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Mathematical Model and Analysis of Corruption Dynamics Incorporating Media Coverage and Anticorruption Agencies

Mohammed Fori & Ibrahim Jaduwa Mallum
Department of Mathematics, College of Education Waka – Biu, PMB 1502, WAKA – BIU

Abstract

This study presents and analyzes a mathematical model that incorporates media coverage and the activities of anticorruption agencies to explain the transmission dynamics of corruption within a population. The model, formulated using a system of nonlinear differential equations, divides the population into compartments representing susceptible aware, susceptible unaware, corrupted, Jail, Reformed, and Anti – corruption Agencies, with the influence of media campaigns and punitive actions by anticorruption agencies explicitly integrated as control measures. The positivity and boundedness of solutions are established, ensuring the model's biological relevance. The basic reproduction number (R_0) is derived using the next-generation matrix method, serving as a threshold parameter that determines the potential for corruption spread. The model exhibits two equilibrium points: a corruption-free equilibrium and an endemic equilibrium. Analysis reveals that the corruption-free equilibrium is locally and globally asymptotically stable when $R_0 < 1$, indicating that corruption can be eradicated from the population under such conditions. Conversely, when $R_0 > 1$, the endemic equilibrium is locally and globally asymptotically stable, signifying the persistence of corruption. Sensitivity analysis identifies key parameters influencing R_0, highlighting the importance of enhancing the effectiveness of media campaigns and the efficiency of anticorruption agencies.

Keywords: *Mathematical Model, Corruption Free, Corruption Endemic and Sensitivity Analysis*

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I. INTRODUCTION

Corruption, defined as the abuse of entrusted power for private gain, represents a significant threat to the rule of law, democratic institutions, economic development, and social justice across the globe, particularly in developing nations. To combat this pervasive issue, mathematical modeling has emerged as a powerful tool for understanding the transmission dynamics of corruption [7]. This article presents a comprehensive analysis of a mathematical model for corruption dynamics that incorporates the critical roles of media coverage and anticorruption agencies, drawing upon recent advancements in the field.

Nowadays, mathematical modelers are becoming more realistic based on many social problems. Some mathematical modelers consider socio-economic problems such as rumor spreading, poverty, crime, prostitution, corruption, etc. as epidemics [13]. Furthermore, several studies have proposed mathematical models to understand and analyze the dynamics of corruption [7]. For instance, [2] developed a mathematical model using a deterministic model approach with constant recruitment rates and a standard incidence rate for the transmission dynamics of corruption as a disease. [11] proposed a mathematical model for corruption by considering awareness created by anti-corruption efforts and counseling in jail. [3] proposed and analyzed a mathematical model of the transmission dynamics of corruption among the populace. [5] proposed a nonlinear deterministic model for the dynamics of corruption and analyzed qualitatively using the stability theory of differential equations. [12] developed and studied a deterministic model for the spread of corrupt morals that involves a group of people who are going through a counseling and guidance procedure. [15] developed a mathematical model for the dynamics of corruption transmission incorporating media coverage. [1] proposed a deterministic mathematical model that explains the transmission dynamics of corruption by considering social influence on honest individuals and analyzed the model. [14] developed mathematical models and analyzed it to investigate the transmission of corruption within population. [6] proposed to combat corruptions in Nigeria system by examine the dynamics of corruption and control measures. The dynamics of the corruption model were described by the susceptible – Exposed – Corrupt – Jail – Honest (SECJH) model using linear ODEs. [4] investigate the impact of corruption by developing a mathematical model that describe it dynamics. [7] proposed a deterministic mathematical model that explained the dynamic of corruption using epidemiological models' approach by considering Loss of Immunity of Ex – Convict. Furthermore, [8] proposed a mathematical model and analysis of the dynamics of corruption incorporating anti – corruption agencies.

In this study, we have taken into cognizance the role of media coverage and anti-corruption agencies in combating corruption. The rest of the paper is organized as follows: Section II, is dedicated to model formulation. Section III, is dedicated to the basic properties of the model, Section IV, is devoted to the equilibrium points of the model. Section V, is dedicated to sensitivity analysis of the model. Lastly, summary and conclusion are presented in Section VI.

II. MODEL FORMULATION

The model is formulated as a system of nonlinear differential equations, dividing the total population into five distinct compartments: unaware Susceptible compartment $S_u(t)$ and aware Susceptible compartment $S_a(t)$, who are both vulnerable to corrupt practices; Corrupt individuals compartment $C(t)$, who are actively involved in corrupt activities; Individuals serving jail term $J(t)$, Reformed individual $R(t)$, who have served punishment due to participation in corruption, and Anti – corruption Agencies $A(t)$ who are responsible for detecting and arrest of corrupt individuals. The model assumes a constant recruitment rate (Λ) into the susceptible class through birth or immigration, with a proportion (ϕ) of new individuals. A natural death rate (μ) is applied uniformly across all compartments. The transmission of corruption occurs through contact between susceptible individuals and corrupted individuals at a rate (β), while those who do not become corrupted move to the reformed class at the rate η. Jailed individuals are reformed from the system through incarceration and punishment, transitioning to the recovered class at a rate (ω). A critical component of this model is the integration of media coverage and anticorruption agencies as control mechanisms. Media coverage is modeled as a factor that reduces the transmission rate (β) by increasing public awareness and fostering a culture of integrity. The effectiveness of media campaigns is represented by a parameter (δ), which quantifies the reduction in the contact rate due to increased awareness. Similarly, the impact of anticorruption agencies is modeled through a control variable (m), representing the rate at which corrupted individuals are identified, prosecuted, and punished. In addition to the above, we also consider the following:

i. i. Anti – corruption agencies do not participate in the act of corruption

ii. ii. Reformed individual will never indulge in corruption again

iii. iii. An individual can be corrupt only through contacts with corrupt individuals

