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Different Deterioration Rates Two Warehouse Inventory Model with Time and Price Dependent Demand under Inflation and Permissible Delay in Payments and Shortages

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ABSTRACT- An inventory model with different deterioration rates under inflation and permissible delay in payments for two level storage is developed. Demand is considered as function of price and time. Holding cost is time dependent. Shortages are allowed and completely backlogged. Numerical case is given to represent the model. Affectability investigation is likewise done for parameters.

KEYWORDS – Two warehouse, Different deterioration, Time dependent demand, Price dependent demand, Shortages, Inflation, Permissible Delay in Payments

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I. Introduction

Many researchers studied inventory models for deteriorating items in past. An inventory model with constant rate of deterioration was developed by Ghare and Schrader [7]. Covert and Philip [6] extended the model by considering variable rate of deterioration. The model was further extended by Shah [22] by considering shortages. An inventory model for stock dependent consumption rate was considered by Mandal and Phaujdar [14]. Patel and Parekh [17] developed an inventory model with stock dependent demand under shortages and variable selling price. An inventory model with stock and price dependent demand under shortages was developed by Sheikh and Patel [23]. The related works are found in (Nahmias [15], Raffat [18], Goyal and Giri [9], Ruxian et al.[19]).

Generally it is assumed that retailer must pay off as and when items are received. But it will not be always true in today's competitive market. The supplier often offers his retailer certain delay in time period for making payment for the items he has received. It is an effective way of attracting new customers. An economic order quantity model under the condition of permissible delay in payments was developed by Goyal [8]. Goyal's [8] model was extended by Aggarwal and Jaggi [1] to consider the deteriorating items. Aggarwal and Jaggi's [1] model was further extended by Jamal et al. [12] to consider shortages. The related works are found in (Chung and Dye [4], Salameh et al. [20], Chung et al. [5], Chang et al. [3]).

Many time retailers decide to buy goods exceeding their Own Warehouse (OW) capacity to take advantage of price discounts. Therefore an additional stock is arranged as Rented Warehouse (RW) which has better storage facilities with higher inventory holding cost. Hartley [10] first developed a two warehouse inventory model. Sarma [21] developed an inventory model with finite rate of replenishment with two warehouses. Yang [24] considered a two warehouse inventory problem for deteriorating items with constant rate of demand under inflation in two alternatives when shortages are completely backordered. Bhunia et al. [2] deals with a deterministic inventory model for linear trend in demand under inflationary conditions with different rates of deterioration in two warehouses. For non-instantaneous deteriorating items with two storage facilities under inflation was developed by Jaggi et al. [11]. Liang and Zhou [13] considered a two warehouse inventory models for deteriorating items under conditionally permissible delay in payments. Deteriorating item inventory models for two warehouses with linear demand under inflation and permissible delay in payments was developed by Parekh and Patel [16].

A two warehouse inventory model with different deterioration rates is developed. Demand function is price and time dependent. Holding cost is linear function of time. Shortages are allowed and completely backlogged. Numerical case is given to represent the model. Affectability investigation is likewise done for parameters.

II. Assumptions And Notations

NOTATIONS:

The following notations are used for the development of the model:

D(t) : Demand rate is a function of time and price ($a+bt-\rho p$, $a>0$, $0<b<1$, $\rho>0$)

HC(OW) : Holding cost is function of time t (x_1+y_1t, $x_1>0$, $0<y_1<1$) in OW.

HC(RW) : Holding cost is function of time t (x_2+y_2t, $x_2>0$, $0<y_2<1$) in RW.

A : Ordering cost per order

c : Purchasing cost per unit

p : Selling price per unit

c_2 : Shortage cost per unit

T : Length of inventory cycle

$I_0(t)$: Inventory level in OW at time t

$I_r(t)$: Inventory level in RW at time t

I_e : Interest earned per year

I_p : Interest paid in stocks per year

R : Inflation rate

Q_1 : Inventory level initially

Q_2 : Shortage of inventory

Q : Order quantity

t_r : Time at which inventory level becomes zero in RW.

W : Capacity of own warehouse

θ : Deterioration rate in OW, during $\mu_1<t<\mu_2$, $0<\theta<1$

θt : Deterioration rate in OW during, $\mu_2 \le t \le t_0$, $0<\theta<1$

π : Total relevant profit per unit time.

ASSUMPTIONS:

The following assumptions are considered for the development of the model.

- The demand of the product is declining as a function of time and price.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and completely backlogged.
- OW has fixed capacity W units and RW has unlimited capacity.
- The goods of OW are consumed only after consuming the goods kept in RW.
- The unit inventory cost per unit in the RW is higher than those in the OW.
- Deteriorated units neither be repaired nor replaced during the cycle time.
- During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

III. The Mathematical Model And Analysis

At time t=0, Q units enters into the system of which W are stored in OW, Q_2 units are used to fulfil previous cycles backlog and rest (Q_1-W) are stored in RW. During interval ($0,t_r$) level of inventory in RW of depletes due to demand and reaches to 0 at time tr and inventory in OW remains W. During the interval (t_r,μ_1) inventory depletes in OW due to demand, during interval (μ_1, μ_2) inventory depletes in OW due to deterioration at rate θ and demand. During interval (μ_2, t_0) inventory in OW depletes due to joint effect of deterioration at rate θt and demand. By time t_0 both the warehouses are empty. Shortages occur during (t_0, T) of size Q_2 units.

