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Deriving ε_0 and μ_0 from First Principles and Defining the Fundamental Electromagnetic Equations Set

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Abstract:- In 1941, renowned physicist Julius Adams Stratton wrote in his seminal work titled 'Electromagnetic Theory' : "*In the theory of electromagnetism, the dimensions of* ε_0 *and* μ_0 *that link respectively D and E and B and H, in vacuum, are perfectly arbitrary*." This assertion is still valid today in the context of classical electrodynamics since these two constants have never been derived from first principles. It will be shown in this paper how they can be derived from first principles.

Keywords:- Permittivity constant, permeability constant, first principles, speed of light, Maxwell equations, Lorentz equation, fundamental electromagnetic dimensions Cms.

I. BRIEF HISTORY

The electrostatic permittivity constant of vacuum $\boldsymbol{\varepsilon_0}$ was defined and then experimentally accurately measured by means of an experiment carried out with a parallel-plates capacitor of known area "A" with fixed distance plates separation "d" across which a known voltage V was maintained, which allowed calculating the exact corresponding capacitance "C" involved. This allowed an exact calculation of $\boldsymbol{\varepsilon_0}$ in terms of very accurately measured quantities, as clearly described in ([2], Section 30.2):

$$\varepsilon_0 = \frac{Cd}{A}$$

However, despite having been measured in this manner with the highest precision, $\boldsymbol{\varepsilon_0}$ remains to this date an *ad hoc* value that never was derived from any underlying theory.

The magnetic permeability constant of vacuum $\boldsymbol{\mu_0}$ on its part, was assigned the arbitrary value $4\pi \bullet 10^{-7}$ and the current used to define the Ampere with the current balance experiment that was used in the process, was adjusted by convention so that $\boldsymbol{\mu_0}$ would have the exact value needed in the following equation, as clearly laid out in ([2], Section 34.1 to 34.4):

$$\frac{F}{l} = \frac{\mu_0 i^2}{2\pi d}$$

In this case also, constant $\boldsymbol{\mu_0}$ never was derived to this date from any underlying theory, on top of having had its value arbitrarily assigned. All electromagnetic phenomena mathematically described with the help of both constants demonstrate out of any doubt that these constants are exact and required, even if no theory can currently mathematically explain their origin.

Many attempts have been made to derive them from known theories but with inconclusive results.

II. THE SPEED OF LIGHT CALCULATED FROM MAXWELL'S EQUATIONS

Let us examine the equation allowing calculating the speed of light by means of these two fundamental constants ([1], p.689), an equation that was elaborated by Maxwell as he discovered more than 150 years ago that the reciprocal of the product of the permittivity and permeability constants was equal to the square of the speed of light:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \quad (1)$$

where $\boldsymbol{\varepsilon_0}$ is the electrostatic permittivity constant, with a value of

$$\boldsymbol{\varepsilon_0} = 1/(4\pi c^2 \bullet 10^{-7}) = 8.854187817E\text{-}12 \text{ Farad per meter.}$$

This constant was experimentally measured ([2], p. 746).

Constant $\boldsymbol{\mu_0}$ on its part is the magnetic permeability constant, which was calculated to be exactly:

$\mu_0 = 4\pi\bullet 10^{-7}$ = 1.256637061E-6 Henri per meter

from Ampere's Law ([2], p. 848):

$$\oint \mathbf{B}\cdot \mathrm{dl} = \mu_o i \qquad (2)$$

It is very interesting to note that the permittivity constant (ε_0) is in reality a measure of ***transverse capacitance per meter*** (usually symbolized by capital C), related to the "presence of electric energy" in electrodynamics. The permeability constant (μ_0) on the other hand is in reality a measure of ***transverse inductance per meter*** (usually symbolized by capital L), related to the "presence of magnetic energy" in electrodynamics.

