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Generalized Triangular Fuzzy Numbers In Intuitionistic Fuzzy Environment

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Abstract:- In this paper, a more general definition of triangular intuitionistic fuzzy number (TIFN) is proposed by removing the `normality' in the definition of intuitionistic fuzzy number. The basic arithmetic operations of generalized triangular intuitionistic fuzzy numbers (GTIFNs) and the notion of (α, β)-cut sets are defined. Also a nearest interval approximation method is described to approximate a GTIFN to a nearest interval number. Moreover the average ranking index is introduced to find out order relations between two GTIFNs. Numerical examples are also provided for illustration.

Keywords:- Intuitionistic fuzzy set(IFS), TIFN, GTIFN, Nearest interval approximation, Average ranking index.

I. INTRODUCTION

Fuzzy set theory introduced by Zadeh [1], has been researched widely in many fields and various modifications methods and generalization theories have been appeared in different directions. One of them is the concept of intuitionistic fuzzy set(IFS), first introduced by Atanassov [3, 4], which can be viewed as an appropriate/alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. Out of several higher-order fuzzy sets, IFSs have been found to be compatible to deal with vagueness. In fuzzy sets the degree of acceptance is considered only but IFS is characterized by a membership function and a non-membership function so that the sum of both the values is less than one [3, 4]. Intuitionistic fuzzy numbers (IFN) in one way seem to suitably describe the vagueness and lack of precision of data. To the best of our knowledge, Burillo [5] proposed the definition of intuitionistic fuzzy number and the first properties of the correlation between such numbers. Recently, the research on IFNs has received a little attention and several definitions of IFNs and ranking methods have been proposed. Mitchell [6] interpreted an IFN as an ensemble of fuzzy numbers and introduced a ranking method. Nayagam et al. [7] described IFNs of a special type and introduced a method of IF scoring that generalized Chen and Hwang's [8] scoring for ranking IFNs. Wang and Zhang [9] defined the trapezoidal IFN and gave a raking method which transformed the ranking of trapezoidal IFNs into the ranking of interval numbers. Shu et al. [10] defined a triangular intuitionistic fuzzy number (TIFN) in a similar way, as the fuzzy number introduced by Dubois and Prade [11]. Very recent Li [12, 13] introduced a new definition of the TIFN which has an logically reasonable interpretation and some applications are given in [14, 15]. Seikh et al.[16] proposed few non-normal operations on TIFN and also investigated on inequality relations between two TIFNs. In this paper, GTIFNs are introduced in a more general way, by adding an additional non-membership function, which can consider the degree rejection also. Arithmetic operations of proposed GTIFNs are evaluated. Moreover a GTIFN is approximated to a nearest interval by using (α, β) -cut sets. A new ranking method is proposed to find inequality relations between two GTIFNs by defining average ranking index.

This paper is organized as follows. In Section 2 we present basic concept of GTIFN and described the arithmetic operations of such numbers. In Section 3 an approximation method is described to approximate a GTIFN to a nearest interval number. In Section 4 inequality relations between two GTIFNs are proposed by defining average ranking index. Section 5 describes a short conclusion.

II. INTUITIONISTIC FUZZY SETS

The intuitionistic fuzzy set introduced by Atanassov [2, 3] is characterized by two functions expressing the degree of belongingness and the degree of non-belongingness respectively.

Definition 1 *Let* $U = \{x_1, x_2, \cdots, x_n\}$ *be a finite universal set. An intuitionistic fuzzy set* $\tilde{A}$ *in a given universal set* U *is an object having the form*

$$\tilde{A} = \{\langle x_i, \mu_{\tilde{A}}(x_i), \nu_{\tilde{A}}(x_i)\rangle : x_i \in U\} \qquad (1)$$

where the functions

$$\mu_{\tilde{A}}: U \to [0,1]; \quad i.e., x_i \in U \to \mu_{\tilde{A}}(x_i) \in [0,1]$$
$$and \quad \nu_{\tilde{A}}: U \to [0,1] \quad i.e., x_i \in U \to \nu_{\tilde{A}}(x_i) \in [0,1]$$

define the degree of membership and the degree of non-membership of an element $x_i \in U$, *such that they satisfy the following conditions :*

$$0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1, \forall x_i \in U$$

which is known as intuitionistic condition. The degree of acceptance $\mu_{\tilde{A}}(x)$ *and of non-acceptance* $\nu_{\tilde{A}}(x)$ *can be arbitrary.*