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

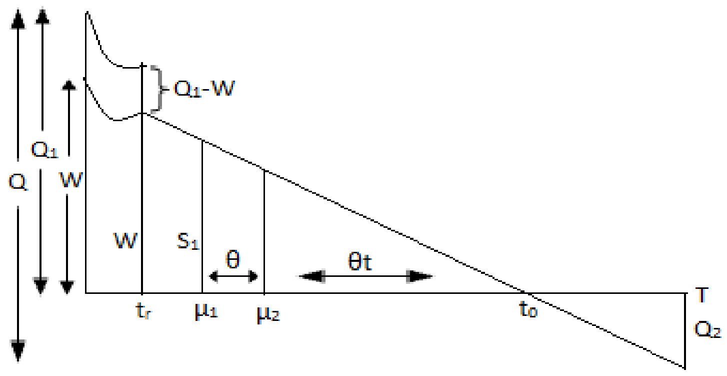


Figure 1

Hence, the inventory level at time t in RW and OW are governed by the following differential equations:

$$\frac{dI_r(t)}{dt} = -(a + bt - \rho p), \qquad 0 \le t \le t_r \qquad (1)$$

$$\frac{dI_0(t)}{dt} = 0, \qquad 0 \le t \le t_r \qquad (2)$$

$$\frac{dI_0(t)}{dt} = -(a + bt - \rho p), \qquad t_r \le t \le \mu_1 \qquad (3)$$

$$\frac{dI_0(t)}{dt} + \theta I_0(t) = -(a+bt - \rho p), \qquad \mu_1 \le t \le \mu_2 \qquad (4)$$

$$\frac{dI_0(t)}{dt} + \theta t I_0(t) = -(a+bt - \rho p), \qquad \mu_2 \le t \le t_0 \qquad (5)$$

$$\frac{dI_0(t)}{dt} = -(a + bt - \rho p), \qquad t_0 \le t \le T \qquad (6)$$

with initial conditions $I_0(0) = W$, $I_0(\mu_1) = S_1$, $I_0(t_r) = W$, $I_r(t_r) = 0$, $I_r(0) = Q_1$-W, $I_r(t_r) = 0$, $I_0(t_0)=0$ and $I_0(T) = -Q_2$.
Solving equations (1) to (6) we have,

$$I_r(t) = Q_1 - W - (a - \rho p)t - \frac{1}{2}bt^2$$
(7)

$$I_0(t) = W \qquad (8)$$

$$I_0(t) = S_1 + (a - \rho p)(\mu_1 - t) + \frac{1}{2}b\left(\mu_1^2 - t^2\right) \qquad (9)$$

$$I_0(t) = \begin{bmatrix} a(\mu_1 - t) - \rho p(\mu_1 - t) + \frac{1}{2}a\theta\left(\mu_1^2 - t^2\right) - \frac{1}{2}\rho p\theta\left(\mu_1^2 - t^2\right) + \frac{1}{2}b\left(\mu_1^2 - t^2\right) \\ + \frac{1}{3}b\theta\left(\mu_1^3 - t^3\right) - a\theta t(\mu_1 - t) + \rho pt(\mu_1 - t) - \frac{1}{2}b\theta t\left(\mu_1^2 - t^2\right) \end{bmatrix} + S_1\left(1 + \theta(\mu_1 - t)\right) \qquad (10)$$

$$I_0(t) = \begin{bmatrix} a(t_0 - t) - \rho p(t_0 - t) + \frac{1}{6}a\theta\left(t_0^3 - t^3\right) - \frac{1}{6}\rho p\theta\left(t_0^3 - t^3\right) + \frac{1}{2}b\left(t_0^2 - t^2\right) \\ + \frac{1}{8}b\theta\left(t_0^4 - t^4\right) - \frac{1}{2}a\theta t^2(t_0 - t) + \frac{1}{2}\rho p\theta t^2(t_0 - t) - \frac{1}{4}b\theta t^2\left(t_0^2 - t^2\right) \end{bmatrix} \qquad (11)$$

$$I_0(t) = \left[a(t_0 - t) - \rho p(t_0 - t) + \frac{1}{2}b\left(t_0^2 - t^2\right)\right] \qquad (12)$$

(by neglecting higher powers of θ)

Putting $t = t_r$ in equation (7), we get

$$Q_1 = W + (a - \rho p)t_r + \frac{1}{2}bt_r^2 \qquad (13)$$

Putting $t = t_r$ in equations (8) and (9), we get

$$I_0(t_r) = W \qquad (14)$$

$$I_0(t_r) = S_1 + (a - \rho p)\left(\mu_1 - t_r\right) + \frac{1}{2}b\left(\mu_1^2 - t_r^2\right) \tag{15}$$

So from equations (14) and (15), we have

$$S_1 = W - (a - \rho p)\left(\mu_1 - t_r\right) - \frac{1}{2}b\left(\mu_1^2 - t_r^2\right) \tag{16}$$

Putting $t = \mu_2$ in equations (10) and (11), we get

$$I_0(t) = \left[\begin{array}{l} a\left(\mu_1 - \mu_2\right) - \rho p\left(\mu_1 - \mu_2\right) + \frac{1}{2}a\theta\left(\mu_1^2 - \mu_2^2\right) - \frac{1}{2}\rho p\theta\left(\mu_1^2 - \mu_2^2\right) + \frac{1}{2}b\left(\mu_1^2 - \mu_2^2\right) \\ + \frac{1}{3}b\theta\left(\mu_1^3 - \mu_2^3\right) - a\theta t\left(\mu_1 - \mu_2\right) + \rho p t\left(\mu_1 - \mu_2\right) - \frac{1}{2}b\theta t\left(\mu_1^2 - \mu_2^2\right) \end{array}\right] + S_1\left(1 + \theta(\mu_1 - \mu_2)\right) \tag{17}$$

$$I_0(t) = \left[\begin{array}{l} a\left(t_0 - \mu_2\right) - \rho p\left(t_0 - \mu_2\right) + \frac{1}{6}a\theta\left(t_0^3 - \mu_2^3\right) - \frac{1}{6}\rho p\theta\left(t_0^3 - \mu_2^3\right) + \frac{1}{2}b\left(t_0^2 - \mu_2^2\right) \\ + \frac{1}{8}b\theta\left(t_0^4 - \mu_2^4\right) - \frac{1}{2}a\theta\mu_2^2\left(t_0 - \mu_2\right) + \frac{1}{2}\rho p\theta\mu_2^2\left(t_0 - \mu_2\right) - \frac{1}{4}b\theta t^2\left(t_0^2 - \mu_2^2\right) \end{array}\right] \tag{18}$$

