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Effect of Load on the Characteristics of Linear Motor Used in Free Piston Free Displacer Miniature Cryocoolers

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Abstract:—The free piston Stirling miniature cryocoolers of small capacity, as discussed earlier are being used for cooling infrared sensors of night vision equipments for defense use and space borne applications. To make them work for long periods without maintenance, linear electromagnetic drivers, which are also called linear motors, have been used for high reliability. Usually such motors employ a single moving coil. Recent trends, however favour a double coil configuration for higher motor efficiency. The paper describe the effect on the performance of the cryocooler with respect to the operating parameters of the cryocooler. By changing the operating chracteristics depending upon the cooling capacity required the optimum efficiency can be achieved.

Keywords:—Free piston free displacer, analysis, Stirling cryocooler, Current, Input power to the linear motor, Output power from the linear motor, Efficiency of the motor.

I. INTRODUCTION

Since the miniature Stirling cryocooler is driven by linear motor, it does not have fixed amplitude of motion as in the case of a crank driven system. So, it is necessary to analyze the motor characteristics and study the dynamics of the cryocooler coupled with the electromagnetic forces of the motor.

II. ANALYSIS OF THE LINEAR MOTOR

The linear motor consists of two identical in series electric coils, fixed to the piston shaft, free to reciprocate in two annular working gaps formed by a stationary assembly of an axially symmetric permanent magnet and pole pieces. A radial magnetic field exists in each gap and, if the gap width is small enough compared to its mean diameter and height, then the magnetic field can be considered to be fairly uniform. The required oscillatory motion is generated by passing an alternating current through the coils. Since the magnetic flux lines in one gap flow in a radially opposite way to those in the other gap; for the coils to act in tandem, they have to be wound with an opposite orientation. The mean position of the coil/piston assembly is controlled by a mechanical spring. Figure 1 given below shows a schematic diagram of double coil linear motor and its representative circuit. Analysis of double coil linear motor as suggested by [1] is being analyzed in detail here.

