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Solving Multi-Objective Transportation Problem by using the Revised Simplex Method

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Abstract: *The Multi-Objective Transportation Problem (MOTP) is a challenging optimization issue that encompasses various conflicting objectives. This research investigates the process of converting MOTP into a single-objective problem by employing the Weight Sum Method. Three separate conditions were analyzed to achieve the transformation from a multi-objective to a single-objective framework. The Revised Simplex Method was then applied to solve the problem under each of these conditions. The results of the study confirm the effectiveness of the Weight Sum Method in reconfiguring multi-objective problems into a solvable single-objective format and emphasize the role of the Revised Simplex Method in yielding optimal solutions for complex transportation challenges.*
Keywords:*Multi-ObjectiveTransportationProblem,WeightSumMethod,RevisedSimplex Method*

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I. Introduction:

One significant area that uses linear programming is the transportation of goods and services from multiple supply areas to multiple demand centers. A TP that is expressed in terms of an LP model can also be resolved using the simplex method. Even if a TP includes a lot of variables and restrictions, solving it with simplex methods takes a long time. The structure of theTPconsistsofmanyshippingroutesfromdifferentsupplysitestodifferentdemandregions [17, 21].The goal is to establish shipping routes between supply and demand hubs to fulfill the demand for a certain amount of products or services at each destination location with the supply of those same goods or services at each supply location at the lowest possible transportationexpense.DifferentexamplescorrespondtodifferenttypesofTP.MOTPisatype of special type of TP. The problem is called a multi-objective transportation problem when it includesmultipleobjectivefunctions[8].Intheactualworld,everycompanywantstodeliver goodswhileaccomplishingseveralgoals,suchreducingexpenses,time,distance,risk,etc.The first TP model was developed in 1941 by Hitchcock [15].

Theintricacyofthesocialandeconomicbackdropinreal-worldsituationsrequirestheexplicit considerationofaspectsotherthancost,whichcanbeachievedbyredefiningclassicalTPas

MOTPmodels. In 1961, Charnes and Cooper [28] first suggested a number of approaches for addressing management-level problems with numerous conflicting goals. Zangiabadi [29] addressed MOTP in 2007 by using fuzzy goal programming. A fuzzy compromise programmingmethodwasputupbyLi[20]forMOTP.Kaur[2]offersasimpletechniquefor findingthebestcompromisesolutionforthelinearMOTP.Afuzzyprogrammingapproach was usedinthestrategythatGeorge[13]suggested.Doke[5]usedthearithmeticmeanoftheglobal assessmenttosolvethethreeobjectivelinearTPs.In2016,Bharathi[3]employedevolutionary methods fortheMOTP. Singh [26]addressed MOTPin a fuzzy environment using geometric methods.The multi-objective transportation problem is studied by Khan [19] using an S-type membershipfunction.Usingfuzzytechniqueandthec-program,Kavita[14]proposedthenew row maximum method to MOTP. Singh [27] developed a new method known as the Matrix MaximaMethodtosolveaMOTPutilizingParetoOptimalityCriteria.In2022,Ekanayake [11] introduced the ant colony optimization technique and geometric mean method to address MOTPinfuzzysituations.Moreover,manyresearchersproposedseveralapproachestosolve MOTP. Different type of algorithms was proposed to resolve MOTP [1,4,6,7,9,10,11,16,18,29,24,25].

Thegoalofthisworkistodevelopanovelalternativealgorithmthatusesthegeometricmean in conjunction with the penalty technique to solve the multi-objective transportation issue. Finally, using illustrated instances of MOTP with multiple targets, the suggested method is compared with several current methods.

II. Preliminaries

Inthissectionsomebasicdefinitionsarereviewed

FormulationofTransportation probleminLinearProgrammingProblem

Given m origins and n destinations, the transportation problem can be formulated as the following linear programming problem model:

$$\text{Minimize:} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij}$$

Subjecttoconstraint:

$$\sum_{j=1}^{n} x_{ij} \leq \quad a_i \qquad i=1, 2, ... m$$

$$\sum_{i=1}^{m} x_{ij} \geq \quad b_j \qquad j=1, 2,…..........n$$

$$x_{ij} \geq 0 \qquad \text{foralliandj}$$

Where $x_{ij}$is the amount of units of shipped from origin I to destination j and $c_{ij}$is the cost of shipping one unit from origin i to destination j. The amount of supply at origin is ai and the amount of destination j is bj. The objective is to determine the unknown xij that will the total transportation cost while satisfying all the supply and demand constraints.

