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A Mathematical Model to Calculate Contact Stresses In Artificial Human Hip Joint

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Abstract:- In this paper a mathematical model of the hip joint is developed to calculate the contact stresses in artificial hip joint. The stresses in any joint play important role in the restoration of the joint after surgery. Mathematical modelling is a non invasive method of determining stresses in the hip joint. The contact stresses in the artificial hip joint are predicted using Hertz contact stress theory. The high value of contact stresses in UHMWPE cup increases the wear. It was found that the radius of femoral head has significant influence on the peak contact stresses in the acetabular cavity. In addition to femur head dimensions, polymeric cup thickness and clearance are the important factors that affect the contact stresses in human hip joint.

Keywords:- UHMWPE, Hip joint, Contact stresses, Femur Head, Mathematical model

I. INTRODUCTION

Treatments for many orthopaedic disorders involve implants that are attached to bone. The implants may be metal on metal, ceramic on ceramic but the most common are the metal on Ultra high molecular weight polyethylene (UHMWPE). When an implant is attached to a bone, the load transfer through the bone is altered [1,3]. The geometry and the material properties of the implant and the bone determine how the load is shared between the bone and the implant. Load transfer also occurs between different components of the implant. Joint replacements must provide motion while transmitting large forces across the joint. Hence these joints are similar to other machine elements like rolling contact bearings, gear teeth and cams. In these types of joints forces are transferred across surfaces that are rolling or sliding with each other. As the forces are transferred across the surfaces, contact pressure or contact stresses are developed at the mating surfaces. These contact stresses increase if the joint becomes less conforming. The increase in contact stresses increases the wear rate of the UHMWPE cup [2,5]. In total hip prostheses structural failure of acetabular cups is rare, but surface wear of UHMWPE cup causes the loosening and failure of prostheses in longer duration [6]. Bartel et al have studied the effects of conformity and plastic thickness on metal backed plastic implants [1]. Different artificial hip joint designs are proposed in the past. However the combined effects of the hip joint parameters are still unclear [5,7]. Mathematical modelling is a non invasive method to investigate contact stresses in the human hip joints. According to the method the mathematical models can be divided into two main categories, analytical models and numerical models [7,8]. Past literature shows various analytical models developed for calculation of contact stresses over the surface. The most favourite of the numerical methods is the finite element method [4,9].
In this paper attempt is made to find the effect of various design parameters on the calculated contact stresses in the artificial hip joint using the simple elasticity solution given by Bartel. The parameters investigated are the radius of femoral head, the radial clearance, the metal backing thickness and the material of acetabular cup.

II. METHODOLOGY

The model considered in this study is an axis symmetric contact problem of a metal ball indenting a metal backed polyethylene cup. The model is a good approximation of an acetabular cup, where the contact surfaces are nearly conforming. The Hertz contact theory gives a good approximation to this type of problems. All contact problems are inherently non-linear [9,10]. The contact problems of bone implant systems have some fundamental characteristics. First, when the load increases, the contact area and the stress both increase. Second, if the diameter of sphere decrease (contact is less conforming), the contact area decreases and contact pressure increases. Finally if the elastic modulus increases, then the contact area decreases and the maximum pressure increase [12].

The materials for the implants are assumed to be linear and isotropic. Both the femoral head and metal backing are assumed to be rigid, since the elastic modulus for metallic materials is usually 100 to 200 times that of UHMWPE cup. A quasistatic condition for the joint was assumed and the effect of friction is neglected between the metallic femur head and UHMWPE cup. This investigation is done by using low stiffness Ti alloy as femur head instead of traditional femur heads made of SS and Co Cr Mo for un-cemented hip joint under static loading conditions. Also, this study considers the effects of femur head dimensions on the resultant contact pressure at the hip joint cup in the presence of radial clearance between ball and cup. The clearance values are taken as 0.05 mm, 0.1 mm, 0.5 mm between the femur head and UHMWPE cup. The clearance disturbs the conformity and reduces the contact area, thus increasing the stresses. The effects of the presence of a metal backing shell of different thicknesses on the UHMWPE contact stress are also studied. The main aim of this study is to reduce the contact stresses at the UHMWPE cup that are thought to play an important role in the long term clinical performance of the artificial hip joint prostheses [11,13].

