

## **Relativistic effects on the nonlinear propagation of electron plasma waves in dense quantum plasma with arbitrary temperature**

Swarniv Chandra and Basudev Ghosh

*Department of Physics, Jadavpur University, Kolkata-700032, India.*

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**Abstract**—*Relativistic effects on the linear and nonlinear properties of electron plasma waves are investigated using the one-dimensional quantum hydrodynamic (QHD) model for a two-component electron-ion dense quantum plasma with an arbitrary temperature and streaming motion. It is shown that the relativistic effects significantly affect the linear and nonlinear properties of electron plasma waves. Depending on the value of electron degeneracy parameter and the streaming velocity both compressive and rarefactive solitons can be excited in the model plasma under consideration. The importance of the result is also pointed out.*

**Keywords**—*Quantum plasma, Quantum hydrodynamic model, finite temperature, solitary structure, relativistic effect*

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### **I. INTRODUCTION**

Many authors have studied the linear and nonlinear properties of plasma waves. However, most of these investigations are confined to classical non-relativistic plasma. Particle velocities in some plasmas may become comparatively high; in some cases it may even approach the speed of light. For such plasmas it becomes important to consider the relativistic effects. In fact relativistic effect may significantly modify the linear and nonlinear behaviour of plasma waves. Relativistic plasma can be formed in many practical situations e.g. in space-plasma phenomena [1], the plasma sheet boundary of earth's magnetosphere [2], Van Allen radiation belts[3] and laser-plasma interaction experiments[4]. The relativistic motion in plasmas is assumed to exist during the early period of evolution of the universe [5]. Regarding the relativistic effects on ion-acoustic solitary waves a number of works have been reported. Das and Paul [6] first investigated the ion-acoustic solitary waves in relativistic plasma and showed that relativistic effect is important on the formation of ion-acoustic solitary waves in presence of streaming motion of plasma particles. Subsequently, many authors considered various parameters together with the relativistic effect for the study of ion-acoustic solitary waves and obtained some fascinating results which are important in laboratory and space plasma. Recently, Saeed et al [7] have shown that in electron-positron-ion plasma increase in the relativistic streaming factor causes the soliton amplitude to thrive and its width shrinks. El-Labany et al [8] have shown that relativistic effect can modify the condition of modulational instability of ion-acoustic waves in warm plasma with nonthermal electrons. Han et al [9] have studied the existence of ion-acoustic solitary waves and their interaction in weakly relativistic two-dimensional thermal plasma. Electron acoustic solitons in relativistic plasma with nonthermal electrons has been studied by Sahu and Roy Chowdhury [10]. Gill et al [11] have studied the amplitude and width variations of ion-acoustic solitons in relativistic electron-positron-ion plasma. Regarding the relativistic effects on electron plasma waves only a very few works can be found in the literature. Recently Bharuthram and Yu [12] have shown that relativistic electron plasma waves can propagate as quasi-stationary nonlinear waves as well as solitary waves. All of the above works on the relativistic effects on plasma waves have been reported for classical plasma. But in plasmas, where the particle density is high and the temperature is low quantum phenomena becomes important. Such quantum plasma is ubiquitous in white dwarfs, neutron stars, galactic plasma, metal nanostructures, intense laser-solid interaction and in many other environments. In recent years propagation of various electrostatic modes such as ion-acoustic waves, electron-acoustic waves, dust-acoustic waves, dust ion-acoustic waves etc. in quantum plasma have been studied by many authors. Quantum effects in plasmas are usually studied with the help of two well-known formulations, viz. the Wigner-Poisson and the Schrödinger-Poisson formulations. The Wigner-Poisson model is often used in the study of quantum kinetic behaviour of plasma. The Schrödinger-Poisson model describes the hydrodynamic behaviour of plasma particles in quantum scales. The quantum hydrodynamic (QHD) model is derived by taking velocity space moments of the Wigner equations. The QHD model generalizes the classical fluid model for plasma with the inclusion of a quantum correction term also known as the Bohm potential [13]. The model incorporates quantum statistical effects through the equation of state. Because of simplicity, straight forward approach and numerical efficiency the QHD model has been widely used by several authors [13-18]. Different approaches for modelling quantum plasmas in electrostatic limit have been reviewed by Manfredi [14]. The QHD model as used by most authors is valid for quantum plasmas in the ultra-cold limit. But in most practical cases the plasma temperature is finite and not approaching zero. Recently Eliasson and Shukla [19] have developed nonlinear fluid equations taking into account the moments of the Wigner equation and by using the Fermi Dirac equilibrium distribution for electrons with an arbitrary temperature. The model thus developed is expected to describe a finite temperature quantum plasma. The linear and nonlinear properties of electron plasma waves in a quantum plasma have been studied by a few authors in the ultra-cold limit by using QHD model [14-18]. To the best of our knowledge no one has studied this problem including finite temperature effects. The motivation of the present paper is to study the relativistic