A. Model Diagram

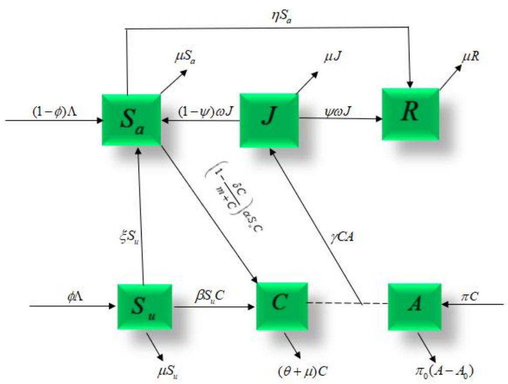

Figure 1: Schematic diagram of the model

Base on the assumptions we consider, the dynamics of corruption is described by the following system of differential equations:

$$
\left.\begin{aligned}
\frac{dS_u}{dt} &= \phi\Lambda - \beta S_u C - \xi S_u - \mu S_u \\
\frac{dS_a}{dt} &= (1-\phi)\Lambda + \xi S_u + (1-\psi)\omega J - \eta S_a - \left(1 - \frac{\delta C}{m+C}\right)\alpha S_a C - \mu S_a \\
\frac{dC}{dt} &= \beta S_u C + \left(1 - \frac{\delta C}{m+C}\right)\alpha S_a C - \gamma CA - (\theta + \mu)C \\
\frac{dJ}{dt} &= \gamma CA - (\omega + \mu)J \\
\frac{dR}{dt} &= \eta S_a + \psi\omega J - \mu R \\
\frac{dA}{dt} &= \pi C - \pi_0 (A - A_0)
\end{aligned}\right\} \qquad (1)
$$

where, $S_u(0) > 0$, $S_a(0) > 0$, $C(0) \geq 0$, $J(0) \geq 0$, $A(0) > 0$, $0 < \psi < 1$, $0 < \phi < 1$, and $\frac{\delta\psi C}{m+C}$ is the reduced rate of contact with corrupt individual due to media coverage.

Table I: Variables of the Model

Variable	Description of Variables
$S_u(t)$	Susceptible individuals' population at time t
$S_a(t)$	Susceptible individuals' population at time t
$C(t)$	Corrupt individuals' population at time t
$J(t)$	Jailed individuals' population at time t
$R(t)$	Reformed individuals' population at time t
$A(t)$	Anticorruption agencies population at time t

Table II: Parameters of the Model

Variable	Description of Parameters
Λ	Constant recruitment rate of susceptible individuals
ξ	Rate at which susceptible unaware individuals become susceptible aware
m	Media coverage
δ	Measures efficacy of media coverage
β	Transmission rate from unaware susceptible individuals to corrupt
μ	Natural death rate
γ	Rate at which corrupt individuals move to jail
θ	Corruption related death rate

ω	Rate at which individual that served jail time become reformed
α	Rate at which aware susceptible will become corrupt
η	Rate at which aware susceptible individual become reformed due to good moral
π	Recruitment rate of anti – corruption agencies
π_o	Decay rate of anti – corruption
A_o	Baseline anticorruption population size
ϕ, ψ	Proportions

III. BASIC PROPERTIES OF THE MODEL

Existence, positivity and boundedness of solution of the model

The mathematical model is well posed. Since we are dealing with human population, we should ensure positivity and boundedness of solutions.

Theorem 1: The solution of system (1) is positive

Proof
From the first equation of the model system (1), we have

$$\begin{aligned}\frac{dS_u}{dt} &= \phi\Lambda - \beta S_u C - \xi S_u - \mu S_u \\ &\geq -\beta S_u C - \xi S_u - \mu S_u \quad (\phi \text{ is positive}) \\ &= -\left(\beta C + \xi + \mu\right) S_u\end{aligned}$$

$$\Rightarrow \frac{dS_u}{S_u} \geq -\left(\beta C(t) + \xi + \mu\right) dt$$

After integration,

$$S_u(t) \geq S(0) e^{\int -(\beta C(t) + \xi + \mu) dt} \qquad (2)$$

Which is positive for all time $t \geq 0$
The positivity of the remaining state variables can be proved in the same way.

Boundedness
Let N be the total population, $N(t) = S_u + S_a + C + J + R$. Consider the set

$$\Omega = \left\{ \left(S_u.S_a, C, J, R, A\right) \in \mathbb{R}_+^6 : 0 < N \leq \frac{\Lambda}{\mu}, 0 \leq A \leq A_o \right\}$$

Theorem 2 below establishes the boundedness of the solution.

