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Unknown Input Estimation Using Full Order Observer in the Domain of Generalized Matrix Inverse

Ashis De[1], Avijit Banerjee[2], Gourhari Das[3]
[1]*Department of Electrical Engineering (P.G., Control System, 2014), Jadavpur University, Kolkata-32, India*
[2]*Department of Electrical Engineering (P.G., Control System, 2013), Jadavpur University, Kolkata-32, India*
[3]*Professor, Department of Electrical Engineering, Jadavpur University, Kolkata-32, India*

ABSTRACT

This paper presents a simple unknown input estimation technique using generalized matrix inverse. The full order observer constructed by g-inverse is extended and implemented for this purpose. The necessary and sufficient condition has been introduced for existence of such an estimator. The proposed method is illustrated by numerical example (Two loop autopilot in pitch plane) and the MATLAB simulation results.

Keywords:- Unknown Input Estimation(UIE), Linear Time Invariant(LTI) System, Unknown Input Observer (UIO), Full Order Observer, Unknown Input, Generalized Matrix Inverse, Missile Autopilot.

I. INTRODUCTION

Unknown input estimation has a special importance in many industrial applications like fault detection and identification, cryptography or parameter identifiability, track seeking, railway signaling etc. Several researchers have designed different observers that have been extended to develop UIO and to estimate unknown input. D. G. Luenberger first designed an observer in 1971 which approximately reconstructed missing state variable information [3]. Prof. Das and Prof. T. K. Ghoshal jointly proposed the method of construction of reduced order observer by using generalized matrix inverse [4]. Now to estimate the unknown input, it is necessary to design an unknown input observer (UIO). In [5], a constructive solution to the problem of designing a reduced order Luenberger observer for linear systems subject to arbitrary unknown inputs has been presented. A direct full-order observer design procedure for a linear system with unknown inputs is presented in [6]. Avijit Banerjee, Partha Pratim Mondal and Prof. Gourhari Das constructed a full order observer using the concept of generalized matrix inverse [10]. Alexander Stotsky and Ilya Kolmanovsky presented a paper on unknown input estimation techniques for automotive applications [11]. Unknown input estimation technique for linear discrete-time systems has been given in [7]. The problem of simultaneous estimation of states and unknown inputs for a class of non linear chaotic systems has been addressed in [12]. Avijit Banerjee and Prof Gourhari das presented a method of estimating the unknown input using reduced order Das and Ghoshal observer [4] in [13]. However, in this paper the full order observer constructed in [10] has been extended for estimation of unknown input.

Notations: In this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n dimensional Euclidean space and the set of all n×m real matrices respectively. I and $\emptyset$ are the identity and null matrices respectively with appropriate dimensions. The superscripts "T" represents the transpose of a matrix and the superscript "g" indicates the Moore-Penrose generalized matrix inverse of a matrix [8], [1].

II. MATHEMATICAL PRELIMINARIES

If $A \in \mathbb{R}^{m\times n}$ is a matrix then there exists a unique matrix $A^g \in \mathbb{R}^{n\times m}$, which satisfies the following conditions:

$$(AA^g)^T = AA^g \quad (1)$$
$$(A^g A)^T = A^g A \quad (2)$$
$$A A^g A = A \quad (3)$$
$$A^g A A^g = A^g \quad (4)$$

Consider a system described by linear equation,

$$Ax=y \quad (5)$$

where matrix $A \in \mathbb{R}^{m\times n}$, known vector $y \in \mathbb{R}^m$ and unknown vector $x \in \mathbb{R}^n$. Eq. (5) is consistent if and only if,

$$AA^g y = y \quad (6)$$

Now, if eq. (5) is consistent then the general solution of eq. (5) is given by

$$x = A^g y + (I - A^g A)r \quad (7)$$

([1] Graybill 1969 p. 104). Where $r \in \mathbb{R}^n$ is an arbitrary vector.

III. PROBLEM FORMULATION

Consider an LTI system described by

$$\dot{x} = Ax + Bu + Ev \tag{8}$$

$$y = Cx \tag{9}$$

where the state vector $x \in \mathbb{R}^n$, known input vector $u \in \mathbb{R}^{m1}$ and the unknown input vector $v \in \mathbb{R}^{m2}$. $y \in \mathbb{R}^m$ is the output vector. A, B, C, and E are known constant matrices with appropriate dimensions. It is assumed that the system is state observable.