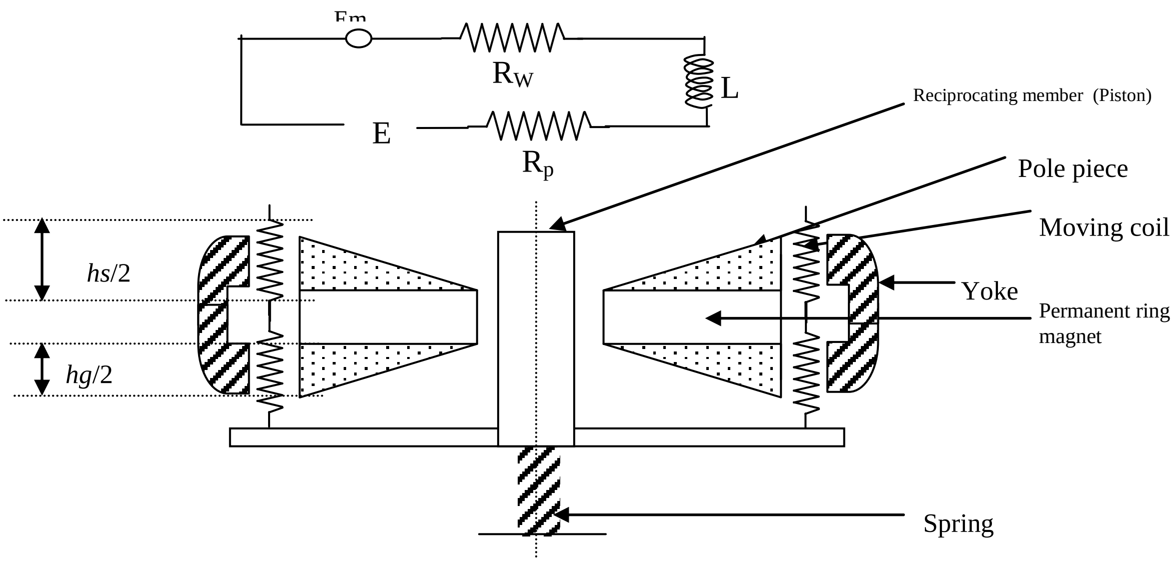

Figure 1 Schematic diagram of double coil linear motor

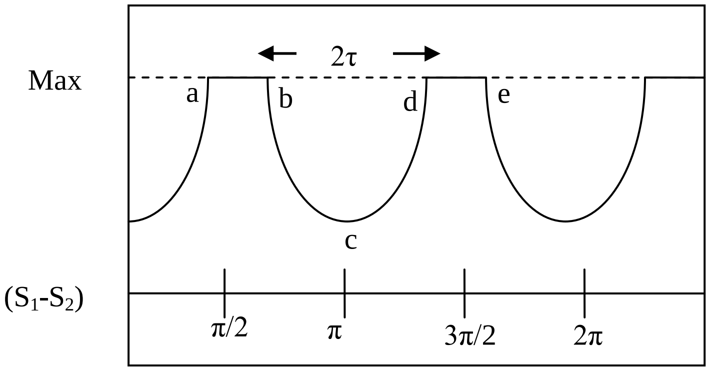

Figure 2 Variation of *hv*/*hg* as a function of β

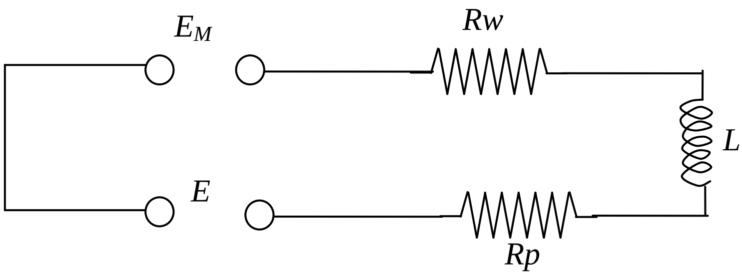

Figure 3 Electric circuit of the linear motor

A Assumptions

The following are the assumptions for the theoretical analysis of the in series double coil linear motor.

1. The magnetic field in each gap is radial, constant and uniform, with no leakage of flux outside the gap.
2. The impressed voltage has only a first harmonic given as

   $E = E_0 \, Cos \, (\beta + \phi)$ (1)

   Where,

   ϕ = Motor phase shift

   E_0= Amplitude of impressed Voltage
3. The motion of the piston is of a simple harmonic nature and is given as:

   $x_p = X_p \, Cos \, \beta$ (2)

   Where,

   X_p is the piston amplitude
4. The self-inductance and resistance of each coil are constant and independent of coil position.
5. Eddy current and hysteresis losses are negligible.

B Active Wire Length and Active Coil Height

At any give instant of time, the part of the total wire length, which is under influence of the magnetic field, may be defined as the active wire length *lv* at that instant. The variation of active wire length plays an important role in determining the expressions for the input and output powers of the motor. For a given number of turns per unit length, wire length is proportional to the coil height:

$$lv = ls * \frac{hv}{hs} = ls \frac{(hv/hg)}{(hs/hg)} \quad (3)$$

Where,

$\frac{hv}{hg}$ = Normalized active coil height and is a periodic function of the angle β

$\frac{hs}{hg}$ = Normalized coil height

Figure 2 shows a generalized representation of *hv*/*hg* as a function of *β.* The angle τ denotes the angular measure of that fraction of a quarter cycle for which only a part of the maximum utilizable magnetic flux is actually used.

