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Performance Evaluation of a Queueing Model with Two Component Mixture of Doubly Truncated Exponential Service Times

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Abstract:-In this paper, we introduce and analyze a new queueing model with the assumption that the service time of the customer follows a two component mixture of doubly truncated exponential distribution. The doubly truncated two component mixture distribution is capable of characterizing the heterogeneous and finite range nature of the service time. This service time distributions also includes two component mixture of exponential service time, doubly truncated exponential time and exponential service time queuing models as limiting/particularcases. Assuming the arrival process follows a Poison process and using the embedded markov chain technique the system behaviour is analyzed. Explicit expressions for the system performance like average number of customers in the system and in the queue, average waiting time of a customer in the queue, the throughput of the service station, the probability of the idleness of the service etc,. are derived. Through numerical illustration, the sensitivity of the model performance measures with respect to the changes in the model parameter is also studied.

Keywords:-Queueing,Truncated,Exponential Service times,Poisson Model

I. INTRODUCTION

Queueing models create lot of interest due to their ready applicability in wide varity of situations prevailing at places like, transportation, communication networks, production processes, reliability analysis, computer performance evaluations studies, etc,.(Boxima O.J.,(1988), Bunday.B.D.(1996)).A queue is a waiting line of units demanding service at a service facility (Andreas Brandt,2008). Congestion is a natural phenomenon in everyday life. Queueing is a mechanism that is used to handle congestion. A system consisting of service facility process of a service and a process of arrival of customer is called a queueingsystem(Burke. P.J. (1975)). In queueing models it is customary to assume that the arrival and service processes follow Poisson processes such that the inter arrival times and the inter service times follow exponential distributions.(Haviv.M(2007),Guy.L.Curry(2008),K.Srinivasa Raoet.al,2008). This assumption of inter-service times being follow an exponential distribution is valid only when the service time required by a customer is having infinite range and long upper tail. (Hiroyuki Masuyama, Tetsuya Takine (2003)).However, in many practical situations arising at places like communication systems the service time required by the customer is finite. Hence, to have an approximate model for this sort of situations, the service time of the customer is to be approximated with truncated distributions. (Altiok,T.(1982), Patrick,M.,(1999), Guodong Pan (2010)). Very little work has been reported regarding queueing models with truncated service time distribution. Hence in this paper, in order to have efficient congestion control we developed and analyzed a queueing model with the assumption that the service time required by the packet follows a two component mixture of a doubly truncated exponential distribution. Some practical situations at places, like telecommunications, the service time required by the packet may not be homogeneous. Theheterogeneity of service times can be modeled by considering two component mixture of distribution.Here, it is assumed that the arrival of packets follows a Poisson process. Using the imbedded Markovchain technique, the probability generating function of the number of packets in the system is derived. The system characteristics like, the probability of emptiness of the system, the mean number of packets in the queue,theLaplace transformation of the waiting time distribution of a packet, the average waiting time of apacket, the variance of the number of packet in the queue, the variance of the waiting time of a packetetc,. are obtained. The sensitivity of the model with respect to the parameters is also studied. This model also includes some of the earlier models as particular casesfor specific or limiting values of the parameters.

II.POISSON QUEUEING MODEL WITH TWO COMPONENT MIXTURE OF DOUBLY TRUNCATED EXPONENTIAL SERVICE TIMES

Weconsider a single server Poisson Queueing system in which the arrival of packets follows a Poisson process with parameter λ.It is also assumed that theinter-service times follow a two component mixture of doubly truncated exponential distribution with parameters $\mu_1, \mu_2,$ p, a and b and having differences. This assumption is made since the service time required for completing any activity is finite and is bound by finite values of a and b.

The probability density function of the inter service times is,

$$b(t)=\left[\frac{p\mu_1}{e^{-\mu_1 a}-e^{-\mu_1 b}}\right]e^{-\mu_1 t}+\left[\frac{(1-p)\mu_2}{e^{-\mu_2 a}-e^{-\mu_2 b}}\right]e^{-\mu_2 t};a<t<b \quad (1)$$

The mean service time is,

$$E(T)=\frac{p\left[\left(1+a\mu_1\right)e^{-\mu_1 a}-\left(1+b\mu_1\right)e^{-\mu_1 b}\right]}{\mu_1(e^{-\mu_1 a}-e^{-\mu_1 b})}+\frac{(1-p)\left[\left(1+a\mu_2\right)e^{-\mu_2 a}-\left(1+b\mu_2\right)e^{-\mu_2 b}\right]}{\mu_2(e^{-\mu_2 a}-e^{-\mu_2 b})} \quad (2)$$

Following the heauristic argument given by Gross and Harris (1974) the steady state behavior of the model is studied. The imbedded stochastic process $X(t_i)$, where, X denotes the number in the system and $t_{1,}t_{2,}t_{3,}..$,are the successive times of completion of service.

Since,t_i is the completion time of the ith packet, then X(t_i) is the number of packets the ith packet leaves behind as he departs. Since, the state space is discrete, X_i represents the number of packets remaining in the system as the $i$thpacket departs. Then forall n>0 one can have

$$X_{n+1}=\begin{cases}X_n-1+A_{n+1} & ; \quad X_n\geq 1\\ A_{n+1} & ; \quad X_n=0\end{cases} \quad (3)$$

where,$X_n$is the number in the system at the nth departure point and $A_{n+1}$is the number of packets who arrived during the service time,$S^{(n+1)}$, of the (n+1)st packet.