Multi–objectiveTransportationProblem(MOTP)

Inreallifesituations,allthetransportationproblemsarenotsingleobjective.Thetransportation problems which are characterized by multiple objective functions are considered here. A special type of linear programming problem in which constraints are of equality type and all the objectives are conflicting with each other, are called MOTP. Similar to a typical transportationproblem,inaMOTPproblemaproductistobetransportedfrom$m$sourcesto$n$ destinations and their capacities are $a_1,a_2,\cdots a_m$and $b_1,b_2,\cdots b_n$respectively. In addition, thereisapenaltyc_{ij}associatedwithtransportingaunitofproductfromi^{th}andj^{th}destination. Thispenaltymaybecost ordeliverytimeorsafety ofdeliveryor etc.Avariable x_{ij}represents theunknownquantitytobeshippedfromi^{th}sourcetoj^{th}destination.Amathematicalmodel of MOTP with r objectives, m sources and n destinations can be written as: Mathematically, the problem can be stated as

$$\text{Minimize}Z_1=\sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^{r}x_{ij} \qquad (2.1)$$

$$\text{Subject to} \sum_{j=1}^{n} x_{ij} \geq a_i, = 1,2,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} \leq b_j, \quad j = 1,2,\ldots,n \qquad (2.2)$$

$$x_{ij} \geq 0 \quad \forall i,$$

Atransportationproblemissaidtobebalancediftotalsupplyfromallsourcesequalstothe totaldemandinalldestinations$\sum^{m} a_{i=1} = \sum_{j=1}^{n} b_j$. Otherwise,itiscalledunbalanced.

WeightedSum

Weights are assigned $w_{1,2,}w_{3,}\cdots w_r$to each objective to reflect their relative importance according to requirement of customers to change the multiple objectives into the single objective. The weight must be satisfied the following condition:

$$w_1 + w_{2,} + \cdots + w_r = 1$$

i.e

$$\sum_{l=1}^{r} w_r = 1 \qquad (2.3)$$

Multiple objective change into the single objective by using assign weights, which know asweighed objective function. The new objective function to be minimized or maximized i.e.

$$Z = w_1.\sum_{i=1}^{m}\sum_{j=1}^{n} c^1 q_{ij} + w_2.\sum_{i=1}^{m}\sum_{j=1}^{n} c^2 q_{ij} + w_3.\sum_{i=1}^{m}\sum_{j=1}^{n} c^3 q_{ij} + \cdots + w_r.\sum_{i=1}^{m}\sum_{j=1}^{n} c^r q_{ij} \qquad (2.4)$$

RevisedSimplexAlgorithm:

Therevisedsimplexalgorithm canbeexpressed inthefollowing steps:

Step1:Expressthegivenprobleminstandardfrom:

Expressthegivenproblemintherevisedsimplexfrombyconsideringtheobjectivefunction as one of the constraints and slack and surplus variable, if needed, to inequalities to convert them into equalities.

Step2:Obtaininitialbasicfeasible solution:

Startwithinitialbasicmatrix$B = I_m$andfind$B_1^{-1}$and$B_1^{-1}b$tofromtheinitialrevised simplextableasshowninTable1

Variablein Basic **B**	Solution Value $b=(x_1^{(1)})$	BasicInverse B_1^{-1}				$y_k^{(1)}$
		$\beta_0^{(1)}(=Z)$	$\beta_1^{(1)}(=s)_1$	$\cdots$	$\beta_m^{(1)}(=s)_m$	
Z	0	1	0	$\cdots$	0	c_k-z_k
$x_{B_1}=s_1$	b_1	0	1	$\cdots$	0	y_{1k}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$x_{B_m}=s_m$	b_m	0	1	$\cdots$	0	y_{mk}

Step3:Selectavariabletoenterintothebasic(keycolumn):

Foreach non-basic variable,calculate$c_j-z_j$by using theformula$c_j-z_j=c_j-c_{B1}B^{-1}a_j^{(1)}$

Where,$^{-1}a_j^{(1)}$representtheproductofthefirstrowofB_1^{-1}andsuccessivecolumnsofAnot in B_1^{-1}

i) Ifall $c_j-z_j\leq 0$,thenthecurrentbasicsolutionisoptimal.Otherwisegoingto Step 4.

ii) Ifoneormorec_j-z_jarepositive,thenvariabletoenter intothebasicmaybe selectedbyusingtheformula

$$c_j-z_j=Ma\{c_j-z_j;c_j-z_j>0\}$$

Step4:Selectavariabletoleavethebasic(Keyrow)

Calculate$y_k^{(1)}=B_1^{-1}a_k^{(1)}=a_k^{(1)};(k=1)$where$a_k^{(1)}=[-c_k,a_k]$.ifall$y_{ik}\leq 0$,the optimal solution is unbounded. But if at least one $y_{ik}>0$, then variable to be removed from the basic is determined by calculating the ratio

$$\frac{x_{Br}}{y_{rk}}=Min_i\{\frac{x_{Bi}}{y_{ik}};_{ik}>0\}$$

That is,thevector$\beta_r^{(1)}$isselectedtoleave the basicandgoto step5.

Iftheminimum ratio is not unique,i.e.the ratio is sameformorethanone row, thenresulting basic feasible solution will be degenerate. To avoid cycling to occur, the usual method of resolving the degeneracy is applied.

Step5:Updatethecurrentsolution

Updatetheinitialtablebyintroducinganon-basicvariable$x(=a_k^{(1)})$intobasicand removingbasicvariable$x(=\beta_r^{(1)})$fromthebasic.

Repeat Steps 3 to 5 until an optimal solution is obtained or there is an indication for an unbounded solution.

