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Water Wave Generation Due to Initial Disturbance in Presence of an Inertial Surface in an Ocean with Porous Bed

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Abstract:- The proposed research paper is concerned with the generation of water waves due to initial disturbances at the upper surface of a fluid assumed to be covered by an inertial surface. The inertial surface is composed of thin but uniform distribution of non-interacting floating material. The fluid is assumed to be of uniform finite depth but the bed is considered to be porous. The Fourier and Laplace transform techniques have been used as the mathematical analysis to obtain the depression of the inertial surface in the form of an infinite integral. The inertial surface depression is evaluated asymptotically for large time and distance using method of stationary phase for an initial disturbance concentrated at a point. They are also depicted graphically and appropriate conclusions are drawn.

Keywords:- initial boundary value problem, initial disturbance, inertial surface, Fourier transformation, Laplace transformation, free surface depression, method of stationary phase, porous bed, velocity potential.

I. INTRODUCTION

The classical two-dimensional problem of water wave generation due to an initial disturbance in the free surface of the ocean was studied in treatises of Lamb[1] and Stoker[2] using linear theory of water waves. They used the method of Fourier transform and obtained the free surface elevation in the form of infinite integrals which were then evaluated approximately for large time and distance from the source of disturbance.

This problem is referred to as Cauchy-Poisson problem in the water wave literature.

Kranzer and Keller[3] discussed the problem of water wave generated by explosions in case of three dimensional motion in water of finite depth due to an initial surface impulse or initial surface elevation on a circular area. They also compared their theory with the available experimental results. Chaudhuri[4] extended these results in case of any initial surface impulse and elevation across arbitrary regions. In this respect, Wen[5] derived asymptotic results using the method of stationary phase as applied to double integrals. Debnath and Guha[6] worked out asymptotic analysis to the integral solutions while investigating the form of the axially symmetric free surface response of an inviscid stratified fluid in presence of an initial displacement of the free surface of the fluid of finite or infinite depth. Mandal[7] considered water wave generated by disturbance at an inertial surface. He employed Laplace transform technique to solve the initial value problem describing waves generated by a disturbance created at the surface of water covered by an inertial surface composed of a thin but uniform distribution of floating particles. Ghosh et.al[8] examined water wave due to initial disturbance at the inertial surface in a stratified fluid of finite depth. Dolai[9] considered generation of interface waves due to initial disturbances in the form of an initial depression or impulse at the interface when the upper fluid extends infinitely upwards and the lower fluid extends infinitely downwards. Dolai and Banerjea[10] considered the problem of generation of capillary-gravity waves due to initial interface disturbance at the interface between two superposed fluids, in which the upper fluid extends infinitely upwards and the lower fluid extends infinitely downwards. Ghosh[11] considered generation of interface waves due to initial axially symmetric disturbances at the inertial surface between two superposed fluids wherein the lower fluid is of uniform finite depth while the upper fluid extends infinitely upwards. Maity and Mandal [12,13] threw considerable light on the problem of generation of surface waves caused by an initial disturbance at the upper surface of the ocean with an elastic ice-cover. They applied Laplace-Fourier transform to solve the problem and derived the elevation of the upper surface and also computed numerical results leading to graphs depicting the variations of surface elevation for large values of time and distance. In the present paper, the Cauchy-Poisson problem is considered, the upper surface of the ocean being covered by an inertial surface while the bed of the ocean is assumed to be porous in nature.

Study of problems involving inertial surface has gained importance as sometimes in cold regions the ocean is covered by broken ice, which constitutes an inertial surface.

Fourier and Laplace transform techniques are employed to solve the problem. After invoking the inverse transforms, the potential function describing the wave motion is obtained and then the form of the inertial surface is deduced in terms of an infinite integral. This is then evaluated asymptotically for a large value of time and distance by using the method of stationary phase when the initial disturbance at the inertial surface is concentrated at the origin. The asymptotic form of the inertial surface is graphically presented. The figures depict variations of the non-dimensional inertial surface depression at a fixed point x' for different time t', and at a fixed time t' for different x', x' and t' being the non-dimensional horizontal distance from the origin and time respectively.

Plots are considered for various values of the non-dimensionalized inertial surface parameter E' and for different values of the non-dimensionalized porous parameter $G'h$.