The random variable $S^{(n+1)}$by assumption is independent of previous service times and the length of the queue. Since arrivals are Poissonian, the random variable A_{n+1} depends only on S and not on the queue are on the time of service initiation. Then

$$\Pr\{A=a\}=\begin{cases}\displaystyle\int_0^\infty \frac{e^{-\lambda t}(\lambda t)^{j-i+1}}{(j-i+1)!}\,dB(t)\ , & (j\geq i-1, i\geq 1)\\ 0 & , \ (j<i-1, i\geq 1)\end{cases} \quad (4)$$

Letp_{ij} denote the probability that the system size immediately after a departure point is j given that the system size after previous departure was i. $k_n$is the probability that there are n arrivals during a service time t.Then

$p_{ij} = P_r$ {System size immediately after a departure point is j| system size after previousDeparturewas }

$=P_r \{X_{n+1}=j|X_n=i\}$

where,

$$p_{ij}==\int_a^b e^{-\lambda t}\,\frac{(\lambda t)^n}{n!}\left(\frac{p\mu_1}{e^{-\mu_1 a}-e^{-\mu_1 b}}\,e^{-\mu_1 t}+\frac{(1-p)\mu_2}{e^{-\mu_2 a}-e^{-\mu_2 b}}\,e^{-\mu_2 t}\right)dt \quad (5)$$

This implies that p_{ij} is equals tok_{j-i+1} and

$P=\left[pij\right]$

Assuming that the system is in steady state, and $p_{ij} = \pi_j$

Then,

$$p_i=\pi_0 k_i+\sum_{j=1}^{i+1}\pi_j k_{i-j+1} \qquad \left(i=0,1,2,...\right) \quad (6)$$

where, π_j is the probability that there arejpackets in the system at a departure point after steady state is reached.

Let $P(z)=\sum_{i=0}^{\infty}\pi_i z^i$ $(|z|\leq 1)$ (7)

$K(z)=\sum_{i=0}^{\infty}k_i z^i$ are the generating functions of π_n and k_n respectively

Hence, $$K(z)=\sum_{n=0}^{\infty}\int_a^b e^{-\lambda t}\,\frac{(\lambda t)^n}{n!}\,z^n\left(\frac{p\mu_1}{e^{-\mu_1 a}-e^{-\mu_1 b}}\,e^{-\mu_1 t}+\frac{(1-p)\mu_2}{e^{-\mu_2 a}-e^{-\mu_2 b}}\,e^{-\mu_2 t}\right)dt \quad (8)$$

After simplification, we get

$$K(z) = \frac{p\mu_1}{e^{-\mu_1 a} - e^{-\mu_1 b}} \left[\frac{e^{-[\lambda(1-z)+\mu_1]a} - e^{-[\lambda(1-z)+\mu_1]b}}{\mu_1 + (1-z)\lambda} \right]$$

$$+ \frac{(1-p)\mu_2}{e^{-\mu_2 a} - e^{-\mu_2 b}} \left[\frac{e^{-[\lambda(1-z)+\mu_2]a} - e^{-[\lambda(1-z)+\mu_2]b}}{\mu_2 + (1-z)\lambda} \right] \qquad (9)$$

$$K'(1) = \frac{\lambda p\left[(1+a\mu_1)e^{-\mu_1 a} - (1+b\mu_1)e^{-\mu_1 b}\right]}{\mu_1(e^{-\mu_1 a} - e^{-\mu_1 b})}$$

$$+ \frac{\lambda(1-p)\left[(1+a\mu_2)e^{-\mu_2 a} - (1+b\mu_2)e^{-\mu_2 b}\right]}{\mu_2(e^{-\mu_2 a} - e^{-\mu_2 b})} \qquad (10)$$

$$P(z) = \left[1 - \frac{\lambda p\left[(1+a\mu_1)e^{-\mu_1 a} - (1+b\mu_1)e^{-\mu_1 b}\right]}{\mu_1(e^{-\mu_1 a} - e^{-\mu_1 b})} - \frac{\lambda(1-p)\left[(1+a\mu_2)e^{-\mu_2 a} - (1+b\mu_2)e^{-\mu_2 b}\right]}{\mu_2(e^{-\mu_2 a} - e^{-\mu_2 b})} \right]$$

$$(1-z)\left(\frac{p\mu_1}{e^{-\mu_1 a} - e^{-\mu_1 b}} \left[\frac{e^{-[\lambda(1-z)+\mu_1]a} - e^{-[\lambda(1-z)+\mu_1]b}}{\mu_1 + \lambda(1-z)} \right] + \frac{(1-p)\mu_2}{e^{-\mu_2 a} - e^{-\mu_2 b}} \left[\frac{e^{-[\lambda(1-z)+\mu_2]a} - e^{-[\lambda(1-z)+\mu_2]b}}{\mu_2 + \lambda(1-z)} \right] \right)$$

$$\left\{ \left(\frac{p\mu_1}{e^{-\mu_1 a} - e^{-\mu_1 b}} \left[\frac{e^{-[\lambda(1-z)+\mu_1]a} - e^{-[\lambda(1-z)+\mu_1]b}}{\mu_1 + \lambda(1-z)} \right] + \frac{(1-p)\mu_2}{e^{-\mu_2 a} - e^{-\mu_2 b}} \left[\frac{e^{-[\lambda(1-z)+\mu_2]a} - e^{-[\lambda(1-z)+\mu_2]b}}{\mu_2 + \lambda(1-z)} \right] \right) - z \right\}^{-1} \quad (11)$$

III.SYSTEM CHARACTERISTICS

The performance measures of the model are developed by Expanding the above equation (11) and collecting the constant terms we get the

Probability that the system size is emptyas

$$P_0 = 1 - \frac{\lambda p\left[(1+a\mu_1)e^{-\mu_1 a} - (1+b\mu_1)e^{-\mu_1 b}\right]}{\mu_1(e^{-\mu_1 a} - e^{-\mu_1 b})}$$

$$- \frac{\lambda(1-p)\left[(1+a\mu_2)e^{-\mu_2 a} - (1+b\mu_2)e^{-\mu_2 b}\right]}{\mu_2(e^{-\mu_2 a} - e^{-\mu_2 b})} \qquad (12)$$

