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Row and Column Concatenation methods for Stability Testing of Two Dimensional Recursive Filters

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Abstract—The stability testing of first quadrant quarter-plane (QP) two dimensional recursive digital filters had been a classical problem for the last two decades. In literature, the two major types of stability testing methods available are algebraic and mapping methods. Even though the algebraic methods can determine the stability in finite number of steps, it has a few practical limitations in its implementations and it consumes very large time to find the exact stability as it requires larger number of calculations. Moreover, the accuracy of the algebraic methods is also affected by the finite word length effects of the computer. The mapping methods, in general cannot guarantee the stability of given recursive digital filter in finite number of steps as it can determine the stability only in infinite number of steps. Out of the mapping methods, Jury's row and column algorithms have been considered as highly efficient, even though they still run short of accuracy due to the undefined length of the FFT used. Hence, the best mapping method is not yet available, though some researchers like Bistritz have been working on this problem even now.The main aim of this paper is to find the simple and fast solution for testing the stability of first quadrant quarter-plane two dimensional recursive digital filters. In this work, it is assumed that the given transfer function is devoid of non-essential singularities of second kind. A new mapping stability test procedure for two dimensional quarter-plane recursive digital filter (QP) is proposed which is primarily based on the row and column concatenation method. One important advantage of the proposed test procedure is that it requires single 1-D polynomials to be tested for stability for two dimensional recursive digital filters of any order.This method gives a simple and best solution procedure even for barely stable or barely unstable recursive digital filters. Sufficient examples are taken from the literature for two dimensional recursive digital filters and the same accurate results as that found in the literature are also obtained using this method.

Index Terms- Stability ,Mapping methods, Quarter Plane filters(QP),Non-symmetric half plane filters(NSHP).

I. INTRODUCTION

The stability testing methods and stabilization procedures of 2-D recursive filters have been available in the literature as a classical, important and interesting area of research for the past two decades. The problem of testing the 2-D recursive filters, devoid of non essential singularities of second kind, has attracted the attention of many researchers. T.S.Huang [6] has given many useful, effective and important methods for the stability of 2-D recursive filters Still many researchers are working on this stability testing problem and have suggested many solution methods based on complex integrals,[29] telepolation of 2-D tabular tests..etc. In this work, we present a simple and best mapping based stability testing procedure for the first quadrant QP 2-D recursive digital filters. Then, it is extended to NSHP and multidimensional recursive digital filters.

In this paper, section II deals with the review of 2 D recursive digital filters and also about the multidimensional digital signal processing. In section III, we deal the mapping methods for testing the stability of 2-D recursive digital filters and the need for the optimum L value in row and column concatenation methods. In section IV**,** the simplified form of concatenation methods with optimum value of L is proposed by the authors and sufficient examples are provided. In section V, the extension of the proposed stability test procedure for testing the NSHP filters is presented. In section VI, stability testing of multidimensional recursive filters is dealt with and a few examples are given and the final section VII lists the various inferences and conclusions on this paper followed by the references.

II. REVIEW OF 2-D RECURSIVE FILTERS

In this section, we briefly review the 2-D recursive digital filters and their two types namely, QP and NSHP filters. In multidimensional digital signal processing, digital filters are more specifically concerned with the processing of signals represented in the form of multidimensional arrays. These are sampled simultaneously from many input sensors and represented as sampled images or sampled waveforms. The field of two-dimensional signal processing has received resurgence during last two decades because of its use in many image and array processing applications like seismic record processing, medical X-ray processing, image de-blurring, weather predictions, processing of RADAR and SONAR arrays etc.,.

Two dimensional (2-D) recursive filters are widely used in many applications because of their reduced computational time and less storage requirements provided linear phase is not of great concern. Recursive digital filter is useful only if it is stable, i.e. their outputs remain bounded for all bounded inputs (BIBO stable). Many earlier design procedures do not ensure this necessary stability condition for recursive filters where some later designs incorporate the stability checking procedures at the end of iterations in the computer aided filter design. For the one-dimensional filters, the difference equations which are used to describe the digital recursive filters are totally ordered, whereas for the multidimensional filters they are only partially ordered. The sampling rate of a band limited signal of one-dimensional filters can be adjusted whereas for the multidimensional ones, the geometric arrangements of samples should also be adjusted along with sampling rate. In multi-dimensional case, the polynomials cannot be factored whereas in one-dimension, polynomials can be factorized.