Equation (1) in fact, turns out to be the only means ever discovered for calculating the speed of light from theory. This equation is actually arrived at from identities drawn from Faraday's equation and Maxwell's forth equation:

$$\oint \mathbf{E}\cdot \mathrm{dl} = -\,\mathrm{d}\mathbf{\Phi}_B/\mathrm{dt} \quad \text{and} \quad \oint \mathbf{B}\cdot \mathrm{dl} = \mu_o\left(i + \varepsilon_o \mathrm{d}\mathbf{\Phi}_E/\mathrm{dt}\right) \qquad (3)$$

Without going into full description, let us mention that from these two equations, equivalent second partial derivatives of the instantaneous magnetic field of a propagating electromagnetic wave in vacuum with respect to distance and time can be equated in the following manner (note that the same relation can be arrived at for the related electric field):

$$\frac{\partial^2 \mathbf{B}}{\partial x^2} = \mu_0\,\varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} \quad \text{and/or} \quad \frac{\partial^2 \mathbf{E}}{\partial x^2} = \mu_0\,\varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad (4)$$

The similarity of this form with the equation giving the velocity of a transverse wave on an elastic cord was obvious:

$$\frac{\partial^2 y}{\partial x^2} = \frac{m_L}{F}\frac{\partial^2 y}{\partial t^2} \quad \text{which resolves to} \quad \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad (5)$$

Where F is the tension applied to the cord, expressed in Newtons. m_L is the lineic mass (mass per meter) and y = A cos(kx-ωt + f), that is, the instantaneous transverse amplitude of the travelling wave. Since $F/m_L = v^2$, which is the square of the longitudinal velocity of the wave along the rope, the similarity was obvious with the wave equation derived from Maxwell's equations and so it is by similarity that equation (1) came about, since the product $\varepsilon_0\mu_0$ will obviously be equal to the inverse of a squared velocity, which in this case is $1/c^2$.

Consequently, since the local conditions existing in all of vacuum that determine the values of ε_0 and μ_0 can be assumed to be universally constant, this is sufficient in and of itself to conclude that the speed of light can only be constant in vacuum, a state of fact that was extensively confirmed by experiment.

Note also that simple dimensional analysis of the ratio m_L/F of equation (5) directly leads to the ratio $1/c^2$. First, from equation $E=mc^2$, m can obviously be redefined as $m=E/c^2$, which gives m the dimensions Joules/c^2. This gives m_L the dimensions Joules/(c^2 m).

The Newton on is part, which is the dimension of F ultimately resolves to J/m (Joules per meter). Here is how J/m can be derived from the usual Newton kg•m/s^2 definition. The kg is a measure of mass, and from $m=E/c^2$, kg = J/m^2/s^2, that is J•s^2/m^2. If we replace kg by this equivalent in kg•m/s^2, we obtain J•s^2•m/s^2•m^2, which, upon simplifying, becomes J/m, which is Joules per meter.

So the dimensions of m_L/F resolve to J/(c^2 m) ÷ J/m, that is (J m)/(c^2 m j), and simplifying: $1/c^2$.

III. DIMENSIONAL ANALYSIS OF ε_0 AND μ_0

Obviously, ε_0 and μ_0 are rather abstract values and it is somewhat difficult to imagine what causes their product to remain constant. The traditional unit of ε_0 (farad per meter) and that of μ_0 (Henry per meter) give no idea of the real dimensions of these constants.

But if we resolve them down to their International System (SI) dimensions, we find that ε_0 is an expression involving charge in Coulomb, mass in kg, time in seconds and distance in meters (**$Q^2 \bullet s^2/kg \bullet m^3$**). Similarly, we observe that μ_0 also involves into charge in Coulomb, mass in kg, and distance in meters (**$kg \bullet m/Q^2$**). Upon simplifying, we obtain m/s as the end result:

$$c = \frac{1}{\sqrt{\left(\frac{Q^2}{kg}\right)\times\left(\frac{kg}{Q^2}\right)\times\frac{s^2}{m^2}}} = \frac{m}{s} \qquad (13)$$

This level of detail allows much better understanding of why the first equation presented in this chapter allows calculating a velocity in meters per second, that is, the speed of light. But it may seem surprising that a unit of mass, the kilogram, should be part of the SI dimensions of two constants meant to determine the velocity of free electromagnetic energy in vacuum!

