

Assessment of Blood Pressure Level of Inhabitants of Ibadan and Its Metropolis, Using Log-Normal Model

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Abstract:- Several papers written by many medical researchers have disclosed that hypertension and its related complications are major health problems not only in Nigeria but in Africa at a large. Thus, this paper attempted at assessing the expected blood pressure level of the hypertensive patients inhabited in Ibadan city and its environments. Precisely, the diastolic blood pressures of 30 randomly selected patients visited University College Hospital, Ibadan between 23/07/2008 and 12/08/2008 for medical check-up are the data used for the assessment, and the evaluation was done using 2- parameter log- normal model. It is deduced from the analysis that the model is deemed fit for the data used following the confirmatory kolmogorov Smirnov one sample test carried out. Also the result of the test disclosed that the expected diastolic blood pressure of patients visited the hospital between the period under study is 91.525 mmhg with a standard deviation of 16.8154 mmhg. Finally, we observed that the expected diastolic blood pressure obtained is greater than that of World Health Organization standard (90mmhg), meaning that the patients under study are hypertensive patients.

Keywords:- hypertension, blood pressure, risk factor, 2-parameter log- normal model, Kolmogorov Smirnov, Kurtosis, skewness.

I. INTRODUCTION

Biologically, there are two types of blood pressure measures, namely: systolic and diastolic. The table below shows the group of blood pressure in mmhg (millimeters of mercury) among human beings

Category		Systolic	Diastolic
Normal		Less than or equal to 120	Less than or equal to 80
Pre hypertension		120-139	80-89
Hypertension	A	140-159	90-99
	B	160 ⁺	100 ⁺

A – Stage 1

B – Stage 2

Note: There is an exception to the above definition of high blood pressure, if the systolic and diastolic readings are too low to 120:80 respectively. This condition is called hypotension which at times emanated as a result of agglutination of blood in mammals.

Medically, hypertension means blood pressure of 140/90 mmhg (millimeters of mercury) or more, based on at least two readings on separate occasions (Mlunde, 2007; Ali A.A and Jimoh. A, 2011). Distinction is made between primary and secondary hypertension (Mlunde, 2007; Jimoh, 1992). The primary hypertension is the most common type which has no identifiable causes. Primary hypertension is deep-rooted in genetic, socio-economic and environmental factors, secondary hypertension may be due to renal, endocrine and cardiovascular causes (Mlunde, 2007). Although hypertension is asymptotic, a times, attributed to severe health problems such as congestive heart failure, cardiovascular disease, renal failure, stroke, cognitive decline, dementia and even death (Hansson et al. 2000).

Many studies have unfolded the relationship between occupational stressors and elevation of blood pressure (Theorell et al. 1991; Schnall et al. 1992; Theorell et al. Melmaed et al. 1998). Increased risks of high blood pressure are connected with job strain (Landsbergis et al. 2003; Markovitz et al., 2004).

There is strong evidence to suggest that hypertension and its associated complications are major health challenges of the 21st century. As of year 2000, more than 900 million people were living with hypertension worldwide (Kearney et al. 2005; Ali A.A and Jimoh A, 2001).

It has been predicted that this number could jump to more than 1.5 billion in 2025 if drastic measures are not taken to control hypertension (Kearney et al. 2005).

Developing countries experiencing epidemiological transition from communicable to non-communicable chronic diseases often bear the burnt of hypertension (Dodu, 1988; WHO, 1993; Kusuma, 2009a).

In Sub-Saharan Africa, hypertension affects over 20 million people and remains a leading cause of high mortality rate (World Hypertension League, 2003). In Nigeria, hypertension is one of the most common deadly diseases (Akinkugbe, 1992, Ali A.A and Jimoh A. 2011) with more than 11% of adult population living with the illness in the African most populous country (Kadiri, 2001). More reported cases are unfolding among people living in urban areas of Nigeria such as Ibadan (Odotola and Amu, 1997).

In this paper, we focused on the assessment of blood pressure level of people living in Ibadan and its metropolis. The 2-parameter log-normal model is used to determine the mean and standard deviation of diastolic blood pressure level of patients under study. Later, the estimated values are compared with WHO standard.