Multi dimensional digital signal processing is quite different from one-dimensional digital signal processing. This is due to the following three factors [1].

1. Two-dimensional problems generally involve considerably more data than one-dimensional ones.
2. The mathematics for handling multidimensional system is less complete than the mathematics for handling one-dimensional system.
3. Multidimensional systems have many degrees of freedom, which gives a system designer more needed flexibility that is not encountered in the one-dimensional systems.

The recursive 2-D digital filters represented by recursive filter array b (m, n) are stable if and only if the impulse response is absolutely summable, that is $\sum\sum |h(m,n)| < \infty$. Even if these impulse responses are rotated by a multiple of 90º, transposed or flipped off and the resulting sequences are also still absolutely summable [2].

In other words, the stability of the recursive digital filter is determined by the properties of its z-transform of the denominator polynomial in the transfer function. In one dimensional filter, this property is given as no zeros of the denominator z-transform polynomial should lie out side the unit circle [2]. That is, the denominator polynomial must have the "minimum phase property". In two dimensional cases, the recursive digital filter is stable if and only if

$B(z_1, z_2) \neq 0$ for $|z_1| \geq 1$ and $|z_2| \geq 1$ (1)

where $B(z_1, z_2)$ is the denominator z-transform polynomial of the recursive digital filter transfer function $H(z_1, z_2)$, assuming that there are no non-essential singularities of the second kind [6].

For testing the stability of a two dimensional recursive digital filters, direct check of the absolute summability condition is very difficult. Also for non - trivial filters, checking the condition, $B(z_1, z_2) = 0$ for $|z_1| \geq 1$ and $|z_2| \geq 1$ is not so easy as compared with that of one dimensional recursive digital filter. This is due to the fact that the fundamental property of algebra namely, factorizability does not extend to two dimensional polynomials.

Due to the initial conventions and historical reasons, to test 2-D recursive filters for stability the researchers have defined the z-transform with positive powers of z. Due to this, condition given in equation (1) gets changed to

$B(z_1, z_2) \neq 0$ for $|z_1| \leq 1$ and $|z_2| \leq 1$ (2)

Based on this, several equivalent theorems appear in the literature [6] for stability testing. One such theorem is as follows

A Theorem 1:

The 2-D Polynomial quarter plane polynomial is stable iff

(a) $B(z, 0) \neq 0$ for $|z| \leq 1$

(b) $B(z_1, z_2) \neq 0$ for $|z_1| = 1$, $|z_2| \leq 1$

In general, it is much easier to check the one dimensional condition (a) above by determining whether one dimensional polynomial is a minimum phase, that is, whether all its zeros are out side the unit circle. For this type of testing, Jury's table test [9] is the well defined and most efficient procedure. The major computational effort for testing is required for checking whether $B(z_1, z_2)$ satisfies condition (b).

B Region of support:

According to the region of supports, 2-D filters are classified into three types. They are

1. Quarter plane causal recursive filter (QP-filter)
2. Non symmetric half plane recursive filter (NSHP filter)
3. General Recursive filters.

A two dimensional linear shift invariant filter with an impulse response h (m, n) is called as quarter plane filter (QP-filter), if h (m, n) has the support only in any one of the quadrants. A two dimensional recursive filter is characterized by the difference equation which relates the input and output as,

$$y(m,n) = \sum_{k,l}\sum_{\in\alpha} a(k,l)\,x(m-k,n-l) - \sum_{k,l}\sum_{\in\beta-(0,0)} d(k,l)\,y(m-k,n-l) \quad (3)$$

where $a(k, l)$ and $d(k, l)$ are real finite extent arrays with respective lattice supports of α and β, x(m, n) and y(m, n) are the respective input and output arrays . The support of an array value is defined as the set of all ordered integer pairs where the array is not zero. Further more b (0, 0) = 1 and that β is contained in a lattice sector with vertex (0, 0) of angle less than π. The support of d(k, l) is shaped so that there exists a half plane of lattice points defined by a line passing through the origin such that the half plane intersects β only at (0,0). These conditions guarantee that for all finite extent input arrays and a class of infinite extent inputs, equation (3) can be solved by incrementing the values of the indices (m, n) in such a fashion that all values of the output can be computed in turn from a given set of initial conditions. If the zero conditions are assumed in equation (3) and then impulse response of the system represented by equation (3) becomes

$$h(m,n) = a(m,n) - \sum_{k,l} \sum_{\in \beta-(0,0)} d(k,l)\, h(m-k, n-l) \quad (4)$$