effects arising out of streaming motion on the linear and nonlinear properties of electron plasma waves in a finite temperature quantum plasma by using a finite temperature quantum hydrodynamic model [19].

## II. FINITE TEMPERATURE QHD MODEL EQUATIONS

The model as developed by Eliasson and Shukla [19] is based on 3D Fermi-Dirac equilibrium distribution for electrons with an arbitrary temperature. Propagation of plane longitudinal electron plasma waves in a collisionless quantum plasma leads to adiabatic compression along one dimension only and hence to a temperature anisotropy of the electron distribution. In quantum picture the classical incompressibility of phase fluid is violated by quantum tunneling. However to a first approximation one may assume the incompressibility of the electron phase fluid. It may also be assumed that the chemical potential ( $\mu$ ) remains constant during the nonequilibrium dynamics of plasma. Based on these assumptions one may consider the following nonequilibrium particle distribution function:

$$f(x, \vec{v}, t) = \frac{2(m/2\pi\hbar)^3}{\exp\left\{(\beta m/2)\left[(v_x - v_{ex})^2 \eta + v_y^2 + v_z^2\right] - \beta\mu\right\} + 1} \quad (1)$$

where  $m$  is the electronic mass,  $\hbar$  is the Plank constant divided by  $2\pi$ ,  $\beta = 1/k_B T_{e0}$ ,  $k_B$  is the Boltzman constant and  $T_{e0}$  is the background temperature,  $\mu$  is the chemical potential.  $v_{ex}$  is the mean velocity of the particles given by

$$v_{ex}(x, t) = \langle v_x \rangle = \frac{1}{n_e} \int v_x f d^3v \quad (2)$$

and  $\eta_{ex}(x, t) = T_{e0}/T_{ex}(x, t) = [n_0/n_e(x, t)]^2$  is the temperature anisotropy of the distribution function which is defined from the number density variations where

$$n_0 = \frac{1}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \int_0^\infty \frac{E^{1/2} dE}{\exp[\beta(E - \mu) + 1]} = -\frac{1}{2\pi^2 \beta^{3/2}} \left(\frac{2m}{\hbar^2}\right)^{3/2} \Gamma\left(\frac{3}{2}\right) Li_{3/2}[-\exp(\beta\mu)] \quad (3)$$

$Li_j(y)$  is the polylogarithm function. In the ultracold limit i.e  $T \rightarrow 0$ , we have  $\beta \rightarrow \infty$  and  $\mu \rightarrow E_F$ . where

$$E_F = (3\pi^2 n_0)^{2/3} (\hbar^2 / 2m) \quad (4)$$

Now using the zeroth and first moments of the Wigner equation with the Fermi-Dirac distribution function and assuming that the Bohm potential is independent of the thermal fluctuations in a finite temperature plasma one can derive the continuity and momentum equation in the following form:

$$\frac{\partial n_e}{\partial t} + \frac{\partial(n_e v_{ex})}{\partial x} = 0 \quad (5)$$

$$\frac{\partial v_{ex}}{\partial t} + v_{ex} \frac{\partial(v_{ex} \gamma)}{\partial x} = \frac{e}{m_e} \frac{\partial \phi}{\partial x} - \frac{n_0 V_{Te}^2}{n_e} G \frac{\partial(n_e/n_0)^3}{\partial x} + \frac{\hbar^2}{2m_e^2 \gamma^2} \frac{\partial}{\partial x} \left[ \frac{1}{\sqrt{n_e}} \frac{\partial^2 \sqrt{n_e}}{\partial x^2} \right] \quad (6)$$

where  $n_e$  and  $v_{ex}$  are respectively the particle density and fluid velocity of electron;  $\phi$  is the electrostatic wave potential and