The average number of packets in system can be obtained as,

$$L = P\left\{ \frac{\lambda\left[(1+a\mu_1)e^{-\mu_1 a} - (1+b\mu_1)e^{-\mu_1 b}\right]}{\mu_1(e^{-\mu_1 a} - e^{-\mu_1 b})} + \frac{\lambda^2\left[(a^2\mu_1^2 + 2a\mu_1 + 2)e^{-\mu_1 a} - (b^2\mu_1^2 + 2b\mu_1 + 2)e^{-\mu_1 b}\right]}{2\mu_1\left[\mu_1(e^{-\mu_1 a} - e^{-\mu_1 b}) - \lambda\left[(1+a\mu_1)e^{-\mu_1 a} - (1+b\mu_1)e^{-\mu_1 b}\right]\right]} \right\}$$

$$+(1-P)\left\{ \frac{\lambda\left[(1+a\mu_2)e^{-\mu_2 a} - (1+b\mu_2)e^{-\mu_2 b}\right]}{\mu_2(e^{-\mu_2 a} - e^{-\mu_2 b})} + \frac{\lambda^2\left[(a^2\mu_2^2 + 2a\mu_2 + 2)e^{-\mu_2 a} - (b^2\mu_2^2 + 2b\mu_2 + 2)e^{-\mu_2 b}\right]}{2\mu_2\left[\mu_2(e^{-\mu_2 a} - e^{-\mu_2 b}) - \lambda\left[(1+a\mu_2)e^{-\mu_2 a} - (1+b\mu_2)e^{-\mu_2 b}\right]\right]} \right\} \quad (13)$$

The average number of packets in the system is,

$$Lq = P\left\{ \frac{\lambda^2\left[(a^2\mu_1^2 + 2a\mu_1 + 2)e^{-\mu_1 a} - (b^2\mu_1^2 + 2b\mu_1 + 2)e^{-\mu_1 b}\right]}{2\mu_1\left[\mu_1(e^{-\mu_1 a} - e^{-\mu_1 b}) - \lambda\left[(1+a\mu_1)e^{-\mu_1 a} - (1+b\mu_1)e^{-\mu_1 b}\right]\right]} \right\}$$

$$+(1-P)\left\{ \frac{\lambda^2\left[(a^2\mu_2^2 + 2a\mu_2 + 2)e^{-\mu_2 a} - (b^2\mu_2^2 + 2b\mu_2 + 2)e^{-\mu_2 b}\right]}{2\mu_2\left[\mu_2(e^{-\mu_2 a} - e^{-\mu_2 b}) - \lambda\left[(1+a\mu_2)e^{-\mu_2 a} - (1+b\mu_2)e^{-\mu_2 b}\right]\right]} \right\} \quad (14)$$

The variance of the number of packets in the system is,

$$
\begin{aligned}
Var(N) = P\Bigg\{ & \left\{ \frac{\lambda\left[(1+a\mu_1)e^{-\mu_1 a}-(1+b\mu_1)e^{-\mu_1 b}\right]}{\mu_1(e^{-\mu_1 a}-e^{-\mu_1 b})} \left[\frac{\mu_1(e^{-\mu_1 a}-e^{-\mu_1 b}) - \lambda\left[(1+a\mu_1)e^{-\mu_1 a}-(1+b\mu_1)e^{-\mu_1 b}\right]}{\mu_1(e^{-\mu_1 a}-e^{-\mu_1 b})} \right] \right\} \\
& + \left\{ \frac{\left[\lambda^2\left[\left(a^2\mu_1^{\,2}+2a\mu_1+2\right)e^{-\mu_1 a}-\left(b^2\mu_1^{\,2}+2b\mu_1+2\right)e^{-\mu_1 b}\right]\right]}{2\mu_1\left[\mu_1(e^{-\mu_1 a}-e^{-\mu_1 b}) - \lambda\left[(1+a\mu_1)e^{-\mu_1 a}-(1+b\mu_1)e^{-\mu_1 b}\right]\right]} \right\} \\
& \left\{ \frac{3\mu_1(e^{-\mu_1 a}-e^{-\mu_1 b}) - 2\lambda\left[(1+a\mu_1)e^{-\mu_1 a}-(1+b\mu_1)e^{-\mu_1 b}\right]}{\mu_1(e^{-\mu_1 a}-e^{-\mu_1 b})} + \frac{\left[\lambda^2\left[\left(a^2\mu_1^{\,2}+2a\mu_1+2\right)e^{-\mu_1 a}-\left(b^2\mu_1^{\,2}+2b\mu_1+2\right)e^{-\mu_1 b}\right]\right]}{2\mu_1\left[\mu_1(e^{-\mu_1 a}-e^{-\mu_1 b}) - \lambda\left[(1+a\mu_1)e^{-\mu_1 a}-(1+b\mu_1)e^{-\mu_1 b}\right]\right]} \right\} \\
& + \left\{ \frac{3\lambda^3\left[a^2\mu_1^{\,2}+2\mu_1 a+2)e^{-\mu_1 a}-(b^2\mu_1^{\,2}+2\mu_1 b+2)e^{-\mu_1 b}\right] - \left(\mu_1^{\,3}\left[\lambda^3(a^3e^{-\mu_1 a}-b^3e^{-\mu_1 b})\right]\right)}{3\mu_1^{\,2}\left[\mu_1(e^{-\mu_1 a}-e^{-\mu_1 b}) - \lambda\left[(1+a\mu_1)e^{-\mu_1 a}-(1+b\mu_1)e^{-\mu_1 b}\right]\right]} \right\} \Bigg\} \\
+(1-P)\Bigg\{ & \left\{ \frac{\lambda\left[(1+a\mu_2)e^{-\mu_2 a}-(1+b\mu_2)e^{-\mu_2 b}\right]}{\mu_2(e^{-\mu_2 a}-e^{-\mu_2 b})} \left[\frac{\mu_2(e^{-\mu_2 a}-e^{-\mu_2 b}) - \lambda\left[(1+a\mu_2)e^{-\mu_2 a}-(1+b\mu_2)e^{-\mu_2 b}\right]}{\mu_2(e^{-\mu_2 a}-e^{-\mu_2 b})} \right] \right\} \\
& + \left\{ \frac{\lambda^2\left[\left(a^2\mu_2^{\,2}+2a\mu_2+2\right)e^{-\mu_2 a}-\left(b^2\mu_2^{\,2}+2b\mu_2+2\right)e^{-\mu_2 b}\right]}{2\mu_2\left[\mu_2(e^{-\mu_2 a}-e^{-\mu_2 b}) - \lambda\left[(1+a\mu_2)e^{-\mu_2 a}-(1+b\mu_2)e^{-\mu_2 b}\right]\right]} \right\} \\
& \left\{ \frac{3\mu_2(e^{-\mu_2 a}-e^{-\mu_2 b}) - 2\lambda\left[(1+a\mu_2)e^{-\mu_2 a}-(1+b\mu_2)e^{-\mu_2 b}\right]}{\mu_2(e^{-\mu_2 a}-e^{-\mu_2 b})} + \frac{\lambda^2\left[\left(a^2\mu_2^{\,2}+2a\mu_2+2\right)e^{-\mu_2 a}-\left(b^2\mu_2^{\,2}+2b\mu_2+2\right)e^{-\mu_2 b}\right]}{2\mu_2\left[\mu_2(e^{-\mu_2 a}-e^{-\mu_2 b}) - \lambda\left[(1+a\mu_2)e^{-\mu_2 a}-(1+b\mu_2)e^{-\mu_2 b}\right]\right]} \right\} \\
& + \left\{ \frac{3\lambda^3\left[a^2\mu_2^{\,2}+2\mu_2 a+2)e^{-\mu_2 a}-(b^2\mu_2^{\,2}+2\mu_2 b+2)e^{-\mu_2 b}\right] - \mu_2^{\,3}\left(\lambda^3(a^3e^{-\mu_2 a}-b^3e^{-\mu_2 b})\right)}{3\mu_2^{\,2}\left[\mu_2(e^{-\mu_2 a}-e^{-\mu_2 b}) - \lambda\left[(1+a\mu_2)e^{-\mu_2 a}-(1+b\mu_2)e^{-\mu_2 b}\right]\right]} \right\} \Bigg\} \qquad (15)
\end{aligned}
$$

