International Journal of Engineering Research and Development
***e**-ISSN: 2278-067X, **p**-ISSN: 2278-800X, www.ijerd.com*
Volume 3, Issue 7 (September 2012), PP. 11-14

On the State Feedback Stabilization of Systems

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Abstract—This paper presents a technique for designing state controller for dynamic systems with uncertain parameters. The designed compensator ensures that the resulting closed-loop system will remain stable while the system matrices values vary throughout their respective intervals

Keywords— stability, controller, uncertain parameters, state feedback, dynamic system

I. INTRODUCTION

Stability analysis for the systems considering the real parametric uncertainties has been carried out and various results and techniques have been developed [1-3]. The important task of feedback design is to provide robustness of closed-loop control systems when a plant includes parametrical uncertainties and/or is affected by disturbances. If the uncertainties can be described by intervals with known lower and upper bounds then the plant can be presented as interval dynamical system.

There has been considerable progress towards developing robust design methods for multiple-input multiple-output systems [4,5,6,7,8]. In terms of the classical control concepts, Kalman's inequality (9) implies that single-input single-output LQ regulators possess 60 degree phase margin, infinite gain margin and 50% gain reduction tolerance.In [8], it has been shown that the multiple loop linear quadratic state-feedback (LQSF) regulators also have excellent robustness properties when measured by the classical criteria of gain and phase margin and can undergo simultaneous phase perturbations of up to +60 in each input channel, or simultaneous gain perturbations from 50% of nominal to infinite in each input channel. In practice, however, the LQSF regulator is merely an ideal case because not all states are available. When combined with an observer or state estimator the robustness of the whole system may considerably degrade. The LQ technique offers a good base for robust MIMO control system design. In this paper a procedure for designing robust controller is presented. This is based on the work of Bialas [10], concerning the stability of interval polynomials and on the work of Pearson [11] on dynamic compensator design.

Bialas [10] presented necessary and sufficient conditions for the stability of interval matrices and has given a theorem concerning the stability of interval polynomials. Based upon this result an algorithmic technique for determining a feedback compensator which will stabilize the interval dynamic system is presented in this paper. The algorithm essentially determines a feedback which ensures positivity of all the closed-loop Hurwitz determinants. Example has been presented which illustrate the substantially improved robustness which results from the algorithm.

Theorem

$S[A_{\min}, A_{\max}] \in G^n$ iff $T[A_{\min}, A_{\max}] \in G^n$

The set $T[A_{\min}, A_{\max}]$ contains 2^n polynomial and G^n is the set of all stable polynomials of degree n.

Proof of this theorem is given in [10].

II. COMPENSATOR DESIGN WITH STATE FEEDBACK

Given the plant to be controlled

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = C^T x(t) \qquad (1)$$

and given $A \in N[P,Q]$, $B \in N[R,V], C \in N[W,Z]$ where P and Q are n x n real matrices, R and V are n x m real matrices and W and Z are q x n matrices. Let the most likely values of A, B and C as A_0, B_0 and C_0 respectively.

The proposed design procedure is as follows:

(i) Using A_0, B_0, C_0 apply the LQ technique to determine the optimal feedback gain K_0

(ii) Adjust the feedback gain K so that the closed-loop system will remain stable while the parameters vary in the given intervals. Specifically, search for K to minimize $\|K - K_0\|$ such that the feedback interval dynamical system A+BK is stable when $A \in N[P,Q]$, $B \in N[R,V], C \in N[W,Z]$.

(iii) Check the performance of the feedback system with the selected K when the parameters take on their most likely values i.e $A = A_0, B = B_0, C = C_0$.

Remark 1

The parameters are random variables and are not time-varying stochastic processes. That is, A, B and C are treated as time invariant matrices which take their values in known intervals.

Remark 2

Minimization of $\|K - K_0\|$ is done in order to keep the feedback gains as close as possible to optimal so as not to alter the performance too much when the parameters take on their most likely values.

Remark 3

In order to solve step (i), assume that the plant is controllable and the weighting matrix Q_1 in the performance index is such that $[A_0, Q_1^{1/2}]$ is observable when the parameters are at their most likely values. Further, assume that the plant is controllable when the parameters are in their intervals.

III. COMPENSATOR DESIGN WHEN ALL THE STATES ARE NOT AVAILABLE

It is often the case that not all the states are available. Since we do not know the parameters precisely we cannot design an observer to reconstruct the states, and a general dynamic compensator, as described by [12,13,14] can be used. [13] have shown that for a controllable and observable plant a dynamic compensator can be designed which is sufficient to achieve arbitrary pole placement in the system consisting of the plant and the compensator in cascade.

For simplicity the procedure is presented for SISO regulator design. We assume that the plant to be controlled is both controllable and observable when its parameters vary in the given intervals.