Definition 2 $(\boldsymbol{\alpha}, \boldsymbol{\beta})$***-cuts)*** *: A set of* (α, β)*-cut, generated by IFS* $\tilde{A}$*, where* $\alpha, \beta \in [0,1]$ *are fixed numbers such that* $\alpha + \beta \leq 1$ *is defined as*

$$\tilde{A}_{\alpha,\beta} = \begin{cases} (x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)); & x \in U \\ \mu_{\tilde{A}}(x) \geq \alpha, \nu_{\tilde{A}}(x) \leq \beta; & \alpha, \beta \in [0,1], \end{cases}$$

where (α, β)*-cut, denoted by* $\tilde{A}_{\alpha,\beta}$*, is defined as the crisp set of elements* x *which belong to* $\tilde{A}$ *at least to the degree* α *and which does belong to* $\tilde{A}$ *at most to the degree* β*.*

2.1 Triangular Intuitionistic Fuzzy Number

Intuitionistic fuzzy number was introduced by Burillo et al. [4]. The definition of IFN is given below

Definition 3 (**Intuitionistic Fuzzy Number**): An intuitionistic fuzzy number (Fig. 3) $\tilde{A}^i$ is

i. an intuitionistic fuzzy subset on the real line

ii. normal i.e. there exists $x_0 \in \Re$ such that $\mu_{\tilde{A}^i}(x_0) = 1$ (so $\nu_{\tilde{A}^i}(x_0) = 0$)

iii. convex for the membership function $\mu_{\tilde{A}^i}$ i.e.

$$\mu_{\tilde{A}^i}(\lambda x_1 + (1 - \lambda x_2)) \geq \min\{\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)\}; \ \forall x_1, x_2 \in \Re, \lambda \in [0,1]$$

iv. concave for the non-membership function $\nu_{\tilde{A}^i}$ i.e.

$$\nu_{\tilde{A}^i}(\lambda x_1 + (1 - \lambda x_2)) \leq \max\{\nu_{\tilde{A}^i}(x_1), \nu_{\tilde{A}^i}(x_2)\}; \ \forall x_1, x_2 \in \Re, \lambda \in [0,1].$$

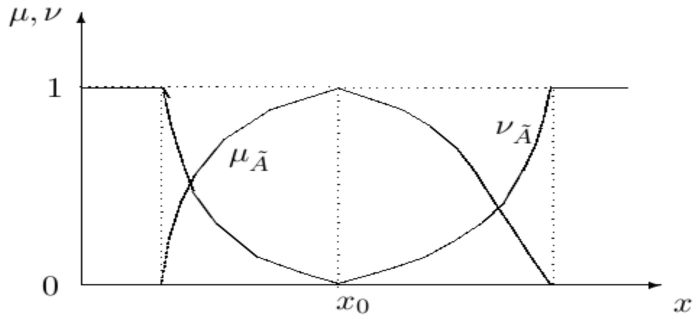

Figure 1: Membership and non-membership functions of $\tilde{A}^i$

Accoding to Seikh et al.[15] we have defined GTIFN in a more general way.

Definition 4 (**Generalized Triangular Fuzzy Number**)

A generalized triangular intuitionistic fuzzy number(GTIFN) $\tilde{\tau}_a = \langle (a, l_\mu, r_\mu; w_a), (a, l_\nu, r_\nu; u_a) \rangle$ is a special intuitionistic fuzzy set on a real number set $\Re$, whose membership function and non-membership functions are defined as follows (Fig. 2):

$$\mu_{\tilde{\tau}_a}(x) = \begin{cases} \frac{x - a + l_\mu}{l_\mu} w_a; & a - l_\mu \leq x < a \\ w_a; & x = a \\ \frac{a + r_\mu - x}{r_\mu} w_a; & a < x \leq a + r_\mu \\ 0; & otherwise \end{cases}$$

$$and \quad \nu_{\tilde{\tau}_a}(x) = \begin{cases} \dfrac{(a - x) + u_a(x - a + l_\nu)}{l_\nu}; & a - l_\nu \leq x < a \\ u_a; & x = a \\ \dfrac{(x - a) + u_a(a + r_\nu - x)}{r_\nu}; & a < x \leq a + r_\nu \\ 1; & otherwise \end{cases}$$

where $l_\mu, r_\mu; l_\nu, r_\nu$ are called the spreads of membership and non-membership function respectively and a is called mean value. w_a and u_a represent the maximum degree of membership and minimum degree of non-membership respectively such that they satisfy the conditions $0 \le w_a \le 1, 0 \le u_a \le 1 \quad and \quad 0 \le w_a + u_a \le 1.$

