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β-Zero Sets and Their Properties in Topology

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ABSTRACT: *In 1990 , Malghan et al. have defined and studied the concepts of almost p-regular, p-completely regular and almost p-completely regular spaces. In 1997 & 2004 ,Malghan et al. have defined and studied the concepts of almost s-completely regular spaces and s-completely regular spaces . In 2010, Navalagi introduced the concepts of pre-zero sets and co-pre-zero sets to characterize the concepts of p-completely regular spaces and almost p-completely regular spaces. Recently, Navalagi introduced and studied the concepts of α-zero sets , co-α-zero sets , α-completely regular spaces and almost α-completely regular spaces. In this paper, we offer some new concepts of β-zero sets , co-β-zero sets , β-completely regular spaces and almost β-completely regular spaces . We also characterize their basic properties via β-zero sets .*
KEYWORDS: β-open sets , α-continuity , pre-continuity, semicontinuity, β-continuity,zero sets , pre-zero sets , semi-zero sets, α-zero sets , β-zero sets ,s-completely regularity and p-completely regularity.

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I. INTRODUCTION

In the literature zero sets and co-zero sets due to Gilman and Jerison [13] were used to characterize the concepts like completely regular spaces and almost completely regular spaces by Singal , Arya and Mathur in topology , See [28 & 29] . In [17] and [18] , Malghan et al have defined and studied the concepts of semi-zero sets and co-semi-zero sets in topology to characterize the properties of s-completely regular spaces and almost s-completely regular spaces using semicontinuous functions due to N.Levine [16] . In [23] , Navalagi has defined and studied the concepts pre-zero sets and co-pre zero sets in topology by using precontinuous functions due to Mashhour et al [20] to characterize the properties of p-completely regular spaces and almost p-completely regular spaces due to Malghan et al [19]. Recently in [24] , Navalagi introduced and studied the concepts of α-zero sets , co-α-zero sets , α-completely regular spaces and almost α-completely regular spaces. In this paper, we offer some new concepts of β-zero sets , co-β-zero sets , β-completely regular spaces and almost β-completely regular spaces . We also characterize their basic properties via β-zero sets .

II. PRELIMINARIES

Throughout this paper , we let (X , τ) and (Y,σ) be topological spaces (or simply X and Y be spaces) on which no separation axioms are assumed unless explicitly stated.Let A be a subset of a space X . Let Cl(A) and Int(A) denote the closure and the interior of subset A.

We need the following definitions and results in the sequel of the paper.

DEFINITION 2.1: A subset A of a space X is said to be :
(i) preopen [20] if $A \subset$ Int Cl(A) .
(ii) semiopen [16] if $A \subset$ Cl Int(A) .
(iii) regular open [30] if A = Int Cl(A) .
(iv) α-open [25] if $A \subset$ Int Cl Int(A).
(v) β-open[1] if $A \subset$ Cl Int Cl(A).
(vi) δ-open set [31] if for each $x \in A$,there exists a regular open set G such that $x \in G \subset A$.

The complement of a preopen (resp. semiopen , regular open, α-open , β-open ,δ-open) set of a space X is called preclosed [10] (resp. semiclosed [7], regular closed[30],α-closed[21] ,β-closed [2] , δ-closed [31]) set. The family of all preopen (resp. semiopen , regular open , α-open , β-open and δ-open) sets of X is denoted by PO(X) (resp. SO(X), RO(X) , αO(X) , βO(X) and δO(X)) and that of preclosed (resp. semiclosed , regular closed, α-closed , β-closed and δ-closed) sets of X is denoted by PF(X) (resp. SF(X) , RF(X) , αF(X) ,βF(X) and δF(X))

DEFINITION 2.2 : A function f :X →Y is called :

(i) precontinuous [20] if the inverse image of each open set U of Y is preopen set in X.
(ii) semicontinuous [16] if the inverse image of each open set U of Y is semiopen set in X.
(iii) α-continuous [21] if the inverse image of each open set U of Y is α-open set in X.
(iv) β-continuous [1] if the inverse image of each open set U of Y is β-open set in X.