$V_{Te} = \sqrt{k_B T_{Te} / m_e}$  is the themal speed and  $\gamma = (1 - v_{ex}^2/c^2)^{-1/2}$  is the relativistic factor for electrons where 'c' is the velocity of light in free space.  $G$  is the ratio of two polylogarithm functions given by:

$$G = \left[ Li_{5/2}(-\exp[\beta\mu]) \right] / \left[ Li_{3/2}(-\exp[\beta\mu]) \right] \quad (7)$$

The system is closed under the Poisson equation,

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e (n_e - n_i) \quad (8)$$

We now introduce the following normalization:

$$x \rightarrow x \omega_{pe} / V_{Fe}, t \rightarrow t \omega_{pe}, \phi \rightarrow e\phi / 2k_B T_{Fe}, n_j \rightarrow n_j / n_0 \text{ and } u_j \rightarrow u_j / V_{Fe}$$

where  $\omega_{pe} = \sqrt{4\pi n_0 e^2 / m_e}$  the electron plasma oscillation frequency and  $V_{Fe} = \sqrt{2k_B T_{Fe} / m_e}$  is the Fermi speed of electrons. Using the above normalization Eqs. (4)- (6) can be written as:

$$\frac{\partial n_e}{\partial t} + \frac{\partial(n_e v_{ex})}{\partial x} = 0 \quad (9)$$

$$\left( \frac{\partial}{\partial t} + v_{ex} \frac{\partial}{\partial x} \right) v_{ex} \gamma = \frac{\partial \phi}{\partial x} - 3G \alpha^2 n_e \frac{\partial n_e}{\partial x} + \frac{H^2}{2\gamma^2} \frac{\partial}{\partial x} \left[ \frac{1}{\sqrt{n_e}} \frac{\partial^2 \sqrt{n_e}}{\partial x^2} \right] \quad (10)$$

$$\frac{\partial^2 \phi}{\partial x^2} = (n_e - n_i) \quad (11)$$

where  $H = \hbar \omega_{pe} / 2 k_B T_{Fe}$  is a nondimensional quantum parameter proportional to the quantum diffraction and  $\alpha = (V_{Te} / V_{Fe})$ . The parameter H is proportional to the ratio between the plasma energy  $\hbar \omega_{pe}$  (energy of an elementary excitation associated with an electron plasma wave) and the Fermi energy  $k_B T_{Fe}$ .

### III. DISPERSION CHARACTERISTICS

In order to investigate the nonlinear behaviour of electron plasma waves we make the following perturbation expansion for the field quantities  $n_e$ ,  $v_{ex}$ ,  $n_{ec}$  and  $\phi$  about their equilibrium values:

$$\begin{bmatrix} n_e \\ v_{ex} \\ \phi \end{bmatrix} = \begin{bmatrix} 1 \\ u_0 \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} n_e^{(1)} \\ v_{ex}^{(1)} \\ \phi^{(1)} \end{bmatrix} + \varepsilon^2 \begin{bmatrix} n_e^{(2)} \\ v_{ex}^{(2)} \\ \phi^{(2)} \end{bmatrix} + \dots \quad (12)$$

Substituting the expansion (12) in Eqs. (9)-(11) and then linearizing and assuming that all the field quantities vary as  $e^{i(kx - \omega t)}$ , we get for normalized wave frequency  $\omega$  and wave number  $k$ , the following linear dispersion relation :

$$(\omega - ku_0)^2 \gamma_3 = 1 + 3G\alpha^2 k^2 + \frac{H^2 k^4 \gamma_2}{4} \quad (13)$$

where  $\gamma_3 = 1 + \frac{3u_0^2}{2c^2}$  and  $\gamma_2 = 1 - \frac{u_0^2}{2c^2}$ .

