution

Table 3: Shapiro-Wilk Test for the AMZN Daily Log Returns

	Null Hypothesis H_0	Stats	p-value	Test Result
Shapiro-Wilk test for normality	The series come from a normally distributed population	0.88	<2.2e-16	Reject H_0 , the series does not come from a normal distribution

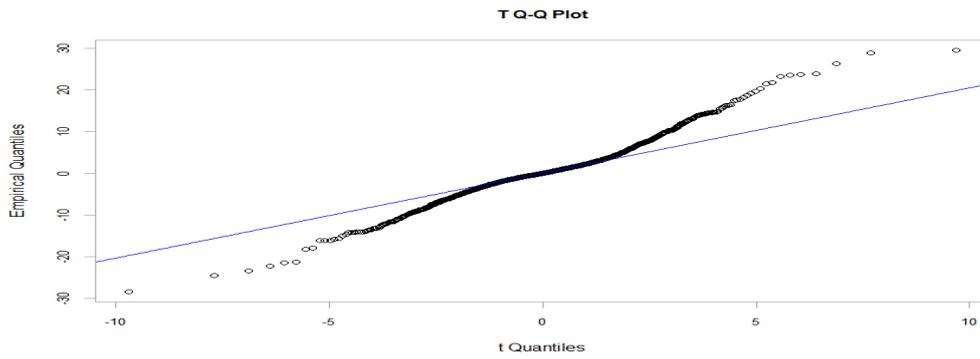


Fig. 3: QQ-plot of AMZN daily log returns against the t-student distribution

D. Test for Correlations

Autocorrelation Coefficient Function(ACF) and Partial Auto-correlation Coefficient Function (PACF) are very useful in helping us to describe random processes. We plot the autocorrelation function and partial autocorrelation function of the daily log returns $\{r_t\}$. Fig. 4 shows that the time series $\{r_t\}$ of daily log returns of AMZN does not have strong autocorrelations.

We can apply Ljung-Box test [17] for serial correlation of daily log returns to confirm this result. In this test, the null and alternative hypothesis is defined as the following,

$$H_0 : \rho_1 = \rho_2 = \dots = \rho_m = 0 \quad \text{vs} \quad \rho_i \neq 0 \quad \text{for some } i \in \{1, 2, \dots, m\}$$

where ρ_l is the sample autocorrelation function at lag l, and m is the number of lags being tested, and the Ljung-Box Q test statistics is commonly represented as Q(m).

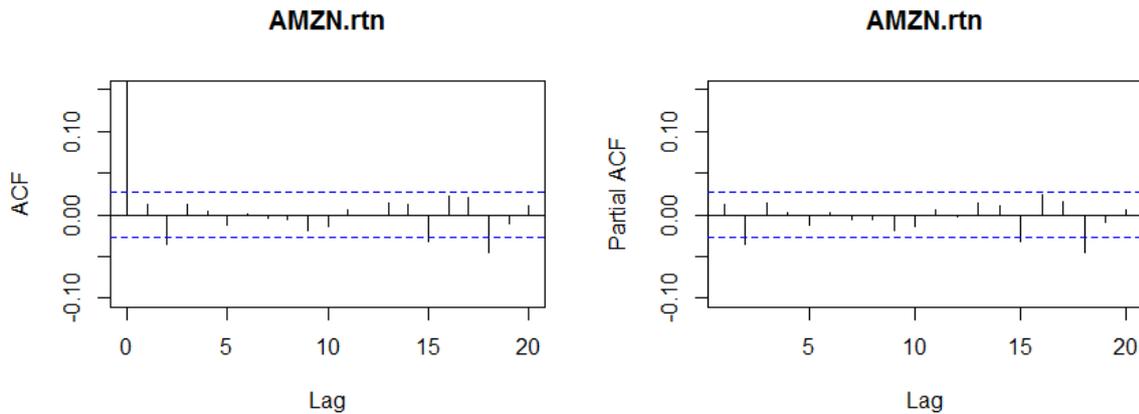


Fig. 4: The left panel is sample autocorrelation coefficients for AMZN log returns, the right panel is sample partial autocorrelation coefficients for AMZN log returns