The novelty involved in the current paper lies in the fact that the natural seepage of the ocean bed is taken into account as the bottom of the ocean is considered to be permeable unlike the previous authors who considered the bed to be impermeable.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

We consider the upper surface of the fluid to be consisting of an inertial surface composed of non-interacting floating material of area density ρE where ρ is the density of the liquid and $E = 0$ corresponds to a liquid with a free surface. The ocean is assumed to be of uniform finite depth h but the bed is assumed to be porous where $G' = \frac{\alpha}{\sqrt{\nu}}$ is the porous effect parameter. The quantity α is a dimensionless constant which depends on the structure of the porous medium and ν is permeability of the porous medium.

The motion of the fluid is generated due to an initial disturbance prescribed on the inertial surface in the form of an initial depression of the inertial surface. We choose a rectangular co-ordinate system such that $y = 0$ denotes the undisturbed inertial surface and $y = h$ denotes the ocean bed, the y-axis pointing vertically downwards. Two-dimensional motion is considered and the motion of the fluid remains irrotational since it is assumed to start from rest. The velocity potential $\varphi(x, y; t)$ describing the motion of the fluid satisfies the following initial value problem given by:

$\nabla^2\varphi = 0, 0 < y < h, t \geq 0$ (1)

$\frac{\partial^2}{\partial t^2}\left(\varphi - E\varphi_y\right) - g\varphi_y = 0$ on $y = 0, t > 0$ (2)

$\frac{\partial\varphi}{\partial y} - G'\varphi = 0$ on $y = h, t \geq 0$ (3)

where g is the acceleration due to gravity.

The initial conditions when the initial disturbance is in the form of a prescribed depression of the inertial surface are:

$\varphi - E\varphi_y = 0$ at $t = 0$ for $y = 0$ (4)

$\frac{\partial}{\partial t}\left(\varphi - E\varphi_y\right) = gf(x)$ at $t = 0$ for $y = 0$ (5)

where $f(x)$ is the prescribed initial depression of the inertial surface.

III. METHOD OF SOLUTION

Let $\bar{\varphi}(k, y; t)$ denote the Fourier transform of $\varphi(x, y; t)$ given by $\bar{\varphi}(k, y; t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\varphi(x, y; t)e^{-ikx}\,dx$

On applying Fourier transform, equations (1) – (5) reduce to

$$\left.\begin{aligned} &\bar{\varphi}_{yy} - k^2\bar{\varphi} = 0, \quad 0 < y < h, \ t \geq 0 \\ &\frac{\partial^2}{\partial t^2}\left(\bar{\varphi} - E\bar{\varphi}_y\right) - g\bar{\varphi}_y = 0 \quad on\ y = 0, t > 0 \\ &\frac{\partial\bar{\varphi}}{\partial y} - G'\bar{\varphi} = 0 \ \text{ on } y = h, t \geq 0 \\ &\bar{\varphi} - E\bar{\varphi}_y = 0 \ \text{ on } y = 0 \ \text{ at } \ t = 0 \\ &\frac{\partial}{\partial t}\left(\bar{\varphi} - E\bar{\varphi}_y\right) = g\bar{f}(k) \text{ on } \ y = 0 \ \text{ at } \ t = 0 \end{aligned}\right\} \qquad (6)$$

where $\bar{f}(k)$ denote the Fourier transform of $f(x)$.

Next, let $\bar{\bar{\varphi}}(k, y; p)$ denote the Laplace transform in time defined by

$\bar{\bar{\varphi}}(k, y; p) = \int_0^{\infty}\bar{\varphi}(k, y; t)e^{-pt}\,dt, \quad p > 0$

Applying Laplace transform in time, equations in (6) reduce to

$\bar{\bar{\varphi}}_{yy} - k^2\bar{\bar{\varphi}} = 0, 0 < y < h$ (7)

$p^2\bar{\bar{\varphi}} - (g + Ep^2)\bar{\bar{\varphi}}_y = g\bar{f}(k)$ on $y = 0$ (8)

$\frac{\partial\bar{\bar{\varphi}}}{\partial y} - G'\bar{\bar{\varphi}} = 0$ on $y = h$ (9)

Equation (7) suggests:

$\bar{\bar{\varphi}} = A_1 e^{-ky} + A_2 e^{ky}$; $\frac{\partial \bar{\bar{\varphi}}}{\partial y} = -A_1 k e^{-ky} + A_2 k e^{ky}$

Using conditions (8) and (9) we obtain:

$(p^2 + kg)A_1 + A_2 p^2 (1 - Ek) = g\bar{f}(k)$ (10)

and $\frac{A_2}{A_1} = \frac{k+G'}{k-G'} e^{-2kh}$ (11)

