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Dimensional Analysis in Rainfall Intensity Modeling (A Case of Selected Cities in South-South Nigeria)

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Abstract:- Most models of rainfall intensity are empirical equations which are not dimensionally balanced. Some researchers had come up with relationships of rainfall intensity with duration which could predict very well but dimensionally, such models are not balanced. This research took cognizance of this problem and applied dimensional analysis to develop models of rainfall intensity for the cities of Warri, Benin City and Port Harcourt in South South Nigeria. The statistical method of regression was used to fit the data collected. These models have limitations of three-parameter models; rainfall amount and duration must be known before rainfall intensity can be evaluated, but the models are scientifically more acceptable because of their being dimensionally balanced. The models also relate the field data very well.

Keywords:- Rainfall intensity, Dimensional Analysis, Regression.

I. INTRODUCTION

Dimensional analysis forms the basis for the design and operation of physical scale models which are used to predict the behavior of the corresponding prototypes. Such models, which are generally geometrically similar to the prototype, are used in the design of pumps, spillways, river and estuary engineering works, etc. Mathematical modeling techniques have progressed rapidly due to the advent of high-speed digital computers, enabling equations of motion coupled with semi-empirical relationships to be solved for complex situations. (Featherstone and Nalluri, 1998).

Without the technique of dimensional analysis, experimental and computational progress in fluid mechanics would have been considerably retarded. The basis of dimensional analysis is to condense the number of separate variables involved in a particular type of physical system into a smaller number of non-dimensional groups of the variables. The arrangement of the variables in the groups is generally chosen so that each group has a physical significance. All physical parameters can be expressed in terms of a number of basic dimensions. In engineering, the basic dimensions, mass (M), length (L) and time (T) are sufficient for this purpose (Featherstone and Nalluri, 1998).

Rainfall intensity is a parameter that can as well be expressed in terms of basic dimensions. Its unit of mm/hr can be employed to ascertain that its formula is also expressed to be dimensionally compliant to the basic units of length and time. That is the objective of this work.

Different characteristics of rainfall are important to specialists in various fields and therefore the number of ways of analyzing rainfall data is virtually unlimited. The method chosen depends upon the nature of the available data and the purpose of the study. At most stations, only daily totals of rainfall are measured. These totals may refer to several storms during a day or to a part of one storm that bridges two measuring periods. In this case the records are a somewhat artificial and inaccurate description of precipitation. The only characteristics of rainfall that can be obtained from such data are those pertaining to specified intervals of time, such as days, months, seasons, or years. A relatively small amount of rainfall-measuring stations are equipped with continuous recording gauges, which yield data on the characteristics of individual storms such as duration and intensity as well as total amount (Thomas and Luna, 1978).

Another consideration in choosing appropriate methods of analysis is the particular planning problem for which a description of the rainfall regime is required. Ecologists and Agronomists may be interested specifically in seasonal totals, the frequency of small amounts of rain or the probability of droughts for their studies of crops or natural plant populations. Civil engineers concentrate on the intensity, duration and areal extent of the large, infrequent storms that challenge the design of their structures (Thomas and Luna, 1978).

The major difficulty encountered by engineers and hydrologists in planning and design of water resources structures in the developing nations such as Nigeria is the unavailability of the required long term rainfall data. While rainfall records of many years and consequently, rainfall frequency atlas are commonly available in developing countries such as United States of America, Nigeria cannot boast of a consistent 30 year rainfall record (Nwaogazie and Duru, 2002)

The first extensive rainfall frequency analysis in the United States of America was made by Yarnell (1936). This study yielded a set of 56 isohyetal maps of the continental United States, covering the range of durations of 5, 10, 30, 60 and 120 minutes for 2 year frequency. With the passage of time more rainfall data were collected and thus, detailed rainfall frequency atlas of the United States was published by U.S weather Bureau (Hershfield, 1961).

In 1983, the Federal Government of Nigeria through the ministry of water resources sponsored a study on monitoring and management of hydrometric stations in Hadejia-Jama'are hydrological area-region 3. Intensity-duration-frequency model was developed for seven meteorological stations of Bauchi, Jos, Kano, Katsina, Samaru, Potiskun, and Nguru for the region 3 (Nwaogazie and Duru, 2002).