The issue is resolved if we keep in mind that, as we saw at the end of **Section II**, the kilogram is itself a composite unit that ultimately resolves to J•s^2/m^2, that is Joules (a unit of energy) divided by a velocity squared. The same obscurity problem occurs with the unit of impedance of vacuum (Z_0), which is the Ohm. By means of the SI units that we just resolved, we can already determine the value of the Ohm in SI units:

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{\mathrm{kg\cdot m}}{Q^2}\frac{\mathrm{kg\cdot m^3}}{Q^2\cdot s^2}} = \sqrt{\frac{\mathrm{kg^2\cdot m^4}}{Q^4\cdot s^2}} = \frac{\mathrm{kg\cdot m^2}}{Q^2\cdot s} = \Omega \text{ (Ohm)} \tag{14}$$

But resolving now kg to its more elementary units: kg = J•s^2/m^2, and substituting reveals already that we are really dealing with Joules second per charge squared!

$$Z_0 = \Omega = \frac{\mathrm{kg\cdot m^2}}{Q^2\cdot s} = \frac{\mathrm{J\cdot s^2}}{\mathrm{m^2}}\frac{\mathrm{m^2}}{Q^2\cdot s} = \frac{\mathrm{J\cdot s}}{Q^2} \tag{15}$$

Now the mystery deepens again if we resolve Z_0 from the pi related values of $\boldsymbol{\varepsilon_0}$ and $\boldsymbol{\mu_0}$:

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{16\pi^2c^2}\cdot 10^{-7} = 4\pi c\cdot 10^{-7} = 376.7303135\,\mathrm{m/s} = \Omega \tag{15b}$$

which resolves the impedance of vacuum in Ohms as being meters per second (m/s), that is, a velocity! We will resolve this apparent incompatibility in **Section IX**.

IV. FORCE AT THE PAIR DECOUPLING RADIUS

The manner in which a photon of energy 1.022 MeV can logically decouple into a pair of electron and positron was thoroughly analyzed in a previous paper ([5]).

Given the permanently fixed amount of rest energy that the electron presents at the relative decoupling distance as the separation becomes complete, it is not excluded that the "unit charge" of the electron could be an aspect not yet clarified of kinetic energy motion at this decoupling distance. The expanded tri-spatial geometry ([3]) that allows explaining how this decoupling can mechanically proceed definitely gives hints in this direction.

We will now use the well known classical force equation F=ma, where a=v^2/r to initiate this new stage in our analysis. We know the rest mass of the electron, its velocity in electrostatic space at the decoupling radius(r_d), that is c, as well as its decoupling radius proper as analyzed in ([5]):

$$r_d = \frac{\lambda_c}{2\pi} = r_o\alpha = \sqrt{\frac{K}{E_e}} = 3.861592642\,\mathrm{E}-13\,\mathrm{m} \tag{16}$$

In line with our hypothesis, let us calculate the force that would apply at this decoupling radius of the pair:

$$F_e = \frac{m_e\,c^2}{r_d} = 0.2120136660\,\mathrm{N} \tag{17}$$

V. MAXIMUM TRANSVERSE DECOUPLING VELOCITY

In a separate paper (**[3], Section 7.5**), we verified that the maximum transverse velocity of the part of a photon's energy that is cycling between electrostatic and magnetostatic spaces was reached when half of its energy had left either space during each cycle.

Knowing that the corresponding part of the electron energy has the same pulsating motion between magnetostatic and normal spaces and having just determined the force that applies at the decoupling radius, we are in a position to calculate that maximum transverse velocity of the oscillating energy, a maximum transverse velocity that applies by definition to photons also, given the identity of structure that this model reveals between electrons and photons.

We know from the electron structure in the tri-spatial geometry as described in previous paper (**[5], Section 2**) that only half of its energy dynamically pulsates so we must use m = m_e/2 as the mass that must be considered.

Let us note that, as is traditionally done with constants $\boldsymbol{\varepsilon_0}$ and $\boldsymbol{\mu_0}$, we will use mass as a convenient place-holder for the energy of the electron as we introduce the concept of transverse acceleration since the acceleration equation is traditionally deemed to apply to mass.