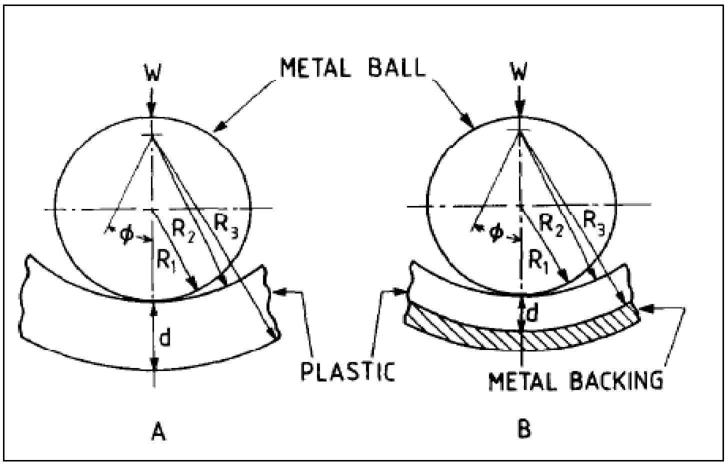

Fig.1. Axisymmetric model of Hip-joint

The governing equations for the simple elasticity analysis are given below

$$c + \delta \ll R_1 \qquad (1)$$

$$u_r = (c + \delta)cos\emptyset - c \qquad (2)$$

$$\cos\theta = \frac{c}{c + \delta} \qquad (3)$$

$$\omega = \pi R_1^2 p_0 \int_0^\theta f(\emptyset) sin2\emptyset d\emptyset \qquad (4)$$

Where $f(\emptyset)$ defines the correct pressure variation along the cup,

$$f(\emptyset) = \left(1 + \frac{c}{\delta}\right) cos\emptyset - \frac{c}{\delta} \qquad (5)$$

Furthermore the stiffness is given as

$$\frac{p}{u_r} = \frac{E\left[\frac{1}{1-2\nu} + \frac{1}{1+\nu}\left(\frac{R_3}{R_2}\right)^3\right]}{R_2\left[\left(\frac{R_3}{R_2}\right)^3 - 1\right]} \qquad (6)$$

The main parameters to be investigated in the present study of elastic contact include the radius of femoral head, the metal backing thickness, and the radial clearance. The material properties are assumed as per given in table I [4]. The Diameter of femoral head is chosen between 22 mm to 36 mm. A single load of 3 kN is assumed corresponding to the subject body weight of 70 kg. The range of radial clearance is taken as 0, 0.05, 0.1, 0.5 mm. The UHMWPE thickness is assumed to be 15 mm. The metal backing thickness of Stainless Steel plate is assumed to be in the range of 0 to 3 mm in the intervals of 0.5 mm. The metal backing plate helps to fix the UHMWPE cup to the bone properly [8,10,11].

The variation of contact stresses with femoral head diameter is also calculated from Finite Element Method (FEM). For calculating the FEM results ANSYS Workbench V12 is used. The CAD Model was developed in Pro-E software and then imported into the ANSYS environment [4].

Table I: Material Properties for the Hip joint

Material	Elastic Modulus GPa	Poisson's Ratio
Titanium-Aluminum Alloy	58	0.3
Stainless Steel	210	0.3
UHMWPE	1.2	0.33

III. RESULTS AND DISCUSSION

A simple iteration method is used for calculating the contact stresses from the mathematical model. The results are calculated for variation of maximum contact stresses with the femoral head diameter. Then the variation of contact stresses with metal backing thickness is calculated for femur head diameters. The effect of clearance on contact stresses is shown in the last graph. The results are given in three different sections.

A. Variation of Maximum Contact Stress with Femur Head Diameter

The maximum contact stress variation with femoral head diameter is shown in Fig.2. The curves are plotted for simple elasticity analysis and FEM analysis. For analytical analysis the maximum contact stress varies from 8.3 MPa for 22 mm femur head to 1.05 MPa for femur head of 36 mm. The contact stress values from FEM analysis vary from 22.4 MPa to 14.12 MPa for the same range. In simple elasticity values are considerably less than the FEM analysis. The reasons behind this may be that the elasticity solution considers only radial movement of UHMWPE cup and neglect the lateral expansion. The FEM analysis gives the values of von Mises stresses.

The contact stresses decrease as the femur head diameter increases. This is because as the femur head diameter increases the contact area and the load carrying area increases. Hence with the load remaining constant the contact stresses decrease.