DEFINITION 2.3 : A space X is said to be :

(i) p-completely regular [19] (resp. s-completely regular[17] , α-completely regular [24]) if for each closed set F and each point x ∈ (X \ F) , there exists a precontinuous (resp.a semicontinuous , an α-continuous) function f : X→ [0,1] such that f(x) = 0 and f(y) = 1 for each y∈ F .
(ii) almost p-completely regular [19] (resp. almost s-completely regular[18] , almost α-completely regular [24]) if for each regular closed set F and each point x ∈ (X \ F) , there exists a precontinuous(resp. a semicontinuous ,an α-continuous) function f : X→[0,1] such that f(x) = 0 and f(y) = 1 for each y∈F.
(iii) submaximal [8] if every dense subset of it is open (i.e. if PO(X) = τ [12]).
(iv) an extremally disconnected (E.D.) [32] if closure of each open set is open in it (i.e. if A ∈ τ for each A ∈ RF(X) [15]).
(v) PS-space [5]if every preopen set of X is semiopen in X.
(vi) α-space [11] if every α-open set of X is open in X(i.e. τ = αO(X)).
(vii) β-regular space [4 &26] if each pair of a point and a closed set not containing the point can be separated by disjoint β-open sets.

It is well-known that a subset A of a space X is called a zero set [13] if there exists a continuous functions f : X → **R** such that A = { x∈ X | f(x) = 0} . The complement of a zero set of a space X is called a co-zero set of X.

REMARK 2.4 : If f : X→**R** is continuous function may be denoted by Z(f). Thus, we write Z(f) = {x∈ X | f(x) = 0 } . Thus , Z(f) is a zero set of X . Therefore, it is clear that if A is a zero set in X then it can be expressed as A = Z(f) , where f is continuous function.

DEFINITION 2.5 : A subset A of a space X is said to be semi-zero set [17] of X if there exists a semicontinuous function f :X → **R** such that A = { x∈ X | f(x) = 0}.

DEFINITION 2.6 : A subset A of a space X is said to be co-semizero set [17] of X if its complement is a semi-zero set.

REMARK 2.7: If f : X→**R** is semicontinuous function may be denoted by SZ(f). Thus, we write SZ(f) = {x∈ X | f(x) = 0 } . Thus , SZ(f) is a semi-zero set of X . Therefore, it is clear that if A is a semi-zero set in X then it can be expressed as A = SZ(f) , where f is semicontinuous function.

DEFINITION 2.8 : A subset A of a space X is said to be pre-zero set [23] of X if there exists a precontinuous function f :X → **R** such that A = { x∈ X | f(x) = 0}.

DEFINITION 2.9 : A subset A of a space X is said to be co-prezero set [23] of X if its complement is a pre-zero set.

REMARK 2.10: If f : X→**R** is precontinuous function may be denoted by PZ(f). Thus, we write PZ(f) = {x∈ X | f(x) = 0 } . Thus , PZ(f) is a pre-zero set of X . Therefore, it is clear that if A is a pre-zero set in X then it can be expressed as A = PZ(f) , where f is precontinuous function.

DEFINITION 2.11: A subset A of a space X is said to be α-zero set [24] of X if there exists a α-continuous function f :X → **R** such that A = { x∈ X | f(x) = 0}.

DEFINITION 2.12: A subset A of a space X is said to be co-α-zero set [24] of X if its complement is a α-zero set.

REMARK 2.13 : If f : X→**R** is α-continuous function may be denoted by αZ(f). Thus, we write αZ(f) = {x∈ X | f(x) = 0 } . Thus , αZ(f) is a α-zero set of X . Therefore, it is clear that if A is a α-zero set in X then it can be expressed as A = αZ(f) , where f is α-continuous function.

RESULT 2.14 [1]: Let X be a space and A and U be subsets of X,If U∈αO(X) and A∈βO(X) then A∩U ∈βO(U).

III. β-ZERO SETS

We define the following.

DEFINITION 3.1 : A subset A of a space X is said to be β-zero set of X , if there exists a β-continuous function f : X → **R** such that A = { x ∈ X | f(x) = 0} .

A subset A of a space X is said to be co-β-zero set of X if its complement is β-zero set.

NOTE 3.2 : Every zero set in X is a β-zero set in X.

REMARK 3.3: Let X be a space . If f : X → **R** is a β-continuous function then the set {x ∈ X | f(x) = 0 } is a β-zero set. If g : X → **R** is also a β-continuous function then { x ∈ X | g(x) = 0 } is also a β-zero set of X.

REMARK 3.4 : If f : X→**R** is β-continuous function may be denoted by βZ(f). Thus, we write βZ(f) = {x∈ X | f(x) = 0 } . Thus , βZ(f) is a β-zero set of X . Therefore, it is clear that if A is a β-zero set in X then it can be expressed as A = βZ(f) , where f is β-continuous function.