The known decoupling radius also allows easily determining that half the distance between maximum amplitude and the tri-spatial junction will be:

$$d = r_d/2 = \lambda/4\pi \tag{18}$$

Once again, from F=mv^2/r, we can derive F=mv^2/d and

$$v = \sqrt{\frac{Fd}{m}} = 299{,}792{,}45\ \ 7.8 \text{ m/s} \tag{19}$$

Thus interestingly confirming that maximum transverse velocity of a photon's or electron's energy during its internal cyclic pulsating motion is precisely the speed of light, and that this velocity is reached at precisely half of its amplitude in either electrostatic and magnetostatic space.

VI. DEFINING THE REQUIRED 3-SPACES MAJOR UNIT VECTORS SET

Before proceeding to the actual derivation, it is useful to put in perspective the major and minor unit vectors set required in the 3-spaces expanded geometry to allow proper representation. This unit vectors set was completely described and justified in previous paper ([3]), but we will summarily reproduce the unit vectors base here for convenience.

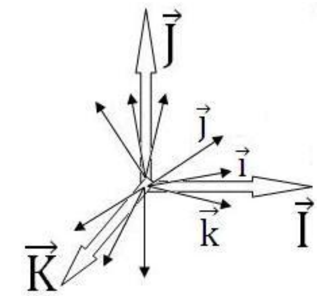

Fig.1: The 3-spaces unit vectors set

The traditional $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ unit vectors set was of course defined to represent vectorial properties in normal space since electromagnetic phenomena have up to now been perceived as occurring entirely within normal 3-D space. But the expanded 3-spaces geometry involves two more spaces normal to normal space and to each other, each of which also requiring its own internal minor unit vectors set.

The three mutually orthogonal spaces (normal, electrostatic and magnetostatic) also need to be represented by a special unit vectors set. It was defined as a new superset of major unit vectors that identify the three orthogonal spaces as $\hat{\mathbf{I}}$, $\hat{\mathbf{J}}$ and $\hat{K}$, or to make notation easier, **I**, **J** and **K** (**I**bar, **J**bar and **K**bar), so each local $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ unit vectors set becomes subordinated to the major unit vector specific to its local space, all 12 resulting unit vectors (9 minor and 3 major) being of course drawn from the same origin O.

Each of the three orthogonal minor unit vectors subsets (represented in **Fig.1** as being half folded), that is **I-i**, **I-j**, **I-k** for normal space, **J-i**, **J-j**, **J-k** for electrostatic space, and **K-i**, **K-j**, **K-k** for magnetostatic space, allow defining the vectorial magnitude of energy in any one of the three orthogonal coexisting spaces.

VII. DERIVING ε_0 AND μ_0 FROM THE TRANSVERSE ACCELERATION EQUATION

But let's come back to equation $F=mv^2/d$, that we can now formulate $F=mc^2/d$ in the case of the electron at rest to account for the velocity being the speed of light at the decoupling radius (equation (17)), that is

$$F = mc^2/d \tag{20}$$

which becomes after replacing d with its proper value determined in equation (18):

$$F = \frac{m}{} \frac{c^2}{\lambda/4\pi} \quad \text{and finally} \quad F = \frac{m}{} \frac{4\pi c^2}{\lambda} \tag{21}$$

We have now measured a force that applies in normal space (**I**), produced by an acceleration that acts orthogonally in electrostatic space (**J**), that is, transversally with respect to the direction of motion of the particle in normal space:

$$\overrightarrow{\mathbf{F_e}}\mathbf{I} = \frac{m}{} \frac{\overrightarrow{4\pi c^2}}{\lambda}\mathbf{J} \tag{22}$$

The energy having reached its maximum velocity, that is c, after having reached half its amplitude in electrostatic space, it will then start decelerate and come to a complete stop as it reaches its maximal amplitude, as described in paper ([3]).