The future management of hydrological systems must be informed by climate change projections at relevant time horizons and at appropriate spatial scales. Furthermore the robustness of some decisions is dependent on both the uncertainty inherent in future climate change scenarios and the natural climate system (Burton et al,2010).

Most runoff and erosion models require rainfall intensity data of high temporal resolution, which restricts their application and predictive potential (Van Dijk et al,2005).

The engineering application of rainfall intensity is mainly in the estimation of design discharge for flood control structures (Nwoke et al,2012). With the recent devastations caused by flood, this study becomes very necessary because most drainage structures have been built without the required rainfall intensity values.

II. METHODOLOGY

2.1 Study Area and Data Collection

The study area is the South-South Nigeria with three selected cities, namely; Warri, Benin City and Port Harcourt.

Warri, a commercial and oil rich city in Delta State is located at latitude $05^0$31'N and longitude $05^0$44'E at an altitude of 2.4m. Benin City, the capital of Edo State lies at latitude $06^0$19'N, longitude $05^0$36'E and at an altitude of 79.23m.Port Harcourt, the capital city of Rivers State lies at latitude $04^0$51'N, longitude $07^0$01'E and at an altitude of 17.6m.

Rainfall amounts and durations constitute the major data used in the study. The data were obtained from Nigeria meteorological office in Oshodi, Lagos State, Nigeria. The office has the data base for most climatic elements in Nigeria. The rainfall data of the areas were carefully studied and the maximum rainfall values were extracted and tabulated as shown in Tables 1, 2 and 3.

Table 1: Rainfall Data of Warri

Average Amount (mm)	14.93	17.15	27.82	32.29	36.40	53.89	58.23	69.75
Duration (hr)	0.2	0.33	0.5	0.67	0.83	1.0	1.5	2.0
Intensity (mm/hr)	74.63	51.98	55.64	48.20	43.86	53.89	38.82	34.87

Table 2: Rainfall Data of Benin City

Average Amount (mm)	9.43	17.20	23.31	32.73	33.46	53.87	55.53	70.79
Duration (hr)	0.2	0.33	0.5	0.67	0.83	1.0	1.5	2.0
Intensity (mm/hr)	47.17	52.12	46.62	48.84	40.31	53.87	37.02	35.40

Table 3: Rainfall Data of Port Harcourt

Average Amount (mm)	11.95	16.72	27.88	35.61	30.45	56.52	63.35	76.18
Duration (hr)	0.2	0.33	0.5	0.67	0.83	1.0	1.5	2.0
Intensity (mm/hr)	59.77	50.68	55.75	53.15	36.68	56.52	42.24	38.09

2.2 Method of Analysis

Dimensional analysis is applied to rainfall intensity to obtain the required relationship. Rainfall intensity is a function of Rainfall Amount, A; Duration, D and Return Period, R; which can be expressed mathematically as: $i = f(A, D, R)$ (1)

For a dimensionally balanced equation to be derived, the Mass, Length and Time (MLT) system is applied to Equation (1). Table 4 shows the dimensions of applicable variables of Equation (1) in the MLT system.

Table 4: Dimensions of Variables

Physical Variables	**Symbols**	**Dimensions**
Rainfall Intensity	i	LT^{-1}

Rainfall Amount	A	L
Duration	D	T
Return Period	R	T

2.2.1 Buckingham's π-Theorem

This theorem states that if there are n variables in a problem involving physical system and these variables contain m primary dimensions, the equation relating the variables will contain (n-m) dimensionless independent groups called $\pi_{s.}$ This theorem is chosen for the analysis of the rainfall intensity relations. Hence Equation (1) can be written as:

$$f(i, A, D, R,) = 0 \quad (2)$$

From Equation (2), we can deduce the following;
Total number of variables, n = 4
Number of fundamental dimensions, m = 2
(because there is no M term in the variables)
Number of π -terms = n-m = 4-2 = 2
Hence the number of π-terms in the equation can be written as:

$$f(\pi_1, \pi_2) = 0 \quad (3)$$

$$\pi_1 = i^a D^b A \quad (4)$$

$$\pi_2 = i^a D^b R \quad (5)$$

For π_1 –model, we have;

$$L^0T^0 = (LT^{-1})^a T^b L$$

Equating the indices of L and T yields a = b = -1, thus;