Table 4: Ljung-Box Test for The AMZN Negative Daily Log Returns

χ -squared	m	p-value
9.0447	5	0.1073
12.08	10	0.2798
19.138	15	0.2076

Under the assumption of $\{r_t\}_{t=1}^n$ is an independent identically distributed (iid), the distribution of statistics Q(m) can be approximated as the $100(1-\alpha)$ the percentile of chi-squared distribution with m degrees of freedom.

In the case of significance level $\alpha = 5\%$, if the p-value is less than 5%, then the null hypothesis that the sequence does not have autocorrelation is rejected, in other words, the series has autocorrelation; and if the p-value is more than 5%, the series does not have autocorrelation. If $Q(m) > \chi^2_\alpha$ for significance level α , we can also reject the null hypothesis.

The test results are shown in the Table 4: all the p-values are more than significance level 5% on lag 5, 10, 15, we can determine that the daily log returns are not autocorrelation.

Based a series tests on the above, we conclude that the sample set of log returns of AMZN is neither a normal distribution nor a t-student distribution but it is a stationary and uncorrelated time series. Therefore, the assumption of the series $\{r_t\}$ with normal distribution or t-distribution does not fit the real case even though many researchers [18] [19] have used this assumption to compute VaR. Instead of using any of assumption to the distribution of the log returns of AMZN, we apply the Extreme Value Theory to approach the VaR of daily log returns.

III. VALUE AT RISK WITH EXTREME VALUE APPROACHING

A. VaR of a Time Series

There are extreme risks in all areas of financial investment, credit and insurance, where extreme disasters can take a large toll. Therefore, for investors and risk managers, the loss estimation and probability prediction of extreme loss are very important.

Extreme Value Theory (EVT)[20][21][22] is a powerful tool for the tail distribution even if the Historical or Monte Carlo simulation methods also does the same approach, but they are not inefficient as EVT.

Assume a random variable X , we first fix some high threshold μ and consider the distribution of excess values $Y = X - \mu$, which is defined as:

$$F_\mu(y) = \Pr(X - \mu \leq y | X > \mu) = \frac{F(\mu + y) - F(\mu)}{1 - F(\mu)} \quad (2)$$

where F is the underlying distribution of X , F_μ is the conditional excess distribution function. In fact, Pickands III(1975)[7] introduced the GPD as a two parameter family of distributions for exceedance over a threshold μ .

Extreme Value Theory[23] Assume $\{X_t\}$ is a sequence of stationary, uncorrelated random variables with distribution F . For any $\mu > 0$, let F_μ be the conditional excess distribution function, for random variables defined in (2) with

$Y_t = X_t - \mu$. Let $\omega_F = \sup\{x : F(x) < 1\}$, then

$$\lim_{\mu \rightarrow \omega_F} F_\mu(y) = H_{\sigma_\mu, \xi}(y)$$

Where $H_{\sigma_\mu, \xi}(y)$ is called GPD, specified as

$$H_{\sigma_\mu, \xi}(y) = 1 - \left(1 + \xi \frac{y}{\sigma_\mu}\right)_+^{-1/\xi} \quad (3)$$

The parameters of GPD are the scale parameter σ_μ and the shape parameter ξ .

Although the distribution of each random variable X_t is not known, EVT describes the tail distribution in detail. The shape parameter can reflect the tail fatness of a distribution:

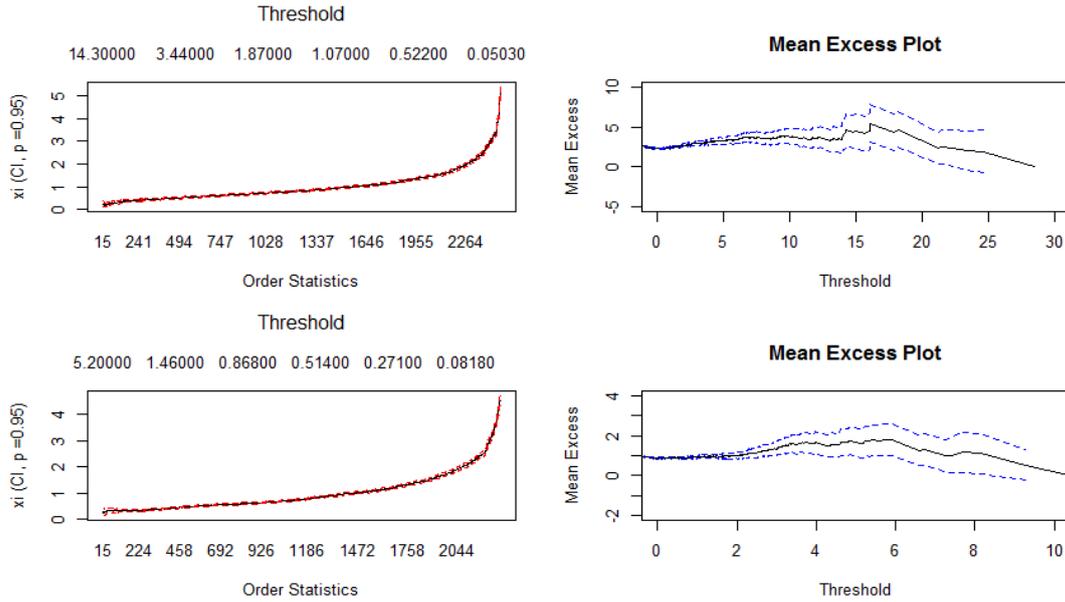


Fig. 5: Hill plots (left) and mean excess plots (right) for the AMZN daily negative logreturns (top) and SPY daily negative logreturns (lower) with 95% asymptotic confidence bounds (dotted line).

- $\xi < 0$ refers to thin tails;
- $\xi = 0$ implies that the kurtosis is 3 as for a standard normal distribution;
- $\xi > 0$ implies fat tails.

The extreme value theory address that the distribution of GPD can be used to approximate all of the excess distribution function that are greater than μ if the threshold μ is larger enough. Therefore, in order to evaluate the VaR, we have to find a sufficiently large value of μ , and estimate the parameters of the distribution of GPD, then use the EVT method to compute the quantile of the tail of the distribution to estimate VaR.

How to choose an optimal threshold μ , that becomes a very important work for us. Here we combine the Mean Excess plot and Hill plot together, using a graphical method to select a reasonable threshold.

IV. VAR ANALYSIS OF AMZN AND SPY

A. Modeling the Distribution of AMZN and SPY Negative Daily Log Return

We adopt One-day-ahead VaR forecasts along with the significance level of 5%, 1% and 0.1% in the empirical investigation. For comparison, we use AMZN and SPY daily negative log returns to compute the VaR and related statistical properties.

The daily negative log returns is defined by

$$\tilde{r}_t = -r_t \quad (4)$$

it is the opposite of the daily log returns r_t .

As we have pointed out in section 3, before applying the extreme value method to the VaR on the returns, we choose a threshold μ at first by Mean Excess plot and Hill plot. Fig.5 shows, with 95% confidence interval, a reasonable threshold should around 5 and around 2 for the daily negative log returns of AMZN and SPY, respectively. Assume that $\{r_t\}$ have a high enough threshold μ , the number of exceedance of the threshold is N_μ . For the daily negative log returns of AMZN,

Table 5: GPD Tests for the AMZN Daily Negative Log Returns

Daily negative log returns from 1997-10-31 to 2017-10-30	AMZN	SPY
Threshold μ	4.975259	1.918341
Exceedances N_μ	289	262
Shape parameter ML estimator ξ	0.1101152	0.2282867
	2.9135830	0.7628415

Scale parameter ML estimator $\hat{\sigma}_\mu$		
VaR(T = 1 day, $\alpha = 5\%$)	5.374776	1.949535
VaR(T = 1 day, $\alpha = 1\%$)	10.592159	3.447035
VaR(T = 1 day, $\alpha = 0.1\%$)	19.849102	6.815119

$N_\mu=289$, with corresponding threshold $\mu=4.975259$; for the daily negative log returns of SPY, $N_\mu=262$, with corresponding threshold $\mu=1.918341$ would be reasonable. See Table 5.