II. MATERIAL AND METHODS

The data used in this paper is produced by University College Hospital, Ibadan. The data are the diastolic blood pressure levels of hypertensive patients treated between 23/07/2008 and 19/08/2008.

The most common log-normal distributions in use are the two-parameter log-normal distribution (2P-LND) and the three-parameter log-normal distribution (3P-LND). But in this paper, we used 2-parameter log – normal distribution. A random variable X is said to have the 2P-LND, with location and scale parameters (mean and standard deviation) respectively, if $\ln X$ (the natural logarithm of X) has the normal distribution with mean and standard deviation μ and σ . Equivalently, $X = \exp(Y)$, where Y is normally distributed with mean μ and standard deviation σ . While the parameter (mean) can be any real number, the parameter (standard deviation) must be a positive real number.

The log-normal density function of a random variable X, with two parameters in the probability density function is given by:

$$f(x) = \frac{e^{-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2}}{\sqrt{2\pi\sigma^2 x^2}}, \quad x > 0, -\infty < \mu < \infty, \sigma^2 > 0$$

Below are the maximum likelihood estimates for the parameters μ and σ^2

$$\mu = \frac{\sum \ln x_i}{n} \quad \text{and} \quad \sigma^2 = \frac{\sum (\ln x_i - \mu)^2}{n}$$

The log-normal distribution is uni-modal and has a positive asymmetry. It is one of the simplest examples of a distribution that is not defined uniquely by its moments. The properties of a log-normal distribution are determined by the properties of the corresponding normal distribution.

The maximum likelihood estimator (mle) method is one among the many methods of estimating the parameters of a probability density function (pdf). First proposed by Sir R A Fisher in 1921, it has been accepted as the best rationale for estimating the unknown parameters of a given pdf. It is ‘perhaps the most widely used method and the resulting estimators have some (mainly asymptotic) optimum properties’ (Bamett, 1973). That is, method yields sufficient estimators that are Asymptotically-Minimum-Variance Unbiased- Estimators (AMVUE).

For a random variable X with parameter θ , the MLE method estimates θ by the value $\hat{\theta}$ which the likelihood function, $L_x(\theta)$, is a maximum, for a given data x. the estimator $\hat{\theta}$ is called the maximum likelihood estimator (MLE).

The general expression of the likelihood function is given as:

$$L(\theta_1, \theta_2, \dots, \theta_k / X_1, \dots, X_n) = \prod_{i=1}^n f(x_i; \theta_1, \theta_2, \dots, \theta_k), \quad \text{where } (\theta_1, \theta_2, \dots, \theta_k) \text{ denote the parameters of the function which we want to estimate.}$$

For the 2P-LND in this paper, the likelihood function in general form is given by:

$$L(\mu, \sigma^2 / \ln X_1, \dots, \ln X_n) = \prod_{i=1}^n f(\ln x_i; \mu, \sigma^2).$$

The MLE method consists of the maximization of the likelihood function. From the general condition probability function maximization, we know that the maximum value of a function is that value where the first derivative of the function, with respect to the parameters μ and σ^2 are zero. Thus we take the partial derivative of the likelihood function, with respect to μ and σ^2 to zero and solve for μ and σ^2 . The values of μ and σ^2 thus obtained are the MLE of the population parameters μ and σ^2 . Often, it is easier to maximize the log likelihood function $k(\mu, \sigma^2) = \ln L(\mu, \sigma^2)$ than the likelihood function $L(\mu, \sigma^2)$. Thus it is usual to consider the log likelihood function.

In this study, the 2P_LND of a random variable X is given as $f(X, \mu, \sigma^2) = \frac{e^{-\frac{1}{2}\left(\frac{\ln, x - \mu}{\sigma^2}\right)^2}}{\sqrt{2\pi\sigma^2 x^2}}, \dots \dots \dots i$

The log likelihood function of equation (i) is

$$L(\mu, \sigma^2) = e^{-\frac{1}{2} \sum_{i=1}^n \left(\frac{\ln, x - \mu}{\sigma^2}\right)^2} \times \prod_{i=1}^n (2\pi\sigma^2 x^2)^{-\frac{1}{2}} \dots \dots \dots ii$$