Based in the geometrical relationship between the coil and gap heights and the amplitude of motion, the angle τ can be defined as:

$$Cos\ \tau = \frac{|1-hs/hg|}{4X_p/hg} \quad \text{for} \quad \frac{|1-hs/hg|}{4X_p/hg} \le 1 \tag{4}$$

$$Cos\ \tau = 1 \quad \text{for} \quad \frac{|1-hs/hg|}{4X_p/hg} > 1 \tag{5}$$

For the rest of the quarter cycle (i.e. from τ to *π*/2), *hv*/*hg* takes a constant value, *MAX* given as

MAX = 1 for *hs*/*hg* ≥ 1 (6)

MAX = *hs*/*hg* for *hs*/*hg*< 1 (7)

The quarter cycle variation of *hv/hg* described above is symmetric over the quarter cycles (i.e. from *β = π/2* to *π*) and this pattern is repeated for the latter half cycle (i.e. from *β* = *π* to 2*π*). Hence it would be sufficient to define *hv/hg* over the first quarter of the cycle only in a general form as described below:

Since,

hs/*hg*≥ 1 for *MAX* = 1

and,

hs/*hg*≤ 1 for *MAX* = *hs*/*hg*

So the arrange value of that part which is not dependent on *β* can be evaluated with the help of constant MAX. Let it be represented by S_1

$$S_1 = \frac{[\,MAX(\text{for}\ hs/hg \ge 1) + MAX(\text{for}\ hs/hg < 1)\,]}{2}$$

$$= \frac{[1 + hs/hg]}{2} \tag{8}$$

Now, the value of variable, which is a function of phase angle *β*, can be calculated with the help of the Figure 2.

(0 – a) is the is the portion, which is varying as per phase angle *β,* so that the average value of this part can be

$$S_2 \quad = \frac{0 + 4X_p/hg}{2} = \frac{2X_p}{hg} \tag{9}$$

So, *hv*/*hg* can be represented as

$$hv/hg \quad = \frac{(1 + hs/hg)}{2} - \frac{2X_p Cos\beta}{hg} \tag{10}$$

$$hv/hg = f(\beta) = S_1 - S_2\ Cos\ \beta \quad for\ 0 \le \beta < \tau \tag{11}$$

$$= MAX \quad for\ \tau \le \beta < \pi/2 \tag{12}$$

C Normalized Active Coil Height

Normalized active coil height, *hv/hg* can be analyzed with the help of Fourier series, so it can be written as:

$$hv/hg = [\, a_0 + a_1 Cos\ \beta + a_2 Cos\ 2\beta + \_\_\_\_\_ + a_n Cos\ n\beta + \dots\,] + [b_0 + b_1 Sin\ \beta + b_2\ Cos\ 2\beta + \_\_\_\_ + b_n Cos\ n\beta + \dots\,] \tag{13}$$

Where, a_0, $a_1 \dots\dots a_n$ *and* b_0, b_1, $\dots\dots b_n$ are constants evaluated in the following expressions:

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\beta)\,d\beta = \frac{4}{2\pi}\int_0^{\pi/2} f(\beta)\,d\beta$$

$$= \frac{2}{\pi}\{(S_1 - MAX)\tau - S_2 \sin\tau\} + MAX \tag{14}$$

$$a_{2n-1} = \frac{1}{2\pi}\int_0^{2\pi} f(\beta) Cos(2n-1)\beta\, d\beta = \frac{1}{2\pi}\int_0^{2\pi} (s_1 - s_2 Cos\beta) Cos(2n-1)\beta\, d\beta$$

$$= \frac{1}{2\pi}\left\{\frac{S_1}{2n-1}\left(\sin(2n-1)\beta\right)_0^{2\pi} - \frac{S_2}{2}\left(\frac{\sin 2n\beta}{2n} + \frac{\sin 2(n-1)\beta}{2(n-1)}\right)_0^{2\pi}\right\} = 0 \quad (15)$$

$$b_n = \frac{1}{2\pi}\int_0^{2\pi}\left(s_1 - s_2\cos\beta\right)\sin n\beta d\beta$$

$$= \frac{1}{2\pi}\left\{-\frac{S_1}{n}\left(\cos n\beta\right)_0^{2\pi} - \frac{S_2}{2}\left(\frac{1}{n+1}\cos(n+1)\beta + \frac{1}{n-1}\cos(n-1)\beta\right)_0^{2\pi}\right\}$$

$$= \frac{1}{2\pi}\left\{-\frac{S_1}{n}\left(1-\cos 2n\pi\right) + \frac{S_2}{2}\left(\frac{\left(\cos 2(n+1)\pi\right)}{n+1} - 1 + \frac{\left(\cos 2(n-1)\pi\right)}{n-1} - 1\right)\right\} = 0$$