Step 1

For the plant with its most likely values of parameters, apply the technique presented by [12] to design an (n-1)th order compensator with transfer function

$$G_{e0}(s) = \frac{-U(s)}{Y(s)} = \frac{\sum_{j=0}^{n-1} \beta_{j0} s^j}{s^{n-1} + \sum_{i=0}^{n-2} \alpha_{i0} s^i} \tag{2}$$

Step 2

Search for $\alpha_i (i = 0,1,...n-2), \beta_j = (j = 0,1,...n-1)$ so that the minimization of

$$\sum_{j=0}^{n-1} (\beta_j - \beta_{j0})^2 + \sum_{i=0}^{n-2} (\alpha_i - \alpha_{i0})^2 \tag{3}$$

is achieved, subject to the constraint that the closed-loop interval system consisting of the plant and the compensator with coefficients α_i, β_j is stable.

Check the performance of the closed-loop system designed.

IV. EXAMPLE

Given the plant to be controlled

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) \\ y(t) &= C^T x(t) \end{aligned} \tag{4}$$

where

$$A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C^T = \begin{bmatrix} 2 \\ -c_2 \end{bmatrix} \tag{5}$$

with $a \in [0.5, 1.5]$, $c \in [-1.7, 0.5]$ and their most likely values being $a_0 = 1$ and $c_{20} = -1$. We want to design a compensator so that the closed-loop system is stable for all possible values of a and c_2. It is easy to check that the plant is both controllable and observable if $a \neq 0$.

Step 1

Using the technique given by [11], first determine a dynamic compensator for a and c_2 at their most likely values. Since n = 2, this dynamic compensator should be first order i.e.

$$\frac{-u(s)}{y(s)} = \frac{\beta_{10} s + \beta_{00}}{s + \alpha_{00}} \tag{6}$$

Introducing u_2, the derivative of input u (t) and writing the state equation of the plant for $a_0 = 1$ and $c_{20} = -1$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{u}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ u_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2 \tag{7}$$

and considering the performance index

$$J = \int (x_1^2 + u_1^2) dt \tag{8}$$

we can apply the optimal control design technique of Kalman to obtain the optimal state feedback

$$u_2 = -K^T (x_1, x_2, u)^T \tag{9}$$

with $K^T = [1 \quad 2 \quad 3]$

Now choose the coefficients α_{00} , β_{00} and β_{10} in (7) so that the same u_2 is obtained.

From (7)

$$su + \alpha_{00} u = -(\beta_{10} C_0^T A_0 + \beta_{00} C_0^T) x - (\alpha_{00} + \beta_{00} C_0^T b) u \tag{10}$$

Comparing (11) with (10) ,we get

α_{00} =3.25, $\beta_{00} = 0.5$ and $\beta_{10} = 1.25$

With this compensator, the closed-loop system will have very good performance when $\alpha = a_0 = 1$ and $c_2 = c_{20} = -1$ .However, with this compensator the closed-loop system will not be stable when $\alpha = 0.5$ and $c_2 = -1.7$. If the intervals of α and c_2 are $a \in [0.5, 1.5]$, $c \in [-1.7, 0.5]$ then the above controller will not work well. So the next step is to modify the compensator parameters such that the system is stable for all possible α and c_2 .

Step 2

Search for the parameters of dynamic compensator

$$\frac{-u(s)}{y(s)} = \frac{\beta_1 s + \beta_0}{s + \alpha_0} \tag{11}$$

such that minimization of $(\beta_1 - \beta_{10})^2 + (\beta_0 - \beta_{00})^2 + (\alpha_0 - \alpha_{00})^2$ is achieved subject to the interval closed-loop system being stable.

For the compensator (12) the minimal realization is

$$\begin{aligned} \dot{x}_c(t) &= -\alpha_0 x_c(t) + y(t) \\ -u(t) &= (\beta_0 - \beta_1 \alpha_0) x_c(t) + \beta_1 y(t) \end{aligned} \tag{12}$$

Where $x_c(t)$ is the internal state of the compensator. Thus the state description of the closed-loop system is

$$\begin{aligned} \dot{x}(t) &= (A - b\beta_1 C^T) x(t) - b(\beta_0 - \beta_1 \alpha_0) x_c(t) \\ \dot{x}_c(t) &= C^T x(t) - \alpha_0 x_c(t) \end{aligned} \tag{13}$$

The characteristic polynomial of (14) is

$$\lambda^3 + d\lambda^2 + e\lambda + f \tag{14}$$

Where $d = c_2 \beta_1 + \alpha_0, e = c_2 \beta_0 + 2\alpha\beta_1, f = 2\alpha\beta_0$

When $a \in [0.5, 1.5]$, $c \in [-1.7, 0.5]$ is an interval polynomial. The intervals of its coefficient are

$d \in [d_{\min}, d_{\max}], e \in [e_{\min}, e_{\max}], f \in [f_{\min}, f_{\max}]$