$$\therefore \pi_1 = i^{-1} D^{-1} A = \frac{A}{i\,D} \quad (6)$$

Similar to π_1 – model, π_2 – model becomes,

$$L^0 T^0 = (LT^{-1})^a T^b T$$

Equating the indices of L and T, gives a = 0 and b = -1, thus;

$$\therefore \pi_2 = i^0 D^{-1} R = \frac{R}{D} \quad (7)$$

Substituting values of π_s to Equation (3) yields;

$$f\left(\frac{A}{iD}, \frac{R}{D}\right) = 0 \quad (8)$$

$$\frac{A}{iD} = \left(\frac{R}{D}\right)^a$$

$$\ln\frac{A}{iD} = a \ln\frac{R}{D} \quad (9)$$

This could be represented by a linear model as;

$$y = ax \quad (10)$$

where:

$$y = \ln\frac{A}{iD} \quad (11a)$$

$$x = \ln\frac{R}{D} \quad (11b)$$

Using rainfall data for calibration to obtain the value of 'a' was not possible because ln $\frac{A}{iD}$ yielded zero values. Alternatively, we adopted the method proposed by Nwaogazie (2011) thus;

$$[i] = [A]^a [D]^b [R]^c \quad (12)$$

$$LT^{-1} = [L]^a [T]^b [T]^c \quad (13)$$

Equating the indices of L and T gives a = 1, b = c = -1, thus;

$$i = \frac{A}{D} \quad (14a)$$

or

$$i = \frac{A}{R} \quad (14b)$$

Equation (14a) could also take the form:

$$i = c\, A^{a1} D^{a2} \quad (15)$$

In Equation (15); c, a_1 and a_2 are constants. This equation can be linearized or transformed to multiple regression equivalent by taking the logarithm of both sides, thus;

$$\log i = \log c + a_1 \log A + a_2 \log D \quad (16)$$

This is similar to the linear regression solution (Nwaogazie, 2011);

$y = a_0 + a_1x_1 + a_2x_2$ (17)

where;

$y = \log i$, $a_0 = \log c$, $x_1 = \log A$, $x_2 = \log D$

III. RESULTS AND DISCUSSION

The calibration of Equation (15) necessitated the transformation of the values of Intensity, Amount and Duration in Tables 1, 2 and 3 to their logarithmic values. The constants, c, a_1 and a_2 were evaluated using regression methods in Microsoft Excel Software. The resulting equations are given as;

Warri: $- 0.001781 + 0.99883x_1 – 0.99992x_2$ (18)

Benin City: $- 0.00157 + 1.000883x_1 – 1.00061x_2$ (19)

Port Harcourt: $- 0.00426 + 1.00231x_1 – 1.0023x_2$ (20)

Returning the constants to their positions in Equation (15) by taking the antilog of a_0 to obtain c, results in the following dimensionally balanced models;

Warri: $i = 1.004\ A^{0.99883}\ D^{-0.99992}$ (21)

Benin City: $i = 0.9964\ A^{1.000883}\ D^{-1.00061}$ (22)

Port Harcourt: $i = 0.9902\ A^{1.00231}\ D^{-1.0023}$ (23)

Equations (21), (22) and (23) are the models that represent rainfall intensities of the three cities in area of study. These models are dimensionally balanced in the sense that intensity is measured in mm/hr and the parameters, A and D have their units as mm and hr respectively. These models also gave very close representation of the field values when compared.

To verify this, in an earlier study on rainfall intensity in Warri (Nwoke and Okoro, 2012), a model was derived as $i = \frac{8130.08}{t+60.59}$ (24)

where i is rainfall intensity in mm/hr and t is duration in minutes. For a duration of 60 minutes, Equation (24) will produce intensity of 67.42 mm/hr while Equation (21) will give 53.85 mm/hr which is closer to the field value.

IV. CONCLUSION

The design of flood control structures requires information about the rainfall intensity in the area where such a structure will be located. The process of determining this rainfall intensity is normally from models formulated from historic rainfall events. However, most of the empirical models formulated for this purpose have been found to be dimensionally incorrect. This reason prompted the research into the possibility of deriving models of rainfall intensity that are dimensionally balanced. Dimensional analysis was applied in formulating models while least square method of analysis was used to get equations of best fit from the available data. The models derived are dimensionally balanced and relate the field data very well.

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