This energy will then re-accelerate in magnetostatic space (**K**) until the speed of light is again reached in that space, that is, when half of the energy will have left electrostatic space. We will thus have:

$$\overrightarrow{\mathbf{F}_m}\text{Ɨ} = \frac{m\,\overrightarrow{4\pi c^2}}{\lambda}\text{Ꝁ} \tag{23}$$

We have thus identified two accelerations in two spaces orthogonal to each other (Ɉ and Ꝁ) that result in two forces acting opposite to each other in normal space (Ɨ), which is orthogonal to both of them. Let's see what happens when we execute an inverse product (to take into account the mutual orthogonality of both electrostatic and magnetostatic spaces) of these two equations term for term:

$$\frac{\overrightarrow{\mathbf{F_m}}\text{Ɨ}}{\overrightarrow{\mathbf{F_e}}\text{Ɨ}} = \frac{\overrightarrow{4\pi c^2}\text{Ꝁ}}{\overrightarrow{4\pi c^2}\text{Ɉ}} \tag{24}$$

The two scalar quantities $\boldsymbol{\lambda}$ and **m** will of course simplify term for term to unity, but the vectorial quantities remain.

In **Section II**, we mentioned the values involving $\boldsymbol{\pi}$ of fundamental constants $\boldsymbol{\varepsilon_0}$ and $\boldsymbol{\mu_0}$, that is:

$$\boldsymbol{\varepsilon_0} = 1/(4\pi c^2 \bullet 10^{-7}) \quad \text{and} \quad \boldsymbol{\mu_0} = 4\pi \bullet 10^{-7} \tag{25}$$

Don't we recognize here values having the form of accelerations identical to those that we just derived from the fundamental acceleration equation? Note that the 10^{-7} factor which is part of these constants is a simple conversion factor that was introduced to allow proper conversion between the rationalized absolute CGS units system and the MKS units system that we are using here ([1], p. 24.).

The reason why this 10^{-7} factor remains outstanding in the MKS system is that contrary to the CGS gram which is a natural fraction of the MKS kilogram, and to the CGS centimeter which is a natural fraction of the MKS meter, the CGS erg is a unit different from the MKS Joule, which renders impossible its direct integration (1 erg = 10^{-7} Joule).

So let's add mutually reducible occurrences of these values to our equation, since we are using the MKS system here:

$$\frac{\overrightarrow{\mathbf{F_m}}\text{Ɨ}}{\overrightarrow{\mathbf{F_e}}\text{Ɨ}} = \frac{\overrightarrow{4\pi c^2}\text{Ꝁ}}{\overrightarrow{4\pi c^2}\text{Ɉ}}\,\frac{10^{-7}}{10^{-7}} \tag{26}$$

which, when converted to scalar quantities gives:

$$1 = \frac{4\pi c^2 10^{-7}}{4\pi c^2 10^{-7}}, \text{ and rearranging: } 1 = \left(\frac{1}{4\pi c^2 10^{-7}}\right)\left(4\pi 10^{-7}\right)c^2 \tag{27}$$

which allows clearly revealing the actual $\boldsymbol{\pi}$ related definitions of fundamental constants $\boldsymbol{\varepsilon_0}$ and $\boldsymbol{\mu_0}$, as explained in **Section II**:

$$1 = \left(\frac{1}{4\pi c^2 10^{-7}}\right)\left(4\pi 10^{-7}\right)c^2 = \varepsilon_0\mu_0 c^2 \tag{28}$$

Which, after isolating c^2 and extracting the square root gives the following relation, which is identical to equation (1) previously derivable only from Maxwell's equations:

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \tag{29}$$

VIII. CYCLIC TRANSVERSE ACCELERATION CONSTANTS $\boldsymbol{\varepsilon_0}$ AND $\boldsymbol{\mu_0}$

Haven't we just completely clarified here the origin up to now so mysterious of these two constants? Let's recall what Julius Adams Stratton wrote in 1941: "*In the theory of electromagnetism, the dimensions of* $\boldsymbol{\varepsilon_0}$ *and* $\boldsymbol{\mu_0}$ *that link respectively **D** and **E** and **B** and **H** in vacuum, are perfectly arbitrary*." ([1], p. 17).

This may have been the case in traditional electromagnetism, but haven't we just observed that in this model, they definitely have the form of mutually orthogonal accelerations, respectively in electrostatic space, and in magnetostatic space, in perfect harmony with the obliged dynamic motion of any quantum of kinetic energy in the 3-spaces model?