IV. WAITING TIME DISTRIBUTION OF THE MODEL

We derive the waiting time distribution of the single server Poisson arrival queueingmodel with two component mixture ofdoublytruncated exponential service times. Consider the queueing discipline of the system as first in first out.

Let $B^*(s)$ be theLaplace transformation of the inter-service time distribution and $W^*(s)$ be the Laplace transform of the waiting time distribution. Then we have

$$
\begin{aligned}
B^*(s) = \; & \left\{ \frac{p\mu_1}{\mu_1+s} \left(\frac{\left[e^{-(s+\mu_1)a} - e^{-(s+\mu_1)b}\right]}{e^{-\mu_1 a}-e^{-\mu_1 b}} \right) \right\} \\
& + \left\{ \frac{(1-p)\mu_2}{\mu_2+s} \left(\frac{\left[e^{-(s+\mu_2)a} - e^{-(s+\mu_2)b}\right]}{e^{-\mu_2 a}-e^{-\mu_2 b}} \right) \right\} \qquad (16)
\end{aligned}
$$

The Laplace transformation of waiting time distributionis

$$W^*[\lambda(1-z)] = \frac{[1-K'(1)](1-z)B^*[\lambda(1-z)]}{B^*[\lambda(1-z)]-z} \qquad (17)$$

writing $\lambda(1-z)=\theta,$ we get $z = 1-\dfrac{\theta}{\lambda}$

Therefore,

$$W^*(\theta) = \frac{[1-K'(1)]\theta B^*(\theta)}{\theta-\lambda[1-B^*(\theta)]} \qquad (18)$$

$$W^*{}_q(s) = \frac{[1-K'(1)]s}{s-\lambda[1-B^*(s)]} \qquad (19)$$

where, $B^*(s)$ is the Laplace transformation of the service time distribution and $K'(1)$ is as given in equation (10).

The mean waiting time of the packet in queue is,

$$Wq = p \left\{ \frac{\lambda\left[(a^2\mu_1^{\,2}+2\mu_1 a+2)e^{-\mu_1 a}-(b^2\mu_1^{\,2}+2\mu_1 b+2)e^{-\mu_1 b}\right]}{2\mu_1\left[\mu_1(e^{-\mu_1 a}-e^{-\mu_1 b})-\lambda\left[(1+a\mu_1)e^{-\mu_1 a}-(1+b\mu_1)e^{-\mu_1 b}\right]\right]} \right\} \qquad (20)$$

$$+\,(1-p)\left\{ \frac{\lambda\left[(a^2\mu_2^{\,2}+2\mu_2 a+2)e^{-\mu_2 a}-(b^2\mu_2^{\,2}+2\mu_2 b+2)e^{-\mu_2 b}\right]}{2\mu_2\left[\mu_2(e^{-\mu_2 a}-e^{-\mu_2 b})-\lambda\left[(1+a\mu_2)e^{-\mu_2 a}-(1+b\mu_2)e^{-\mu_2 b}\right]\right]} \right\}$$