If a (m, n) and d (m, n) are first quadrant arrays, i.e. both a (m, n) and d (m, n) are zero for any negative values of m or n, then the recursive filter recourses in the (+m, +n) direction. The output has to be computed using previously computed values which are to the left or below the present output point. If a (m, n) and d (m, n) are second quadrant arrays, that is both a (m, n) and d (m, n) have supports contained in the second quadrant, then recourses in the (-m, n) direction. Third and fourth quadrant filters can be defined accordingly. A first quadrant filter belongs to the (++) class, second-quadrant (−+) class, third quadrant (− −) class and fourth quadrant (+ −) class [1].

In non-symmetric half-plane filters the filter coefficients a (m, n) and d (m, n) have supports in any of the two adjacent quadrants with vertex (0, 0) and with the sector angle less than π. The main difference between QP and NSHP filters is that the way the output masks are defined. The NSHP filters are superior to QP due its output mask which is more general than that of QP filters. Totally, there are eight classes of non-symmetric half plane (NSHP) recursive digital filters based on the region of support. The NSHP Filters are more frequently used because these filters have positive definite magnitude characteristics. In general filter, the only restriction is the filter coefficients a (m, n) and d (m, n) have the supports with vertex (0, 0) and with the sector angle less than π

III. MAPPING METHODS FOR TESTING THE STABILITY OF 2-D RECURSIVE DIGITAL FILTERS

In this section, we describe briefly the important stability testing methods of 2-D recursive digital filters called as mapping methods.

The mapping methods need infinite number of steps to determine stability of the digital filter exactly which is refereed as approximate methods in the literature. In mapping methods, checking the condition B (z_1, z_2) $\neq 0$ for $|z_1| = 1, |z_2| \leq 1$, is to effectively map the circle $|z_1| = 1$ into z_2 plane based on B (z_1, z_2) = 0 and to determine whether the image lies out side the disc $|z_2| \leq 1$. Here the condition is satisfied iff none of these curves intersects the unit disc. If any of these curves do intersect the unit disc, implies that there exists a B (z_1, z_2) = 0, for $|z_1| = 1$, $|z_2| \leq 1$. Therefore, in theory, the stability of a filter can be determined exactly by checking the zero distribution of the one dimensional polynomial in z_2, B(z_1, z_2) for each fixed z_1 with $|z_1| = 1$. Since z_1 has unity magnitude, we will alternatively specify these one-dimensional polynomials with a parameter u where $z_1 = e^{ju}$ for $0 \leq u < 2\pi$ as

$$B\left(e^{-ju}, z_2\right) = \sum_{n=0}^{N} \left[\sum_{m=0}^{N} b(m,n)\, e^{-jum} \right] z_2^n \quad (5)$$

The methods which attempt to obtain stability of two dimensional recursive filters by examining $B\left(e^{-ju}, z_2\right)$ for $0 \leq u \leq 2\pi$ is called mapping methods. For each fixed u_1, $B\left(e^{-ju_1}, z_2\right)$ is a one dimensional polynomial with complex coefficients. If $B\left(e^{-ju_1}, z_2\right)$ has no zeros inside or on the unit circle for all u_1, that is, $B\left(e^{-ju_1}, z_2\right)$ must be stable one dimensional polynomial. Therefore, for each value of u, any root distribution method which can handle polynomials with complex coefficients may be used to check the stability of $B\left(e^{-ju}, z_2\right)$ [4].

