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Bayesian approach for spare parts replenishment policies under uncertainties

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Abstract: The legislative constraints, the need to optimize the dismantling process, the introduction of recycled parts on the spare parts market are reinforced since the systems in end-of-life phase have become increasingly profitable. There are few works that treat recycled spare parts integration problem in economic models of inventory control. These works do not consider uncertainty. In order to manage more realistically the inventory control of spare parts, we propose a probabilistic model formalized by a Bayesian network. The model is used to identify the best purchase policy. More precisely, it allows choosing the best proportions between new spare parts (NSP) and recycled spare parts (RSP) by taking into account the traditional criteria of inventory control and the availability of the spare parts on the market. The proposed method provides a decision-making tool for manufacturers who are interested both in reducing the costs of stocks and guaranteeing a minimal availability in an uncertain environment.

Keywords: Inventory control, Bayesian networks, Decision-aided tool, Modeling, Recycling, Sustainability

I. INTRODUCTION

For several years, the end of life cycle of the industrial systems takes an increasing importance both in conception and in flow management. This importance is due to industrialization, environmental legislative constraints and awareness of the economic perspectives offered by the dismantled product's valorization. We can find end of life components and parts with 50% to 75% less expensive than similar new parts [1].

Traditionally, four categories of revalorization activities associated to three ends of life options may be developed:

- Functional recycling which consists in re-using a product in a functional system (an electric alternator of a heat engine),
- Material recycling which provides the re-use of a product after transformation (recycling of plastics).
- Energetically recycling which consists in using products to produce energy (used oil as complementary fuel for cement works),
- The packaging for the landfill that meets a number of environmental constraints (waste resulting of incineration processes).

The awareness of the manufacturers about profitability of the end of life products recycling, allowed the development of recycled spare parts [2]. This market is characterized by difficulties in anticipating the availability of these spare parts.

In literature few works treat integration of the recycled spare parts in the economic models of inventory control. In order to contribute to the management of economical parameters, we propose a probabilistic model formalized by a Dynamic Bayesian Network (DBN). This model allows to evaluate and to obtain the best policies of purchasing the spare parts [3]. More precisely, it gives the possibility to choose the best ratio between the NSP and RSP by taking into account the availability of the recycled spare parts on the market as well as the traditional criteria of inventory control. Modeling and resolution of this problem provides a decision-aid tool in spare parts inventory control and guarantee low costs and stock availability to manufacturers .

II. STATE OF ART

There are studies that treat the problem of SP supply chain but there not many reflect the uncertainty of variables such as inventory management work [4] [5] [6].

In [7] is proposed an original use of DBN for the evaluation of pathways reclamation of products to disassemble in the presence of uncertainties. This uncertainty is mainly in requirements from processes of disassembly and arrivals of end of life. As in this article, we discuss a problem of uncertainty in the spare parts supply management, the uncertainty treated relates mainly: the lead time, demand and the parts purchase price.

In [8] is addressed the problem of determining the economic quantity of SP over the different cycle phases of the product life.

From our point of view, the problem of SP supply chain induces three research directions: the definition of optimal safety stock level so as to avoid stock-outs, determining optimal amounts of SP to order with uncertainties in demand, price and lead time, and formalism of probabilistic and predictive tools to help manage the inventory of SP.