The variance of the waiting time of a packetin the queue is,

$$\begin{aligned}
V(Tq) = p\Bigg(&\frac{\lambda}{12}\left\{4\left[1-\frac{\lambda\left[(1+a\mu_1)e^{-\mu_1 a}-(1+b\mu_1)e^{-\mu_1 b}\right]}{\mu_1(e^{-\mu_1 a}-e^{-\mu_1 b})}\right]\right.\\
&\left[\frac{3(a^2\mu_1^{\,2}+2\mu_1 a+2)e^{-\mu_1 a}-(b^2\mu_1^{\,2}+2\mu_1 b+2)e^{-\mu_1 b}-\mu^3(a^3e^{-\mu_1 a}-b^3e^{-\mu_1 b})}{\mu_1^{\,3}(e^{-\mu_1 a}-e^{-\mu_1 b})}\right]\\
&\left.+\;3\lambda\left(\frac{(a^2\mu_1^{\,2}+2\mu_1 a+2)e^{-\mu_1 a}-(b^2\mu_1^{\,2}+2\mu_1 b+2)e^{-\mu_1 b}}{\mu_1^{\,2}(e^{-\mu_1 a}-e^{-\mu_1 b})}\right)^2\right\}\\
&\left[\frac{\mu_1(e^{-\mu_1 a}-e^{-\mu_1 b})-\lambda\left[(1+a\mu_1)e^{-\mu_1 a}-(1+b\mu_1)e^{-\mu_1 b}\right]}{\mu_1(e^{-\mu_1 a}-e^{-\mu_1 b})}\right]^{-2}\Bigg)
\end{aligned}$$

$$\begin{aligned}
+(1-p)\Bigg(&\frac{\lambda}{12}\left\{4\left[1-\frac{\lambda\left[(1+a\mu_2)e^{-\mu_2 a}-(1+b\mu_2)e^{-\mu_2 b}\right]}{\mu_2(e^{-\mu_2 a}-e^{-\mu_2 b})}\right]\right.\\
&\left[\frac{3(a^2\mu_2^{\,2}+2\mu_2 a+2)e^{-\mu_2 a}-(b^2\mu_2^{\,2}+2\mu_2 b+2)e^{-\mu_2 b}-\mu^3(a^3e^{-\mu_2 a}-b^3e^{-\mu_2 b})}{\mu_2^{\,3}(e^{-\mu_2 a}-e^{-\mu_2 b})}\right]\\
&\left.+\;3\lambda\left(\frac{(a^2\mu_2^{\,2}+2\mu_2 a+2)e^{-\mu_2 a}-(b^2\mu_2^{\,2}+2\mu_2 b+2)e^{-\mu_2 b}}{\mu_2^{\,2}(e^{-\mu_2 a}-e^{-\mu_2 b})}\right)^2\right\}\\
&\left[\frac{\mu_2(e^{-\mu_2 a}-e^{-\mu_2 b})-\lambda\left[2(1+a\mu_2)e^{-\mu_2 a}-(1+b\mu_2)e^{-\mu_2 b}\right]}{\mu_2(e^{-\mu_2 a}-e^{-\mu_2 b})}\right]^{-2}\Bigg) \qquad (21)
\end{aligned}$$

The waiting time of apacket in the system is,

$$W_s = p\left\{ \frac{\lambda\left[(a^2\mu_1^{\,2}+2\mu_1 a+2)e^{-\mu_1 a}-(b^2\mu_1^{\,2}+2\mu_1 b+2)e^{-\mu_1 b}\right]}{2\mu_1\left[\mu_1(e^{-\mu_1 a}-e^{-\mu_1 b})-\lambda\left[(1+a\mu_1)e^{-\mu_1 a}-(1+b\mu_1)e^{-\mu_1 b}\right]\right]} \right\}$$

$$+\,(1-p)\left\{ \frac{\lambda\left[(a^2\mu_2^{\,2}+2\mu_2 a+2)e^{-\mu_2 a}-(b^2\mu_2^{\,2}+2\mu_2 b+2)e^{-\mu_2 b}\right]}{2\mu_2\left[\mu_2(e^{-\mu_2 a}-e^{-\mu_2 b})-\lambda\left[(1+a\mu_2)e^{-\mu_2 a}-(1+b\mu_2)e^{-\mu_2 b}\right]\right]} \right\}$$

$$+\left\{ \frac{p\left[(1+a\mu_1)e^{-\mu_1 a}-(1+b\mu_1)e^{-\mu_1 b}\right]}{\mu_1(e^{-\mu_1 a}-e^{-\mu_1 b})} + \frac{(1-p)\left[(1+a\mu_2)e^{-\mu_2 a}-(1+b\mu_2)e^{-\mu_2 b}\right]}{\mu_2(e^{-\mu_2 a}-e^{-\mu_2 b})} \right\} \qquad (22)$$

V.NUMERICALILLUSTRATION

The performance of the queueing model is discussed through a numerical illustration. Different values of the parameter are considered for arrival rate λ and service time parametersμ_1, μ_2, p, a and b.For given values of λ = 1,2,3,4,5;μ_1 = 5,7,9,11, μ_2 = 7,8,9,10, p = 0.1,0.2,0.3,0.4, a = 0.1,0.2,0.3,0.4 and b = 0.1, 0.2, 0.3, 0.4,the probability that the system is empty and the probability that the server is busy are computed and presented in table 1.The relationship between the parameters and the probability of the idealness are shown in figure 1.

From table 1 it is observed that the probability of emptiness is highly influenced by the model parameters. As the mean arrival rate λvaries from 1 to 5. The probability of emptiness in the system is decreasing from 0.858 to 0.289 when other parameter are fixed at μ_1= 10,μ_2= 8, p = 0.8, a = 0.1 and b = 0.2.