We know since Maxwell that free electromagnetic energy cannot exist without intersecting electric and magnetic field, and that such intersection has to be cyclic for the energy to exist at all, which necessarily involves cyclic accelerations, since stable transverse velocity would prevent repeated intersection.

IX. THE FUNDAMENTAL C, m, s DIMENSIONS SUBSET

Let us now consider the new equation to calculate energy derived from Paul Marmet's work in a separate paper ([6], Equation (10)), and let us give it the form of a quantity being submitted to an acceleration, that is, by converting it to a quantity being multiplied by a velocity squared and divided by a length in meter. This requires converting $\boldsymbol{\varepsilon_0}$ to its $\boldsymbol{\pi}$ related value as follows:

$$E = \frac{e^2}{2\varepsilon_0 \alpha\lambda} = \frac{e^2 4\pi c^2 10^{-7}}{2\alpha\lambda} = \left(\frac{e^2 10^{-7}}{}\right)\left(\frac{2\pi c^2}{\lambda\alpha}\right) \qquad \text{with dimensions } J = \frac{C^2 m}{s^2} \tag{30}$$

Examining this relation, we observe that both forms of this definition of energy naturally involve a squared unit charge (e^2), that is C^2 (Coulomb squared), which allows equating the Joule (J), which is the traditional unit of energy, with the dimensions Coulombs^2 • meters per second^2, when combined with the dimensions of c^2 and λ, the two remaining dimensioned variables, which clearly reveals a pair of charges in the process of being accelerated.

Let us now recall that we found earlier two apparently contradictory dimensional definitions of the Ohm in relation with the impedance of vacuum (Z_0), (See Equations (15) and (15b) above), that is:

$$Z_0 = \Omega = \frac{J \cdot s}{Q^2} = \frac{m}{s} \tag{31}$$

If we now substitute the new definition of the Joule that we just established with equation (30), we effectively verify after simplifying that both definitions of the Ohm are equivalent:

$$Z_0 = \Omega = \frac{J \cdot s}{Q^2} = \frac{Q^2 \cdot m}{s^2}\frac{s}{Q^2} = \frac{m}{s} \tag{32}$$

Consequently, to succeed in finally clarifying the relations between charges (in Coulombs), amplitude of a photon's energy (in meters) and time (in seconds), for the remainder of this paper we will use this new definition of the Joule that is C^2m/s^2, which will allow highlighting an underlying acceleration process present in all classical equations related to energy.

One more thing that the analysis of Z_0 in **Section III** revealed was the fact that the conversion factor 10^{-7} present in both $\boldsymbol{\varepsilon_0}$ and $\boldsymbol{\mu_0}$, is not part of the value that the square root is to be extracted of (See equation(15b)), but that it must be applied to the square root result for the correct impedance to be obtained. By the same token, this highlights the fact that here also this factor has to be external to the squared value of the charge.

We will see that this infinitesimal constant quantity (the squared charge), that seems made up of 2 unit quantities, in conformity with de Broglie's hypothesis and that the equations present to us as if they were charges, appears to be found at the heart of each photon.

X. THE FUNDAMENTAL CHARGES ACCELERATION EQUATION

To simplify notation, and to clearly highlight the similarity in structure with the traditional acceleration equation, let us identify the pair of squared charges with a capital **X**, and identify by **a** the associated acceleration involving the square of the speed of light on a transverse distance equal to the integrated amplitude of the absolute wavelength of the associated particle, which in context corresponds to half this integrated amplitude on either side of the Y-x axis (along which the photon is assumed to be moving in vacuum). So from now on, we will use **X** and **a** with the following definitions:

$$X = e^2 \cdot 10^{-7} = 2.56696941\ 5E-38\ C^2 \cdot 10^{-7} \qquad \text{and} \qquad a = \frac{2\pi c^2}{\lambda\alpha} \tag{33}$$

We will convert here some of the major electrodynamics equations to more easily perceive the progressive complexification of these equations from that for energy to that for energy density, and observe that all of them involve the "**Xa**" transverse acceleration relation that we just identified.

XI. DEFINING THE FUNDAMENTAL EM EQUATION SET

Before going further, let us say a few words regarding what must be understood by "integrated wavelength", and "integrated amplitude" of the energy of a particle.