The system dynamics in the absence of unknown input can be represented as

$$\dot{x}_u = Ax_u + Bu \tag{10}$$

$$y_u = Cx_u \tag{11}$$

Now, subtracting eq. (10) from eq. (8) and eq. (11) from eq. (9) respectively the state space description of the auxiliary system can be obtained as,

$$\dot{x}_v = Ax_v + Ev \tag{12}$$

$$y_v = Cx_v \tag{13}$$

where $x_v = x - x_u$ is the system response due to unknown input only and $y_v = y - y_u$ is the output response of the system due to unknown input only.

The state of the auxiliary equation can be obtained from eq. (12) and (13). Now the general solution of eq. (13) is

$$x_v = C^g y_v + (I - C^g C)h \tag{14}$$

where $h \in \mathbb{R}^n$ whose elements are arbitrary function of time.

Form eq. (12) and (13) we have

$$\dot{y}_v = CAx_v + CEv \tag{15}$$

From eq. (14) and (15) we have

$$\dot{y}_v = CA(I - C^g C)h + CAC^g y_v + CEv \tag{16}$$

Putting x_v from eq. (14) into eq. (12) we get,

$$(I - C^g C)\dot{h} = A(I - C^g C)h + AC^g y_v + Ev - C^g \dot{y}_v \tag{17}$$

The general solution of eq. (17) for $\dot{h}$ is given by,

$$\dot{h} = (I - C^g C)A(I - C^g C)h + (I - C^g C)AC^g y_v + (I - C^g C)Ev + C^g Cz \tag{18}$$

where $z \in \mathbb{R}^n$ is a vector whose elements are arbitrary function of time.

From eq. (18) and (16), the observer dynamic equation can be written as,

$$\dot{\hat{h}} = [(I - C^g C)A(I - C^g C) - KCA(I - C^g C)]\hat{h} + [(I - C^g C)AC^g - KCAC^g]y_v + [(I - C^g C)E - KCE]v + C^g Cz + K\dot{y}_v \tag{19}$$

where K is the observer gain and $\hat{h} \to h$ as $\hat{x} \to x$.

In order to eliminate the first derivative of y_v explicitly present in (15) the following substitution has been made.

$$\hat{h} = \hat{q} + Ky_v \tag{20}$$

and the observer dynamics can be written as

$$\dot{\hat{q}} = \{(I - C^g C)A(I - C^g C) - KCA(I - C^g C)\}\hat{q} + \{(I - C^g C)E - KCE\}v + \{(I - C^g C)AC^g - KCAC^g + (I - C^g C)A(I - C^g C)K - KCA(I - C^g C)K\}y_v + C^g C \tag{21}$$

Without loss of generality, the arbitrary vector z may be chosen as a null vector for simplicity and less computation, the observer dynamics can be represented as,

$$\dot{\hat{q}} = \{(I - C^g C)A(I - C^g C) - KCA(I - C^g C)\}\hat{q} + \{(I - C^g C)E - KCE\}v + \{(I - C^g C)AC^g - KCAC^g + (I - C^g C)A(I - C^g C)K - KCA(I - C^g C)K\}y_v \tag{22}$$

Condition for existence of UIO:

To nullify the effect of the unknown input from observer dynamics the observer gain parameter K can be designed such that

$$(I - C^g C)E - KCE = 0 \tag{23}$$

The general solution of K is given by

$$K = (I - C^g C)E(CE)^g + K_v\{I - (CE)(CE)^g\} \tag{24}$$

where K_v is an arbitrary matrix and the arbitrariness of K is also depends on K_v. Putting the value of K from eq. (24) into eq. (23) we get,

$$(I - C^g C)E(CE)^g(CE) = (I - C^g C)E \tag{25}$$

Eq. (25) is the consistency condition of eq. (23). Putting eq. (24) into eq. (22), it can be shown that the observer matrix would be (A_v - $K_v C_v$) where,

$$A_v = (I - C^g C)A(I - C^g C) - (I - C^g C)E(CE)^g CA(I - C^g C) \tag{26}$$

and $$C_v = \{I - (CE)(CE)^g\}CA(I - C^g C) \tag{27}$$

The arbitrary matrix K_v should be chosen such that the real part of the eigen values of observer matrix $(A_v - K_v C_v)$ become negative.
Then the UIO for auxiliary system, given by eq. (12) and (13) can be obtained as