1. Spare Parts inventory control

Spare parts inventory control is an important challenge when spare parts of many systems change over time. To maintain the performance of complex systems, spare parts inventory control must be a priority. When a manufacturer starts to market a new product, it is committed to providing the necessary spare parts for replacement of failed parts in the future. This commitment creates a real problem of spare parts inventory control, sizing and location. To meet the demand of spare parts from maintenance services, which have generally a stochastic nature, the manufacturer must set an effective strategy of inventory control. The object of this management is to provide in time and at the lowest cost the spare parts needed for clients. In this context, [9] provide a model for an application for global demand of spare parts generated by a single product with a growing number of parts in a homogeneous Poisson process. Their model is a special case where the time to failure of the product follows the exponential distribution; the results found by this method are means and variances of demand for spare parts. Using a multi-resolution based on their model, a policy of restocking dynamics (Q, R) has been formulated and solved. Finally, they illustrate the model by two numerical examples to demonstrate the relevance of their model in spare parts inventory control under a service level constraint. A study of effectiveness of this method was carried out through simulation.
Preventive maintenance or corrective maintenance is still behind the need for inventory management of spare parts. Determining the required amount of spare parts for maintenance service is always difficult through the predictions based on historical data. Therefore, obtaining an optimal policy of stock management is difficult. Jointly optimizing the management stock spare parts and preventive maintenance were presented by [10].In fact, in the presence of random nature of plant failures, they develop a cost model for stochastic spare parts inventory control and maintenance, and are based on a dynamic programming to find the joint optimal solutions on a finite time horizon.

To construct the probabilities of the number of failures and the number of the defective items identified at a preventive maintenance epoch, [10] use the delay-time concept developed for inspection modeling which has not been used in this type of problems before. They are in the case of a policy for periodic review of inventory management to meet the needs of a maintenance service. They demonstrate their models through a numerical example.

Many studies deal with the issue of spare parts, but there are few that take into account the uncertainty that has been on variables such as inventory management work [5] [6]. [7] proposes an original use of the Bayesian network tool for the evaluation of beneficial reuse of products to disassemble in the presence of uncertainty. This uncertainty mainly concerns the requirements for products from the processes of disassembly and arrivals of the products' end of life. addresses the problem of determining the economic lot of SP to be produced, taking into account the different phases of the life cycle of the product.

In this paper we will consider first the solutions used to define levels of optimum safety stocks to move to analyze the methods used to determine the optimal amounts of SP to provide. The good choice of optimal procurement policy rests well on the choice of level of safety stock but also on the amount of SP to provide. Unfortunately, the Economic Order Quantity method does not have the effect of uncertainty in its formulas; however, uncertainties about demand, purchase prices and lead times are really in the SP inventory control.
In the next section, we will present the four SP replenishment policies existing in the literature.

2. Spare parts replenishment policies

New spare parts used in the replenishing industry are expensive, they have a high demand rate in the maintenance sector and the consumed quantity is variable. In literature there are four inventory basic policies for replenishing inventory [11] [12].We distinguishes in these replenishment policies, the policy for a continuous review system and the policy for a periodic review system. For the policies with continuous review system, we find two different policies: the policy (s,Q) which is characterized by a fixed quantity of supply and varying periodicity, and the policy (s,S) which is characterized by varying quantity of supply and at a varying periodicity. For the policies with a review periodic system, we also find two policies: (T, S) and (T, s, S) policies, which are characterized both by a fixed periodicity and a variable quantity of supply. Both policies are summarized in the table above:

T :Periodicity Q :Quantity	Fixe	Variable
Fixe	-	(s,Q)
Variable	(T, S) ; (T,s,S)	(s,S)

Table 1: Replenishment policies in inventory control

For continuous review systems:
The (s, Q) Policy: Whenever the inventory position (items on hand plus items on order) drops to a given level, s, or below, an order is placed for a fixed quantity, Q.
The (s, S) Policy: Whenever the inventory position (items on hand plus items on order) drops to a given level s, or below, an order is placed for a sufficient quantity to bring the inventory position up to a given level, S.
For periodic review systems:
The (T, S) Policy: Inventory position (items on hand plus items on order) is reviewed at regular instants spaced at time intervals of length T. At each review, an order is placed for a sufficient quantity to bring the inventory position up to a given level, S.
The (T, s, S) Policy: Inventory position (items on hand plus items on order) is reviewed at regular instants spaced at time intervals of length T. At each review, if the inventory position is at level s or below, an order is placed for a sufficient quantity to bring the inventory position up to a given level S. If the inventory position is above s, no order is placed (Figure 1).
The indicators Q, s, S, and T are defined as follows:

Q = order quantity
S = order-up-to level
s = reorder point
T = review period

The (T, s, S) policy can be regarded as a periodic version of the (s, S) policy. In fact the (s, S) policy is a special case of the (T, s, S) policy in which T = 0 [12].