Where

$$\begin{aligned} d_{\min} &= \min_{c_{\min} \ge c_2 \ge c_{\max}} (c_2 \beta_1 + \alpha_0), d_{\max} = \max_{c_{\min} \ge c_2 \ge c_{\max}} (c_2 \beta_1 + \alpha_0) \\ e_{\min} &= \min_{\substack{c_{\min} \ge c_2 \ge c_{\max} \\ a_{\min} \ge a \ge a_{\max}}} (c_2 \beta_0 + 2\alpha\beta_1), e_{\max} = \max_{\substack{c_{\min} \ge c_2 \ge c_{\max} \\ a_{\min} \ge a \ge a_{\max}}} (c_2 \beta_0 + 2\alpha\beta_1) \\ f_{\min} &= \min_{a_{\min} \ge a \ge a_{\max}} (2\beta_0 \alpha), f_{\max} = \max_{a_{\min} \ge a \ge a_{\max}} (2\beta_0 \alpha) \end{aligned} \tag{15}$$

and $\alpha_{\min} = 0.5, \alpha_{\max} = 1.5, c_{2\min} = -1.7, c_{2\max} = 0.5$

With reference to the Theorem, the interval polynomial (14) will be stable if all the polynomials are stable.

The parameters of the compensator, $\alpha_0, \beta_0, \beta_1$ are not known so without loss of generality we can constrain $\alpha_0 \ge 0, \beta_0 \ge 0, \beta_1 \ge 0$ then construct the Hurwitz determinants for all the polynomials and search for $\alpha_0, \beta_0, \beta_1$ to

Minimize $(\alpha_0 - \alpha_{00})^2 + (\beta_0 - \beta_{00})^2 + (\beta_1 - \beta_{10})^2$

subject to

$$\begin{aligned} &-f_{\min} + \varepsilon_1 \le 0 \\ &-d_{\min} + \varepsilon_1 \le 0 \\ &-(d_{\min} e_{\min} - f_{\max}) + \varepsilon_1 \le 0 \\ &-\beta_1 \le 0 \\ &-\alpha_0 \le 0 \end{aligned} \tag{16}$$

The small positive scalar ε_1 is used in order to make the feasible set $(\alpha_0, \beta_0, \beta_1)$ closed and can, to some extent, represent a stability margin to accommodate unconsidered uncertainties. Problem cited is a standard optimization problem with non-linear constraints and several methods are available for its solution [14].

In this example, set $\varepsilon_1 = 0, \alpha_{\min} = 0.5, \alpha_{\max} = 1.5, c_{2\min} = -2.5, c_{2\max} = -0.5$ and the solution is $\alpha_0 = 3.31778, \beta_0 = 0.13357, \beta_1 = 1.12405$

Now check whether the new compensator will make the closed-loop system stable while still having satisfactory performance over the range of α and c_2.For this example, Fig. 1 show the performance of the closed-loop system with thedesigned compensator.

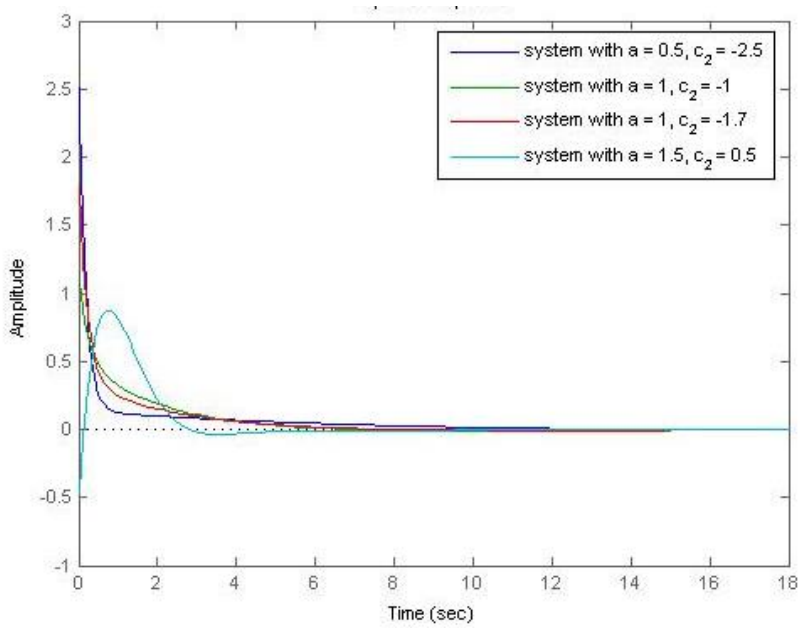

Figure 1.Impulse responses of the closed-loop systems with compensator $\frac{1.12405s + 0.13357}{s + 3.31778}$

V. CONCLUSION

A technique for designing robust feedback compensator has been presented. A simple example illustrates the method; however, the method is directly applicable to multivariable systems. The approach taken here is to allow the system parameters to vary within prescribed intervals then design a controller which guarantees closed-loop system stability for the known range of parameter variation.

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