$$\dot{\hat{q}} = (A_v - K_v C_v)\hat{q} + \{(I - C^g C)AC^g - KCAC^g + (I - C^g C)A(I - C^g C)K - KCA(I - C^g C)K\}y_v \tag{28}$$

and the estimated state of the auxiliary system can be expressed as

$$\hat{x}_v = (I - C^g C)\hat{q} + \{C^g + (I - C^g C)K\}y_v \tag{29}$$

The general solution for estimated unknown input can be expressed from eq. (12),

$$\hat{v} = E^g\left(\dot{\hat{x}}_v - A\hat{x}_v\right) + (I - E^g E)h_0 \tag{30}$$

where h_0 is any arbitrary vector. From the theory of generalized matrix inverse it can be concluded that if the matrix E is of full rank the unknown input can be estimated as

$$\hat{v} = E^g\left(\dot{\hat{x}}_v - A\hat{x}_v\right), \quad \text{since } E^g E = I. \tag{31}$$

IV. NUMERICAL EXAMPLE

Consider the system of a class of flight path rate demand missile autopilot in state space model as described in [9]. In this example the states of the state vector *x* are:

x_1 = Flight path rate($\dot{\gamma}$)
x_2 = Body rate(q)
x_3 = Elevator deflection(η)
x_4 = Rate of change of elevator deflection($\dot{\eta}$).

The state space model of a classical two loop autopilot has been given below:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} -\frac{1}{T_a} & \frac{1+\sigma^2\omega_b^2}{T_a} & -\frac{K_b\sigma^2\omega_b^2}{T_a} & -K_b\sigma^2\omega_b^2 \\ -\frac{1+\omega_b^2 T_a^2}{T_a(1+\sigma^2\omega_b^2)} & \frac{1}{T_a} & \frac{(T_a^2-\sigma^2)K_b\omega_b^2}{T_a(1+\sigma^2\omega_b^2)} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_a^2 & -2\zeta_a\omega_a \end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ K_q\omega_a^2\end{bmatrix}u + Ev \tag{32}$$

and $$y = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \tag{33}$$

The system has the initial condition, $x_0 = \begin{bmatrix}20\\ 0.25\\ 1.43\\ 100\end{bmatrix}$.

The following numerical data for a class of guided missile have been taken for MATLAB simulation:
T_a=2.85 sec; $\boldsymbol{\sigma}^2$ =0.00142 sec^2 ; $\boldsymbol{\omega}_b$ =5.6 rad/sec; ζ_a =0.6; K_b = -0.1437 per sec; v = 3000 m/sec; K_p = 28.99; K_q= -1.40; $\boldsymbol{\omega}_a$ =180 rad/sec;

We have taken the unknown input matrix, $E = \begin{bmatrix}1\\ 0\\ 1\\ 0\end{bmatrix}$ and the arbitrary matrix K_v is chosen as, $k_v = \begin{bmatrix}-5 & 7\\ 2 & 7\\ 5 & 8\\ 5 & 9\end{bmatrix}$.

The above system is observable and the existence condition of unknown input has been satisfied. For simulation the known input is taken unit step and the unknown input is taken as $v = 500e^{-50t}sin1000t$. Simulation responses for estimation of states and the responses of the system with and without unknown input are given in Fig.4.1-4.4 below, where black doted lines denote the estimated signals, red firm lines indicate the response of the state with unknown input and green firm lines indicate the response of the state without unknown input.

In Fig. 4.1 to 4.4, x_i denote the system states with unknown input, xu_i denote system states without unknown input and $\hat{x}_i$ denote corresponding estimated states with unknown input for i =1, 2, 3, 4. In Fig. 4.5, v denotes unknown input and $\hat{v}$ denotes corresponding estimated signal. From the simulation responses it can be seen that the estimated signal tracks well the high frequency unknown signal 'v', i.e. '$\hat{v}$' follows 'v'.