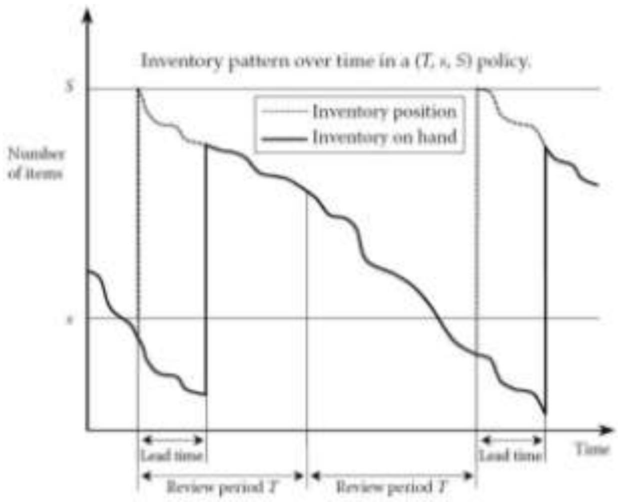

Figure 1: Inventory pattern over time in a (T, s, S) policy

Since the demand for spare parts is variable, and is depending on the maintenance needs and the time from when an order for replenishment is placed until the replenishment arrives (lead time) is also a random variable with a fix variance, we placed our work in the (T, s, S) policy case. In such a system, the period of review is fixed and ordered quantity changes as per demand or rate of consumption. The period of review T is decided such that the ordered quantity is economical to purchase the items.

For all these reasons, modeling and simulation to be presented in the rest of this paper concern this politique (T, s, S).
When the supplier puts the restriction on minimum order quantity, the variable order quantity is decided based on the (S, s) policy, where maximum level of inventory is S and minimum safety stock is s.

The replenishment level is in between S and s value. The (T, S) policy represents a special case of the (T, s, S) policy in which s = S

Since the demand for spare parts is variable, and is depending on the maintenance needs and the time from when an order for replenishment is placed until the replenishment arrives (lead time) is also a random variable with a fix variance, we placed our work in the (T, s, S) policy case. In such a system, the period of review is fixed and ordered quantity changes as per demand or rate of consumption. The period of review T is decided such that the ordered quantity is economical to purchase the items.

The next section will be dedicated to the SP inventory control problem.

3. Spare parts inventory control problems

The management of spare parts is a particular inventory management which poses many problems. The use of a data processing by integrating it to the software package of the company is essential. The choice of the latter can then be made more difficult.

We can manage spare parts with other products that already manage by the company (semi-finished products, raw materials, consumables, components...). In this framework, spare parts management encountered by several problems with four main:

- The lot sizing problem of new spare parts and recondition which can satisfy demand;
- Cost forecasting problem of these parts according to market fluctuations;
- Lead times problem which become more uncertain following the introduction of recycled spare parts;
- Availability spare parts problem from suppliers;

a. Lot sizing problem

Spare parts management is a predictive management of supply inventories. In design, spare parts management analysis is focused on their maintenance level based on the criteria of the life cycle cost, efficiency and availability that we gave. For each spare part, we must, for example ask if it should be repairable or not and at what level. During operation equipment, it is subject to analyze the maintenance tasks to be carried out at a given level, the number of parts required and quantity to stock (eg. safety threshold) [13] [14]. Equipment suppliers offer lot formation lists of initial spares but often the design is not adequate [15] [16] [17].

Thus, the most important issue in the sizing of spare parts [18], [19], [20] in the maintenance workshops is to match up the predictive model of spare parts stock levels to the realities of the company and its dynamics [21], [22].