So from equations (17) and (18), we have

$$t_0 = \frac{1}{b\left(\theta\mu_2^2 - 2\right)}\left(\begin{array}{l} -a\theta\mu_2^2 + \rho p\theta\mu_2^2 + 2a - 2\rho p \\ + \sqrt{\begin{array}{l} -4b\rho p\theta\mu_2^2 + 4b\theta\mu_2^2\rho p t_r - 2a\theta^2\mu_2^4\rho p + 8a\theta\mu_2^2\rho p - 2b\theta^2\mu_2^2\rho p\mu_1^2 \\ - 4b\theta^2\mu_2^2 W\mu_1 + 4b\theta^2\mu_2^2\rho p t_r\mu_1 + 2ab\theta^2\mu_2^2\mu_1^2 + 2b\theta^2\mu_2^4\rho p \\ - 4ab\theta^2\mu_2^2 t_r\mu_1 + 4ab\theta^2\mu_2^3 t_r - 8bW\theta\mu_2 - 4ab\theta\mu_1^2 + 4ab\theta\mu_2^2 \\ - 8b\rho p t_r + 4b\theta\rho p\mu_1^2 - 8b\theta\mu_2\rho p t_r\mu_1 + 8ab\theta t_r\mu_1 + 8b\theta\rho p t_r\mu_2 \\ - 8ab\theta t_r\mu_2 + 4b\theta^2\mu_2^3 W - 2b^2\theta\mu_2^2 t_r^2 - 4\rho^2 p^2\theta\mu_2^2 + a\rho^2 p^2\theta^2\mu_2^4 \\ + 8bW\theta\mu_1 - 4b\theta\mu_2^2 W - 4b\theta^2\mu_2^3\rho p t_r - 4ab\theta\mu_2^2 t_r + a^2\theta^2\mu_2^4 \\ - 4a^2\theta\mu_2^2 - 8a\rho p + b^2\theta^2\mu_2^6 + 8abt_r - 2b^2\theta\mu_2^4 + 4a^2 + 4\rho^2 p^2 \\ + 4b^2 t_r^2 + 8bW \end{array}} \end{array}\right) \tag{19}$$

From equation (19), we see that t_0 is a function of W and t_r, so t_0 is not a decision variable.
Putting t=T in equation (12), we have

$$Q_2 = \left[a\left(T - t_0\right) - \rho p\left(T - t_0\right) + \frac{1}{2}b\left(T^2 - t_0^2\right)\right]. \tag{20}$$

Based on the assumptions and descriptions of the model, the total annual relevant profit(π), include the following elements:

(i) Ordering cost (OC) = A (21)

$$\text{(ii) } HC\left(OW\right) = \int_0^{t_r}\left(x_1 + y_1 t\right)e^{-Rt}I_0(t)\,dt + \int_{t_r}^{\mu_1}\left(x_1 + y_1 t\right)e^{-Rt}I_0(t)\,dt + \int_{\mu_1}^{\mu_2}\left(x_1 + y_1 t\right)e^{-Rt}I_0(t)\,dt + \int_{\mu_2}^{t_0}\left(x_1 + y_1 t\right)e^{-Rt}I_0(t)dt \tag{22}$$

$$\text{(iii) } HC\left(RW\right) = \int_0^{t_r}\left(x_2 + y_2 t\right)e^{-Rt}I_r(t)\,dt \tag{23}$$

$$\text{(iv) } DC = c\left(\int_{\mu_1}^{\mu_2}\theta e^{-Rt}I_0(t)\,dt + \int_{\mu_2}^{t_0}\theta t e^{-Rt}I_0(t)\,dt\right) \tag{24}$$

$$\text{(v) } SC = -c_2\left(\int_{t_0}^{T}e^{-Rt}I_0(t)\,dt\right) \tag{25}$$

$$\text{(vi) } SR = p\left(\int_0^{T}\left(a + bt - \rho p\right)e^{-Rt}dt\right) \tag{26}$$

(by neglecting higher powers of θ)

To determine the interest earned, there will be two cases i.e.
Case I: ($0 \le M \le t_0$) and Case II: ($M > t_0$).

Case I: ($0 \le M \le t_0$): In this case the retailer can earn interest on revenue generated from the sales up to M. Although, he has to settle the accounts at M, for that he has to arrange money at some specified rate of interest in order to get his remaining stocks financed for the period M to t_0. So

(vii) Interest earned per cycle:

$$IE_1 = pI_e \int_0^M (a + bt - \rho p)\, te^{-Rt} dt \tag{27}$$

Case II: ($M > t_0$):

In this case, the retailer earns interest on the sales revenue up to the permissible delay period. So

(viii) Interest earned up to the permissible delay period is:

$$IE_2 = p\, I_e \left[\int_0^{t_0} (a + bt - \rho p)\, t\, e^{-Rt} dt + (a + bt_0 - \rho p)\, t_0 (M - t_0) \right] \tag{28}$$

To determine the interest payable, there will be five cases i.e.