Estimation of Mean and Variance of the 2p_Lnd

The first and second moments about the mean of any random variable X with a pdf f(X) are, respectively, the mean and variance of the distribution. Thus, the mean and variance of the 2P_LND are given as:

$$E(X) = \beta_1 = e^{\mu + \sigma^2/2}, \text{ and}$$

$$Var(X) = \beta_2 = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)$$

We also state, without proof, the median and mode, respectively, of the 2P-LND as:

$$\tilde{X} = e^\mu, \text{ and } M_0 = e^{\mu + \sigma^2}$$

Skewness and Kurtosis of the 2p-Lnd

The graphic features of the distribution of a random variable, say X, are described in terms of the two properties called skewness (S) and kurtosis (K). Skewness measures the extent of asymmetry the distribution while kurtosis measures the flatness of the distribution. For the model in the study, skewness and kurtosis are expressed in terms of the first four moments about the mean of the model as follows:

$$S = \frac{\beta_3}{\beta_2^{3/2}} = \frac{e^{3\mu + \frac{3}{2}\sigma^2} (e^{\sigma^2} + 2)(e^{\sigma^2} - 1)}{[e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)]^{3/2}}$$

$$= \frac{e^{3\mu + \frac{3}{2}\sigma^2} (e^{\sigma^2} + 2)(e^{\sigma^2} - 1)}{e^{3\mu + \frac{3}{2}\sigma^2} (e^{\sigma^2} - 1)^{3/2}} = (e^{\sigma^2} + 2)(e^{\sigma^2} - 1)^{1/2}$$

$$K = \frac{\beta_4}{\beta_2^2} = \frac{e^{4\mu + \sigma^2} (e^{6\sigma^2} - 4e^{3\sigma^2} + 6e^{\sigma^2} - 3)}{[e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)]^2} = \frac{e^{4\mu + 2\sigma^2} (e^{6\sigma^2} - 4e^{3\sigma^2} + 6e^{\sigma^2} - 3)}{e^{4\mu + 2\sigma^2} (e^{\sigma^2} - 1)^2}$$

$$= e^{4\sigma^2} + 2e^{3\sigma^2} + 3e^{2\sigma^2} - 3$$

Kolmogorov - Smirnov One Sample Goodness - of - Fit Test

The Kolmogorov Smirnov one sample goodness-of-fit test is one among the many tests used in verifying distributional assumption of a given set of data. Its choice in this paper is based on the fact that the test assumes an underlying continuous distribution, and it treats individual observations separately, and need not lose information through combining of categories, although it may be convenient to use groupings of variables. The test is concerned with the degree of agreement between the distributions of a set of sample values and some theoretical distribution. It determines whether the weights in the sample are thought to have come from the population having the theoretical distribution.

III. DATA PRESENTATION AND ANALYSIS

The statistical method stated earlier is applied on the data, and the proposed 2- parameter log-normal distribution reviewed earlier is fitted for the data.

S/No.	Hospital No.	mmhg B.P	S/No.	Hospital No.	mmhg B.P
1	1132436	88	11	1122117	90
2	1093038	94	12	1122018	102
3	1129068	70	13	1121382	90
4	1126332	102	14	1120464	110
5	1127061	100	15	1123181	90
6	1127063	88	16	1036985	90
7	127655	110	17	1050532	70
8	1126772	90	18	1138412	110
9	1125774	50	19	1139050	100
10	1125776	80	20	1138613	110
21	1138613	90	26	1119091	106
22	1139206	102	27	1131175	74
23	1139901	92	28	1131175	82
24	1139900	100	29	1118236	60
25	1140291	100	30	1141520	102

From table above, the ordinary values of mean and standard deviation are 4.50 and 0.1822mmHg respectively.

Estimation of Mean and standard deviation of 2- Parameter Log - Normal

The estimates of mean and variance of the distribution under study are thus: $\mu = 91.525\text{mmHg}$ and $\sigma = 16.815\text{mmHg}$. Also, the value of Skewness and Kurtosis are respectively 0.5574 (this shows the departure of data used from normality) and 3.5574 (this revealed the peakednes of the distribution).