(for $n = 1, 2, 3, 4$ ……….. ∞) (16)

It is evident from the above expression that due to nature of the function (i.e. its symmetry and periodicity and from the periodicity of $Sin\beta$), the even function of a can be calculated as given further.

$$a_{2n} = \frac{4}{\pi}\int_0^{\pi/2} f(\beta)\cos(2n\beta)d\beta \quad (17)$$

$$= \frac{4}{\pi}\left[\frac{S_1}{2n}\sin 2n\beta - \frac{S_2}{2}\left(\frac{\sin(2n+1)\beta}{2n+1} + \frac{\sin(2n-1)\beta}{2n-1}\right)_0^{\tau} + \left(\frac{MAX\cos 2n\beta}{2n}\right)_{\tau}^{\pi/2}\right]$$

$$= \frac{2}{\pi}\left[\frac{\left(S_1 - MAX\right)\sin 2n\tau}{n} - S_2\left(\frac{\sin(2n+1)\tau}{2n+1} + \frac{\sin(2n-1)\pi}{2n-1}\right)\right] \quad (18)$$

Hence the Normalized active height of the coil

$$\frac{hv}{hg} = a_0 + a_2 Cos\ 2\beta + a_4 Cos\ 4\beta + \_\_\_\_\_ a_{2n} Cos\ 2n\beta$$

$$= \sum_{n=0}^{\infty} a_{2n} Cos(2n\beta) \quad (19)$$

D Analysis of Current

Let us consider the electric circuit of the linear motor as given in Figure 3.

Equating the voltage drops therein with the source voltage

$$E = Em + i(Rw + Rp) + L\frac{di}{dt} \quad (20)$$

Where,

$L\frac{di}{dt}$ = self induced voltage

As per assumption

$E = E_0\ Cos\ (\beta + \phi)$

Let us denote the multiplication by Bg, lv and x'_p equal to Em

$Em = Bg\ lv\ x'_p$ (21)

$x_p = X_p\ Cos\ \beta = X_p\ Cos(\alpha - \theta) = X_p\ Cos(\omega t - \theta)$ (22)

$x'_p = -X_p Sin\ (\omega t - \theta)\ \omega = -X_p\omega\ Sin\beta$ (23)

$E_m = -Bg\ lv\ X_p\ \omega\ Sin\beta$ (24)

Total resistance

$R_T = Rw + Rp$ (25)

Eqn. 20 becomes

$$E = -BglvX_p\omega Sin\ \beta + iR_T + L\frac{di}{dt} \quad (26)$$

So,

$$E_0\ Cos(\beta + \phi) = -BglvX_p\omega Sin\ \beta + iR_T + L\frac{di}{dt} \quad (27)$$

Effective coil quality factor R_L is given by:

$$R_L = \frac{2L(1-cc)\omega}{R_T} \quad (28)$$

Where Cc (mutual inductive coupling coefficient) can take the values between 0 and 1 depending upon the geometry and the magnetic permeability of the various magnetic materials. (We take $cc = 0.5$)
Therefore,

$$R_L = \frac{2L(1-1/2)\omega}{R_T} = \frac{L\omega}{R_T} \tag{29}$$

So,

$$L = \frac{R_T R_L}{\omega} \tag{30}$$

Substituting these values in the eqn. 27 gives,

$$E_0\, Cos(\beta + \phi) = -Bg\, lv\omega\, X_p\, Sin\beta + iR_T + \frac{R_L R_T}{\omega}\frac{di}{dt} \tag{31}$$

Since, $\beta = \omega t - \theta$ and $\frac{d\beta}{dt} = \omega$ (32)