From (10) and (11) we solve for A_1 and A_2 and get:

$A_1 = \frac{g\bar{f}(k)}{B(k)} \cdot \frac{1}{p^2+\lambda^2}$; (12)

$A_2 = \frac{k+G'}{k-G'} e^{-2kh} \cdot \frac{g\bar{f}(k)}{B(k)} \cdot \frac{1}{p^2+\lambda^2}$ (13)

where $\lambda^2 = \frac{kg}{B(k)}$ (14)

and $B(k) = 1 + (1 - Ek) \cdot \frac{k+G'}{k-G'} e^{-2kh}$ (15)

Therefore we have,

$$\bar{\bar{\varphi}}(k, y; p) = \frac{g\bar{f}(k)}{B(k)} \left[\frac{1}{p^2+\lambda^2} e^{-ky} + \frac{\frac{k+G'}{k-G'} e^{-2kh}}{p^2+\lambda^2} \cdot e^{ky} \right] \quad (16)$$

If the initial displacement of the inertial surface is assumed to be concentrated at the origin, then we can take $f(x) = \delta(x)$ or, $\bar{f}(k) = \frac{1}{\sqrt{2\pi}}$ where $\delta(x)$ is Dirac's delta function.

Hence, $\bar{\bar{\varphi}}(k, y; p) = \frac{g}{\sqrt{2\pi} B(k)} \left[e^{-ky} + \frac{k+G'}{k-G'} \cdot e^{-2kh} \cdot e^{ky} \right] \cdot \frac{1}{p^2+\lambda^2}$ (17)

Taking inverse Laplace transform we get,

$\bar{\varphi}(k, y; t) = \frac{g}{\sqrt{2\pi} B(k)} \left[e^{-ky} + \frac{k+G'}{k-G'} \cdot e^{-2kh} \cdot e^{ky} \right] \cdot \frac{\sin\lambda t}{\lambda}$ (18)

The Fourier inversion then yields

$\varphi(x, y; t) = \frac{g}{\pi} \int_0^\infty \frac{1}{\lambda B(k)} \left[e^{-ky} + \frac{k+G'}{k-G'} \cdot e^{-2kh} \cdot e^{ky} \right] \sin\lambda t \cos kx dk$ (19)

The depression of the inertial surface at time t is then given by

$\eta(x, t) = \frac{1}{g} \varphi_t(x, 0, t)$

$= \frac{1}{\pi} \int_0^\infty \frac{1}{B(k)} \left[1 + \frac{k+G'}{k-G'} e^{-2kh} \right] \cos\lambda t \cos kx \, dk$

$= \frac{1}{\pi} \int_0^\infty C(k) \cos\lambda t \cos kx \, dk,$ (20)

where $B(k)C(k) = 1 + \frac{k+G'}{k-G'} e^{-2kh}$ (21)

IV. ASYMPTOTIC ANALYSIS : STATIONARY PHASE METHOD

Equation (20) is an oscillatory integral which can be approximated by employing the method of stationary phase for large x and t such that $\frac{x}{t}$ remains finite.

We can write, $\eta(x, t) = \frac{1}{2\pi} Re\left[\int_0^\infty C(k) \{ e^{it\,S(k)} + e^{it\,S(-k)} \} dk \right.$ (22)

where, $S(k) = \lambda - k \cdot \frac{x}{t} = \sqrt{\frac{kg}{B(k)}} - \frac{kx}{t}$ (23)

The second term of the integral (22) has no stationary point within the range of integration. The stationary point of the first integral is given by the following equation:

$S'(k) = \frac{ds}{dk} = 0$ (24)

In view of the fact that the function $S'(k)$ is monotone decreasing in nature, the stationary point k_0 is the unique positive real root of the transcendental equation (24) and the integral (20) representing the free surface elevation can be approximated according to the stationary phase method as:

$\eta(x, t) \cong C(k_0) \cdot \frac{1}{\sqrt{2\pi t \, |S''(k_0)|}} \cdot \cos\left[tS(k_0) - \frac{\pi}{4} \right]$ (25)

V. GRAPHS

We have considered the case when the initial disturbance is applied in the form of initial depression at the origin by taking $f(x) = \delta(x)$. In this case, the approximate form of the free surface depression is given by asymptotic expression (25).