In order to visualize the model accurately, we use the back test method to the extreme value of the daily negative log returns of AMZN and SPY and the fitness is summarized in Fig. 6 and Fig. 7, respectively. We can see the estimates fit the given negative daily log returns of AMZN and SPY quite well, even in the far end tail. The assumption of an underlying heavy tailed distribution is consistent with the data is confirmed. So it seems very reasonable for the corresponding estimate of the over of 5%, 1% and 0.1% quantile of the VaR.

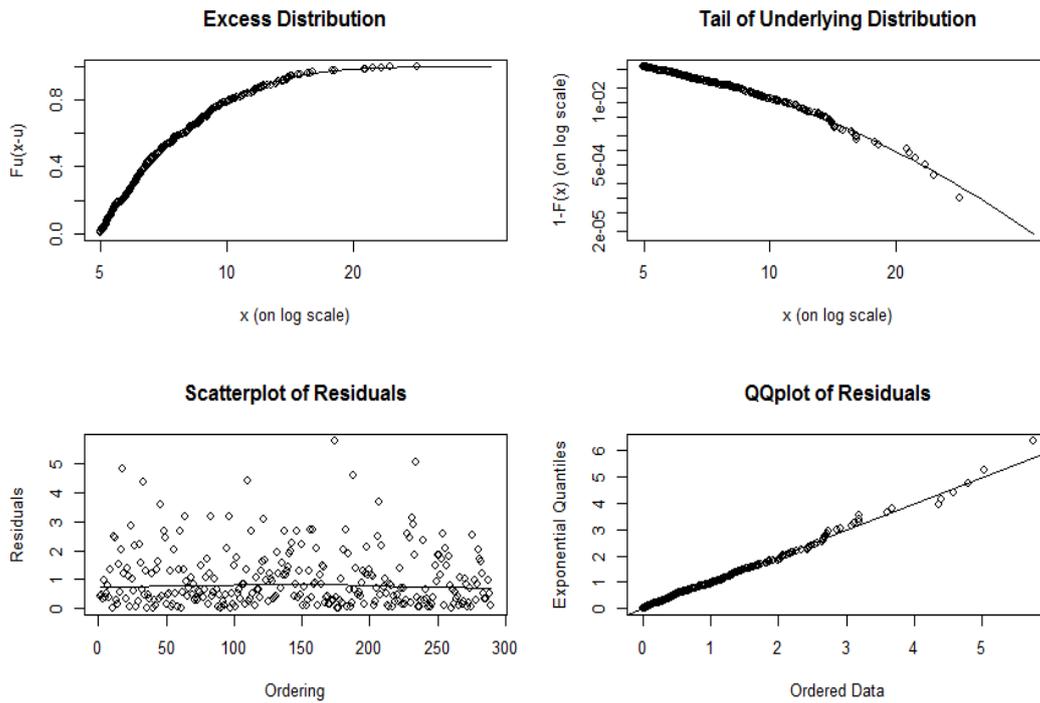


Fig. 6: Diagnostic plots for GPD fit to AMZN daily negative log returns

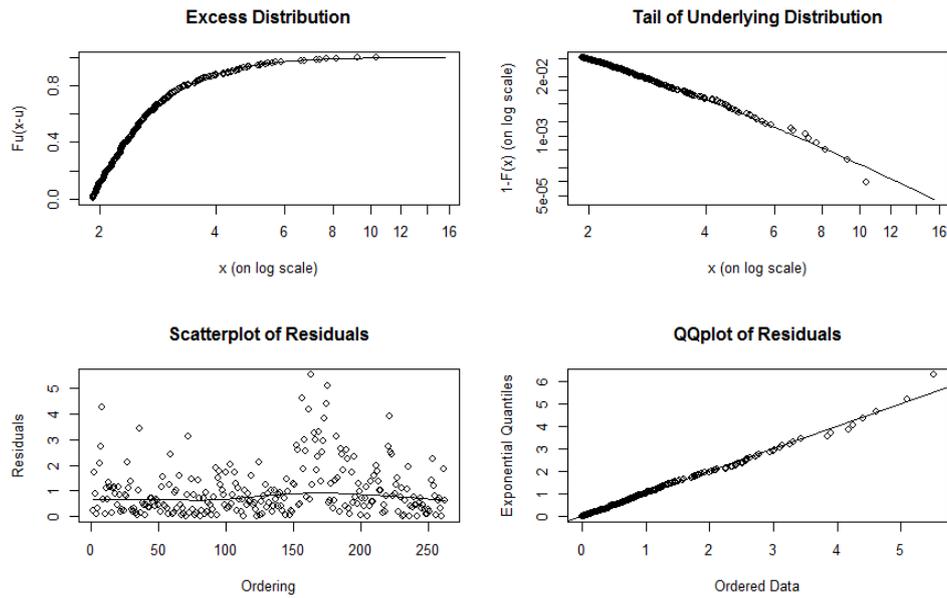


Fig. 7: Diagnostic plots for GPD fit to SPY daily negative log returns

B. Seasonal Effect of AMZN and SPY

Seasonal effects are important in determining stock performance, because the period of three-month on the financial calendar is the basis for reporting stock returns and paying dividends. In order to study the quarterly effects of the system on the stock of AMZN, we divide the sample data $\{\tilde{r}_t\}$ into the following four groups:

$$\{\tilde{r}_t \mid t \in Q_i\}, i = 1, 2, 3, 4$$

they are the four quarters of AMZN daily negative log returns. One-fourth of a year is a quarter and is usually expressed as Q .