Equation 31 becomes (putting the requisite values, rearranging & dividing by R_T)

$$\frac{R_L}{\omega}\frac{di}{d\beta}\frac{d\beta}{dt} + i = \frac{E_0}{R_T} Cos(\beta + \phi) + \frac{Bg lv X_P Sin\beta}{R_T}$$

$$lv = ls\frac{(hv/hg)}{(hs/hg)}$$

$$R\, di/d\beta + i = \frac{E_0}{R_T} Cos(\beta + \phi) + \frac{Bgls\omega Xp(hv/hg)Sin\beta}{R_T(hs/hg)} \tag{33}$$

Denoting,

$$Bg\, ls\, \omega\, X_p = E_s \tag{34}$$

The final form of governing equating for impressed voltage is given by,

$$R_L\frac{di}{d\beta} + i = \frac{E_0}{R_T} Cos(\beta + \phi) + \frac{E_S(hv/hg)Sin\beta}{R_T(hs/hg)} \tag{35}$$

The equation is a first order, linear differential equation governing the relationship between current (i) as a function of piston phase angle (β). The expression for current with the help of Fourier series can be expressed as

$$i = [A_0 + A_1 Cos\beta + A_2 Cos2\beta + \ldots.] + [B_1 Sin\beta + B_2 Sin2\beta + \ldots BnSin(n\beta)] \tag{36}$$

$$= \sum_{n=0}^{\infty} A_n Cos(n\beta) + \sum_{n=1}^{\infty} B_n Sin(n\beta)$$

Substituting the values of i and hv/hg in the eqn. 35, the equation reduces to

$$R_L\frac{d}{d\beta}\{\sum_{n=0}^{\infty} A_n Cos(n\beta) + \sum_{n=1}^{\infty} B_n Sin(n\beta)\} + \sum_{n=0}^{\infty} A_n Cos(n\beta) + \sum_{n=1}^{\infty} B_n Sin(n\beta)$$

$$= \frac{E_0}{R_T}(Cos\beta Cos\phi - Sin\beta Sin\phi) + \frac{E_S Sin\beta}{R_T(hs/hg)}\left(\sum_{n=0}^{\infty} a_{2n} Cos(2n\beta)\right) \tag{37}$$

Simplifying and comparing the coefficients of Cosines and Sines of the angles, we get

$$A_0 = 0 \tag{38}$$

$$A_2 = A_4 = B_2 = B_4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. A_{2n} = B_{2n} = 0 \tag{39}$$

$$A_1 = \frac{1}{R_T(1+R_L^{\ 2})}\left[E_0(Cos\,\phi + R_L Sin\,\phi) - \frac{R_L E_S}{2R_T(hs/hg)}(a_2 - 2a_o)\right] \tag{40}$$

$$B_1 = \frac{1}{R_T(1+R_L^{\ 2})}\left[E_0(R_L\cos\phi - \sin\phi +)\frac{E_S}{2(hs/hg)}(2a_0 - a_2)\right] \tag{41}$$

$$A_{2n-1} = -\frac{E_S(a_{2n-2} - a_{2n})(2n-1)R_L}{2R_T(hs/hg)\left[1+(2n-1)^2 R_L^{\ 2}\right]} \tag{42}$$

$$B_{2n-1} = \frac{E_S\left(a_{2n-2} - a_{2n}\right)}{2R_T\left(hs/hg\right)\left[1+(2n-1)^2 R_L{}^2\right]} \tag{43}$$

Putting $\dfrac{E_S}{2R_T\,(hs/hg)} = c$ (44)

$$A_{2n-1} = \frac{c\left(a_{2n-2} - a_{2n}\right)\left(2n-1\right)R_L}{1+(2n-1)^2 R_L{}^2} \tag{45}$$

$$B_{2n-1} = \frac{c\left(a_{2n-2} - a_{2n}\right)}{1+(2n-1)^2 R_L{}^2} \tag{46}$$

The expression of current in the final form can be written as:

$i= [A_1Cos\,\beta+A_3Cos\,3\beta+\ldots\ldots+A_{2n-1}Cos(2n-1)\beta+\ldots]+ [B_1Sin\,\beta+B_3Sin\,3\beta+\ldots\ldots\ldots+B_{2n-1}\,Sin(2n-1)\,\beta+\ldots]$ (47)