The physical quantities are non-dimensionalized as follows:

$x' = \frac{x}{h}$; $y' = \frac{y}{h}$; $t' = \sqrt{\frac{g}{h}} t$, $\eta' = \frac{\eta}{h}$, $E' = \frac{E}{h}$.

To depict the effect of the inertial surface and the bottom porosity on the wave generation by the initial disturbance, the non-dimensional asymptotic forms of $\eta'(x',t')$ are displayed graphically against x' for fixed t' and against t' for fixed x' in a number of figures. In order to perform it, the unique positive root of $S'(k) = 0$ is obtained first. In view of the fact that the function $S'(k)$ is monotone decreasing in nature, such a root exists uniquely. Clearly, k_0 is a function of x' and t' and other parameters and these are computed numerically for different values of x', t' and other parameters.

To visualize the nature of the wave motion generated, $\eta'(x',t')$ are plotted in Figs.1, 2 against x' with fixed $t' = 20$ where in Fig.1, $G'h = 0$ and in Fig.2, $G'h = 0.1$; $\eta'(x',t')$ are plotted in Figs. 3, 4 against x' with fixed $t' = 30$ where in Fig.3, $G'h = 0$ and in Fig.4, $G'h = 0.1$; in each of the Figs. 1,2,3,4 $E' = 0, 0.1, 0.01$ and $\rho = 0.2$. From Figs. 1,2,3,4 it is observed that for fixed t', when x' increases, amplitude of the wave profiles decreases so that at large distances they die out. This seems feasible as the initial disturbance is concentrated at the origin. From Figs. 1,2,3,4 it is also observed that in absence of porosity i.e., when $G'h = 0$, the curves corresponding to $E' = 0, 0.1, 0.01$ almost coincide while at $G'h = 0.1$ the effect of differences in the values of the inertial surface parameter is more pronounced. This is possibly due to an interaction between porous parameter effect arriving upwards from the bottom and the inertial surface parameter effect arriving downwards from the surface of the water.

In Figs. 5,6 $\eta'(x',t')$ are plotted against t' with fixed $x' = 10$ where in Fig.5, $G'h = 0$ and in Fig.6, $G'h = 0.1$; $\eta'(x',t')$ are plotted in Figs. 7,8 against t' with fixed $x' = 15$ where in Fig.7, $G'h = 0$ and in Fig.8, $G'h = 0.1$; in each of the Figs. 5,6,7,8 $E' = 0, 0.1, 0.01$ and $\rho = 0.2$. From Figs. 5,6,7,8 it is observed that for fixed x', when t' increases, amplitude of the wave motion at the inertial surface increases. This is somewhat unrealistic and is a consequence of the presence of a strong singularity at the origin as has been remarked upon by Stoker[2]. It is further observed from Figs. 5,6,7,8 that there have been a steady increase in the amplitude of the wave profile for $E = 0.01, 0.1, 0$ respectively in all cases.

All the eight figures demonstrate the interaction of the porous bed effect with the floating materials on the surface and both the phenomena affect the wave motion significantly.

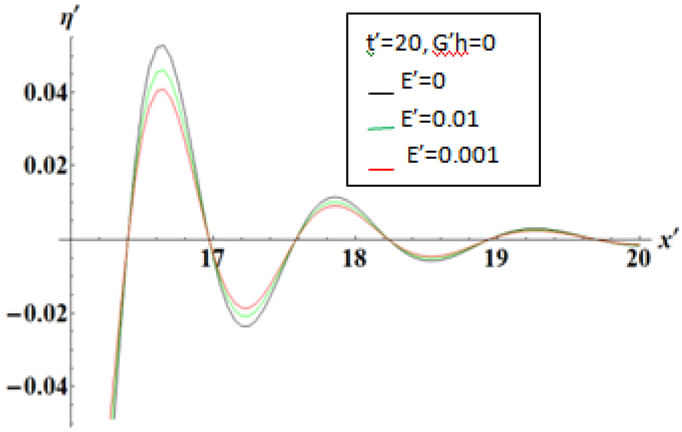

Fig.1 Wave profiles of an inertial surface depression due to an initial disturbance in the form of an initial depression