Interest payable per cycle for the inventory not sold after the due period M is

Case I: ($0 \le M \le t_r$):

$$\text{(ix) } IP_1 = cI_p \int_M^{t_0} I(t)e^{-Rt}dt = cI_p \left(\int_M^{t_r} I_r(t)e^{-Rt}dt + \int_M^{t_r} I_0(t)e^{-Rt}dt + \int_{t_r}^{\mu_1} I_0(t)e^{-Rt}dt + \int_{\mu_1}^{\mu_2} I_0(t)e^{-Rt}dt + \int_{\mu_2}^{t_0} I_0(t)e^{-Rt}dt \right) \tag{29}$$

Case II: ($t_r \le M \le \mu_1$):

$$\text{(x) } IP_2 = cI_p \int_M^{t_0} I(t)e^{-Rt}dt = cI_p \left(\int_M^{\mu_1} I_0(t)e^{-Rt}dt + \int_{\mu_1}^{\mu_2} I_0(t)e^{-Rt}dt + \int_{\mu_2}^{t_0} I_0(t)e^{-Rt}dt \right) \tag{30}$$

Case III: ($\mu_1 \le M \le \mu_2$):

$$\text{(xi) } IP_3 = cI_p \int_M^{t_0} I(t)e^{-Rt}dt = cI_p \left(\int_M^{\mu_2} I_0(t)e^{-Rt}dt + \int_{\mu_2}^{t_0} I_0(t)e^{-Rt}dt \right) \tag{31}$$

Case IV: ($\mu_2 \le M \le t_0$):

$$\text{(xii) } IP_4 = cI_p \int_M^{t_0} I(t)e^{-Rt}dt \tag{32}$$

Case V: ($M > t_0$):

$$\text{(xiii) } IP_5 = 0 \tag{33}$$

(by neglecting higher powers of b and R)

The total profit (π_i), i=1,2,3,4 and 5 during a cycle consisted of the following:

$$\pi_i = \frac{1}{T}\left[SR - OC - HC(RW) - HC(OW) - DC - SC - IP_i + IE_i\right] \tag{34}$$

Substituting values from equations (21) to (33) in equation (34), we get total profit per unit. Putting $\mu_1 = v_1 t_0$, $\mu_2 = v_2 t_0$ and value of S_1 and t_0 from equation (16) and (19) in equation (34), we get profit in terms of tr, T and p for the five cases as under:

$$\pi_1 = \frac{1}{T}\left[SR - OC - HC(RW) - HC(OW) - DC - SC - IP_1 + IE_1\right] \tag{35}$$

$$\pi_2 = \frac{1}{T}\left[SR - OC - HC(RW) - HC(OW) - DC - SC - IP_2 + IE_1\right] \tag{36}$$

$$\pi_3 = \frac{1}{T}\left[SR - OC - HC(RW) - HC(OW) - DC - SC - IP_3 + IE_1\right] \tag{37}$$

$$\pi_4 = \frac{1}{T}\left[SR - OC - HC(RW) - HC(OW) - DC - SC - IP_4 + IE_1\right] \tag{38}$$

$$\pi_5 = \frac{1}{T}\left[SR - OC - HC(RW) - HC(OW) - DC - SC - IP_5 + IE_2\right] \tag{39}$$

The optimal value of t_r*, T*, and p* (say), which maximizes π_i can be obtained by solving equation (35), (36), (37), (38) and (39) by differentiating it with respect to t_r, T and p and equate it to zero

$$\text{i.e. } \frac{\partial \pi_i(t_r,T,p)}{\partial t_r} = 0,\ \frac{\partial \pi_i(t_r,T,p)}{\partial T} = 0,\ \frac{\partial \pi_i(t_r,T,p)}{\partial p} = 0, \quad i=1,2,3,4,5 \tag{40}$$

provided it satisfies the condition

$$\begin{vmatrix} \dfrac{\partial\pi_i^2(t_r,T,p)}{\partial^2 t_r} & \dfrac{\partial\pi_i^2(t_r,T,p)}{\partial t_r \partial T} & \dfrac{\partial\pi_i^2(t_r,T,p)}{\partial t_r \partial p} \\ \dfrac{\partial\pi_i^2(t_r,T,p)}{\partial T \partial t_r} & \dfrac{\partial\pi_i^2(t_r,T,p)}{\partial^2 T} & \dfrac{\partial\pi_i^2(t_r,T,p)}{\partial T \partial p} \\ \dfrac{\partial\pi_i^2(t_r,T,p)}{\partial p \partial t_r} & \dfrac{\partial\pi_i^2(t_r,T,p)}{\partial p \partial T} & \dfrac{\partial\pi_i^2(t_r,T,p)}{\partial^2 p} \end{vmatrix} > 0 \quad i=1,2,3,4,5. \tag{41}$$

IV. Numerical Example

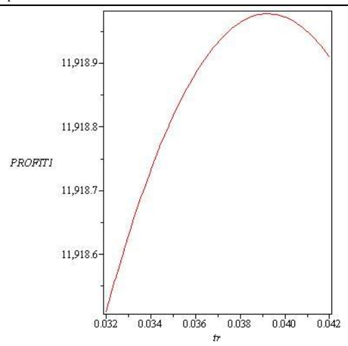
Considering A= Rs.100, W = 40, a = 500, b=0.05, c=Rs. 25, ρ= 5, c_2=Rs. 12, θ=0.05, x_1 = Rs. 2, y_1=0.04, x_2=Rs. 6, y_2=0.08, v_1=0.30, v_2=0.50, R = 0.06, Ie = 0.12, Ip = 0.15 in appropriate units. The optimal values of t_r, T, p, Profit and Q for the five cases are shown in table below.