Table 6 summarizes the basic statistical characteristics for the four quarters of daily negative log returns of AMZN. Next, we carry out the basic tests to examine the four quarters seasonal pattern in daily negative log returns of AMZN. Based on the Shapiro-Wilk tests, all the p-value of four quarters are less than $2.2e-16$, which are less than the significant level 0.1%, so all of the four quarters have no normal distribution.

Before applying the extreme value method to the VaR on our four quarters data sets, we have to select a specific threshold to confine the estimation to those observations that are above the given threshold. As mentioned in section 3, we choose the threshold by Mean Excess plot and Hill plot, which is the graphical procedures.

Table 6: Summary Statistics of AMZN Daily Negative Log Returns by Quarter

	Mean	Range	Std dev	Skewness	Kurtosis	Obs
Q1	-0.05	(-23.57, 21.29)	3.79	-0.61	6.86	1226
Q2	-0.17	(-28.95, 21.50)	3.57	-0.70	9.30	1264
Q3	-0.08	(-21.87, 28.46)	3.66	0.41	10.36	1270
Q4	-0.13	(-29.62, 22.31)	3.97	-0.73	8.20	1271

Table 7: Estimated GPD Parameters and VaR of AMZN Daily Negative Log Returns by Quarter

	ξ	σ_μ	μ	N_μ	$VaR_{0.05}$
Q1	0.0036675	3.0436056	4.445176	91	5.648514
Q2	0.1393965	2.5267834	4.054026	87	4.879855
Q3	0.2846917	2.5921566	4.040345	88	4.040345
Q4	-0.059547	3.414381	5.142519	83	6.046996

Table 8: Estimated GPD Parameters and VaR of AMZN Daily Negative Log Returns by Quarter via the Same Threshold

$$\mu = 5.142519.$$

	ξ	σ_μ	N_μ	$VaR_{0.05}$
Q1	0.091395	2.628499	76	5.713111
Q2	0.200889	2.455344	58	4.933508
Q3	0.2125037	3.2742714	57	4.792961
Q4	-0.059547	3.414381	83	6.046996