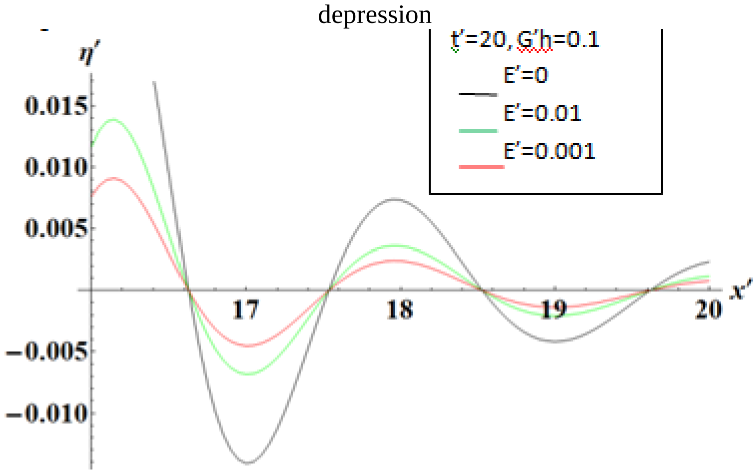

Fig.2 Wave profiles of an inertial surface depression due to an initial disturbance in the form of an initial depression

Fig.3 Wave profiles of an inertial surface depression due to an initial disturbance in the form of an initial depression

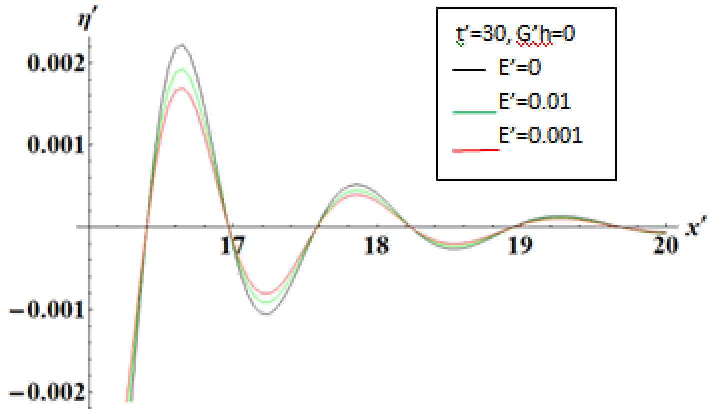

Fig.4 Wave profiles of an inertial surface depression due to an initial disturbance in the form of an initial depression

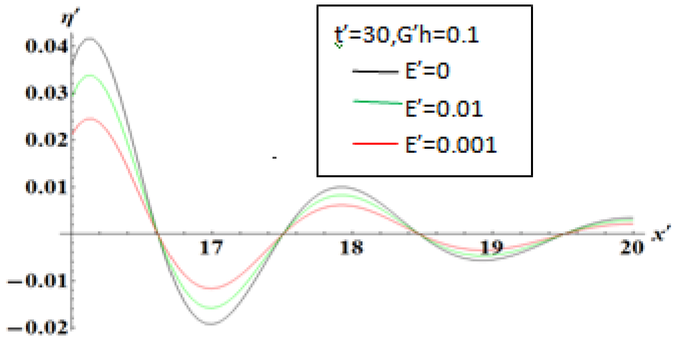

Fig.5 Wave profiles of an inertial surface depression due to an initial disturbance in the form of an initial depression

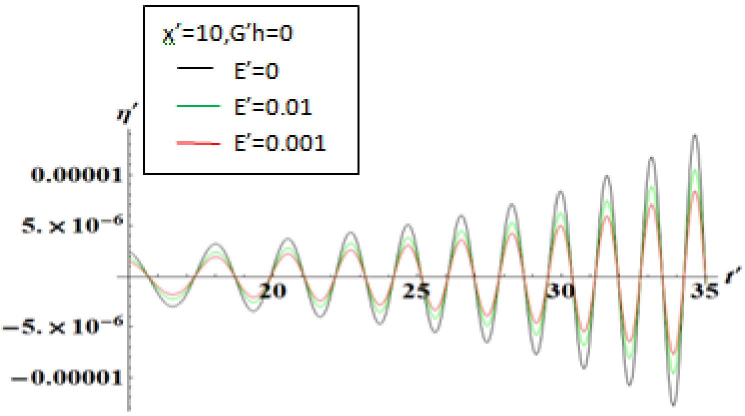

Fig.6 Wave profiles of an inertial surface depression due to an initial disturbance in the form of an initial depression