Theorem 2

All solutions $\left(S_u(t), S_a(t), C(t), J(t), R(t), A(t)\right)$ of the model system (1) are bounded in the region Ω.

Proof

We can easily show that if $N(t) = S_u + S_a + C + J + R$, then it follows that $N(t) \leq \max\left\{N(0), \dfrac{\Lambda}{\mu}\right\}$. On other hand $\dfrac{dA}{dt} = \pi C - \pi_0\left(A - A_0\right)$ implies that $A(t) \leq \max\left\{A_o, A(0)\right\}$

Hence, the solution of the model system is bounded in the region Ω . □

IV. EQUILIBRIUM POINTS

A. Corruption – Free Equilibrium Point (CFE)

Corruption – free equilibrium point E_0 is a steady state solution, where three is no corruption in the society. Thus, CFE of the system (1) is attend when all the variables and parameters related to corruption are zero.

$$\left.\begin{aligned}
&\phi\Lambda - \beta S_u C - \xi S_u - \mu S_u = 0 \\
&\left(1-\phi\right)\Lambda + \xi S_u + \left(1-\psi\right)\omega J - \eta S_a - \left(1 - \frac{\delta C}{m+C}\right)\alpha S_a C - \mu S_a = 0 \\
&\beta S_u C + \left(1 - \frac{\delta C}{m+C}\right)\alpha S_a C - \gamma CA - \left(\theta + \mu\right)C = 0 \\
&\gamma CA - \left(\omega + \mu\right)J = 0 \\
&\eta S_a + \psi\omega J - \mu H = 0 \\
&\pi C - \pi_0\left(A - A_0\right) = 0
\end{aligned}\right\} \tag{3}$$

Substituting $C = 0$ in equation (3), we have:

$$E_0 = \left(\frac{\phi\Lambda}{k_1}, \frac{\Lambda\left(v_1 k_1 + \phi\xi\right)}{k_1 k_2}, 0, 0, \frac{\Lambda\eta(v_1 k_1 + \phi\xi}{\mu k_1 k_2}, A_0\right)$$

where $v_1 = 1 - \phi$, $k_1 = \xi + \mu$ and $k_2 = \eta + \mu$

B. Basic Reproduction Number

The basic reproduction number, R_0 , is define as number of secondary infections

produce by a typical infected individual in a complete susceptible population. We applied the next generation matrix techniques by [9] to calculate the R_0 .

Now, we want to linearize the corrupters system. Therefore, let's set

$$\begin{aligned}
&f_1 = \beta S_u C + \left(1 - \frac{\delta C}{m+C}\right)\alpha S_a C - \gamma CA - \left(\theta + \mu\right)C \\
&f_2 = \gamma CA - \left(\omega + \mu\right)J
\end{aligned}$$

Let matrix F represent the rate of appearance of new corrupters into the compartments and V represent the rate of transaction into (out) of compartments. Then, we have

$$F = \begin{pmatrix} \beta S_u C + \left(1 - \dfrac{\delta C}{m + C}\right)\alpha S_a C \\ 0 \end{pmatrix}$$

$$V = \begin{pmatrix} \gamma CA + (\theta + \mu)C \\ (\omega + \mu)J - \gamma CA \end{pmatrix}$$

The Jacobian matrix at the corruption – free equilibrium points J_F and J_V are

$$J_F(E_0) = \begin{pmatrix} \beta S_u^0 + \alpha S_a^0 & 0 \\ 0 & 0 \end{pmatrix} \text{ and } J_V(E_0) = \begin{pmatrix} \gamma A + (\theta + \mu) & 0 \\ \gamma A & \omega + \mu \end{pmatrix}$$

Where $S_u^0 = \dfrac{\phi\Lambda}{k_1}$ and $S_a^0 = \dfrac{\Lambda(v_1 k_1 + \phi\xi)}{k_1 k_2}$

Now, the inverse of $J_V(E_0)$ is given as

$$J_V(E_0)^{-1} = \begin{pmatrix} \dfrac{1}{\gamma A_0 + \theta + \mu} & 0 \\ \dfrac{\gamma A_0}{(\omega + \mu)(\gamma A_0 + \theta + \mu)} & \dfrac{1}{\omega + \mu} \end{pmatrix}$$

Therefore, NGM is

$$J_F(E_0) J_V(E_0)^{-1} = \begin{pmatrix} \beta S_u^0 + \alpha S_a^0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dfrac{1}{\gamma A_0 + \theta + \mu} & 0 \\ \dfrac{\gamma A_0}{(\omega + \mu)(\gamma A_0 + \theta + \mu)} & \dfrac{1}{\omega + \mu} \end{pmatrix} = \begin{pmatrix} \dfrac{\beta S_u^0 + \alpha S_a^0}{\gamma A_0 + \theta + \mu} & 0 \\ 0 & 0 \end{pmatrix}$$

Hence, the basic reproduction number, R_0, is

$$R_0 = \frac{\Lambda\phi\beta(\eta + \mu) + \Lambda\alpha\left((1 - \phi)(\xi + \mu) + \phi\xi\right)}{(\eta + \mu)(\xi + \mu)(\gamma A_0 + \theta + \mu)} \tag{4}$$

C. Corruption Endemic Equilibrium Point (CEEP)

The corruption endemic equilibrium point represents persistence of corruption in the population. To obtain this equilibrium point we assume that
$S_u^0 > 0$, $S_a^0 > 0$, $C^0 > 0$, $J^0 > 0$, $R^0 > 0$, $A^0 > 0$ in equation (1).