In practice, the one dimensional stability of $B\left(e^{-ju}, z_2\right)$ cannot be checked for all the possible values of u. Only for a finite number of u_j $B\left(e^{-ju_j}, z_2\right)$ can be examined for stability. Once the interval (0, 2π) is sampled on a countable grid the accuracy of the method suffers. For particular u_j, $B\left(e^{-ju_j}, z_2\right)$ has zeros inside or on the unit circle, then the filter is definitely unstable. However, if $B\left(e^{-ju_j}, z_2\right)$, j = 0, 1… L-1 are all stable, this does not imply that B (z_1, z_2) $\neq 0$ on $|z_1| = 1$, $|z_2| \leq 1$. Even if we check $B\left(e^{-ju_j}, z_2\right)$ for a very large set, stability cannot be guaranteed.

Although there exist examples, where the mapping methods will fail, for the vast majority of filters a fine enough sampling of (0, 2π) will yield good results. Consider U_0 and V_0 as a set of all points in the interval (0, 2π). From the fact that the root maps of B (z_1, z_2) =0 form continuous curves and hence if there exists a (u_0, z_0) with $|z_0| < 1$ such that $B\left(e^{-ju_o}, z_0\right)$ =0 then there exists an interval around u_o such that if u Є U_o, $B\left(e^{-ju}, z_2\right)$ will have at least one zero inside or on the unit circle. Therefore, if for one value of L, the number of samples on (0, 2π) is large enough, then u_j will fall in the interval U_o and the test will determine that the filter is unstable. Hence, the number of samples, L, determines the accuracy of the test. If larger L is chosen, more polynomials $B\left(e^{-ju_j}, z_2\right)$ j = 0, 1, 2... L-1, with complex coefficients are checked for stability and hence the test is more time consuming.

The root mapping method is based on B (z_1, z_2) ≠ 0 on $|z_1|$ =1, $|z_2| \leq 1$ iff the root maps of $B\left(e^{-ju_j}, z_2\right)$ as a function of z_2 do not intersect the unit disc $|z_2| \leq 1$. Huang [4]and Shanks [6] implemented this test by calculating the minimum magnitude root of $B\left(e^{-ju_j}, z_2\right)$ for a finite number of u_j. These roots are then plotted in the z_2 plane to form a curve. The filter is unstable if this curve intersects the unit disc. This method uses FFT algorithm which requires L $\log_2$ L operations. This implementation becomes increasingly more efficient as the value of L is increased [2][10].

Jury's row algorithm and column algorithm is computationally less complex, but the algorithm uses the FFT and the size of the FFT determines the accuracy of the method. Suppose L is the size of the FFT used, (L/2 + 1) numbers of second degree one dimensional polynomials of complex coefficients are to be tested for stability using Jury - Marden algorithm. Bistritz method [12] [13] brings down the problem of 2-D filter stability testing to testing a set of 1 D polynomials with real coefficients. But this procedure is time consuming.

The one dimensional sequence associated with column concatenation has a representation of B (λ^N, λ) while row concatenation has one of B (λ, λ^M). These sequences can have both positive and negative indices. If b (m, n) is stable then for any integers M and N,

Ind [B(λ^N, λ)] = Ind[B(λ, λ^M)] = 0 (6)

where Ind indicates the number of encirclements of the origin.

However if b (m, n) is not stable and if Ind [B (λ, 1)] = Ind [B (1, λ)] = 0.Then in most cases, if N or M is large enough in magnitude, then

Ind [B (λ^N, λ)] ≠ 0 and / or Ind [B(λ, λ^M)] ≠ 0. (7)

Therefore if for any integer value of M or N equation (6) does not hold then b (m, n) is unstable. Moreover, if equation (7) holds for very large values of M or N then we can be "almost sure" that b (m, n) is stable.

The accuracy of these tests can be increased to almost any desired level by proper selection of N, M values. In some critical cases, it was found that the most accurate algebraic test may fail when implemented on a computer with its finite precision arithmetic. Moreover, for the filters of order greater than four or so in either dimension, the algebraic methods are difficult to program and consumes more computational time. Hence for the moderate and higher order filters, these mapping methods are the only feasible choice of solution.