We recall the following.

LEMMA 3.5 [3]: If a space X be an E.D., and submaximal space , then τ = βO(X).

LEMMA 3.6 : If X is a E.D and submaximal space then a function f : X→ Y is β-continuous then the inverse image of each member of a basis for Y is β-open set in X.

LEMMA 3.7 : Let X be a E.D and submaximal space . A function f : X → **R** is β-continuous iff for each b∈**R** both the sets $f^{-1}(b , \infty)$ and $f^{-1}(- \infty , b)$ are β-open sets.

LEMMA 3.8 : Let X be an E.D. and submaximal space then the following are equivalent :

(i) f : X→ **R** is β-continuous.
(ii) For each b ∈ **R** , $f^{-1}(-\infty , b)$ and $(-f)^{-1}(-\infty ,-b)$ are β-open sets in X.
(iii) For each b ∈ **R** , $f^{-1}(b,\infty)$ and $(-f)^{-1}(-b , \infty)$ are β-open sets in X.

PROOF : Since (b, ∞) and (-∞ , b) are subbasic open sets for the usual topology on R, thus the proof follows from Lemma –3.6 above.

We need the following.

LEMMA 3.9 : Let X be an E.D. and submaximal space .Let f , g : X→ **R** are β-continuous then,

(i) $|f|^{\alpha}$ is β-continuous for each α ≥ 0 .
(ii) (af + bg) is β-continuous for each pair of reals a and b .
(iii) f . g is β-continuous.
(iv) 1/ f is β-continuous whenever f ≠ 0 on X.

These results can be proved by using the proofs of Lemmas : 2.5 , 2.6 and 2.7 .
See [9 , p.84].

LEMMA 3.10 : If X is an E.D. and submaximal space and if $\{ f_i : X \to \mathbf{R} \}_{i=1}^{k}$ is a finite family of β-continuous functions, then the functions M , m : X→ **R** defined by M(x) =

Max $\{f_i(x)\}^k{}_{i=1}$ and m(x) = Min $\{f_i(x)\}_{i=1}^{k}$ are also β-continuous .

Proof is straight forward and hence omitted.

LEMMA 3.11 : In an E.D. and submaximal space X, the following statements hold for real valued functions :

(i) If A is a β-zero set in X then there exists a β-continuous function g : X→ **R** such that g(x) ≥ 0 for each x ∈ X and A = βZ(g) .
(ii) If A is a β-zero set in X then there is a β-continuous function h : X→ [0,1] such that A = βZ(h).
(iii) Finite union of β-zero sets in X is a β-zero set in X.
(iv) Finite intersection of β-zero sets in X is a β-zero set in X.
(v) If a ∈ R and f :X→ **R** is a β-continuous function then the sets A = {x∈X | f(x) ≥ a} and B = { x ∈ X | f(x) ≤ a} are β-zero sets in X.
(vi) If a ∈ R and f :X→ **R** is a β-continuous function then the sets A = { x ∈ X | f(x) < a } and B = {x ∈ X | f(x) > a} are co-β-zero sets in X.

These results can be proved by using Lemma- 2.8 and 2.9 . See [27 , p. 18].

Next , we give the following.

THEOREM 3.12 : If A and B are disjoint β-zero sets of an E.D. and submaximal space X , there exist disjoint co-β-zero sets U and V such that A ⊂ U and B ⊂ V .

We , prove the following.

THEOREM 3.13 : In an E.D. and submaximal space X every β-zero (resp. co-β-zero) set is β-closed (resp. β-open) set.

PROOF : If A is β-zero set in X then by Lemma -3.11 , we have A = βZ(g) , where g : X → **R** is β-continuous and g(x) ≥ 0 for all x ∈ X . Then , g(x) = 0 for all x ∈ A. Hence , $g^{-1}(\{0\})$ = A . Since {0} is closed in **R** and g is β-continuous , it follows that A is β-closed set in X. The second part is proved similarly.

Next , we give implications of these allied zero sets in the following and so we need following results :

REMARK 3.14: It is known that , τ ⊂αO(X) ⊂SO(X)⊂βO(X) and τ⊂αO(X) ⊂PO(X)⊂βO(X).

THEOREM 3.15 [11]: In an α-space X , we have τ = αO(X).