III. Numerical Example

Hereweconsiderexample,Acompanyshipstruckloadsofgrainfromthreewarehousestofour distributed centres. The supply (in Truckloads) and the demand (also in truckloads) together with the unit transportation costs is per Quintals per kilometre on the different routes and transportation time t_{ij}between source and destination are summarized in the transportation model in table.2.

Table-2

		D1	**D2**	**D3**	**D4**	**Supply**
S1	**Cost**	3	1	4	2	20
	Time	**2**	**3**	**2**	**4**	

S2	**Cost**	2	3	1	5	30
	Time	**3**	**1**	**5**	**2**	
S3	**Cost**	4	2	3	3	25
	Time	**1**	**4**	**3**	**2**	
Demand		15	20	25	15	

Tochangeourmultipleobjectivesintothesingleobjectivewearegoingtouseweightsum method by considering the three different condition and obtained the following single objective:
CASEI:weightageofthefirstandsecondobjectivesare0.6 and0.4respectively,i.e.

$$w_1=0.6 \text{and } w_2=0.4$$

Thenbyusing equation(2.4)weobtained

Table-3

	D1	D2	D3	D4	Supply
S1	2.6	1.8	3.2	2.8	20
S2	2.4	2.2	2.6	3.8	30
S3	2.8	2.8	3	2.6	25
Demand	15	20	25	15	

CASEII:weightageofthefirstandsecondobjectivesare0.6 and0.4respectively,i.e.

$$w_1=0.3 \text{and } w_2=0.7$$

Thenbyusing equation(2.4)weobtained

Table-4

	D1	D2	D3	D4	Supply
S1	2.3	2.4	2.6	3.4	20
S2	2.7	1.6	3.8	2.9	30
S3	1.9	3.4	3	2.3	25
Demand	15	20	25	15	

CASEII:weightageofthefirstandsecondobjectivesare0.6 and0.4respectively,i.e.

$$w_1=0.7 \text{and } w_2=0.3$$

Thenbyusing equation(2.4)weobtained

Table-5

	D1	D2	D3	D4	Supply
S1	2.7	1.6	3.4	2.6	20
S2	2.3	2.4	2.2	4.1	30
S3	3.1	2.6	3	2.7	25
Demand	15	20	25	15	

To get the optimal solution of all three conditions, firstly we are converting above transportationproblem(TableNo-3,4,5)intothelinearprogrammingproblemandobtained
Minimize

$$Z_1=2.6x_{11}+1.8x_{12}+3.2x_{13}+2.8x_{14}+2.4x_{21}+2.2x_{22}+2.6x_{23}+3.8x_{24}+2.8x_{31}+2.8x_{32}+3x_{33}+2.6x_{34}$$

$$Z_2=2.3x_{11}+2.4x_{12}+2.6x_{13}+3.4x_{14}+2.7x_{21}+1.6x_{22}+3.8x_{23}+2.9x_{24}+1.9x_{31}+3.4x_{32}+3x_{33}+2.3x_{34}$$

$$Z_3=2.7x_{11}+1.6x_{12}+3.4x_{13}+2.6x_{14}+2.3x_{21}+2.4x_{22}+2.2x_{23}+4.1x_{24}+3.1x_{31}+2.6x_{32}+3x_{33}+2.7x_{34}$$

Subjecttoconstraints

$$x_{11}+x_{12}+x_{13}+x_{14}\leq 20$$
$$x_{21}+x_{22}+x_{23}+x_{24}\leq 30$$
$$x_{31}+x_{32}+x_{33}+x_{34}\leq 25$$
$$x_{11}+x_{21}+x_{31}\geq 15$$

$$x_{12}+x_{22}+x_{32}\geq 20$$
$$x_{13}+x_{23}+x_{33}\geq 25$$
$$x_{14}+x_{24}+x_{34}\geq 15$$
$$x_{11},x_{12},x_{13},x_{14},x_{21},x_{22},x_{23},x_{24},x_{31},x_{32},x_{33},x_{34}\geq 0$$

Here we observe that all objectives function is different – 2 but subject to constraints are sameforalltheobjective.Nowweapplyingrevisedsimplexmethodobtainedtheoptimal solution is as follows

Table-6:OptimalSolutionbyRevisedSimplexMethod

	Transportation Cost	Transportation Time	Optimal Solution
CaseI	0.6	0.4	$Z_1=200$
CaseII	0.7	0.3	$Z_2=168$
CaseIII	0.3	0.7	$Z_3=190.5$

IV. Conclusion:

This study successfully demonstrates the transformation of a Multi-Objective Transportation Problem into a Single Objective Transportation Problem using the Weight Sum Method, considering three distinct weighting conditions. The Revised Simplex Method efficiently solvestheresultingproblems,providingoptimalsolutionsforeachcase:$Z_1= 200$,$Z_2=168$, and $Z_3 = 190.50$. These findings confirm the robustness and adaptability of the combined methods in solving MOTPs, offering valuable insights for practical applications in transportation logistics and decision-making. Future research could explore alternative weighting strategies and extend the methodology to more complex multi-objective optimization problems.

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