Case	M	t_r	T	p	Profit	Q
I	0.02	0.0392	0.3175	50.3831	11918.9776	78.8678
II	0.05	0.0429	0.3149	50.3541	11938.6837	78.2721
III	0.08	0.0461	0.3114	50.3282	11960.4753	77.4467
IV	0.15	0.0514	0.2993	50.2815	12019.7690	74.5176
V	0.24	0.0562	0.2769	50.2588	12115.3459	68.9844

The second order conditions given in equation (41) are also satisfied. The graphical representation of the concavity of the profit function is also given.

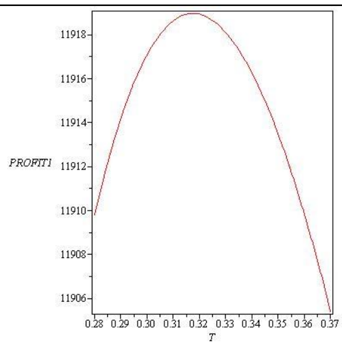
Case I

t_r and Profit



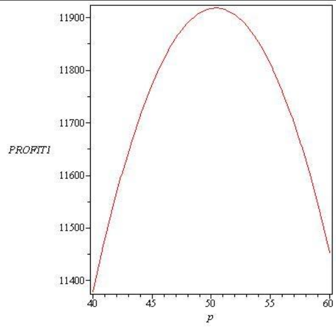
Graph 1

T and Profit



Graph 2

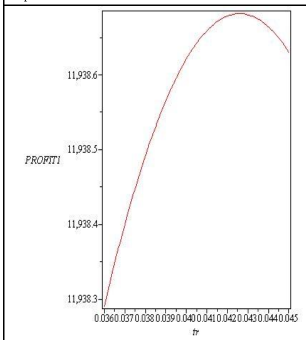
p and Profit



Graph 3

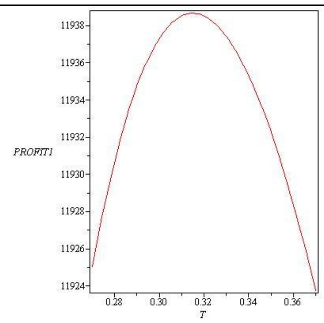
Case II

t_r and Profit



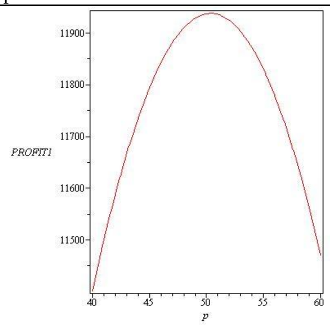
Graph 4

T and Profit

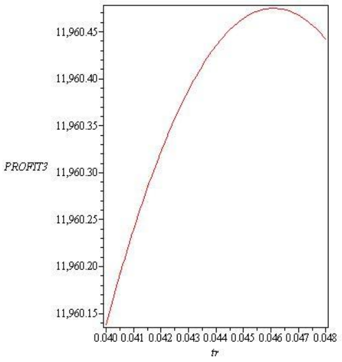
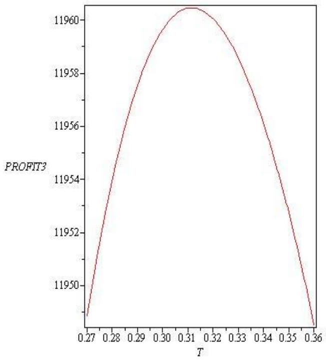
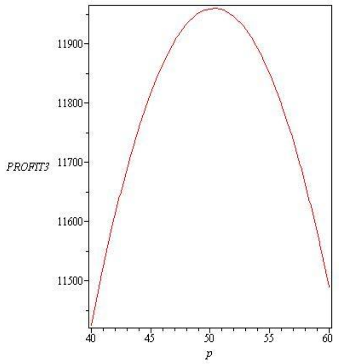


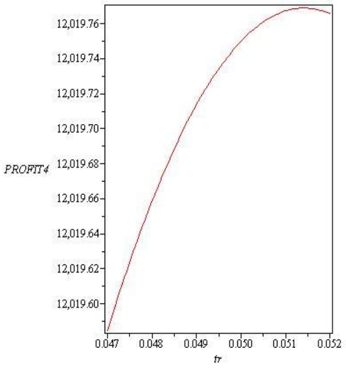
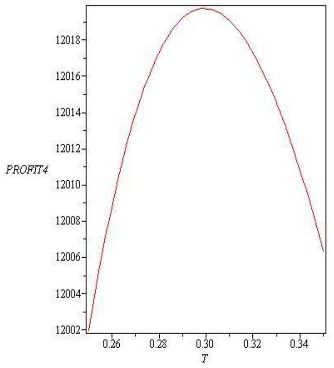
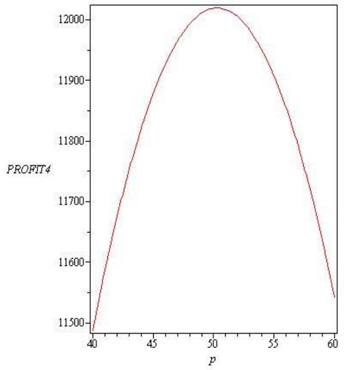
Graph 5