Let $E_1 = \left(S_u^*, S_a^*, C^*, J^*, R^*, A^*\right)$ denotes the corruption endemic equilibrium point of the model, it satisfies the algebraic equations:

$$
\left.\begin{aligned}
&\phi\Lambda-\beta S_u^* C-\xi S_u^*-\mu S_u^*=0\\
&(1-\phi)\Lambda+\xi S_u^*+(1-\psi)\omega J^*-\eta S_a^*-\left(1-\frac{\delta C^*}{m+C^*}\right)\alpha S_a^* C^*-\mu S_a^*=0\\
&\beta S_u^* C^*+\left(1-\frac{\delta C^*}{m+C^*}\right)\alpha S_a^* C^*-\gamma C^* A^*-(\theta+\mu)C^*=0\\
&\gamma C^* A^*-(\omega+\mu)J^*=0\\
&\eta S_a^*+\psi\omega J^*-\mu H^*=0\\
&\pi C^*-\pi_0(A-A_0)=0
\end{aligned}\right\} \tag{5}
$$

After simplifications we obtain $CEEP$ in terms of C^* as

$$S_u^*=\frac{\phi\Lambda}{\beta C^*+\tau+\mu}$$

$$S_a^*=\frac{(1-\phi)\Lambda\pi_0(\omega+\mu)(\beta C^*+\xi+\mu)+\phi\xi\Lambda+(1-\psi)\omega\gamma C^*(\pi C^*+\pi_0 A_0)(\beta C^*+\xi+\mu)}{\pi_0(\omega+\mu)\left(\left(1-\frac{\delta C^*}{m+C^*}\right)\alpha C^*+\xi+\mu\right)\left(\beta C^*+\xi+\mu\right)}$$

$$J^*=\frac{\gamma C^*(\pi C^*+\pi_0 A_0}{\pi_0(\omega+\mu)}$$

$$R^*=\frac{\eta\left[(1-\phi)\Lambda\pi_0(\omega+\mu)(\beta C^*+\xi+\mu)+\phi\xi\Lambda+(1-\psi)\omega\gamma C^*(\pi C^*+\pi_0 A)(\beta C^*+\xi+\mu)\right]}{\mu\pi_0(\omega+\mu)\left(\left(1-\frac{\delta C^*}{m+C^*}\right)\alpha C^*+\xi+\mu\right)\left(\beta C^*+\xi+\mu\right)}$$

$$+\frac{\psi\omega\gamma C^*(\pi C^*+\pi_0 A_0)\left(\left(1-\frac{\delta C^*}{m+C^*}\right)\alpha C^*+\eta+\mu\right)\left(\beta C^*+\xi+\mu\right)}{\mu\pi_0(\omega+\mu)\left(\left(1-\frac{\delta C^*}{m+C^*}\right)\alpha C^*+\xi+\mu\right)\left(\beta C^*+\xi+\mu\right)}$$

$$A^*=\frac{\pi C^*+\pi_0 A_0}{\pi_0}$$

D. Stability Analysis of CFEP for the Model

To establish the local stability of the CFEP of system (1), we examine the eigenvalues of the Jacobian matrix of the model evaluated at the CFEP

Theorem 2

The corruption free – equilibrium point E_0 of the model is locally asymptotically stable if $R_0<1$.

Proof

To proof the theorem, first we obtain the Jacobian matrix of system (1) and evaluate it at CFEP. Therefore, from the system (1), let

$$\left.\begin{aligned}
f_1 &= \phi\Lambda - \beta S_u C - \xi S_u - \mu S_u \\
f_2 &= (1-\phi)\Lambda + \xi S_u + (1-\psi)\omega J - \eta S_a - \left(1-\frac{\delta C}{m+C}\right)\alpha S_a C - \mu S_a \\
f_3 &= \beta S_u C + \left(1-\frac{\delta C}{m+C}\right)\alpha S_a C - \gamma C A - (\theta+\mu) C \\
f_4 &= \gamma C A - (\omega+\mu) J \\
f_5 &= \eta S_a + \psi\omega J - \mu R \\
f_6 &= \pi C - \pi_0 (A - A_0)
\end{aligned}\right\}$$

(6)