THEOREM 3.16 [5]: For a space X the following conditions are equivalent :
(i) X is an E.D. - space ,
(ii) SO(X) ⊂ PO(X),
(iii) SO(X) ⊂ αO(X),
(iv) βO(X) ⊂ PO(X).

THEOREM 3.17 [5]: A space X is PS-space if it satisfies the following equivalent conditions :
(i) PO(X) ⊂ SO(X) ,
(ii) βO(X) ⊂ SO(X) ,
(iii) PO(X) ⊂αO(X).

THEOREM 3.18 [14 & 22]]: In an E.D.-space and submaximal-space X , then
τ=αO(X) =SO(X) =PO(X)=βO(X).

IMPLICATION 3.23 : In view of Remark-2.4 ,2.7 , 2.10 , 2.13 , 3.4 and 3.14 , we have (i)every zero set →α-zero set →semi-zero set →β-zero set ,
(ii) every zero set →α-zero set →pre-zero set →β-zero set

IMPLICATION 3.24 : In view of Remark-2.4 ,2.13 and Theorem-3.15 , we have Z(f) = αZ(f).

IMPLICATION 3.25 : In view of Remark-2.7 , 2.10 ,2.13,3.4 and Theorem-3.16, we have (i)every semi-zero set → pre-zero set,
(i)every semi-zero set → α-zero set,
(i)every β-zero set → pre-zero set.

IMPLICATION 3.26 : In view of Remark-2.7 ,2.10 , 2.13 ,3.4 and Theorem-3.17 , we have
(i) every pre-zero set →semi-zero set,
(ii) every β-zero set →semi-zero set ,
(iii) every pre-zero set →α-zero set .

NOTE 3.37 : In view of In view of Remark-2.4 ,2.7 , 2.10 , 2.13 , 3.4 and Theorem-3.18 , we have Z(f) = αZ(f) = SZ(f) =PZ(f) = βZ(f).

IV. β-COMPLETE REGULARITY AND β-ZERO SETS

We , need the following.

DEFINITION 4.1 [1]: Let A be a subset of a space X. Then a subset V of a space X is said to be a β-neighbourhood of A if there exist a β-open set U of X such that A ⊂ U ⊂ V .

If A = {x} for some x ∈ X then V in the above definition is the neighbourhood of the point x .

We define the following.

DEFINITION 4.2 : A space X is said to be β-completely regular if for each closed set F and each point x ∈ (X \ F) , there exists a β-continuous function f : X→ [0,1] such that f(x) = 0 and f(y) = 1 for each y∈ F .

Clearly, every completely regular space is β-completely regular , every α-completely regular space (resp. s-completely regular space , p-completely regular space) is β-completely regular and every β-completely regular space is β-regular space.

Next , we prove the following.

THEOREM 4.3 : Every α-open subspace of an β-completely regular space is β-completely regular.

PROOF : Let X be an β-completely regular space and Y be an α-open subspace of X.Let F be a closed set in Y and x∈Y such that x∉F.Hence , x∉ $Cl_X(F)$.Since X is β-completely regular , there exists a β-continuous function f:X→[0,1] such that f(x) = 0 and f(y) = 1 for each y∈ $Cl_X(F)$.Since the restriction of a α-continuous function to a α-open subspace is β-continuous in view of **Result 2.14** , it follows that f/Y : Y → [0,1] is β-continuous such that (f/Y)(x) = 0 and (f/Y)(y) = 1 for each y∈F. Hence Y is β-completely regular .

Next , we prove the following.

THEOREM 4.4 : Every neighbourhood of a point in an E.D. and submaximal β-completely regular space X contains a β-zero set β-neighbourhood of the point.

PROOF : Let x_o be a point of an E.D. and submaximal β-completely regular space X and N be a neighbourhood of x_o . Then there exists a β-continuous function f : X→[0,1] such that f(x_o) = 0 and f(x) = 1 for each x∈ X\N. Then , V = { x ∈ X | f(x) ≥ ½} , then V is a β-zero set β-neighbourhood of x_o such that V ⊂ N , as x_o ∈ { x ∈ X | f(x) < ½} is β-open by above Lemma- 3.10 above.

Now , we need the following.

DEFINITION 4.5 [6]: A family σ of subsets of a space X is a net for X if each open set is the union of a family of elements of σ.

Now , we give the following.