In the dimensional form the dispersion relation can be written as:

$$(\omega - ku_0)^2 \gamma_3 = \omega_{pe}^2 + 3Gk^2 V_{Te}^2 + \frac{k^4 V_{Fe}^4 H^2 \gamma_2}{4\omega_{pe}^2} \quad (14)$$

The degeneracy parameter G as defined by the Eq. (7) determines the transition between the ultra cold and thermal cases. In the low temperature limit  $\beta\mu \rightarrow \infty$ ,  $\mu \approx E_F \equiv (mV_{Fe}^2)/2$  and  $G \approx 2\beta E_F/5$ , then the dispersion relation (14) takes the form

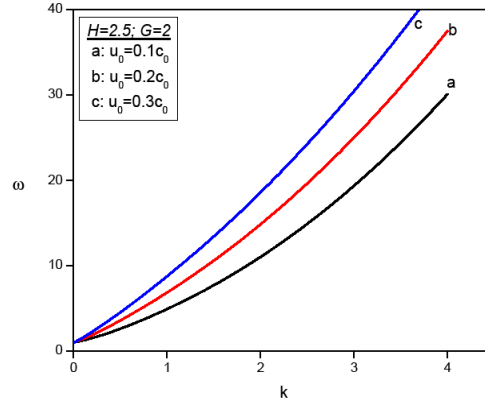
$$(\omega - ku_0)^2 \gamma_3 = \omega_{pe}^2 + \frac{3}{5} k^2 V_{Fe}^2 + \frac{k^4 V_{Fe}^4 H^2 \gamma_2}{4\omega_{pe}^2} \quad (15)$$

which is similar to the dispersion relation for electron plasma waves in a quantum plasma obtained by using one dimensional QHD Model [15]. In the high temperature limit  $\beta\mu \rightarrow -\infty$  so that  $G \rightarrow 1$  and then the dispersion relation (14) reduces to the Bohm-Gross dispersion relation for electron plasma waves in a hot plasma

$$(\omega - ku_0)^2 \gamma_3 = \omega_{pe}^2 + 3k^2 V_{Te}^2 + \frac{k^4 V_{Fe}^4 H^2 \gamma_2}{4\omega_{pe}^2} \quad (16)$$

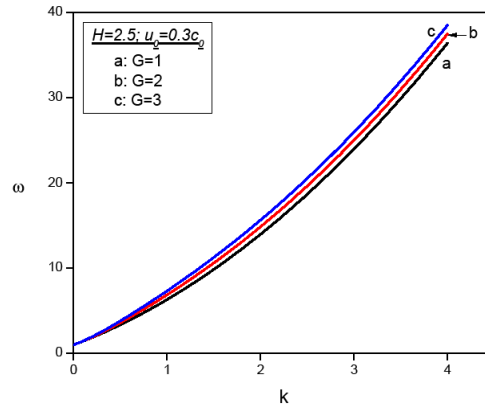
In the high temperature limit the last term on the RHS may be neglected and then one gets the well known Bohm-Gross dispersion relation of electron plasma waves in a hot plasma. For a relativistic classical cold streaming plasma the dispersion relation (15) reduces to  $\omega = ku_0 \pm \omega_{pe} / \sqrt{1 + 3u_0^2/c^2}$ , which indicates two modes of propagation under

the condition  $ku_0 > \omega_{pe} / \sqrt{1 + 3u_0^2/c^2}$ .



**Figure 1.** Dispersion curve for different values of streaming velocity  $u_0$  for fixed values of degeneracy parameter  $G$  and quantum diffraction parameter  $H$ .

The dispersion relation (13) is plotted in Fig.1 both in the nonrelativistic and relativistic limits with the quantum diffraction parameter  $H= 1.5$  and for different values of the streaming velocity. Obviously the wave frequency is enhanced by the relativistic effects.



**Figure 2.** Dispersion curve for different values of degeneracy parameter  $G$  for fixed values of streaming velocity  $u_0$  and quantum diffraction parameter  $H$ .

Fig.2 shows that the wave frequency is also increased with increase in the electron degeneracy parameter  $G$  for fixed values of quantum diffraction parameter  $H$  and streaming velocity  $u_0$ .