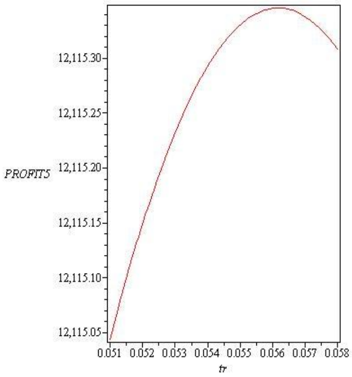
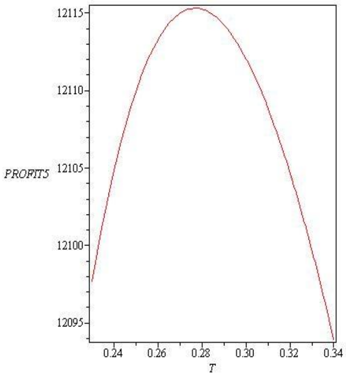
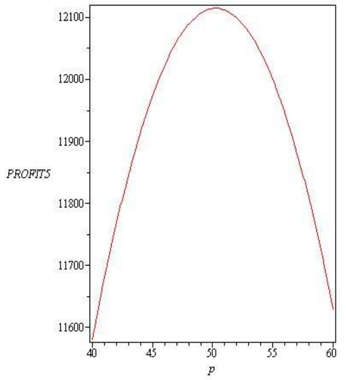
p and Profit



Graph 6

Case III		
t_r and Profit	T and Profit	p and Profit
		
Graph 7	Graph 8	Graph 9

Case IV		
t_r and Profit	T and Profit	p and Profit
		
Graph 10	Graph 11	Graph 12

Case V		
t_r and Profit	T and Profit	p and Profit
		
Graph 13	Graph 14	Graph 15

V. Sensitivity Analysis:

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1 Case I Sensitivity Analysis

Para-meter	% change	t_r	T	p	Profit	Q
a	+20	0.0418	0.2790	60.3329	17335.2413	83.3289
	+10	0.0409	0.2968	55.3563	14501.8022	81.1886
	-10	0.0366	0.3417	45.4141	9586.8840	76.2806
	-20	0.0326	0.3705	40.4507	7505.6779	73.3760
θ	+20	0.0384	0.3171	50.3841	11917.8690	78.7859
	+10	0.0388	0.3173	50.3836	11918.4222	78.8269
	-10	0.0396	0.3177	50.3825	11919.5351	78.9088
	-20	0.0400	0.3179	50.3820	11920.0948	78.9496
x_1	+20	0.0355	0.3161	50.3900	11913.2639	78.5060
	+10	0.0375	0.3168	50.3865	11916.0985	78.6871
	-10	0.0411	0.3182	50.3797	11921.9015	79.0488
	-20	0.0430	0.3189	50.3765	11924.9709	79.2294
x_2	+20	0.0362	0.3152	50.3902	11918.3103	78.2831
	+10	0.0376	0.3163	50.3868	11918.6309	78.5627
	-10	0.0409	0.3188	50.3790	11919.3536	79.1986
	-20	0.0429	0.3202	50.3747	11919.7628	79.5548
A	+20	0.0536	0.3456	50.4259	11858.6606	85.7802
	+10	0.0465	0.3319	50.4050	11888.1793	82.4115
	-10	0.0316	0.3025	50.3600	11951.2363	75.1739
	-20	0.0235	0.2867	50.3357	11985.1832	71.2794
M	+20	0.00397	0.3172	50.3790	11921.4860	78.8004
	+10	0.0395	0.3173	50.3810	11920.2272	78.8218
	-10	0.0390	0.3176	50.3851	11917.7371	78.8893
	-20	0.0387	0.3178	50.3871	11916.5056	78.9354
R	+20	0.0365	0.3095	50.3616	11894.9245	76.7136
	+10	0.0378	0.3134	50.3721	11906.8899	77.8665
	-10	0.0407	0.3218	50.3942	11931.1940	79.9183
	-20	0.0424	0.3264	50.4058	11943.5464	81.0421
ρ	+20	0.0441	0.3274	42.0646	9855.7229	81.1745
	+10	0.0418	0.3228	45.8455	10793.4804	80.1096
	-10	0.0362	0.3114	55.9294	13294.8436	77.4241
	-20	0.0327	0.3043	62.8630	15015.0444	75.7276
c_2	+20	0.0455	0.3092	50.4039	11907.6999	76.7828
	+10	0.0426	0.3131	50.3941	11913.0013	77.7624
	-10	0.0354	0.3227	50.3704	11925.7685	80.1747
	-20	0.0309	0.3287	50.3559	11933.5560	81.7194

Table 2 Case II Sensitivity analysis

Para-meter	% change	t_r	T	p	Profit	Q
a	+20	0.0453	0.2780	60.3039	17360.6160	82.4776
	+10	0.0444	0.2941	55.3273	14524.2310	80.4967
	-10	0.0404	0.3393	45.3851	9604.0704	75.7983
	-20	0.0364	0.3683	40.4215	7520.5266	72.9981
θ	+20	0.0421	0.3145	50.3552	11937.5281	78.1907
	+10	0.0425	0.3147	50.3546	11938.1048	78.2315
	-10	0.0423	0.3151	50.3535	11939.2648	78.3127
	-20	0.0437	0.3153	50.3530	11939.8482	78.3532
x_1	+20	0.0392	0.3136	50.3612	11932.7236	77.9346
	+10	0.0410	0.3143	50.3576	11935.6811	78.1157
	-10	0.0448	0.3156	50.3507	11941.7321	78.4531
	-20	0.0467	0.3162	50.3474	11944.8266	78.6092
x_2	+20	0.0396	0.3125	50.3622	11937.8779	77.6604
	+10	0.0412	0.3137	50.3583	11938.2652	77.9659
	-10	0.048	0.3163	50.3495	11939.1373	78.6288
	-20	0.0469	0.3178	50.3446	11939.6305	79.0111
A	+20	0.0573	0.3433	50.3969	11877.9135	85.2638
	+10	0.0503	0.3294	50.3760	11907.6449	81.8431
	-10	0.0352	0.2998	50.3311	11971.2177	74.5505
	-20	0.0270	0.2838	50.3067	12005.4844	70.6036
M	+20	0.0440	0.3139	50.3451	11945.7141	78.0390
	+10	0.0435	0.3144	50.3495	11942.1198	78.1558
	-10	0.0423	0.3154	50.3587	11935.2556	78.3883
	-20	0.0417	0.3154	50.3634	11931.8852	78.5044
R	+20	0.0402	0.3069	50.3324	11914.8908	76.3167
	+10	0.0415	0.3108	50.3431	11926.7261	77.2700
	-10	0.0444	0.3192	50.3653	11950.7701	79.3232