E Input and Output Power of Linear Motor

We require power input (P_i) and power output (P_o). These expressions can be found as follows.

$$P_i = \frac{1}{2\pi}\int_0^{2\pi} Eid\beta = \frac{4}{2\pi}\int_0^{\pi/2} Eid\beta \tag{48}$$

$$P_i = \frac{E_0}{2R_T\,(1+R_L{}^2)}\left[E_0 - \frac{E_S\left(2a_0 - a_2\right)}{2\left(hs/hg\right)}\left(\sin\phi + R_L\,\cos\phi\right)\right] \tag{49}$$

2.6 Output power (P_o)

The output power of linear motor can be expressed as

$$P_o = \frac{1}{2\pi}\int_0^{2\pi} E_m id\beta \tag{50}$$

Where,

$Em= - Bg\times lv\times x'_p = - Bg\;lv\;X_p\;\omega\;Sin\beta$

$$= - Bg\;ls\;\frac{(hv/hg)}{\left(hs/hg\right)}X_p\;\omega\;Sin\;\beta \tag{51}$$

Where,

$$Es= Bg\times ls\times X_p\times\omega \tag{52}$$

$$E_m= - E_s\,\frac{(hv/hg)}{\left(hs/hg\right)}Sin\beta \tag{53}$$

$$P_o= -\frac{1}{2\pi}\int_0^{2\pi} E_s\frac{(hv/hg)}{\left(hs/hg\right)}\;i\;Sin\beta\;d\beta \tag{54}$$

Substituting the values of (hv/hg) and (i) in eqn. 54 and simplifying, we get

$$P_o= -\frac{E_S}{2\pi(hs/hg)}\left[a_0B_1 + \sum_{n=1}^{\infty}\frac{a_{2n}}{2}\left(B_{2n+1} - B_{2n-1}\right)\right] \tag{55}$$

III. RESULTS AND DISCUSSION

Using the performance parameters as derived above the performance of a typical linear motor used in Phillips 1 W, 80 K miniature free piston free displacer cryocoolers have been analyzed for the effect of load and frequency of oscillation of the motor.

3.1 In the figure 1 the curves show the variation of input power at various frequencies with respect to charge pressure. The lower peak points show the resonance and are the design condition. The resonance frequency and input power increases with increase in the charge pressure. Similarly figure 2 the curves show the variation of output power at various frequencies with respect to charge pressure. The upper peak points show the resonance and are the design condition again. The resonance frequency and output power decreases with increase in the charge pressure.

3.2 In the figure 3 the curves show the variation of input power at various charge pressure with respect to frequencies. The lower peak points show the resonance and are the design condition. The resonance frequency and input power increases with increase in the charge pressure. Similarly figure 4 the curves show the variation of output power at various charge pressure with respect to frequencies. The upper peak points show the resonance and are the design condition again. The resonance frequency and output power decreases with increase in the charge pressure.

The trend of the output shows that the input power increases with the increase in charge pressure and frequency; output power decreases with the increase in charge pressure and frequency. At resonance in each case, the output power is maximum and input power in minimum and thus maximum efficiency. The frequency can be adjusted for each charge pressure to get the resonance condition.

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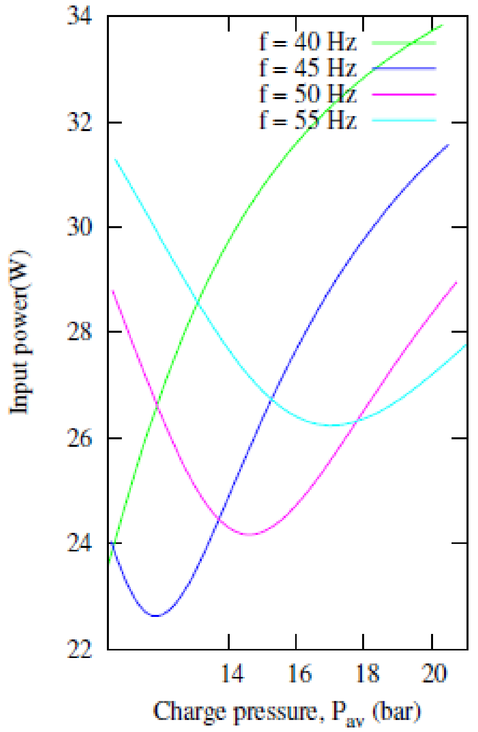