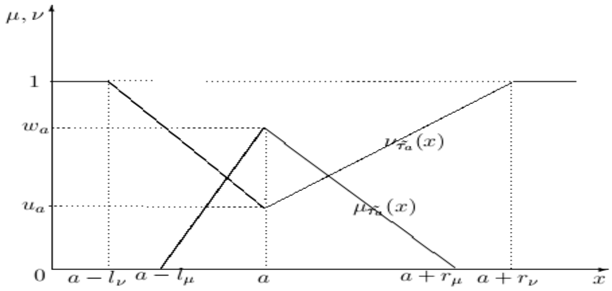

Figure 2: Membership and non-membership functions of $\tilde{\tau}_a$

Observe that for $x \in [a - l_\nu, a - l_\mu]$ and $x \in [a + r_\mu, a + r_\nu]$, it is $\mu_{\tilde{\tau}_a(x)} = 0, \nu_{\tilde{\tau}_a(x)} < 1$. From Fig. 2 and the above definition, we see that $0 \le \mu_{\tilde{\tau}_a}(x) \le 1, 0 \le \nu_{\tilde{\tau}_a}(x) \le 1$.

It is easily shown that $\mu_{\tilde{\tau}_a}(x)$ is convex and $\nu_{\tilde{\tau}_a}(x)$ is concave for $x \in U$. Further, for $a - l_\nu \le x < a - l_\mu, \mu_{\tilde{\tau}_a}(x) = 0$ and $\nu_{\tilde{\tau}_a}(x) \le 1$. Therefore $\mu_{\tilde{\tau}_a}(x) + \nu_{\tilde{\tau}_a}(x) \le 1$.

For $a - l_\mu \le x < a$,

$$\mu_{\tilde{\tau}_a}(x) + \nu_{\tilde{\tau}_a}(x) = \frac{x - a + l_\mu}{l_\mu} w_a + \frac{(a - x) + u_a(x - a + l_\nu)}{l_\nu}$$

When, $x = a - l_\mu, \mu_{\tilde{\tau}_a}(x) + \nu_{\tilde{\tau}_a}(x) < 1$ and when $x = a, \mu_{\tilde{\tau}_a}(x) + \nu_{\tilde{\tau}_a}(x) = w_a + u_a \le 1$.

For $a < x \le a + r_\mu$,

$$\mu_{\tilde{\tau}_a}(x) + \nu_{\tilde{\tau}_a}(x) = \frac{a + r_\mu - x}{r_\mu} w_a + \frac{(x - a) + u_a(a + r_\nu - x)}{r_\nu}$$

$$= (a - x)\left(\frac{w_a}{r_\mu} + \frac{1 - u_a}{r_\nu}\right) + (u_a + w_a).$$

When, $x = a, \mu_{\tilde{\tau}_a}(x) + \nu_{\tilde{\tau}_a}(x) = w_a + u_a \le 1$ and when $x \le a + r_\mu, \mu_{\tilde{\tau}_a}(x) + \nu_{\tilde{\tau}_a}(x) < 1$.

For $a + r_\mu < x \le a + r_\nu, \mu_{\tilde{\tau}_a}(x) = 0$ and $\nu_{\tilde{\tau}_a}(x) \le 1$. So, $\mu_{\tilde{\tau}_a}(x) + \nu_{\tilde{\tau}_a}(x) \le 1$. Therefore for above all cases $0 \le \mu_{\tilde{\tau}_a}(x) + \nu_{\tilde{\tau}_a}(x) \le 1$.

The quantity $\Pi_{\tilde{\tau}_a}(x) = 1 - \mu_{\tilde{\tau}_a}(x) - \nu_{\tilde{\tau}_a}(x)$, is called the measure of hesitation. The set of all these fuzzy numbers is denoted by GTIFN($\mathfrak{R}$).