The group velocity  $c_g = d\omega/dk$  is obtained from the dispersion relation (13) as

$$c_g = \frac{3G\alpha^2 k + (\gamma_2 H^2 k^3 / 2)}{\sqrt{\gamma_3 \left[ 1 + k^2 \left\{ 3G\alpha^2 + (\gamma_2 H^2 k^2 / 4) \right\} \right]}} + u_0 \quad (17)$$

#### IV. DERIVATION OF THE KDV EQUATION

In order to study the nonlinear behavior of electron plasma waves we use the standard reductive perturbation technique and the usual stretching of the space and time variables:

$$\xi = \varepsilon^{1/2} (x - Vt) \quad \text{and} \quad \tau = \varepsilon^{3/2} t \quad (18)$$

where  $V$  is the normalized linear velocity of the and  $\varepsilon$  is the smallness parameter measuring the dispersion and nonlinear effects.

Now writing the Eqs. (9)-(11) in terms of these stretched co-ordinates  $\xi$  and  $\tau$  and then applying the perturbation expansion (12) and solving for the lowest order equation with the boundary condition  $n_e^{(1)}, u_e^{(1)},$  and  $\phi^{(1)} \rightarrow 0$  as  $|\xi| \rightarrow \infty$ , the following solutions are obtained:

$$n_{ex}^{(1)} = \frac{\phi^{(1)}}{3G\alpha^2 - \gamma_3(V-u_0)^2}, v_{ex}^{(1)} = \frac{(V-u_0)\phi^{(1)}}{3G\alpha^2 - \gamma_3(V-u_0)^2}. \quad (19)$$

and then going for the next higher order terms in  $\epsilon$  and following the usual method we obtain the desired Korteweg de Vries (KdV) equation:

$$\frac{\partial \phi}{\partial \tau} + A\phi \frac{\partial \phi}{\partial \xi} + B \frac{\partial^3 \phi}{\partial \xi^3} = 0 \quad (20)$$

$$A = \frac{(3G\alpha^2 + 3\gamma_3(V-u_0)^2 + 2\gamma_1(V-u_0)^2)}{2(V-u_0)(3G\alpha^2 - V^2)} \quad (21)$$

in which  $\gamma_1 = u_0 / c^2$

$$B = \frac{(3G\alpha^2 - \gamma_3(V-u_0)^2)^2 - \gamma_2 H^2 / 4}{2(V-u_0)} \quad (22)$$

To find the solution of Eq. (20) we transform the the independent variables  $\xi$  and  $\tau$  into one variable  $\eta = \xi - M \tau$  where  $M$  is the normalized constant speed of the wave frame. Applying the boundary conditions that as  $\eta \rightarrow \pm \infty$ ;  $\phi, D_\eta \phi, D_\eta^2 \phi \rightarrow 0$  the possible stationary solution of equation (20) is obtained as:

$$\phi = \phi_m \sec h^2 \left( \frac{\eta}{\Delta} \right) \quad (23)$$

where the amplitude  $\phi_m$  and width  $\Delta$  of the soliton are given by:

$$\phi_m = 3 \frac{M}{A} \quad (24)$$

$$\text{and } \Delta = \sqrt{\frac{4B}{M}} \quad (25)$$

For the existence of soliton solution we require  $B > 0$ . It requires that  $3G\alpha^2 < \gamma_3(V-u_0)^2 - (\gamma_2 H/2)$  or  $3G\alpha^2 > \gamma_3(V-u_0)^2 + (\gamma_2 H/2)$ . The nature of the solitary waves i.e. whether the system will support compressive or rarefactive solitary waves depends on the sign of  $A$ . If  $A$  is positive (or negative) a compressive (or rarefactive) solitary wave is excited. Thus for  $3G\alpha^2 < \gamma_3(V-u_0)^2 - (\gamma_2 H/2)$  rarefactive soliton and for  $3G\alpha^2 > \gamma_3(V-u_0)^2 + (\gamma_2 H/2)$  compressive soliton is formed. From Eq (22) it is clear that the dispersive coefficient  $B$  vanishes for two critical values of the diffraction parameter  $H$ , given by