THEOREM 4.6 : For an E.D and submaximal space X , the following statement are equivalent :
(i) X is β-completely regular space.
(ii) Every closed set A of X is the intersection of β-zero sets which are β-neighbourhoods of A.
(iii) The family of all co-β-zero sets of X is a net for the space X.

PROOF . (i)⇒(ii) : Let A be a closed set in X and $x \notin A$. Then from (i) , there is a β-continuous function f_x : $X \rightarrow [0,1]$ such that $f_x(x) = 0$ and $f_x(A) = \{1\}$. Let $G = \{ y \in X \mid f_x (y) \geq 1/3\}$ and $H_x = \{ y \in X \mid f_x(y) < 1/3\}$. Then, $A \subset H_x \subset G_x$, where H_x is β-open and G_x is β-zero set which is β-neighbourhood of A. Further , A = $\bigcap_{x \notin A} G_x$

(ii)⇒ (iii) : Let G be an open set of X. Then , X \ G is closed set in X . Let X \ G = $\bigcap\{B_\lambda \mid \lambda \in \wedge\}$, where B_λ is β-zero set β-neighbourhood of X \ G , for each $\lambda \in \wedge$. Hence , $G = \cup\{X \backslash B_\lambda \mid \lambda \in \wedge\}$, where $X \backslash B_\lambda$ is a co-β-zero set for each $\lambda \in \wedge$. Hence , (iii) holds.

(iii) ⇒ (i) : Let A be a closed set and $x_o \in X \backslash A$. Then , from (iii), as X \ A is open there is a co-β-zero set U such that $x_o \in U \subset X \backslash A$. Let U = X \ βZ(g) , for some β-continuous function $g : X \rightarrow [0,1]$. As $x_o \notin \beta Z(g)$, $| g(x) | = r > 0$. If we define ,
$f : X \rightarrow [0,1]$ by $f(x) = \text{Max}\{ 0, 1-r^{-1}|g(x)|\}$ for some $x \in X$, then f is β-continuous by Lemma -3.9 and 3.10 above and $f(x_o) = 0$ and $f(x) = 1$ for $x \in A$. Hence , X is β-completely regular space.

V. ALMOST β-COMPLETE REGULARITY AND β-ZERO SETS

In this section , we define and characterize the almost β-completely regular spaces using the concepts of β-zero sets and co-β-zero sets in the following .

DEFINITION 5.1 : A space X is said to be almost β-completely regular if for each regular closed set F and each point $x \in (X \backslash F)$, there exists a β-continuous function $f : X \rightarrow [0,1]$ such that $f(x) = 0$ and $f(y) = 1$ for each $y \in F$.

We, prove the following.

THEOREM 5.2: A space X is almost β-completely regular iff for each δ-closed set F and a point $x \in (X \backslash F)$, there is a β-continuous function $f : X \rightarrow [0,1]$ such that $f(x) = 0$ and $f(F) = \{1\}$.

PROOF : Let X be almost β-completely regular space and let A be a δ-closed set not containing a point x.Then there exists an open set G containing x such that $\text{Int Cl}(G) \cap A = \emptyset$.Now, (X-Int Cl(G)) is a regular closed set not containing x. Since X is almost β-completely regular , there exists a β-continuous function $f : X \rightarrow [0,1]$ such that $f(x) = 0$ and $f(X\text{-Int Cl}(G)) = \{1\}$.Since $A \subset (X\text{-Int Cl}(G))$, it follows that $f(A) = \{1\}$.

Converse follows immediately since every regular closed set is δ-closed.

THEOREM 5.3 : For an E.D. and submaximal space X the following are equivalent :

(i) X is almost β-completely regular space.
(ii) Every δ-closed subset A of X is expressible as the intersection of some β-zero sets which are β-neighbourhood of A.
(iii) Every δ-closed subset A of X is identical with the intersection of all β-zero sets which are β-neighbourhoods of A.

(iv) Every δ-open subset of X containing a point contains a co-β-zero set containing that point.