This can become particularly difficult in some sectors such as aeronautics. In fact, in this area, we have mobile workshops that require sometimes unpredictable maintenance or replacements on multiple sites in a predefined schedule. In addition, spare parts unit costs are particularly high. [23] have addressed these problems in order to improve the availability of production sites and to minimize the number of failures and maintenance costs. To achieve this goal, they proposed a modular modeling of maintenance activities in a multi-site through Petri nets. In their paper they consider the NSP. There is therefore no additional uncertainty as availability or cost of RSP.

b. Cost problem

The spare parts cost for major industries constitutes a real obstacle in maintenance of systems because it sometimes exceeds the cost of purchasing the system itself. In this context, many works was started for many decades on identifying ways that reduce maintenance costs. Indeed, to reduce spare parts logistic cost, [24] propose the idea of choosing appropriate BOM (Bill Of Material) configurations, formulates the decision problem, and develop an efficient approach to solve the problem. Suppose a machine has s critical parts, each of which has k vendors. The possible number of BOM configurations may be quite huge (ks). It might take a formidable computation time if they exhaustively evaluate the performance of each BOM configuration by using the evaluation methods developed in literature. To efficiently solve the problem, they propose a GA-NN approach. The NN (neural network) technique is used to efficiently emulate the function of an existing method for evaluating BOM configurations, whereas the GA technique is used to efficiently identify a near-optimal BOM configuration from the huge solution space. Experiment results indicate that the approach can obtain an effective BOM configuration efficiently.

In the same field, [25] have analysed the cost allocation problem in the specific context of repairable spare parts pooling. The focus of their analysis is to obtain a cost allocation policy that is acceptable for all participating companies such that they are all motivated to cooperate in maximizing the benefits of pooling. Their research contributes to the literature on multi-location inventory problems in two important ways. First, using game theoretic models [26] they show that the cost allocation policy influences the final outcome of the game, i.e. whether or not the companies are willing to pool their spare parts inventories. Both the cooperative and competitive game theory models (core and Nash equilibrium concepts) were used throughout our analysis. Second, they have identified that a class of cost allocation policies are able to induce the system-optimal policies [27]. The results of their numerical study demonstrate how a cost allocation policy can give a cost allocation that is in the core and lead to the optimal inventory policy.

Usually, spare parts are expensive, and therefore managerial attention is justified. Spare parts models are desired to be multi-item models, i.e., rather than focusing on the performance for an individual item, they focus on the performance for all items together because that is what the customer really is interested in.

[28] Have developed a multi-item, single-stage spare parts inventory model with multiple groups to study the effect of commonality on spare parts provisioning costs for capital goods. A case study in which they studied several data sets of ASML's company, an original equipment manufacturer in the semiconductor industry, showed that on average 6% reduction can be obtained in spare parts provisioning costs [28] if stocks for different groups are shared. In a numerical experiment with a smaller data set, authors showed that a larger number of groups increases potential benefits considerably. Also, they have seen that the savings obtained by shared stocks are significantly affected by the commonality percentage.

c. Lead time problem

The lead time problem of supply remains a major challenge in spare parts inventory management. As we have mentioned, we will consider in this paper new and recycled spare parts. With the introduction of recycled parts in the supply chain management, estimating lead times for these parts is really difficult because of the critical condition of each item recycled. So the spare parts lead time , new or recycled, is in most industrial cases, a probability factor that modeling is often done via stochastic variables.

In this context, several researchers have addressed this problem in different works.

The aim of the study of [29] is to investigate the desired level of recovery and it is motivated by an industrial practice in a European refinery. The objective of their study was to investigate the desired level of recovery effort for such a system. Their approach integrates tactical and operational decision making. They consider specification of recovery effort and purchasing decisions in a coordinated manner. For the sake of generality and simplicity, they model recovery as a single stage operation with some unit cost and lead time. They consider the expected time spent in recovery operation, i.e., associated expected lead time, as a measure of the amount of recovery effort and focus on the case where the probability of successful recovery increases as the expected lead time increases, i.e., the amount of effort put into recovery [30]. Authors also include the effect of increasing recovery effort on associated unit cost and assume that all of the items completing their usage time (which is stochastic with a known probability distribution) return to the system. They further conclude that information on the number of items currently in use is always available.