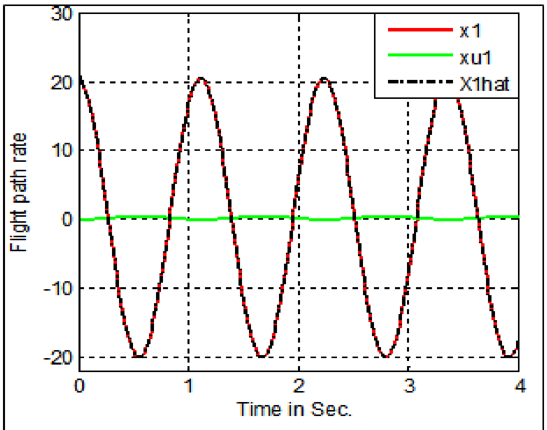

Fig. 4.1: Flight path rate

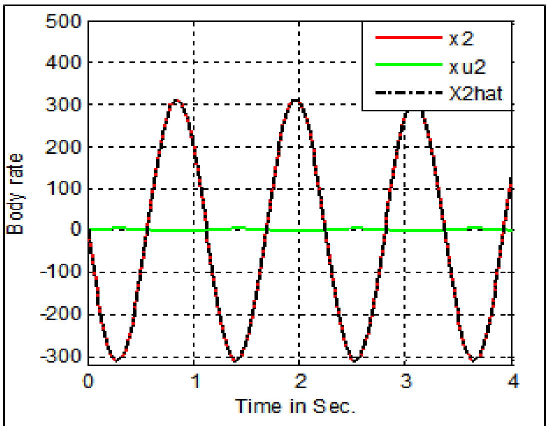

Fig.4.2: Body rate

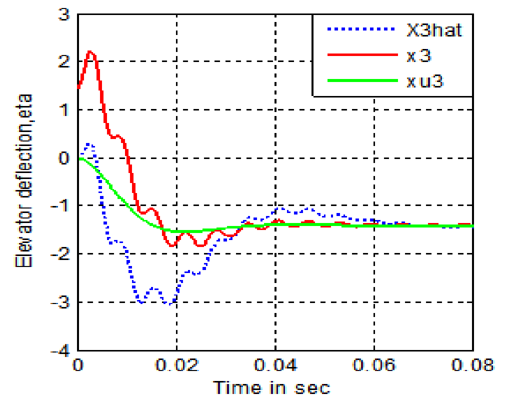

Fig. 4.3: Elevator deflection

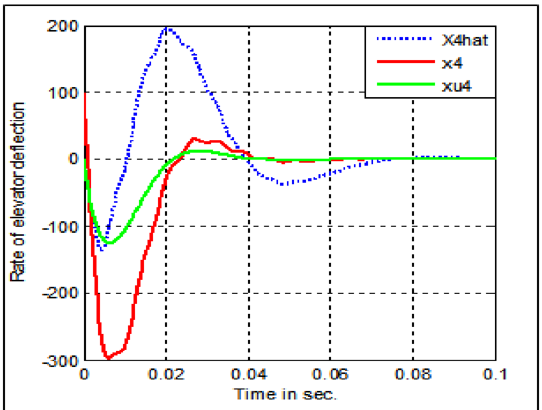

Fig. 4.4: Rate of Elevator deflection

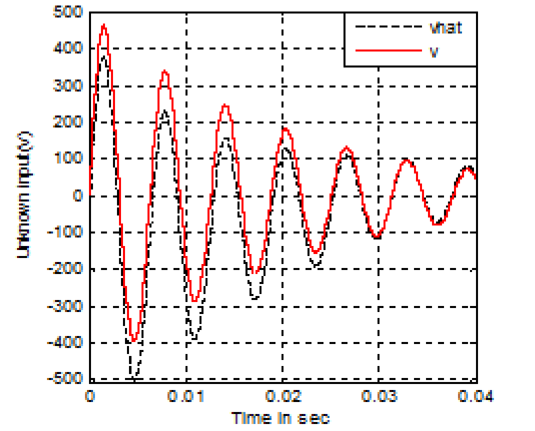

Fig. 4.5: Unknown input

V. CONCLUSION

The unknown input estimator has been presented for LTI system extending the full order observer as reported in [10]. In this process the system is firstly transformed to an auxiliary system. Then using standard methodology an observer has been developed for the auxiliary system. Now by applying simple algebraic equation, estimated unknown input is obtained. The necessary conditions are proposed and solved using generalized matrix inverse. Illustrated numerical example with simulated results show the estimated unknown input which tracks the actual unknown input well enough, even if the signal frequency is very high.

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