PROOF . (i) ⇒(ii) : Let X be an almost β-completely regular space. Let A be a δ-closed set and $x \notin A$. Then there exists a β-continuous function f_x on X into [0,1] such that $f_x(x)= 0$ and $f_x(A) = \{1\}$ by Theorem – 5.2. Let $G_x = \{y\in X \mid f_x(y) \geq 2/3\}$ for every $x\notin A$; G_x is β-neighbourhood of A . Lastly , $A = \cap_{x\notin A} G_x$: We have $A \subset G_x$, for each $x\notin A$, which implies that $A \subset \cap_{x\notin A} G_x$. Further , we claim that $\cap_{x\notin A} G_x \subset A$: Let $z \notin A$. This implies that there is a β-continuous function $f_z : X \to [0,1]$ such that $f_z(z) = 0$ and $f_z(A) = \{1\}$. Also, $G_z = \{y\in X \mid f_z(y) \geq 2/3\}$ .Now, $f_z(z) = 0 < 2/3$. Therefore , $z\notin G_z$. This implies that $z\notin \cap_{x\notin A} G_x$. Therefore, $z \notin A \Rightarrow z\notin \cap_{x\notin A} G_x$. Therefore, $\cap_{x\notin A} G_x \subset A$. Hence , $A = \cap_{x\notin A} G_x$. Therefore, (i) ⇒ (ii) is true.

(ii) ⇒ (iii) : Let us suppose that (ii) holds. Let $A = \cap \{G_\lambda \mid \lambda \in \wedge\}$, where G_λ is a β-zero set which is β-neighbourhood of A for each $\lambda\in \wedge$. Let ρ be the family of all β-zero sets which are β-neighbourhoods of A. Therefore, $\{ G_\lambda \mid \lambda\in\wedge\} \subset \rho$. Therefore, $\cap_{B\in\rho} B \subset \cap_{\lambda\in\wedge} G_\lambda \Rightarrow \cap_{B\in\rho} B \subset A$. Next , we prove that $A \subset \cap_{B\in\rho} B$: Now , B is a β-zero set which is β-neighbourhood of A for each $B\in \rho$ which implies that $A \subset \cap_{B\in\rho} B$. Therefore, $A = \cap_{B\in\rho} B$. Thus, (iii) holds.

(iii) ⇒ (iv) : Suppose (iii) holds.Let G be a δ-open set and $x\in G$. Then, $X \setminus G$ is δ-closed set and $x\notin X \setminus G$. This implies that $X\setminus G = \cap_{\lambda\in\wedge} B_\lambda$ where $\{B_\lambda \mid \lambda \in \wedge\}$ is family of all β-zero sets which are β-neighbourhoods of $X\setminus G$. Now, $x\notin X\setminus G \Rightarrow x\notin B_{\lambda o}$ for some $\lambda_o \in \wedge$, which implies that $x \in X\setminus B_{\lambda o}$. Also, we have $\mathbf{X\setminus G} = \cap_{\lambda\in\wedge} B_\lambda \Rightarrow G = X \setminus \cap_{\lambda\in\wedge} B_\lambda = \cup_{\lambda\in\wedge}(X\setminus B_\lambda)$. Therefore, $(X\setminus B_{\lambda o}) \subset \cap_{\lambda\in\wedge} (X\setminus B_\lambda) = G$. Therefore, $x\in X\setminus B_{\lambda o} \subset G$. Since $B_{\lambda o}$ is β-zero set , $X\setminus B_{\lambda o}$ is a co-β-zero set. Therefore, (iv) holds.

(iv) ⇒ (i) : Suppose (iv) holds. Now , to prove that X is almost β-completely regular space : Let A be a δ-closed set and $x_o \notin A$. Then $X\setminus A$ is a δ-open set containing x_o. Then by (iv) , there exists a co-β-zero set U such that $x_o \in U \subset X\setminus A$. Thus, $X\setminus U$ is a β-zero set . Therefore , there exists a β-continuous function $f : X\to [0,1]$ such that $X\setminus U = \beta Z(f)$. Hence , $X\setminus U = \beta Z(f) = \{ x\in X \mid f(x) = 0\}$. As $x_o \in U$, it follows that $f(x_o) \neq 0$. Hence , $| f(x_o) | = r > 0$. Now , we define $g : X \to [0,1]$ by $g(y)$ $\mathrm{Min}\{ 1 , \frac{1}{r}\left| f(y)\right|\}$, for each $y \in X$. Then g is β-continuous function . Also, $g(x_o) = 1$ and $g(z) = 0$, for each $z\in A$. Let $h = 1/g$. As X is E.D. & submaximal, by Lemma- 3.11 , $h : X\to[0,1]$ is β-continuous such that $h(x_o) = 0$ and $h(a) = \{1\}$ .Hence , X is almost β-completely regular. Hence the theorem.

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