Fig. 1 Variation of input power to the motor with charge pressure with charge pressure

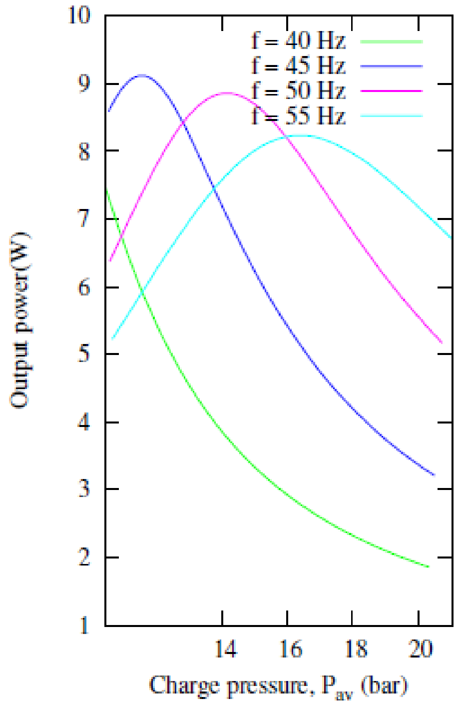

Fig. 2 Variation of output power to the motor

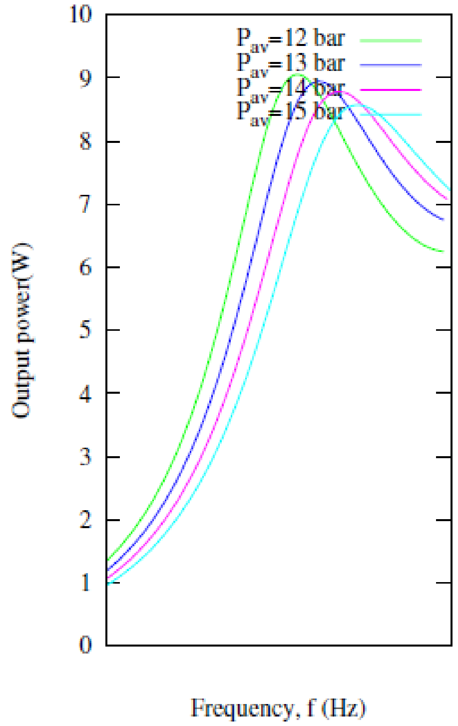

Fig. 3 Variation of output power to the motor with frequency motor with frequency

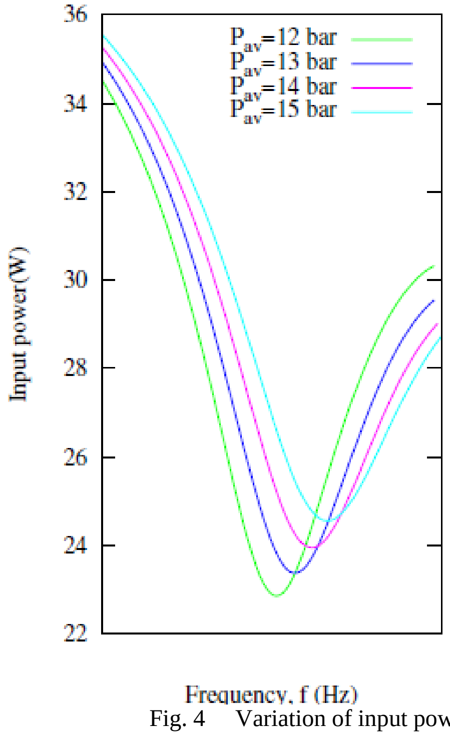

Fig. 4 Variation of input power to the