Para-meter	% change	t_r	T	p	Profit	Q
	-20	0.0460	0.3238	50.3769	11962.9923	80.4477
ρ	+20	0.0479	0.3251	42.0360	9874.4372	80.6645
	+10	0.0456	0.3204	45.8168	10812.6410	79.5691
	-10	0.0398	0.3086	55.9002	13315.2318	76.7728
	-20	0.0361	0.3013	62.8336	15036.3093	75.0205
c_2	+20	0.0488	0.3069	50.3726	11928.4774	76.2637
	+10	0.0460	0.3106	50.3639	11933.2722	77.1925
	-10	0.0393	0.3200	50.3428	11944.8412	79.5526
	-20	0.0352	0.3259	50.3297	11951.9130	81.0349

Table 3 Case III Sensitivity analysis

Para-meter	% change	t_r	T	p	Profit	Q
a	+20	0.0479	0.2715	60.2780	17390.0611	81.1722
	+10	0.0474	0.2901	55.3015	14549.6359	79.4437
	-10	0.0438	0.3362	45.3592	9622.6344	75.1536
	-20	0.0401	0.365	40.3955	7536.2082	72.4949
θ	+20	0.0453	0.3110	50.3295	11959.2735	77.3656
	+10	0.0457	0.3112	50.3289	11959.8732	77.4061
	-10	0.0465	0.3116	50.3276	11961.0796	77.4871
	-20	0.0469	0.3118	50.3270	11961.6862	77.5270
x_1	+20	0.0423	0.3101	50.3356	11954.2718	77.1086
	+10	0.0443	0.3108	50.3319	11957.3505	77.2901
	-10	0.0480	0.3120	50.3247	11963.6467	77.3062
	-20	0.0499	0.3127	50.3213	11966.8651	77.7843
x_2	+20	0.0425	0.3088	50.3373	11959.5342	76.7833
	+10	0.0442	0.3101	50.3330	11959.9867	77.1145
	-10	0.0481	0.3129	50.3231	11961.0044	77.8293
	-20	0.0503	0.3144	50.3176	11961.5794	78.2128
A	+20	0.0607	0.3400	50.3707	11899.0726	84.4935
	+10	0.0535	0.3260	50.3500	11929.1003	81.0452
	-10	0.0383	0.2961	50.3054	11993.3954	73.6726
	-20	0.0300	0.2800	50.2813	12028.1122	69.6977
M	+20	0.0476	0.3091	50.3158	11972.9653	76.8962
	+10	0.0468	0.3103	50.3219	11966.6438	77.1840
	-10	0.0453	0.3124	50.3348	11954.4583	77.6840
	-20	0.0444	0.3134	50.3416	11948.5915	77.9208
R	+20	0.0435	0.3035	50.3067	11936.9893	75.5146
	+10	0.0447	0.3074	50.3173	11948.6715	76.4686
	-10	0.0476	0.3157	50.3395	11972.4071	78.4985
	-20	0.0492	0.3202	50.3510	11984.4740	79.5991
ρ	+20	0.0514	0.3222	42.0108	9894.4679	79.9982
	+10	0.0489	0.3172	45.7913	10833.4614	78.8233
	-10	0.0428	0.3047	55.8740	13338.2455	75.8426
	-20	0.0388	0.2969	62.8068	15060.9051	73.9608
c_2	+20	0.0515	0.3036	50.3447	11951.3482	75.7900
	+10	0.0489	0.3072	50.3370	11955.6330	76.3929
	-10	0.0428	0.3163	50.3182	11965.9930	78.6762
	-20	0.0390	0.3220	50.3065	11972.3405	80.1073

Table 4 Case IV Sensitivity analysis

Para-meter	% change	t_r	T	p	Profit	Q
a	+20	0.0509	0.2550	60.2321	17475.7707	76.3067
	+10	0.0515	0.2759	55.2552	14621.1496	75.6218
	-10	0.0502	0.3259	45.3119	9671.4498	72.9378
	-20	0.0473	0.3570	40.3475	7576.0725	70.9041
θ	+20	0.0506	0.2989	50.2830	12018.4522	74.4363
	+10	0.0510	0.2991	50.2822	12019.1094	74.4766
	-10	0.0518	0.2994	50.2807	12020.4311	74.5320
	-20	0.0522	0.2996	50.2800	12021.0956	74.5719
x_1	+20	0.0477	0.2982	50.2896	12013.0047	74.2273
	+10	0.0495	0.2987	50.2855	12016.3627	74.3596
	-10	0.0532	0.2998	50.2776	12023.2243	74.6497
	-20	0.0551	0.3003	50.2738	12026.7290	74.7808
	+20	0.0474	0.2964	50.2929	12018.5493	73.7747