If $a - l_\nu \ge 0$ then GTIFN $\tilde{\tau}_a = \langle (a, l_\mu, r_\mu; w_a), (a, l_\nu, r_\nu; u_a) \rangle$ is called a positive GTIFN, denoted by $\tilde{\tau}_a \succ \Theta$ and if $a + r_\nu \le 0$ then $\tilde{\tau}_a$ is called negative GTIFN, denoted by $\tilde{\tau}_a \prec \Theta$. Two GTIFNs $\tilde{\tau}_a = \langle (a, l_{\mu_a}, r_{\mu_a}; w_a), (a, l_{\nu_a}, r_{\nu_a}; u_a) \rangle$ and $\tilde{B} = \langle (b, l_{\mu_b}, r_{\mu_b}; w_b), (b, l_{\nu_b}, r_{\nu_b}; u_b) \rangle$ are said to be equal if and only if

$$a = b, l_{\mu_a} = l_{\mu_b}, r_{\mu_a} = r_{\mu_b}, l_{\nu_a} = l_{\nu_b}, r_{\nu_a} = r_{\nu_b}, w_a = w_b \quad and \quad u_a = u_b$$

It is obvious that if $l_{\mu_a} = l_{\nu_a} = l_a$ and $r_{\mu_a} = r_{\nu_a} = r_a$ then $\langle (a, l_{\mu_a}, r_{\mu_a}; w_a), (a, l_{\nu_a}, r_{\nu_a}; u_a) \rangle$ generates to $\langle a, l_a, r_a; w_a, u_a \rangle$ which is just about the triangular intuitionistic fuzzy number. Thus GTIFN is a generalization of the TIFN (Seikh et al.[15]).

The basic arithmetic operations on TFNs are defined in [15]. Based on these operations we extend the basic arithmetic operations on GTIFNs.

2.2 Arithmetic Operations

For two GTIFN $\tilde{\tau}_a = \langle (a, l_{\mu_a}, r_{\mu_a}; w_{a'}), (a, l_{\nu_a}, r_{\nu_a}; u_{a'}) \rangle$ and $\tilde{\tau}_b = \langle (b, l_{\mu_b}, r_{\mu_b}; w_{b'}), (b, l_{\nu_b}, r_{\nu_b}; u_{b'}) \rangle$ and for a real λ, the arithmetic operations are defined as follows:

Addition:

$\tilde{\tau}_a + \tilde{\tau}_b =$
$\langle (a+b, l_{\mu_a} + l_{\mu_b}, r_{\mu_a} + r_{\mu_b}; min\{w_a, w_b\}), (a+b, l_{\nu_a} + l_{\nu_b}, r_{\nu_a} + r_{\nu_b}; max\{u_a, u_b\}) \rangle.$

Subtraction:

$\tilde{\tau}_a - \tilde{\tau}_b =$
$\langle (a-b, l_{\mu_a} + r_{\mu_b}, r_{\mu_a} + l_{\mu_b}; min\{w_a, w_b\}), (a-b, l_{\nu_a} + r_{\nu_b}, r_{\nu_a} + l_{\nu_b}; max\{u_a, u_b\}) \rangle.$

Scalar Multiplication:

$$\lambda\tilde{\tau}_a = \begin{cases} \langle ((\lambda a, \lambda l_{\mu_a}, \lambda r_{\mu_a}; w_a), ((\lambda a, \lambda l_{\nu_a}, \lambda r_{\nu_a}; u_a) \rangle: & for \;\; \lambda > 0 \\ \langle ((\lambda a, -\lambda r_{\mu_a}, -\lambda l_{\mu_a}; w_a), ((\lambda a, -\lambda r_{\nu_a}, \lambda l_{\nu_a}; u_a) \rangle: & for \;\; \lambda < 0. \end{cases}$$