IV. SIMPLIFIED FORM OF CONCATENATION METHODS

In the concatenation method which is briefly mentioned in the previous section, one has to test first 1-D polynomials B (z, 1) and B (1, z) for stability where B (z_1, z_2) is the denominator polynomial of the transfer function H (z_1, z_2). If these two 1-D polynomials do not have zeros inside the unit circle then we form the 1-D column concatenated version of B (z_1, z_2), namely B (z^L,z) and the row concatenated version B(z,z^L) for some suitable value of L and check whether these two 1-D polynomial have zeros inside the unit circle for all values of L until the value taken for L is fairly high. If they all don't have zeros inside the unit circle we may conclude that the 2-D polynomial B (z_1, z_2) is stable.

So this method requires testing of several 1-D concatenated 1-D polynomials before we make any decision regarding its stability. In this section we try to see whether there is optimum value of L which makes it possible for us to test the polynomials B(z^L, z) and B(z, z^L) only for that one value of L and then conclude whether the first quadrant 2-D quarter plane polynomial B(z_1,z_2) is stable or not.

A. Optimum value of L:

To test B (z_1, z_2) for condition 1(b) of Theorem 1, by the above method, let us consider that B(z_1,z_2) =0, for some $z_1 = e^{j\theta_{10}}$($0\leq\theta_{10}\leq 2\pi$) and $z_2 = r^{j\theta 2}$ (r<1, $0\leq \theta_2 \leq 2\pi$) This will be reflected in B(z^L,z)in the following ways since $z_1 = z^L$ and z_2 = z.

1) $\theta_2 = \dfrac{2\pi n + \theta_{10}}{L}$ = θ, where $z_1 = e^{j\theta}$
2) When L = 36,the resolution for the angle is one second.
3) With this one second resolution which is available for θ_2 ,when the value of r if it is less than or equal to 1 (z_2 = inside or on the unit circle) will be certainly revealed in some zero of B ($e^{j\theta_{10}}$, r $e^{j\theta_2}$).

Thus the instability of B (z_1, z_2) will be exposed. So we need to give a value for L as 36 while testing the 1-D polynomials B(z^L,z) and B(z,z^L)to know about the instability of B(z_1,z_2) by not satisfying condition 1(b) of Theorem 1.

Example: 1

Consider the critical polynomial of [17] [page 138, Example 9]

B (z_1, z_2) =$0.2718z_1^2z_2^2-0.72z_1^2z_2+0.6z_1^2-0.75z_1z_2^2+1.8z_1z_2-1.5z_1+0.5z_2^2-1.2 z_2+1$.

- Let $z_1=z^{36}$; z_2=z; then it becomes B (z) =$0.2718z^{74}-0.72 z^{73}+0.6 z^{72}-0.75 z^{38}+1.8 z^{37}-1.5 z^{36}+0.5 z^2-1.2 z+1$.
- Let $z_2=z^{36}$; z_1=z; then it becomes B(z) = $0.2718z^{74}-0.75 z^{73}+0.5 z^{72}-0.72 z^{38}+1.8 z^{37}-1.2 z^{36}+0.6 z^2-1.5 z+1$.

It is found that both the above one dimensional polynomials are stable by having no zeros inside or on the unit circle when tested with L=24. Both the 1-D polynomials are found to be unstable having a pair of complete conjugate zeros inside the unit circle when tested with L=36.

Example: 2
Consider a stable two dimensional polynomial [5]
$B(z_1,z_2)=1.0+0.90441z_1+0.39818z_1^2+0.87742z_2+0.65133z_1z_2+0.21555z_1^2z_2+0.39488z_2^2+ 0.22754z_1z_2^2+0.04207z_1^2z_2^2$.

- Let $z_1=z$; $z_2=z^{36}$; then it becomes
  $B(z,z^{36})=1.0+0.90441z+0.39818z^2+0.87742z^{36}+0.65133z^{37}+0.21555z^{38}+ 0.39488z^{72}+0.22754z^{73}+0.04207z^{74}$
- Let$z_1=z^{36}$; $z_2=z$ then it becomes
  $B(z^{36},z)=1.0+0.90441z^{36}+0.39818z^{72}+0.87742z+0.65133z^{37}+0.21555z^{73}+0.39488z^2+0.22754z^{38}+0.04207z^{74.}$

Both are found to be stable for the value of the sampling factor L=36.