Thus, for the Jacobian matrix for the system (6) is obtain as follow

$$J(X) = \begin{pmatrix}
\frac{\partial f_1}{\partial S_u} & \frac{\partial f_2}{\partial S_a} & \frac{\partial f_3}{\partial C} & \frac{\partial f_4}{\partial J} & \frac{\partial f_5}{\partial R} & \frac{\partial f_6}{\partial A} \\
\frac{\partial f_1}{\partial S_u} & \frac{\partial f_2}{\partial S_a} & \frac{\partial f_3}{\partial C} & \frac{\partial f_4}{\partial J} & \frac{\partial f_5}{\partial R} & \frac{\partial f_6}{\partial A} \\
\frac{\partial f_1}{\partial S_u} & \frac{\partial f_2}{\partial S_a} & \frac{\partial f_3}{\partial C} & \frac{\partial f_4}{\partial J} & \frac{\partial f_5}{\partial R} & \frac{\partial f_6}{\partial A} \\
\frac{\partial f_1}{\partial S_u} & \frac{\partial f_2}{\partial S_a} & \frac{\partial f_3}{\partial C} & \frac{\partial f_4}{\partial J} & \frac{\partial f_5}{\partial R} & \frac{\partial f_6}{\partial A} \\
\frac{\partial f_1}{\partial S_u} & \frac{\partial f_2}{\partial S_a} & \frac{\partial f_3}{\partial C} & \frac{\partial f_4}{\partial J} & \frac{\partial f_5}{\partial R} & \frac{\partial f_6}{\partial A} \\
\frac{\partial f_1}{\partial S_u} & \frac{\partial f_2}{\partial S_a} & \frac{\partial f_3}{\partial C} & \frac{\partial f_4}{\partial J} & \frac{\partial f_5}{\partial R} & \frac{\partial f_6}{\partial A}
\end{pmatrix} \tag{7}$$

$$J(X) = \begin{pmatrix}
-\beta C - \xi - \mu & 0 & -\beta S_u & 0 & 0 & 0 \\
\xi & -\eta - \alpha C + \frac{\delta C}{m+C}\alpha C - \mu & -\alpha S + K & (1-\omega) & 0 & 0 \\
\beta C & \alpha C - \frac{\delta C}{m+C}\alpha C & \beta S_u + \alpha S_a - K - \gamma A - \theta - \mu & 0 & 0 & -\gamma C \\
0 & 0 & \gamma A & -\omega - \mu & 0 & \gamma C \\
0 & \eta & 0 & \theta\omega & -\mu & 0 \\
0 & 0 & \pi & 0 & 0 & -\pi_0
\end{pmatrix}$$

Where, $X = S_u, S_a, C, J, R, A$ and $K = \frac{\delta\mu\alpha S_a C}{(m+C)^2} - \frac{\delta C \alpha S_a}{m+C}$

The Jacobian evaluated at the corruption free equilibrium E_0 is given by

$$J(E_0)=\begin{pmatrix} -K_1 & 0 & -\frac{\beta\phi\Lambda}{K_1} & 0 & 0 & 0 \\ \xi & -K_2 & -\frac{\alpha\Lambda(v_1K_1+\phi\xi}{K_1K_2} & (1-\psi)\omega & 0 & 0 \\ 0 & 0 & \frac{\beta\phi\Lambda}{K_1}+\frac{\alpha\Lambda(v_1K_1+\phi\xi)}{K_1K_2}-\gamma A_0-(\theta+\mu) & 0 & 0 & 0 \\ 0 & 0 & \gamma A_0 & -K_3 & 0 & 0 \\ 0 & \eta & 0 & \psi\omega & -\mu & 0 \\ 0 & 0 & \pi & 0 & 0 & -\pi_0 \end{pmatrix}$$

after simplification, the eigenvalues of the matrix $J(E_0)$ is found as

$\lambda_1=-(\xi+\mu)$, $\lambda_2=-(\eta+\mu)$, $\lambda_3=-(\xi+\mu)(\eta+\mu)(\gamma A_0+\theta+\mu)(1-R_0)$, $\lambda_4=-(\omega+\mu)$, $\lambda_5=-\mu$, $\lambda_6=-\pi_0$

Hence the proof, since all the eigenvalues have negative real part if $R_0<1$.

E. Global Stability Analysis of Corruption Free – Equilibrium Point

We establish the global stability of the CFEP of system (1) base on [10]. Let Y_n

denote compartment of non – corrupt individuals, Y_i denote the compartment of corrupt individuals and $Y\left(E_{0,n}\right)$ is vector at CFCP of the same vector length as Y_n.

According to [10], we shall write our system (1) in the form

$$\left.\begin{aligned} \frac{dY_n}{dt}&=G\left(Y_n-Y_{E_{0,n}}\right)+G_1Y_i \\ \frac{dY_i}{dt}&=G_2Y_i \end{aligned}\right\} \tag{7}$$

where, $Y_n=(S_u,S_a,R,A)$ and $Y_i=(C,J)$

According to [10], the CFEP is globally asymptotically stable if the following conditions holds:

i. G should be a matrix with real negative eigenvalues
ii. G_2 should be Metzler matrix

Theorem 3
The corruption free – equilibrium point is globally asymptotically stable if $R_0<1$.