The relation between the wavelength of a localized particle and calculation of its energy by means of spherical integration is studied in detail in separate paper ([6]). The analysis carried out in that paper allowed defining the **integrated wavelength** of a particle as corresponding to the absolute wavelength of the energy of this particle (λ=hc/E, or λ=h/mc) multiplied by the fine structure (α). The integrated amplitude of a particle will thus be from equation (33):

$$A\alpha = \frac{\lambda\alpha}{2\pi}, \qquad \text{whence} \qquad \mathbf{a} = \frac{c^2}{A\alpha} = \frac{2\pi c^2}{\lambda\alpha} \text{ as defined in (33)} \tag{34}$$

We can now define a general equation equivalent as well to E=mc^2 (for the rest energy of a massive particle) as equal to E=h*f* (for the energy of a photon), considering the similarity in internal structure of both particles, that is

$$E = hf = \frac{hc}{\lambda} = mc^2 = Xa = \left(\frac{e^2 10^{-7}}{}\right)\frac{2\pi c^2}{\lambda\alpha} \quad \text{with dimensions} \quad J = \frac{C^2 m}{s^2} \tag{35}$$

Contrary to the two traditional equations however, the new equation directly highlights the transverse electromagnetic acceleration of two unit charges, as already mentioned.

The expert reader will no doubt immediately see the relation with the Coulomb equation that precisely calculates the force between two separate charges, and the related energy when we divide this energy by the distance between the two charges, and that would directly apply inside a photon since it can now be seen as containing 2 unit charges.

If we consider in context two unit charges, we obtain:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r^2} = \frac{Xa}{d} = \left(\frac{e^2 10^{-7}}{}\right)\frac{4\pi^2 c^2}{\lambda^2\alpha^2} \quad \text{with dimensions} \quad N = \frac{J}{m} = \frac{C^2}{s^2} \tag{36}$$

The magnetic force relation involving an interaction between two electromagnetic particles can also be expressed in this manner ([8], equation (1)). From the definition of the magnetic dipole moment established in paper ([6], Equation (35a)), that is μ=E/2B=e$c\alpha^2\lambda/4\pi$ we can write

$$F = \frac{3\mu_0}{4\pi r^3}\frac{\mu^2}{r} = \frac{Xa}{d}\frac{3\alpha^2}{4} = \left(\frac{e^2 10^{-7}}{}\right)\frac{3\pi^2 c^2}{\lambda^2} \quad \text{with dimensions} \quad N = \frac{J}{m} = \frac{C^2}{s^2} \tag{37}$$

From the equation for energy (35), mass can of course be defined as energy divided by c^2.

$$m = \frac{X}{A\alpha} = \left(\frac{e^2 10^{-7}}{}\right)\frac{2\pi c^2}{\lambda\alpha c^2} = \left(\frac{e^2 10^{-7}}{}\right)\frac{2\pi}{\lambda\alpha} \quad \text{with dimensions} \quad kg = \frac{C^2}{m} \tag{38}$$

<u>This last redefinition of mass is of particular interest since it is the form that allows unifying all classical force equations</u> in separate paper ([7]).

The general equation for mass calculation of down quark, up quark and electron respectively, that we will see in a coming paper, and completely analyzed in ([9], Section 17.10) will thus formulate as:

$$m_{[d,u,e]} = \frac{X}{A_C\alpha}\frac{9}{n^2} = \left(\frac{e^2 10^{-7}}{}\right)\frac{2\pi}{\lambda_C\alpha}\frac{9}{n^2} \quad \text{where (n=1,2,3)} \tag{39}$$

where λ_C is the electron Compton wavelength.