In the same field, [31] treat the problem of how to make an exact policy for the variable lead time by considering seasonal demand which becomes a key factor of reducing the total relevant cost. To the best of authors' knowledge, the problem has not been solved by anyone. For these buyers to implement better the JIT philosophy in their supply chain system by considering seasonal demand and variable lead time [32], authors propose two new models. The first one is an approximate model, which can be easily implemented. Using the proposed approximate model, inventory decision-maker can easily make the approximate optimal purchasing policy for such problem. The second is a precise model, which can obtain the optimal solutions of the seasonal demand inventory problem. The proposed precise model can help inventory decision-maker to obtain solutions closer to global optimum. One should consider the trade-off between ''flexibility of formulation'' and ''global optimal solutions'' in the two models.

The promising results motivate the need for further research on the seasonal demand inventory problem with shortage and back-order problems.

To deal with the problem that suppliers face uncertain fluctuation of customer orders, [33] proposed a two-level supply chain model to analyze how a supplier's delivery lead-time performance can affect its customer's orders in the fluctuating environment. To deal with uncertain fluctuations of high-tech industries, buyers need to hold extra inventory as well as to carefully manage their purchase points when considering the supplier's delivery time. However, these activities can increase costs and reduce operational efficiency. Therefore, how to effectively manage the seasonal demand inventory problem with variable lead time is an important issue for a buyer.

We find also the work of [34] whose consider the single vendor single buyer integrated production inventory problem. They relax the assumption that demand is deterministic and assume that it is stochastic and tackle the lead time issue. They assume a linear relationship between lead time and lot size but take into consideration also nonproductive time in the lead time expression. A solution procedure is suggested for solving the proposed model and numerical examples are used to illustrate the model and explore the effect of key parameters on lot size, reorder point, and expected total cost.

[35] proposed the previous model with two objectives, inventory holding cost and backlogging cost. They try to apply an evolutionary algorithm to find non-dominated solutions for stochastic inventory control systems, the same problem as [36] but in a multi-objective context.
For this reason they reinforce the genetic algorithm with a developed evolutionary algorithm, named electromagnetism-like mechanism (EM) to minimize both costs at the same time.
Ability of their proposed algorithm was investigated through computational experiments using four different metrics. Comparison was done against an outstanding metaheuristic approach which was adapted to this problem, NSGA-II [36]. Out performance of proposed algorithm than NSGA-II revealed its efficiency and absolute superiority.