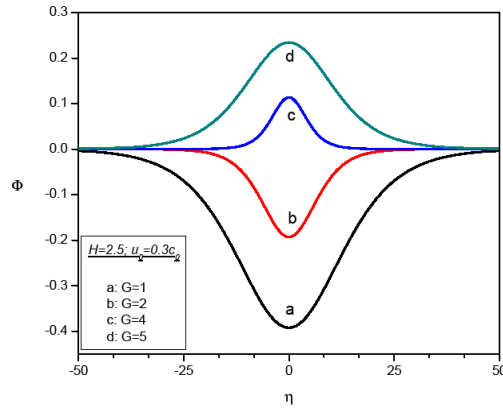
$$H_{c1} = 2(3G\alpha^2 - \gamma_3(V-u_0)^2) / \gamma_2 \quad \text{for } 3G\alpha^2 > \gamma_3(V-u_0)^2 \quad (26a)$$

$$H_{c2} = 2(\gamma_3(V-u_0)^2 - 3G\alpha^2) / \gamma_2 \quad \text{for } 3G\alpha^2 < \gamma_3(V-u_0)^2 \quad (26b)$$

At these values of  $H$  no soliton solution is possible. For  $H < H_{c1}$  compressive solitons and for  $H < H_{c2}$  rarefactive solitons are obtained.

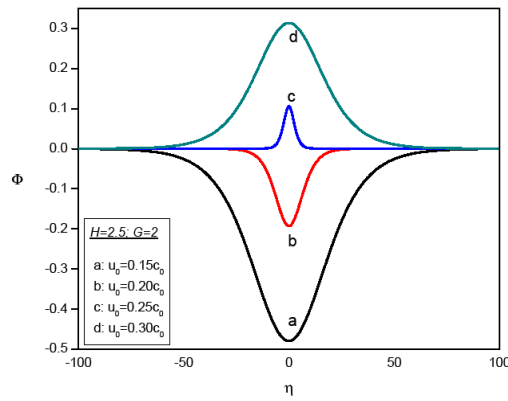
## V. RESULTS AND DISCUSSION

Using the nonlinear quantum fluid equations for electrons with an arbitrary temperature and the standard reductive perturbation technique both the linear and nonlinear properties of electron-plasma waves have been investigated. The electron degeneracy parameter  $G$  and the relativistic effects are shown to influence the linear and nonlinear properties of the electron plasma waves in a significant way. The wave frequency increases with increase in both the degeneracy parameter  $G$  and streaming velocity  $u_0$ . However, the wave frequency is found to be more sensitive to the variation of streaming motion than the electron degeneracy for a fixed value of wavenumber. The model plasma under consideration can support both compressive and rarefactive types of soliton. Soliton amplitude and width are found to depend significantly on the degeneracy parameter  $G$  and streaming velocity  $u_0$  (Figs. 3 and 4).



**Figure 3.** Solitary structures for different values of degeneracy parameter  $G$  for fixed values of streaming velocity  $u_0$  and quantum diffraction parameter  $H$ .

Fig. 3 shows that both the amplitude and width of the compressive solitons increase with increase in  $G$  whereas for rarefactive solitons the amplitude and width decreases with increase in the value of  $G$  for a fixed value of the streaming velocity.



**Figure 4.** Solitary structures for different values of streaming velocity  $u_0$  for fixed values of degeneracy parameter  $G$  and quantum diffraction parameter  $H$ .

Fig. 4 shows that similar to the effect of electron degeneracy an increase in streaming velocity  $u_0$  increases both the amplitude and width of the compressive solitons whereas for rarefactive solitons the amplitude and width decreases with increase in the value of  $u_0$  for fixed value of degeneracy parameter  $G$ . As the degeneracy parameter  $G$  determines the transition from ultracold to thermal cases it is important to have an idea about its value for certain practical plasmas. Table-1 shows the values of  $G$  for certain practical plasmas.

**Table: I:** DEGENERACY PARAMETER OF SOME PRACTICAL PLASMAS

Type of Plasma	Density( $m^{-3}$ )	Temperature(K)	G
Tokamak	$10^{20}$	$10^{18}$	1
Inertial Confinement Fusion	$10^{32}$	$10^8$	1
Metal and Metal clusters	$10^{28}$	$10^4$	1.4
Jupiter	$10^{32}$	$10^4$	1.4
White Dwarf	$10^{35}$	$10^8$	4

Finally we would like to point out that the investigation presented here may be helpful in the understanding of the basic features of long wavelength electron plasma waves in dense and hot plasmas such as can be found in white dwarfs, neutron stars and intense laser-solid plasma experiments.

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