Next, we compute VaR on the seasonal daily negative log returns of AMZN for four quarters using the method of extreme value theory. Table 7 and Table 8 tell us the same results, no matter we choose the different thresholds via Hill plot on the four quarters or using the same threshold, the shape parameter ξ of Q3 returns is the largest returns of AMZN, which indicates the fattest tail behavior. The over of 5% quantile VaR is the largest returns of AMZN in the fourth quarter Q4, and the third quarter Q3 is the smallest returns of the four seasons.

Using the same method, we obtain the VaR of SPY daily negative log returns of four quarters. Table 9 and Table 10 show the results.

After the comparison of SPY returns and AMZN returns, we see that the results of SPY returns are different from AMZN return.

V. CONCLUSION

After a series tests we conclude that the daily log returns of AMZN is stable, but not normal distributed or

Table 9: Estimated GPD Parameters and VaR of SPY Daily Negative Log Returns by Quarter.

	ξ	σ_μ	μ	N_μ	$VaR_{0.05}$
Q1	-0.04821617	0.86020868	1.780453	78	1.986503
Q2	0.1148558	0.5779253	1.581727	77	1.697172
Q3	0.2593656	0.7395883	1.944272	82	2.139780
Q4	0.3350238	0.8553170	1.825313	77	1.994911

Table 10: Estimated GPD Parameters and VaR of SPY Daily Negative Log Returns by Quarter via the Same Threshold

$$\mu = 1.944272.$$

	ξ	σ_μ	N_μ	$VaR_{0.05}$
Q1	-0.02262276	0.82099745	65	1.992357
Q2	-0.01746831	0.78150883	39	1.565409
Q3	0.2593656	0.7395883	82	2.139780
Q4	0.3323455	0.9011683	67	1.992334

t-distributed, therefore we use extreme value theory to approach the VaR. Furthermore, we tested the seasonal effect on the negative daily log returns of AMZN and compared its difference of seasonal effect with SPY stock. Eventually, we found that seasonal behavior of AMZN and SPY are quite different. No matter how we choose the thresholds (the different thresholds or the same threshold), the shape parameter ξ of SPY is largest in the fourth quarter Q4, and the largest VaR over the 5% quantile is in the third quarter Q3. These are the opposite to AMZN returns. On the other hand, the smallest VaR the over of 5% quantile of SPY returns happens in the second quarter Q2, but the same result does not happen to AMZN returns.

Of course, there are many factors affected performance of stocks in financial market, for example, tax-motivated trading, economic, government policies etc., but if we consider these two stocks in the same country, so the above factors should be same affection to all of stocks, therefore, the possible explanations, for the AMZN stock, in Q1 and Q4, the VaR is larger, because there are at least two long holidays, like Thanksgiving and Christmas, there are many coupons and discounts on E-commerce, that drives people to do the online-shopping very crazy. In addition, this is winter time period, people are more likely to stay at home because of the cold weather, so online-shopping in this case becomes favorable, that is why AMZN stock has a larger vibration in Q1 and Q4. But for SPY stocks, its VaR is larger in Q3, mostly because the new product will be announced in this time period, it attracts more people's eyeball, so its VaR is larger than other three seasons.

The above conclusion is very reasonable, it also tells investors what is the good time period to invest their money to which kind of stocks. For E-commerce, like AMZN, investors, if they want to get a larger benefit

from stock investment with higher risk, then they should invest money in Q1 and Q4, other they should choose safe seasons, like Q2 and Q3. But if investors want to invest stock which is not an E-commerce, like SPY, ones should be better to carry out their investment in Q3. Anyway, based on our research, there are a big difference between the stock performance of E-commerce and no E-commerce, that is a basic principle which we have concluded in this paper, investors should be very carefully to treat behaviors of stocks between the very modern stocks, like E-commerce AMZN and traditional stock, like SPY portfolio when they make a plan for their investments.

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