Proof
Consider system (1), we define
$Y_n=\left(S_u,S_a,R,A\right)^T$

$$Y_i = \left(C, J\right)^T$$

$$Y\left(E_{0,n}\right) = \left(\frac{\phi\Lambda}{k_1}, \frac{\Lambda\left(v_1k_1 + \phi\xi\right)}{k_1k_2}, \frac{\Lambda\eta(v_1k_1 + \phi\xi}{\mu k_1k_2}, A_0\right)^T$$

$$E_0 = \left(\frac{\phi\Lambda}{k_1}, \frac{\Lambda\left(v_1k_1 + \phi\xi\right)}{k_1k_2}, 0, 0, \frac{\Lambda\eta(v_1k_1 + \phi\xi}{\mu k_1k_2}, A_0\right)^T$$

Now,

$$Y_n - Y\left(E_{0,n}\right) = \begin{pmatrix} S_u \\ S_a \\ R \\ A \end{pmatrix} - \begin{pmatrix} \dfrac{\phi\Lambda}{k_1} \\ \dfrac{\Lambda\left(v_1k_1 + \phi\xi\right)}{k_1k_2} \\ \dfrac{\Lambda\eta(v_1k_1 + \phi\xi}{\mu k_1k_2} \\ A_0 \end{pmatrix} = \begin{pmatrix} S_u - \dfrac{\phi\Lambda}{k_1} \\ S_a - \dfrac{\Lambda\left(v_1k_1 + \phi\xi\right)}{k_1k_2} \\ R - \dfrac{\Lambda\eta(v_1k_1 + \phi\xi}{\mu k_1k_2} \\ A - A_0 \end{pmatrix}$$

Equation (7) with the model equation (1) is written to the form:

$$\begin{pmatrix} \phi\Lambda - \beta S_u C - (\xi - \mu)S_u \\ \left(1-\phi\right)\Lambda + \xi S_u + \left(1-\psi\right)\omega J - \eta S_a - \left(1 - \dfrac{\delta C}{m+C}\right)\alpha S_a C - \mu S_a \\ \eta S_a + \psi\omega J - \mu R \\ \pi C - \pi_0\left(A - A_0\right) \end{pmatrix}$$

$$= G\begin{pmatrix} S_u - \dfrac{\phi\Lambda}{k_1} \\ S_a - \dfrac{\Lambda\left(v_1k_1 + \phi\xi\right)}{k_1k_2} \\ R - \dfrac{\Lambda\eta(v_1k_1 + \phi\xi}{\mu k_1k_2} \\ A - A_0 \end{pmatrix} + G_1\begin{pmatrix} C \\ J \end{pmatrix}$$

$$\begin{pmatrix} \beta S_u + \left(1 - \dfrac{\delta C}{m+C}\right)\alpha S_a C - \gamma CA - (\theta + \mu)C \\ \gamma CA - (\omega + \mu)J \end{pmatrix} = G_2\begin{pmatrix} C \\ J \end{pmatrix}$$

where the matrix G , G_1 and G_2 make the above equations meaningful. Using the non-corrupt compartment elements of the Jacobian matrix of system (1) and the representation in equation (7) we calculate.

$$G=\begin{pmatrix} -(\xi+\mu) & 0 & 0 & 0 \\ \xi & -(\eta+\mu) & 0 & 0 \\ 0 & \eta & -\mu & 0 \\ 0 & 0 & 0 & -\pi_0 \end{pmatrix}$$

$$G_1=\begin{pmatrix} -\beta S_u & 0 \\ \dfrac{\delta C}{(m+C)^2}\alpha S_a C-\left(1-\dfrac{\delta C}{m+C}\right)\alpha S_a & (1-\psi)\omega \\ 0 & \psi\omega \\ \pi & 0 \end{pmatrix}$$

$$G_2=\begin{pmatrix} \beta S_u \dfrac{\delta C}{(m+C)^2}\alpha S_a C+\left(1-\dfrac{\delta C}{m+C}\right)\alpha S_a-\gamma A-(\theta+\mu) & 0 \\ \gamma A & -(\omega+\mu) \end{pmatrix}$$

Hence, the sufficient conditions are satisfied. Therefore, the corruption free equilibrium point E_0 is globally asymptotically stable if $R_0<1$. □

Local stability of corruption endemic equilibrium point

Theorem 5

If $R_0>1$, then corruption endemic equilibrium point is locally asymptotically stable.

Proof

The stability of the corruption endemic equilibrium point is determined based on the eigenvalues of the Jacobian matrix of the model system at E_1 is given by:

$$J(E_1)=\begin{pmatrix} -a_{11} & 0 & -b_{11} & 0 & 0 & 0 \\ \xi & -d_{11} & -e_{11} & (1-\psi)\omega & 0 & 0 \\ f_{11} & g_{11} & -h_{11} & 0 & 0 & -i_{11} \\ 0 & 0 & j_{11} & -k_{11} & 0 & i_{11} \\ 0 & \eta & 0 & \psi\omega & -\mu & 0 \\ 0 & 0 & \pi & 0 & 0 & -\pi_0 \end{pmatrix}$$