Planck's constant on its part resolves to

$$h = \frac{X2\pi c}{\alpha} = \left(\frac{e^2 10^{-7}}{}\right)\frac{2\pi c^2}{\alpha}\frac{1}{c} \quad \text{with dimensions} \quad J\cdot s = \frac{C^2 m}{s} \tag{40}$$

We can observe that the expression for ħ takes us further away yet from the form of a quantity being submitted to an acceleration :

$$\hbar = \frac{h}{2\pi} = \frac{Xc}{\alpha} = \left(\frac{e^2 10^{-7}}{}\right)\frac{2\pi c^2}{\alpha}\frac{1}{2\pi c} \quad \text{with dimensions} \quad J\cdot s = \frac{C^2 m}{s} \tag{41}$$

The alternate distance based constant that was temporarily named ***electromagnetic intensity constant*** as it was being defined it in paper ([3], Section J) becomes:

$$H = hc = \frac{X2\pi c^2}{\alpha} = \left(\frac{e^2 10^{-7}}{}\right)\frac{2\pi c^2}{\alpha} \quad \text{with dimensions} \quad J\cdot m = \frac{C^2 m^2}{s^2} \tag{42}$$

We can now understand that H is in reality a ***transverse electromagnetic acceleration constant***, possibly the most fundamental constant that can possibly be defined in electromagnetism. One only needs to divide it by the absolute wavelength of any particle, massive or not, to obtain its energy.

By the same token, we can clearly see that h, Planck's constant, necessarily also acts transversally and is in reality a ***transverse electromagnetic action constant*** !

So, If we redefine the first and second Planck radiation constants using transverse electromagnetic acceleration constant (H), they become:

$c_1 = 2\pi hc^2 = 2\pi Hc$ and $c_2 = hc/k = H/k$, and it becomes much more obvious that Planck's black body radiation equation mentioned previously is very directly linked to Maxwell's equations since H is obtained directly from electromagnetic considerations.

We can also easily reformulate the electron ***electrostatic energy induction constant*** (K) that was defined previously in terms of constant H and also associate it to electromagnetic acceleration

$$K = m_0 c^2 \left(\frac{\lambda_C}{2\pi}\right)^2 = H\frac{\lambda_C}{4\pi^2} = \left(\frac{e^2 10^{-7}}{}\right)\frac{c^2}{\alpha}\frac{\lambda_C}{2\pi} \quad \text{with dimensions } J\cdot m^2 = \frac{C^2\cdot m^3}{s^2} \tag{43}$$

The expression of a local electrical field for an individual localized photon described in paper ([6]) drawn from the Lorentz equation becomes:

$$\mathbf{E} = \frac{F}{\alpha e} = \frac{Xa}{A\alpha^2 e} = \left(\frac{e^2 10^{-7}}{}\right)\frac{2\pi c^2}{\lambda\alpha}\frac{2\pi}{\lambda\alpha}\frac{1}{\alpha e} \quad \text{with dimensions } \frac{J}{Cm} = \frac{C}{s^2} \tag{44}$$

The expression for the corresponding local magnetic field will be

$$\mathbf{B} = \frac{Xa}{A\alpha^2 ec} = \left(\frac{e^2 10^{-7}}{}\right)\frac{2\pi c^2}{\lambda\alpha}\frac{2\pi}{\lambda\alpha}\frac{1}{\alpha ec}\ T \quad \text{with dimensions } T = \frac{Js}{Cm^2} = \frac{C}{sm} \tag{45}$$

And the expression for energy density of a localized photon within volume V, as defined in paper ([6], Equation (40i)) becomes:

$$U = \frac{Xa}{V_{\lambda\alpha}} = \left(\frac{e^2 10^{-7}}{}\right)\frac{2\pi c^2}{\lambda\alpha}\frac{2\pi}{\lambda\alpha}\frac{1}{\alpha e}\frac{\pi e}{\alpha^2\lambda^2} \quad \text{with dimensions } \frac{J}{m^3} = \frac{C^2}{s^2 m^2} \tag{46}$$

Let us note also that

$$U = \varepsilon_0 \mathbf{E}^2 = \frac{\mathbf{B}^2}{\mu_0} = \frac{F\pi}{\lambda^2\alpha^4} \quad J/m^3 \tag{47}$$

XII. CONCLUSION

We thus observe conclusively how both $\boldsymbol{\varepsilon_0}$ and $\boldsymbol{\mu_0}$ are directly related to the transverse acceleration of free energy in the expanded 3-spaces geometry and can be derived from the fundamental acceleration equation, which also allows defining the fundamental electromagnetic equations set.

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