2.3 (α, β)-cut set of GTIFN

Definition 5 A (α, β)-cut set of a GTIFN $\tilde{\tau}_a = \langle (a, l_{\mu_a}, r_{\mu_a}; w_a,), (a, l_{\nu_a}, r_{\nu_a}; u_a,) \rangle$ is a crisp subset of $\mathfrak{R}$, which is defined as

$$\tilde{\tau}_a{}^{\alpha,\beta} = \{x: \mu_{\tilde{\tau}_a}(x) \geq \alpha, \nu_{\tilde{\tau}_a}(x) \leq \beta\},$$

where $0 \leq \alpha \leq w_a, u_a \leq \beta \leq 1$ and $0 \leq \alpha + \beta \leq 1$. A α-cut set of a GTIFN $\tilde{\tau}_a$ is a crisp subset of $\mathfrak{R}$, which is defined as

$$\tilde{\tau}_a{}^{\alpha} = \{x: \mu_{\tilde{\tau}_a}(x) \geq \alpha\}; \quad where, \;\; 0 \leq \alpha \leq w_a.$$

According to the definition of GTIFN it can be easily shown that $\hat{\tau}_a{}^{\alpha} = \{x: \mu_{\tilde{\tau}_a}(x) \geq \alpha\}$ is a closed interval, defined by

$$\hat{\tau}_a{}^{\alpha} = [a_L(\alpha), a_R(\alpha)] \qquad (2)$$
$$where \;\; a_L(\alpha) = (a - l_{\mu_a}) + \frac{l_{\mu_a}\alpha}{w_a}; \quad and \;\; a_R(\alpha) = (a + r_{\mu_a}) - \frac{r_{\mu_a}\alpha}{w_a}.$$

Similarly, a β-cut set of a GTIFN $\tilde{\tau}_a = \langle (a, l_{\mu_a}, r_{\mu_a}; w_a,), (a, l_{\nu_a}, r_{\nu_a}; u_a,) \rangle$ is a crisp subset of $\mathfrak{R}$, which is defined as

$$\tilde{\tau}_a{}^{\beta} = \{x: \nu_{\tilde{\tau}_a}(x) \leq \beta\} \quad where \;\; u_a \leq \beta \leq 1.$$

It follows from definition that $\tilde{\tau}_a{}^{\beta}$ is a closed interval, denoted by $\hat{\tau}_a{}^{\beta} = [a_L(\beta), a_R(\beta)]$ which can be calculated as

$$\hat{\tau}_a{}^{\beta} = [a_L(\beta), a_R(\beta)] \qquad (3)$$
$$where \;\; a_L(\beta) = (a - l_{\nu_a}) + \frac{(1-\beta)l_{\nu_a}}{1-u_a}; \quad and \;\; a_R(\beta) = (a + r_{\nu_a}) - \frac{(1-\beta)r_{\nu_a}}{1-u_a}.$$

It can be easily proven that for $\tilde{\tau}_a = \langle (a, l_{\mu_a}, r_{\mu_a}; w_a,), (a, l_{\nu_a}, r_{\nu_a}; u_a,) \rangle \in$ GTIFN($\mathfrak{R}$) and for any $\alpha \in [0, w_a], \beta \in [u_a, 1]$ where $0 \leq \alpha + \beta \leq 1$

$$\tilde{\tau}_a{}^{\alpha,\beta} = \tilde{\tau}_a{}^{\alpha} \wedge \tilde{\tau}_a{}^{\beta}, \qquad (4)$$

where the symbol "$\wedge$" denotes the minimum between $\tilde{\tau}_a{}^{\alpha}$ and $\tilde{\tau}_a{}^{\beta}$.

Example 1 Let $\tilde{\tau}_a = \langle (5,1,2; 0.6), (5,1.5,2.6; 0.3) \rangle$ be a GTIFN. Then for $\alpha \in [0,0.6]$, we get the following α-cut as

$$\tilde{\tau}_a{}^{\alpha} = \left[4 + \frac{\alpha}{0.6}, 7 - \frac{\alpha}{0.3}\right]$$

and for $\beta \in [0.3,1]$, we get following β-cut as

$$\tilde{\tau}_a{}^{\beta} = [3.5 + 2.14(1-\beta), 7.6 - 3.71(1-\beta)].$$

III. NEAREST INTERVAL APPROXIMATION

In this section, we approximate a fuzzy number by a crisp number according to Grzegorzewski [16]. Let $\tilde{\tau}_a$ be an arbitrary triangular fuzzy number with α-cuts $[a_L(\alpha), a_R(\alpha)]$, then by nearest interval approximation method, the lower limit C_L and upper limit C_R of the interval are

$$C_L = \int_0^1 a_L(\alpha)d\alpha \quad and \quad C_R = \int_0^1 a_R(\alpha)d\alpha.$$