Example: 3
Consider a two dimensional polynomial B (z_1, z_2) whose coefficients are given by the 5x5 array b_{mn} (page 168, [5])

$$B(z_1, z_2) = \begin{bmatrix} 1 & z_1 & z_1^2 & z_1^3 & z_1^4 \end{bmatrix} \begin{bmatrix} 1.0 & 1.5 & -1.9 & -0.80 & 1.10 \\ 1.4 & 2.10 & -2.60 & -1.10 & 1.50 \\ -1.80 & -2.40 & 3.30 & 1.30 & -1.60 \\ -0.70 & -0.90 & 1.10 & 0.50 & -0.80 \\ 0.90 & 1.30 & -1.60 & -0.60 & 1.0 \end{bmatrix} \begin{bmatrix} 1 \\ z_2 \\ z_2^2 \\ z_2^3 \\ z_2^4 \end{bmatrix}$$

$B(z_1,z_2)=[1.0+1.4z_1-1.8z_1^2-0.7z_1^3-0.9z_1^4+1.5z_2+2.1z_1z_2-2.4z_1^2z_2-0.9z_1^3z_2+1.3z_1^4z_2- .9z_2^2-2.6z_1z_2^2+3.3z_1^2z_2^2+1.1z_1^3z_2^2-1.6z_1^4z_2^2-0.8z_2^3-1.1z_1z_2^3+1.3z_1^2z_2^3+0.5z_1^3z_2^3- z_1^4z_2^3+1.1z_2^4+1.5z_1z_2^4-.6z_1^2z_2^4-0.8z_1^3z_2^4+1.0z_1^4 z_2^4]$;

$B(z^{36}, z) = [1.0+1.4z^{36}-1.8z^{72}-0.7z^{108}-0.9z^{144}+1.5z+2.1z^{37}-2.4z^{73}-0.9z^{109}+1.3z^{145}-1.9z^2-2.6z^{38}+3.3z^{74}+1.1z^{110}-1.6z^{146}-0.8z^3-1.1z^{39}+1.3z^{75}+0.5z^{111}-.6z^{147}+1.1z^4+1.5z^{40}-1.6z^{76}-0.8z^{112}+1.0z^{148}]$;

$B(z, z^{36}) = [1.0+1.4z-1.8z^2-0.7z^3-0.9z^4+1.5z^{36}+2.1z^{37}-2.4z^{38}-0.9z^{39}+1.3z^{40}-9z^{72}-.6z^{73}+3.3z^{74}+1.1z^{75}-1.6z^{76}-0.8z^{108}-1.1z^{109}+1.3z^{110}+0.5z^{111}-0.6z^{112}+1.1z^{114}+1.5z^{145}-.6z^{146}-0.8z^{147}+1.0z^{148}]$.

It has been found that both the B (z^{36}, z) and B (z,z^{36}) are unstable as they have zeros inside the unit circle.

V. CONCLUSIONS

The method given in this paper presents a simple and best solution test for the stability testing of second or higher order two dimensional recursive digital filters. This test procedure is based on row or column concatenation method where we fix the exact value of L as 36 to use in $B(z^L, z)$ and / $B(z,z^L)$ to test the stability of $B(z_1,z_2)$.

The limitation of the row or column concatenation method which does not give any indication of up to what values of N or M, the polynomials $B(\lambda^N, \lambda)$ and $B(\lambda, \lambda^M)$ have to be tested by testing their 1-D stability while testing any second order 2-D polynomial $B(z_1,z_2)$ is overcome by this proposed simplified test procedure. Sufficient examples along with the accurate stability results have been given to prove this.

The accuracy of this method is evident from the results of the barely stable polynomial in example 1 and for the stable polynomial in example 2.The same method when it is employed to two QP filters, the results are accurate as seen in the examples 1,2 and 3 respectively.

This method is not suffering due to the finite word length effects of the computer as the number of calculations are very much reduced when compared to the algebraic method which involves many calculations of polynomial matrix determinants.

So the method proposed here is simple mainly due to less cumbersome calculations and the need to test the polynomials only for one value of L =36 which is sufficient instead of a range of values for deciding on the stability of the given digital filter.

As we mentioned earlier, our method is as accurate as any other method since it gave the same results as that of the other methods when applied to barely stable or barely unstable recursive filter denominator polynomials of any order and can be extended to any N dimensional recursive filter by the equivalent testing of a set of 2- D recursive filters.

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