Para-meter	% change	t_r	T	p	Profit	Q
x_2	+10	0.0493	0.2978	50.2874	12019.1361	74.1328
	-10	0.0536	0.3008	50.2751	12020.4537	74.9016
	-20	0.0561	0.3025	50.2682	12021.1967	75.3373
A	+20	0.0665	0.3289	50.3219	11956.0902	81.8254
	+10	0.0591	0.3144	50.3021	11987.1799	78.2465
	-10	0.0432	0.2833	50.2600	12054.0962	70.5610
	-20	0.0346	0.2665	50.2374	12090.4705	66.4043
M	+20	0.0527	0.2923	50.2681	12049.0707	72.7974
	+10	0.0521	0.2959	50.2742	12034.1114	73.6830
	-10	0.0505	0.3023	50.2898	12006.0242	75.2491
	-20	0.0495	0.3052	50.2991	11992.8599	75.9547
R	+20	0.0490	0.2917	50.2608	11997.1818	72.6550
	+10	0.0501	0.2954	50.2710	12008.4170	73.5613
	-10	0.0528	0.3033	50.2922	12031.2441	75.4969
	-20	0.0543	0.3073	50.3032	12042.8489	76.5499
ρ	+20	0.0582	0.3129	41.9665	9946.4439	77.7826
	+10	0.0550	0.3067	45.7458	10888.7036	76.3006
	-10	0.0471	0.2908	55.8257	13402.6878	72.4549
	-20	0.0420	0.2807	62.7567	15132.0894	74.3147
c_2	+20	0.0556	0.2923	50.2937	12013.1607	72.7631
	+10	0.0537	0.2955	50.2880	12016.2573	73.5648
	-10	0.0488	0.3036	50.2740	12023.7863	75.5944
	-20	0.0457	0.3087	50.2653	12028.4283	76.8727

Table 5 Case V Sensitivity analysis

Para-meter	% change	t_r	T	p	Profit	Q
a	+20	0.0587	0.2289	60.2258	17623.3266	45.6552
	+10	0.0576	0.2516	55.2402	14740.7490	56.4329
	-10	0.0543	0.3059	45.2819	9746.6420	83.8025
	-20	0.0517	0.3396	40.3105	7634.2950	101.4576
θ	+20	0.0555	0.2767	50.2607	12113.8513	68.9544
	+10	0.0558	0.2768	50.2598	12114.5975	68.9693
	-10	0.0565	0.2770	50.2578	12116.0967	68.9993
	-20	0.0568	0.2771	50.2568	12116.8497	69.0141
x_1	+20	0.0532	0.2766	50.2683	12107.7171	68.8935
	+10	0.0547	0.2768	50.2635	12111.5083	68.9514
	-10	0.0577	0.2771	50.2541	12119.2306	69.0422
	-20	0.0592	0.2772	50.2494	12123.1629	69.0752
x_2	+20	0.0525	0.2745	50.2743	12113.7509	68.3623
	+10	0.0543	0.2757	50.2668	12114.5232	68.6729
	-10	0.0582	0.2783	50.2502	12116.2242	69.3467
	-20	0.0604	0.2798	50.2411	12117.1639	69.7349
A	+20	0.0699	0.3067	50.2894	12046.8081	76.3645
	+10	0.0632	0.2922	50.2743	12080.2052	72.7748
	-10	0.487	0.2608	50.2428	12152.5337	64.9930
	-20	0.0407	0.2437	50.2264	12192.1788	60.7504
M	+20	0.0626	0.2662	50.2660	12173.6049	66.3205
	+10	0.0595	0.2718	50.2617	12143.9755	67.7155
	-10	0.0527	0.2817	50.2571	12087.6613	70.1769
	-20	0.0490	0.2861	50.2567	12060.8747	71.2681
R	+20	0.0542	0.2702	50.2405	12094.2985	67.3405
	+10	0.0552	0.2735	50.2496	12104.7685	68.1502
	-10	0.0573	0.2805	50.2681	12126.0356	69.8679
	-20	0.0584	0.2844	50.2776	12136.8436	70.8254
ρ	+20	0.0641	0.2954	41.9435	10025.0771	73.4860
	+10	0.0603	0.2868	45.7228	10974.8684	71.3992
	-10	0.0516	0.2656	55.8041	13510.3145	66.2156
	-20	0.0466	0.2522	62.7377	15255.6208	62.9175
c_2	+20	0.0584	0.2713	50.2665	12111.9470	67.5835
	+10	0.0573	0.2739	50.2628	12113.5360	68.2339
	-10	0.0548	0.2804	50.2540	12117.4265	69.8601
	-20	0.0532	0.2847	50.2483	12119.8442	70.9362

From the table we observe that as parameter a increases/ decreases average total profit and order quantity increases/ decreases for all five cases.
From the table we observe that as parameter θ increases/ decreases there is very minor change in average total profit and order quantity for all five cases.
From the table we observe that as parameter x_1, x_2, R and c_2 increases/ decreases average total profit and order quantity decreases/ increases for all five cases.
From the table we observe that as parameters A and ρ increases/ decreases average total profit decreases/ increases and order quantity increases/ decreases for all five cases.
From the table we observe that as parameter M increases/ decreases average total profit also increases/ decreases for all five cases but for order quantity almost remains fixed for all five cases.

VI. Conclusion

In this paper, we have developed a two warehouse inventory model for deteriorating items with different deterioration rates under shortages, time and price dependent demand under inflationary conditions. Sensitivity with respect to parameters has been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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