Table 1: VALUES OF Po AND 1-PoFORDIFFERENT VALUESOFλ, μ, a AND b

a	b	p	λ	μ_1	μ_2	Po	1-Po
0.1	0.2	0.8	1	10	8	0.858	0.142
0.1	0.2	0.8	2	10	8	0.716	0.284
0.1	0.2	0.8	3	10	8	0.574	0.426
0.1	0.2	0.8	4	10	8	0.432	0.568
0.1	0.2	0.8	5	10	8	0.289	0.711
0.1	0.2	0.8	3	5	8	0.564	0.436
0.1	0.2	0.8	3	7	8	0.568	0.432
0.1	0.2	0.8	3	9	8	0.574	0.426
0.1	0.2	0.8	3	11	8	0.576	0.424
0.1	0.5	0.8	1	10	8	0.804	0.196
0.2	0.5	0.8	1	10	8	0.714	0.286
0.3	0.5	0.8	1	10	8	0.630	0.370
0.4	0.5	0.8	1	10	8	0.558	0.442
0.6	0.1	0.8	1	10	8	0.800	0.200
0.6	0.2	0.8	1	10	8	0.704	0.296
0.6	0.3	0.8	1	10	8	0.614	0.386
0.6	0.4	0.8	1	10	8	0.530	0.470
0.1	0.5	0.1	3	10	8	0.381	0.619
0.1	0.5	0.2	3	10	8	0.385	0.615
0.1	0.5	0.3	3	10	8	0.390	0.610
0.1	0.5	0.4	3	10	8	0.395	0.605
0.1	0.5	0.8	3	10	7	0.408	0.592
0.1	0.5	0.8	3	10	8	0.413	0.587
0.1	0.5	0.8	3	10	9	0.418	0.582
0.1	0.5	0.8	3	10	10	0.422	0.578

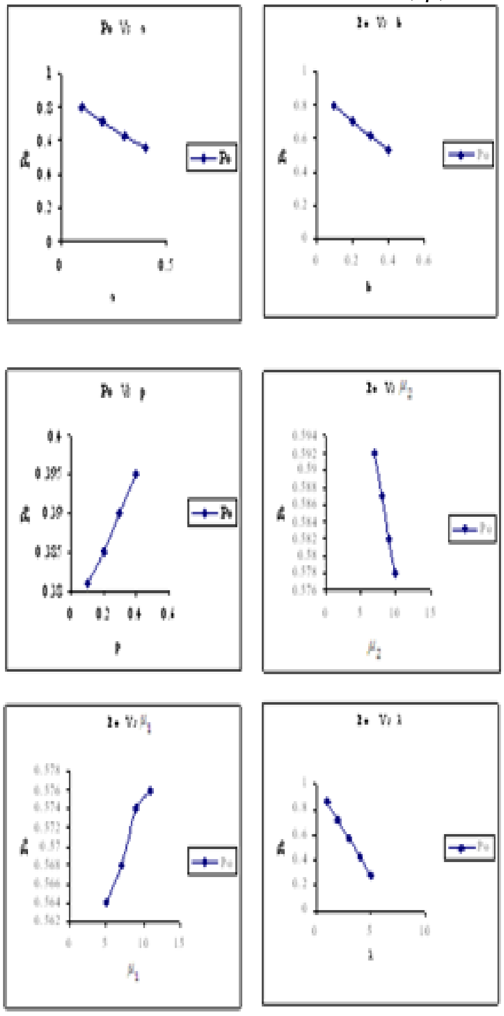

Figure 1:Relationship between probability of Emptinessand input parameters

We have observed that when the service time parameter μ_1 increases from 5 to 11 the probability of emptiness is increasing from 0.564 to 0.576, for fixed values of λ = 3, a = 0.1,b = 0.2, p = 0.8 and μ_2= 8.

When the service time parameter μ_2 increases from 7 to 10 the probability of emptiness is increasing from 0.408 to 0.422, for fixed values of λ = 3, a = 0.1, b = 0.5, p = 0.8 and μ_1=10.

As the truncation parameter of the service time distribution ‘ a ’ increases from 0.1 to 0.4 the probability of emptiness is increases from 0.1 to 0.4 and the probability of emptiness is increasing from 0.804 to 0.558 for fixed values of the other parameters.

As the truncation parameter of the service time distribution ‘b’increases from 0.1 to 0.4 the probability of emptiness is increasing from 0.1 to 0.4 and the probability of emptiness is increasing from 0.800 to 0.530 for given values of the other parameters.

As the truncation parameter of the service time distribution ‘p ’increases from 0.1 to 0.4 the probability of emptiness is increasing from 0.1 to 0.4 and the probability of emptiness is increasing from 0.381 to 0.395 for given values of the other parameters.

For different values of the parameters the average number of packets in the system, the average number of packets in the queue and the variance of the number of packets in the system are computed and presented in table2. The relationship between the parameters and the performance measures is shown in figure 2.

Table 2: VALUES OF L ANDLqANDVar (N)

a	b	p	λ	μ_1	μ_2	L	Lq	Var(N)
0.1	0.2	0.8	1	10	8	0.154	0.012	0.159
0.1	0.2	0.8	2	10	8	0.343	0.059	0.387
0.1	0.2	0.8	3	10	8	0.592	0.165	0.783
0.1	0.2	0.8	4	10	8	0.958	0.389	1.618
0.1	0.2	0.8	5	10	8	1.618	0.907	3.906
0.1	0.2	0.8	3	5	8	0.611	0.175	0.985
0.1	0.2	0.8	3	7	8	0.603	0.171	0.872
0.1	0.2	0.8	3	9	8	0.591	0.165	0.783
0.1	0.2	0.8	3	11	8	0.587	0.163	0.762
0.1	0.5	0.8	1	10	8	0.224	0.028	0.239
0.2	0.5	0.8	1	10	8	0.348	0.061	0.369
0.3	0.5	0.8	1	10	8	0.481	0.111	0.514
0.4	0.5	0.8	1	10	8	0.618	0.176	0.671
0.6	0.1	0.8	1	10	8	0.231	0.031	0.248
0.6	0.2	0.8	1	10	8	0.363	0.067	0.387
0.6	0.3	0.8	1	10	8	0.512	0.126	0.552
0.6	0.4	0.8	1	10	8	0.681	0.211	0.749
0.1	0.5	0.1	3	10	8	1.224	0.605	2.209
0.1	0.5	0.2	3	10	8	1.205	0.590	2.146
0.1	0.5	0.3	3	10	8	1.185	0.575	2.082
0.1	0.5	0.4	3	10	8	1.165	0.560	2.019
0.1	0.5	0.8	3	10	7	1.113	0.521	1.870
0.1	0.5	0.8	3	10	8	1.086	0.499	1.766
0.1	0.5	0.8	3	10	9	1.064	0.482	1.692
0.1	0.5	0.8	3	10	10	1.047	0.469	1.639