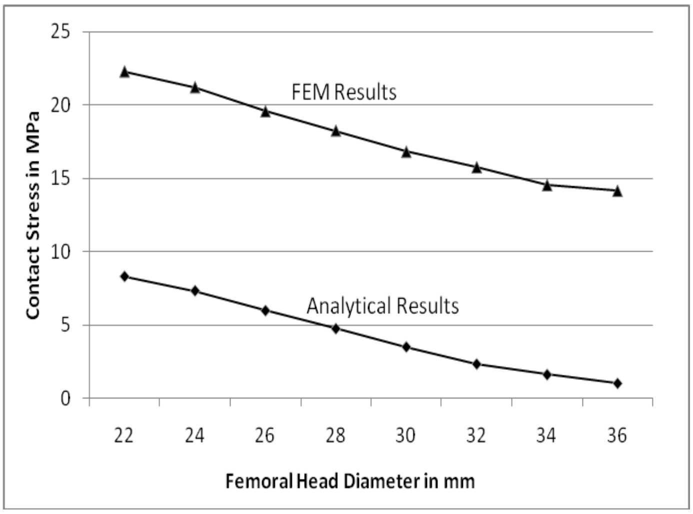

Fig.2: Contact stress variation with femoral head diameter

B. Effect of Metal Backing Thickness

The maximum peak contact stresses for various femoral head diameters and for various metal backing thickness are shown in Fig.3. The UHMWPE is supported by a thin Stainless Steel cup. The thickness of metal backing cup are varied from 0 mm (i.e. no metal backing) to 3 mm. With metal backing present the values of contact stress decrease because of the rise in the stiffness of UHMWPE cup. The metal backing thickness up to 1.5 – 2 mm shows a considerable decrease in contact stress values. After 2 mm the decrease in contact stresses is negligible. This shows that metal backing of up to 2 mm thick should be used in the hip implant. It increases the longevity of the artificial joint.

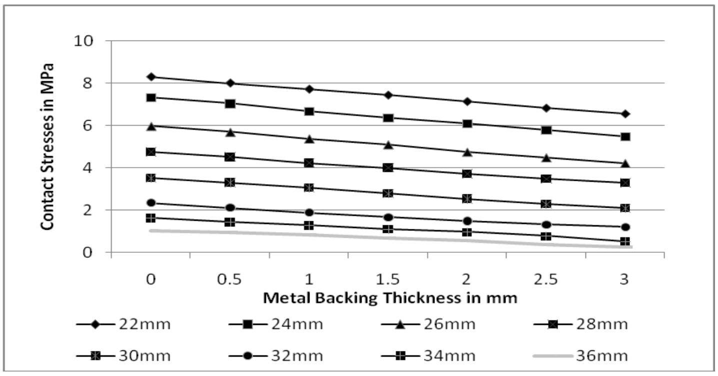

Fig. 3: Contact Stress variation with Metal backing thickness

C. Effect of Clearance

The maximum peak contact stresses for various femoral head diameters and for various clearance values are shown in Fig.4. Small amount of clearance is provided in the acetabular cavity for accommodating the metallic femur head. If this clearance is increased because of wear of acetabular cavity the contact stresses increase exponentially. The contact stress values increase from 8.3 MPa for no clearance to 35.8 MPa for 0.5 mm clearance. Clearance is a dominant parameter in the determination of contact pressure. Hence the clearance in the joint should be kept as minimum as possible.

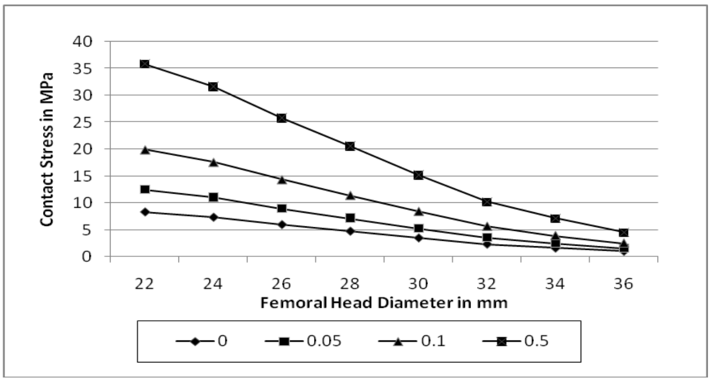

Fig. 4: Contact stress variation with clearance

IV. CONCLUSIONS

The contact stress in the UHMWPE cup is calculated by developing a analytical model. The results are compared with the FEM results. The variation of contact stresses with metal backing thickness and clearance is also studied. The results show that femoral head diameter has major impact on the contact stress values. As the femoral head diameter increases the stress values decrease. The metal backing plate increases the stiffness of UHMWPE cup, thereby reducing the stress values. The increase in clearance increases the stress values exponentially. Hence the contact stress plays important role in the design of artificial hip implants.

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