Here we are to approximate a GTIFN to a nearest interval number. For this let $\tilde{\tau}_a = \langle (a, l_{\mu_a}, r_{\mu_a}; w_a,), (a, l_{\nu_a}, r_{\nu_a}; u_a,) \rangle$ be a GTIFN and it's (α, β)-cut set is defined in section 2.3. Thus the nearest α-cut and β-cut intervals are defined as follows

$$C_d^\alpha(\tilde{\tau}_a) = [C_L^\alpha, C_R^\alpha] = \left[\int_0^{w_a} a_L(\alpha)d\alpha, \int_0^{w_a} a_R(\alpha)d\alpha\right],$$

where $a_L(\alpha)$ and $a_R(\alpha)$ are defined in (2). Using equation (2) and integrating we have

$$C_d^\alpha(\tilde{\tau}_a) \approx \left[(a - \frac{l_{\mu_a}}{2})w_a, (a + \frac{r_{\mu_a}}{2})w_a\right].$$

Similarly

$$C_d^\beta(\tilde{\tau}_a) = \left[C_L^\beta, C_R^\beta\right] = \left[\int_{u_a}^1 a_L(\beta)d\beta, \int_{u_a}^1 a_R(\beta)d\beta\right],$$

where $a_L(\beta)$ and $a_R(\beta)$ are defined in (3). Using equation (3) and integrating we have

$$C_d^\beta(\tilde{\tau}_a) \approx \left[(a - \frac{l_{\nu_a}}{2})(1 - u_a), (a + \frac{r_{\nu_a}}{2})(1 - u_a)\right].$$

If $C_d^\alpha(\tilde{\tau}_a)$ and $C_d^\beta(\tilde{\tau}_a)$ are respectively approximate α-cut and β-cut interval of a TIFN $\tilde{\tau}_a$, then we define the nearest interval approximation for $\tilde{\tau}_a$ as

$$\tilde{\tau}_a{}^I \approx \min\{C_d^\alpha(\tilde{\tau}_a), C_d^\beta(\tilde{\tau}_a)\}. \quad (5)$$

The minimum between two intervals can be determined by usual interval order relations defined in [18, 19]. The set of all such interval numbers is denoted by $I(\mathfrak{R})$.

IV. INEQUALITY RELATIONS OF GTIFNS

The raking order relation between two GTIFNs is a difficult problem. However, GTIFNs must be ranked before the action is taken by the decision maker. In this section we describe a new ranking order relation between two GTIFNs by defining average ranking index. Assume that $\tilde{\tau}_a = \langle (a, l_{\mu_a}, r_{\mu_a}; w_a,), (a, l_{\nu_a}, r_{\nu_a}; u_a,) \rangle$ and $\tilde{\tau}_b = \langle (b, l_{\mu_b}, r_{\mu_b}; w_b,), (b, l_{\nu_b}, r_{\nu_b}; u_b,) \rangle$ be two GTIFNs, $\hat{\tau}_a{}^\alpha, \hat{\tau}_b{}^\alpha$ and $\hat{\tau}_a{}^\beta, \hat{\tau}_b{}^\beta$ be their α-cuts and β-cuts respectively. Let $m_a(\hat{\tau}_a{}^\alpha)$ and $m_a(\hat{\tau}_a{}^\beta)$ be mean values of the intervals $\hat{\tau}_a{}^\alpha$ and $\hat{\tau}_a{}^\beta$, respectively i.e.,

$$m_a(\hat{\tau}_a{}^\alpha) = \frac{2aw_a + (w_a - \alpha)(r_{\mu_a} - l_{\mu_a})}{2w_a}$$

$$and \quad m_a(\hat{\tau}_a{}^\beta) = \frac{2a(1-u_a) + (\beta - u_a)(r_{\nu_a} - l_{\nu_a})}{2(1-u_a)}$$