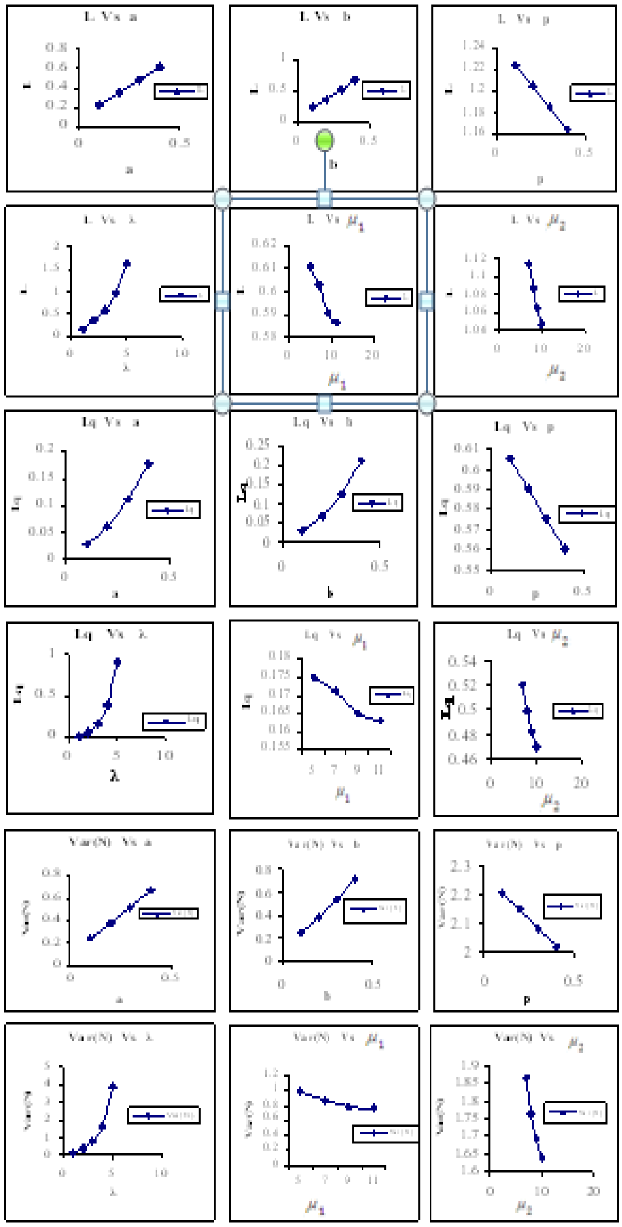

Figure 2: Relationship between parameters L, LqandVar (N)

From table 2 it is observed that the performance measures of the queueing model are significantly influenced by the parameters of the model. As the arrival rate λ increases the average number of packets in the system is increasing. Same phenomenon is observed with respective the average number of packets in the queue for the given values of the other parameters.

When the parameter μ_1 increases from 5 to 11 the average number of packet in the system is decreasing from 0.611 to 0.587 for fixed values of $\lambda = 3$, $\mu_2 = 8$, b = 0.2, $a = 0.1$ and p = 0.8 Similarly the value of average number of packet in the queue is decreasing from 0.175 to 0.163 for the same changes in μ_1 when other parameters remain fixed.

When the parameter μ_2 increases from 7 to 10 the average number of packet in the system is decreasing from 1.113 to 1.047 for fixed values of $\lambda = 3$, $\mu_1 = 10$, b = 0.5, $a = 0.1$ and p = 0.8 Similarly the value of average number of packet in the queue is decreasing from 0.521 to 0.469 for the same changes in μ_2 for fixed values of the other parameters.

As the truncation parameter 'a' is increasing from 0.1 to 0.4 then the average number of packets in the system and the average number of packets in the queue are increasing from 0.224 to 0.618 and 0.028 to 0.176 respectively for fixed values of the other parameters.

As the truncation parameter 'b' is increasing from 0.1 to 0.4 then the average number of packets in the system and the average numbers of packets in the queue are increasing from 0.231 to 0.681 and 0.031 to 0.211 respectively.

As the truncation parameter 'p' is increasing from 0.1 to 0.4 then the average number of packet in the system and the average number of packet in the queue are increasing from 1.224 to 1.165 and 0.605 to 0.560 respectively for fixed values of the other parameters.

It is observed that as λ increases the variance of the number of packets in the system is increasing for given values of the other parameters. When μ_1 is increasing the variance of the number of packets in the system is decreasing for fixed values of the other parameters. When the truncation parameters b is increasing the variance of the number of packets in the system is increasing. When μ_2 is increasing the variance of the number of packets in the system is decreasing for fixed values of the other parameters.

For different values of the parameterstheaverage waiting time of a packet in the system, the average waiting time of a packet in the queue and the variance of the waiting time of a packet in the queue are computed and given in the table 3. The relationship between parameters and the waiting time is shown in figure 3.

From table 3 it is observed that the model parameters have a significant influence on the waiting time of a packet in the system and in the queue. As the mean arrival rate λ is increasing then the average waiting time of a packet in the queue and the average waiting time of a packet in the system are increasing when the other parameters remain fixed.

It is also observed that as the parameterμ_1 is increasing from 5 to 11 the average waiting time of a packet in the system and the average waiting time of a packet in the queue are decreasing from 0.204 to 0.196 and 0.058 to 0.054 respectively for fixed values of the other parameters.

It is also observed that as the parameter μ_2 is increasing from 7 to 10 the average waiting time of a packet in the system and the average waiting time of a packet in the queue are decreasing from 0.371 to 0.349 and 0.174 to 0.156 respectively for fixed values of the other parameters.