Test of Goodness of Fit of the Distribution

Here, Kolmogorov Smirnov one sample test is used to test the goodness of fit to the data. The tested hypotheses are

H_0 : 2- parameter log-normal distribution fit the data (blood pressure levels)

H_1 : 2- parameter log-normal distribution does not fit the data well

Table of analysis

BP interval	Mid-point (X)	Frequency	C. F	$S_n(X)$	$F_n(X)$	$ F_n(X) - S_n(X) $
50 – 60	55	2	2	0.0667	0.0030	0.0637
61 -71	66	2	4	0.1333	0.0409	0.0924
72 – 82	77	3	7	0.2333	0.1949	0.0384
83 – 93	88	9	16	0.5333	0.4522	0.0811
94 – 104	99	9	25	0.8333	0.6985	0.1348
105 – 115	110	5	30	1.0000	0.8643	0.1357

The relative cumulative frequency [$S_n(X_i)$] is obtained from the observed data, while the expected relative cumulative frequency [$F_n(X_i)$] is obtained from the normal cumulative distribution table for the normal variable z. If ,

$$\hat{\mu} = 4.50002$$

$$\hat{\sigma}^2 = 0.033197$$

$$\sigma = 0.1822$$

$$\phi\left[\frac{\ln x - \mu}{\sigma}\right] = Z, \text{ when } X = 55$$

$$\phi\left[\frac{\ln 55 - 4.50002}{0.1822}\right] = \phi(- 2.705)$$

$$= 1 - \phi(2.705)$$

$$= 1 - 0.9970$$

$$= 0.003$$

when $X = 66$

$$\begin{aligned} \phi\left[\frac{\ln 66 - 4.50002}{0.1822}\right] &= \phi(-1.704) \\ &= 1 - \phi(1.704) = 1 - 0.9591 = 0.0409 \end{aligned}$$

when $X = 77$

$$\begin{aligned} \phi\left[\frac{\ln 77 - 4.50002}{0.1822}\right] &= \phi(-0.86) \\ &= 1 - \phi(0.857) = 1 - 0.8051 \\ &= 0.1949 \end{aligned}$$

when $X = 88$

$$\begin{aligned} \phi\left[\frac{\ln 88 - 4.50002}{0.1822}\right] &= \phi(-0.1245) \\ \phi(-0.12) &= 1 - \phi(0.12) = 1 - 0.5478 \\ &= 0.4522 \end{aligned}$$

When $X = 99$

$$\phi\left[\frac{\ln 99 - 4.50002}{0.1822}\right] = \phi[0.52] = 0.6985$$

When $X = 110$

$$\begin{aligned} \phi\left[\frac{\ln 110 - 4.50002}{0.1822}\right] &= \phi(1.1002) \\ &= 0.8643 \end{aligned}$$

The critical region is obtained as D_{30} at 5% level of significance from the statistical table that is $D_{30, 0.05} = 0.2417$. Since Kolmogorov Smirnov test statistic $D = \max |F_n(X_i) - S_n(X_i)|$ i.e. 0.1357 is less than $D_{30, 0.05}(0.2417)$, therefore the Null hypothesis is accepted and conclude that the 2-parameters log-normal distribution model fit the blood pressure level of the patients under study.

IV. FINDINGS

The estimated values for μ and σ^2 of the 2-parameter log-normal are $\hat{\mu} = 5.0002$ and $\hat{\sigma}^2 = 0.033197$, are the respective mean and variance of the natural logarithms of the level of blood pressure of patients under study.

Also, the mean and standard deviation of BP levels have been found to be 91.5256mmhg and 16.8154mmhg respectively.

Thus, 5.0002mmhg and 0.033197mmhg represent the mean and variance of the natural logarithms of the BP levels of patients under study while 91.5256mmhg and 16.8154mmhg are estimates of the true mean and

standard deviation of the BP levels of the patients visited UCH Ibadan for medical check-up during the period under investigation.

V. CONCLUSION

However, based on the conclusion arrived at in this paper, it was observed that the expected diastolic blood pressure of patients treated at UCH Ibadan during the period under study is above the normal diastolic measure of World Health Organization (90mmhg), meaning that majority of people living in Ibadan during the year under study are likely to be hypertensive. Therefore, compulsory awareness on health education, programmes, seminars and conferences must be organized occasionally, so as to enlighten the inhabitants of Ibadan and its metropolis on the dangers in playing with some risk factors contributing to the abnormal rise in blood pressure such as: unnecessary stress, noisy areas, overworking, self medication, overloading the brain and insecurity.

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