$m_b(\hat{\tau}_b{}^\alpha)$ and $m_b(\hat{\tau}_b{}^\beta)$ can be defined similarly. We define average raking index of the membership function $\mu_{\tilde{\tau}_a}(x)$ and the average raking index of the non-membership function $\nu_{\tilde{\tau}_a}(x)$ for the GTIFNs $\tilde{\tau}_a$ as follows:

$$R_\mu(\tilde{\tau}_a) = \int_0^{w_a} m(\hat{\tau}_a{}^\alpha)d\alpha = \frac{w_a}{4}[4a + r_{\mu_a} - l_{\mu_a}].$$

and

$$R_\nu(\tilde{\tau}_a) = \int_{u_a}^1 m(\hat{\tau}_a{}^\beta)d\beta = \frac{1-u_a}{4}[4a + r_{\nu_a} - l_{\nu_a}].$$

respectively. Assume that $\tilde{\tau}_a$ and $\tilde{\tau}_b$ be two GTIFNs. Let $R_\mu(\tilde{\tau}_a), R_\mu(\tilde{\tau}_b)$ be their average ranking index for membership functions and $R_\nu(\tilde{\tau}_a), R_\nu(\tilde{\tau}_b)$ be the same for non-membership function. On the basis of above definition we propose the following inequality relations.

1. If $R_\mu(\tilde{\tau}_a) < R_\mu(\tilde{\tau}_b)$ then $\tilde{\tau}_a$ is smaller than $\tilde{\tau}_b$ and is denoted by $\tilde{\tau}_a \prec \tilde{\tau}_b$.
2. If $R_\mu(\tilde{\tau}_a) = R_\mu(\tilde{\tau}_b)$ then
   (a) If $R_\nu(\tilde{\tau}_a) = R_\nu(\tilde{\tau}_b)$ then $\tilde{\tau}_a$ is equal to $\tilde{\tau}_b$, denoted by $\tilde{\tau}_a \copyright \tilde{\tau}_b$;
   (b) If $R_\nu(\tilde{\tau}_a) < R_\nu(\tilde{\tau}_b)$ then $\tilde{\tau}_a$ is smaller than $\tilde{\tau}_b$, denoted by $\tilde{\tau}_a \prec \tilde{\tau}_b$.

The symbol ``$\prec$" is an intuitionistic fuzzy version of the order relation ``$<$" in the real number set and has the linguistic interpretation as ``essentially less than." The symbols ``$\succ$" and ``$\copyright$" are explained similarly.

Example 2 Let $\tilde{\tau}_a = \langle (5,1,2; 0.6), (5,1.5,2.6; 0.3) \rangle$ and $\tilde{\tau}_b = \langle (6,2,1; 0.6), (6,2.1,1.5; 0.4) \rangle$ be two GTIFNs. Here we have

$$a = 5, \quad l_{\mu_a} = 1, \quad r_{\mu_a} = 2, \quad w_a = 0.6$$
$$l_{\nu_a} = 1.5, \quad r_{\nu_a} = 2.6, \quad u_a = 0.3$$
$$and \qquad b = 6, \quad l_{\mu_b} = 2, \; r_{\mu_b} = 1, \; w_a = 0.6$$
$$l_{\mu_b} = 2.1, \quad r_{\mu_b} = 1.5, \quad u_a = 0.4.$$

Then we have $R_\mu(\tilde{\tau}_a) = 3.15$ and $R_\mu(\tilde{\tau}_b) = 3.45$. Thus from the above discussion we see that $R_\mu(\tilde{\tau}_a) < R_\mu(\tilde{\tau}_b)$ and therefore $\tilde{\tau}_a \prec \tilde{\tau}_b$.

V. CONCLUSION

In general, the theory of IFS is the generalization of fuzzy sets. Therefore it is expected that, IFS could be used to stimulate human decision-making process and any activities requiring human expertise and knowledge which are inevitably imprecise or not totally reliable. In this paper, we have introduced a more general definition of triangular fuzzy number in intuitionistic fuzzy environment such that the degree of satisfaction and rejection are so considered that the sum of both values is always less than one. Basic arithmetic operations and (α, β)-cut sets are defined for this types of fuzzy numbers. Also a GTIFN is approximated to a nearest interval number by using $(\alpha, \beta)-$cut sets. Finally, average ranking index is proposed to find inequality relations between two GTIFNs. This approach is very simple and easy to apply in real life problems. Next, we approximate a GTIFN to a nearest interval number.

However, non-linear arithmetic operations of GTIFNs may be found in near future. A more effective ranking method may be developed which can be effectively applied to decision making problems with intuitionistic fuzzy parameters.

It is expected that the proposed methods and techniques may be applied to solve many competitive decision making problems in the fields such as market share problem, economics, management and biology which are our future aim of work.

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