It is further observed that when the mean arrival rate λ increases the variance of the waiting time of a packet in the system is increasing when other parameters remain fixed. Similarly when the truncation parameters a and b are increasing the variance of the waiting time of the packet in the system is also increasing for fixed values of the other parameters.

It is also observed that as the parameter p is increasing from 0.1 to 0.4 the average waiting time of a packet in the system and the average waiting time of a packet in the queue are decreasing from 0.408 to 0.388 and 0.202 to 0.187respectively for fixed values of the other parameters.

V1. CONCLUSION

This paper deals with a novel queuing model with two component mixture of doubly truncated exponentially distributed service time having poison arrivals. This queuing model is much useful for analyzing and designing the communication networks arising at places like data voice transmission, telecommunications, satellite communications, ATM Scheduling etc,. The traditional assumption of having infinite range service times is modified through doubly truncated nature of the random variable. Since, in reality no customer/ unit requires infinite service time. In addition to this finite range of the service time this model also includes heterogeneous nature of customer service times by considering mixture distribution. Through numerical studies it is established that the truncation as well as mixture distribution parameters have significant influence on system performance measures. This model also includes several of the earlier existing queuing models as particular cases for limiting/specific values of the parameters. This model can also be extended for non-Poison arrivals and servers vacation models.

Table 3: VALUES OF Ws AND Wq AND Var (Tq)

a	b	p	λ	μ_1	μ_2	Ws	Wq	Var(Tq)
0.1	0.2	0.8	1	10	8	0.154	0.012	0.004
0.1	0.2	0.8	2	10	8	0.171	0.029	0.010
0.1	0.2	0.8	3	10	8	0.197	0.055	0.020
0.1	0.2	0.8	4	10	8	0.239	0.097	0.040
0.1	0.2	0.8	5	10	8	0.324	0.181	0.091
0.1	0.2	0.8	3	5	8	0.204	0.058	0.041
0.1	0.2	0.8	3	7	8	0.201	0.057	0.029
0.1	0.2	0.8	3	9	8	0.197	0.055	0.020
0.1	0.2	0.8	3	11	8	0.196	0.054	0.079
0.1	0.5	0.8	1	10	8	0.224	0.028	0.008
0.2	0.5	0.8	1	10	8	0.348	0.061	0.016
0.3	0.5	0.8	1	10	8	0.481	0.111	0.030
0.4	0.5	0.8	1	10	8	0.618	0.176	0.053
0.6	0.1	0.8	1	10	8	0.231	0.031	0.008
0.6	0.2	0.8	1	10	8	0.363	0.067	0.017
0.6	0.3	0.8	1	10	8	0.512	0.126	0.034
0.6	0.4	0.8	1	10	8	0.681	0.211	0.065
0.1	0.5	0.1	3	10	8	0.408	0.202	0.101
0.1	0.5	0.2	3	10	8	0.402	0.197	0.096
0.1	0.5	0.3	3	10	8	0.395	0.192	0.092
0.1	0.5	0.4	3	10	8	0.388	0.187	0.087
0.1	0.5	0.8	3	10	7	0.371	0.174	0.077
0.1	0.5	0.8	3	10	8	0.362	0.166	0.068
0.1	0.5	0.8	3	10	9	0.355	0.161	0.063
0.1	0.5	0.8	3	10	10	0.349	0.156	0.059

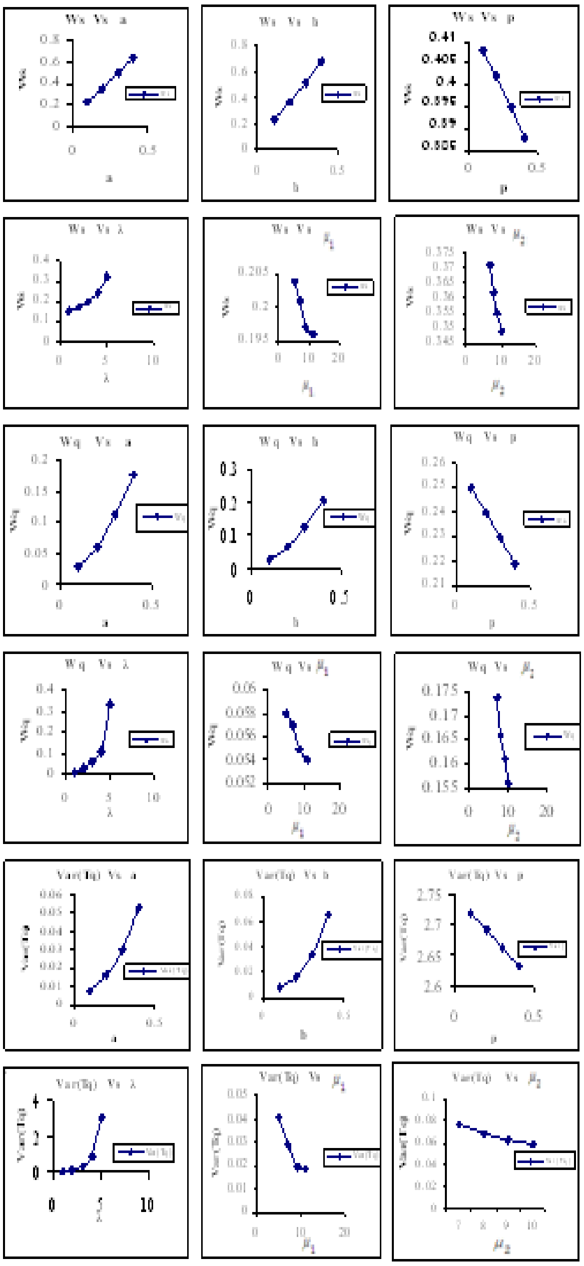

Figure 3: Relationship between the parameters and Ws, Wq and Var(Tq)

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