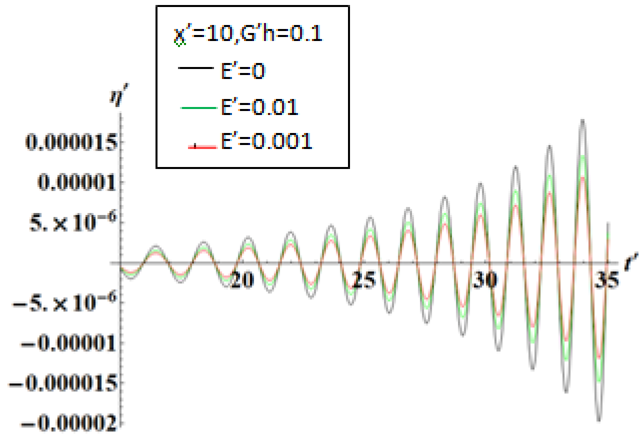

Fig.7 Wave profiles of an inertial surface depression due to an initial disturbance in the form of an initial depression

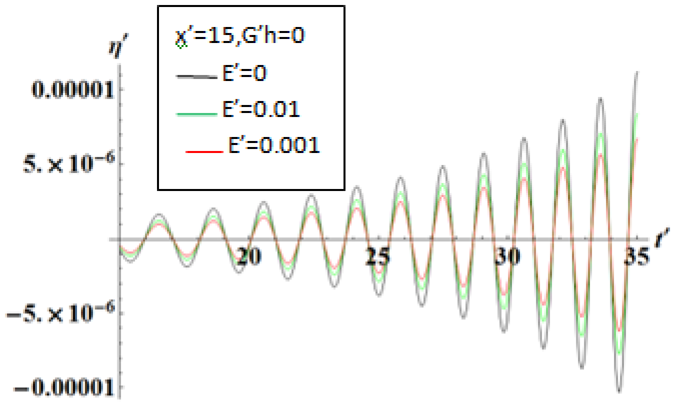

Fig.8 Wave profiles of an inertial surface depression due to an initial disturbance in the form of an initial depression

VI. CONCLUSIONS

The problem of water wave generation due to an initial disturbance in the inertial surface of the ocean with uniform finite depth is investigated using linear theory of water waves. Fourier and Laplace transform techniques are used to solve the problem of the initial disturbance with an inertial surface. The asymptotic form of the non-dimensionalised inertial surface depression is evaluated for a large value of non-dimensionalised time and distance by using the method of stationary phase. Plots of $\eta'(x',t')$ in various cases indicate the possible interaction between bottom porosity and the effect of floating material which composes the inertial surface.

REFERENCES

[1]. H.Lamb, Hydrodynamics, Cambridge University Press, 1932
[2]. J.J.Stoker, Water Waves, Interscience, New York, 1957
[3]. H.C.Kranzer and J.B.Keller, Water wave produced by explosion, J.Appl.Phys., 30, 398-407, 1959
[4]. K.S.Chaudhuri, Water wave in shallow water due to arbitrary surface disturbance, Applied Scientific Research, 19(1), 274-284, 1968
[5]. S.L.Wen, A note on water wave created by surface disturbance, International Journal of Mathematical Education in Science and Technology, 13(1), 55-58, 1982
[6]. L.Debnath, U.B.Guha, The Cauchy Poisson Wave in any inviscid stratified liquid, Applied Mathematics Letter, 2(4), 337-440, 1989
[7]. B.N.Mandal, Water wave generated at the inertial surface, Applied Scientific Research, 45(1), 67-73, 1988
[8]. P.Ghosh, U.Basu, B.N.Mandal, Water Wave due to initial disturbance at the inertial surface in a stratified fluid of finite depth, International Journal of Mathematics and Mathematical Sciences, 24(4), 265-276, 2000
[9]. D.P.Dolai, Waves produced by disturbances at the interface between two superposed fluids. Proceedings of the Indian National Science Academy, 62, 137-147, 1996

[10]. D.P.Dolai and S.Banerjea, On gravity-capillary waves due to interface disturbance, International Journal of Mathematical Education in Science and Technology, 28(1), 75-83, 1997

[11]. P.Ghosh, Waves due to initial axisymmetric disturbances at the inertial interface between two superposed fluids, Indian Journal of Pure and Applied Mathematics 31(9), 1123-1133, 2000

[12]. P.Maiti, B.N.Mandal, Water waves generated by bottom disturbance in an ice-covered ocean, International Journal of Applied Mechanics and Engineering, 12(2), 447-486, 2007

[13]. P.Maiti,B.N.Mandal, Water waves generated due to initial axisymmetric disturbances in water with an ice-cover, Archive of Applied Mechanics, 74(9), 629-636, 2005