Here,

$$a_{11}=\beta C^*+\xi+\mu,\ b_{11}=\beta S_u^*,\quad d_{11}=\left(1-\frac{\delta C^*}{m+C^*}\right)\alpha C^*+\eta+\mu,$$

$$e_{11}=-\left(\frac{\delta m C^*}{(m+C^*)^2}-1-\left(\frac{\delta C^*}{m+C^*}\right)\right)\alpha S_a^*,\ f_{11}=\beta C^*,\quad g_{11}=\left(1-\frac{\delta C^*}{m+C^*}\right)\alpha C^*$$

$$h_{11} = -\beta S_u^* + \left(\frac{\delta m C^*}{(m+C^*)^2} - \left(1 - \frac{\delta C^*}{m+C^*} \right) \right) \alpha S_a^* + \gamma A^* + \theta + \mu ,\ i_{11} = \gamma C^* ,\ j_{11} = \gamma A^*$$

After simplification, the characteristic polynomial is obtained as

$$p(\lambda) = \left(m+\lambda\right)(a_5\lambda^5 + a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 \qquad (8)$$

where

$$a_5 = 1$$
$$a_4 = a_{11} + d_{11} + k_3 + \mu + \pi_0$$
$$\begin{aligned} a_3 = {} & a_{11}d_{11} + a_{11}d_{11} + a_{11}k_3 + a_{11}\mu + a_{11}\pi_0 + d_{11}k_3 + \mu\pi_0 + d_{11}\mu + d_{11}\pi_0 + a_{11}d_{11}k_3 + a_{11}d_{11}\mu + \\ & a_{11}d_{11}\pi_0 + k_3\mu + k_3\pi_0 + i_{11}\pi + e_{11}g_{11} + b_{11}f_{11} \end{aligned}$$

$$\begin{aligned} a_2 = {} & a_{11}k_3\mu + a_{11}k_3\pi_0 + a_{11}\mu\pi_0 + d_{11}k_3\mu + d_{11}k_3\pi_0 + d_{11}\mu\pi_0 + k_3\mu\pi_0 + a_{11}i_{ii}\pi + d_{11}i_{11}\pi + \\ & k_3 i_{11}\pi + a_{11}e_{11}g_{11} + k_3e_{11}g_{11} + \pi_0 e_{11}g_{11} - (1-\psi)\omega g_{11}j_{11} + b_{11}\xi g_{11} + b_{11}f_{11}d_{11} + b_{11}f_{11}k_3 + b_{11}f_{11}\pi_0 \end{aligned}$$

$$\begin{aligned} a_1 = {} & a_{11}d_{11}k_3\mu + a_{11}d_{11}k_3\pi_0 + a_{11}d_{11}\mu\pi_0 + a_{11}k_3\mu\pi_0 + d_{11}k_3\mu\pi_0 + a_{11}d_{11}i_{11}\pi + a_{11}k_3i_{11}\pi + \\ & d_{11}k_3i_{11}\pi + a_{11}k_3e_{11}g_{11} + a_{11}e_{11}g_{11}\pi_0 + k_3e_{11}g_{11}\pi_0 - a_{11}(1-\psi)\omega g_{11}j_{11} - \pi_0(1-\psi)\psi g_{11}j_{11} \\ & -(1-\psi)\omega g_{11}i_{11}\pi + b_{11}\xi g_{11}k_3 + b_{11}\xi g_{11}\pi_0 + b_{11}f_{11}d_{11}k_3 + b_{11}f_{11}d_{11}\pi_0 + b_{11}f_{11}k_3\pi_0 \end{aligned}$$

$$\begin{aligned} a_0 = {} & a_{11}d_{11}k_3\mu\pi_0 + a_{11}d_{11}kg_3i_{11}\pi + a_{11}k_3e_{11}g_{11}\pi_o - a_{11}\pi_0(1-\psi)\omega g_{11}j_{11} - a_0(1-\psi)\omega g_{11}i_{11}\pi + \\ & b_{11}\xi g_{11}k_3\pi_o + b_{11}f_{11}d_{11}k_3\pi_o \end{aligned}$$

One of the eigenvalues of $J(E_1)$ is $\lambda_1 = -\mu$ and the remaining five roots of equation (8) are analyzed by Routh – Hurwiz criteria. The coefficients $a_0, a_1, a_2, a_3, a_4, a_5$ of the characteristic polynomial equation are real positive. Thus, the necessary condition for stability of the equilibrium point is satisfied. From the sufficient condition for stability of the system, the Hurwitz array for the characteristic polynomial is presented as follow:

$$\begin{array}{c|ccc} \lambda^5 & a_5 & a_3 & a_1 \\ \lambda^4 & a_4 & a_2 & a_0 \\ \lambda^3 & b_1 & b_2 & b_3 \\ \lambda^2 & c_1 & c_2 & c_3 \\ \lambda^1 & d_1 & d_2 & d_3 \\ \lambda^0 & e_1 & e_2 & e_3 \end{array}$$

where a_0, a_1, a_2, a_3, a_4 and a_5 are characteristic polynomial coefficients and the rest of the array are computed using the following ways:

$b_1 = \frac{-1}{a_4}\begin{vmatrix} a_5 & a_3 \\ a_4 & a_2 \end{vmatrix} = \frac{a_3a_4 - a_2}{a_4} > 0$, since $a_5 = 1$, $b_2 = \frac{-1}{a_4}\begin{vmatrix} a_5 & a_1 \\ a_4 & a_0 \end{vmatrix} = \frac{a_1a_4 - a_0}{a_4} > 0$, $b_3 = \frac{-1}{a_4}\begin{vmatrix} a_5 & 0 \\ a_4 & 0 \end{vmatrix} = 0$

$c_1 = \frac{-1}{b_1}\begin{vmatrix} a_4 & a_5 \\ b_1 & b_2 \end{vmatrix} = \frac{a_2a_1 - a_4b_2}{b_1} > 0$, $c_2 = \frac{-1}{b_1}\begin{vmatrix} a_4 & a_0 \\ b_1 & b_3 \end{vmatrix} = a_0$,

$$c_3 = \frac{-1}{b_1}\begin{vmatrix} a_4 & 0 \\ b_1 & 0 \end{vmatrix} = 0$$

$$d_1 = \frac{-1}{c_1}\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} = \frac{b_2c_1 - b_1c_2}{c_1} > 0,\ d_2 = \frac{-1}{c_1}\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix} = 0,$$

$$d_3 = \frac{-1}{c_1}\begin{vmatrix} b_1 & 0 \\ 0 & 0 \end{vmatrix} = 0,$$

$$e_1 = \frac{-1}{d_1}\begin{vmatrix} c_1 & c_2 \\ d_1 & d_2 \end{vmatrix} = a_0 > 0\ e_2 = \frac{-1}{d_1}\begin{vmatrix} c_1 & c_3 \\ d_1 & d_2 \end{vmatrix} = \frac{b_2c_1 - b_1c_2}{c_1},\ e_3 = \frac{-1}{d_1}\begin{vmatrix} c_1 & 0 \\ d_1 & 0 \end{vmatrix} = 0$$

The coefficients of the characteristic polynomial equation $a_0, a_1, a_2, a_3, a_4, a_5$ are real positive. Moreover, the first column of the Routh – Hurwitz array have the same positive sign. Hence, by Routh – Hurwitz criteria all eigen values of the characteristics polynomial equation are negative. Therefore, the corruption endemic equilibrium point is locally asymptotically stable if $R_0 > 1$. □

V. SENSITIVITY ANALYSIS

The sensitivity indices of the corruption reproduction number are calculated in order to determine how important each parameter is in the initiation of corruption. We adopted the local sensitivity analysis based on the normalized forward sensitivity index R_0.

Table III: Sensitivity index for the parameters with respect to R_0.

Parameter	Sign	Sensitivity indices
$\chi_{\Lambda}^{R_0}$	+	1
$\chi_{\phi}^{R_0}$	+	0.001883
$\chi_{\beta}^{R_0}$	+	0.001934
$\chi_{\eta}^{R_0}$	−	0.937151
$\chi_{\mu}^{R_0}$	−	0.016708
$\chi_{\alpha}^{R_0}$	+	0.998066
$\chi_{\xi}^{R_0}$	−	0.218837
$\chi_{\gamma}^{R_0}$	−	0.924703
$\chi_{\theta}^{R_0}$	−	0.006605
$\chi_{A_0}^{R_0}$	−	0.924703

Interpretation of Sensitivity Indices

The sensitivity indices of parameters are presented in Table III. Parameters that have positive indices $(\Lambda, \phi, \beta, \alpha)$ have a great impact on expanding the corruption in the community when their values increase. Parameters with negative indices $(\eta, \mu, \xi, r, \theta, A_0)$ minimize the burden of corruption in the community as their values increases. Therefore, the model sensitivity analysis demonstrated that anti -corruption agencies are supposed to decrease positive index parameters to combat corruption in a population.

VI. SUMMARY AND CONCLUSION

In this paper, a mathematical model that incorporates media coverage and the activities of anticorruption agencies to explain the transmission dynamics of corruption within a population was formulated. The model's solution is proven to be positive and bounded for all time, ensuring that the population variables remain biologically meaningful. The basic reproduction number (R_0), which represents the average number of new corrupt individuals generated by a single corrupted individual in a completely susceptible population, is derived using the next-generation matrix method. The value of R_0 serves as a threshold parameter for the system's behavior. When $R_0 < 1$, the corruption-free equilibrium (CFE) is both locally and globally asymptotically stable, indicating that corruption will eventually die out from the population. Conversely, when $R_0 > 1$, the endemic equilibrium (EE) point, where corruption persists at a steady level, becomes locally and globally asymptotically stable, signifying that corruption will become established within the society.

In conclusion, this comprehensive model demonstrates that corruption can be effectively controlled and potentially eradicated through a synergistic approach that leverages the power of public awareness through media coverage and the decisive action of dedicated anticorruption agencies. The findings emphasize that a fragmented or single-pronged strategy is insufficient; instead, a coordinated, integrated effort is essential to reduce the basic reproduction number below one and achieve a corruption-free society. Future research should focus on refining the model with real-world data to quantify the precise impact of different media and enforcement strategies, thereby providing policymakers with a robust, data-driven framework for designing and implementing effective anticorruption policies.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interest regarding the publication of this paper.

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