d. Spare parts availability problem

Spare parts availability remains a key factor in supply management. Different authors tried to estimate parts availability in several ways. Several works have been treated in this area which includes:
[37] Suggest that among the cooperative strategies commercial pooling is more efficient than ad hoc cooperation or cooperative pooling if the service provider is willing to share enough pooling benefits with the members, and the members are ready to trust a mission critical service to an outside party. Ad hoc cooperation is feasible when the airline is cautious about subcontracting the availability service and there is a cooperating airline nearby with close enough fleet composition. A cooperative pool is feasible with a limited number of members under conditions where there is no commercial service provider available or the existing service provider’s pricing is unreasonably high [38]. Going deeper under the general level of contributing factors, the demand structure and the benefit sharing criteria stand out from the other factors. For a cooperative pool authors suggests benefit sharing criteria that drive the demand structure towards a dynamic equilibrium.
[39] Describe how a condition-based maintenance system (CBM) might be operated for operators of assets such as aircraft engines. Authors show that merely having a monitoring and diagnostics system in place is not enough to derive the full, or even the majority of the, benefit from (CBM). They show that to maximize the benefits from CBM for the enterprise, it is as important to focus on the aftermarket supply chain as it is to develop better data gathering, diagnostics, and prognostics techniques. Authors show via simulation that optimizing the value chain results in lower costs and turnaround times, and higher asset availability, spare part availability, and fill rates. As the demands on the aftermarket supply chain become more predictable, they can service the same asset base with lower spares inventory, while meeting increasingly aggressive turn-around-time targets, thus improving the bottom line and customer satisfaction.
[40] present a survey by the US Nuclear Regulatory Commission (NRC) of maintenance practices at US nuclear power plants confirms that spare parts availability is, to some extent, a problem at all but a few plants, and that 57% of the plants sometimes operated in a limiting condition for operation over the last year due to unavailability of spare parts. That's the bad news. The good news is that significant improvement is possible without ballooning inventory. The vast majority of total dollar value of spare parts inventory at power generation stations, whether nuclear or fossil, is rarely used (parts or components that are used 12 or less times per year including many with a history of no use for many years). Inventory Solutions of Akron, Ohio, has developed such a tool, called rarely used inventory stocking logic (RUSL) which is a user-friendly decision support tool for setting spare parts reorder points. Based on statistical computations enhanced to include the most advanced techniques developed for use in aerospace and military combat readiness, RUSL calculates the optimum stocking level to achieve the desired spare part availability at the station. In doing so, it considers the importance of the part in preventing an outage, as well as key economic ordering parameters, such as part cost, carrying cost, and number of parts in service, historical observed usage, anticipated future usage, and lead time to replenish.
[41] present a survey of operational related problems with the 17 years old computerized reactor instrumentation for the TRIGA reactor Vienna and a failure database, based on 47 years of operation, is given. The TRIGA reactor Vienna became critical on March 7, 1962. At that time, the reactor operated with the original General Atomics (GA) electronic tube-type console. In 1968 this console was replaced by a transistorized instrumentation which worked until 1992. In 1990 it was decided to replace this aged instrumentation by a state of the art console. The new computerized reactor instrumentation was ordered in 1990 from GA and installed and tested during summer 1992. The reactor was first critical after a two months shut-down period on November 10, 1992. During the last 17 years of operation, a number of failures occurred which made the reactor inoperable for about 100 days in total. The instrumentation design originates from the mid eighties and most of the electronic equipment especially the console computer and the Data Acquisition Chamber (DAC) computer was already outdated at the time of installation. Due to the rapid development in data acquisition technology, the problem of spare part availability becomes imminent, and led to the decision to replace the existing instrumentation after 17 years. All relevant documents especially all reactor logbooks have been securely stored since 1962, as they contain valuable data not only about normal operation but also about abnormal occurrences

such as automatic reactor shut downs, failure of equipment and maintenance procedures. All these abnormal occurrences have been classified according to a classification proposed at an IAEA Research Reactor ageing meeting in 2008.
In the next section, we will focus in works on uncertainty in inventory management.

4. Uncertainty in inventory management

[42] Has treated in his thesis the problem related to uncertainty in recycled spare parts management. She says producers are required to integrate reverse logistics in their supply chain by government legislation. In addition, the recovery of PRR protects the environment on the one hand and generates economic benefits on the other. It explains why reverse logistics adds complexity to the management techniques of classic parts, insofar as it induces higher levels of uncertainty in the amount, frequency and quality of returned product. His work has presented a flexible and general decision model that integrates detailed design decisions with high-level.
This problem was also treated by Mr. Godichaud in his thesis [43]. He proposed a modeling approach to optimizing disassembly decisions in the presence of uncertainties in particular on product demand and end of life systems arrivals which the spare part will be extracted. Then, it recommends end of life destination and disassembly levels through its RSP forecast stocks. However, it does not consider the cost.
In contrast, A. Alami addresses the same problem of uncertainty about RSP availability by an economic approach [8]. He proposes a system of differential equations that integrates:
- The pricing,
- Determination of the number of NSP or used, respectively, to fabricate or to repackage,
- And the determination of the warranty length to agree on a product component that the company manufactures.

His resolution method is based on the Pontryagin maximum principle, which allows him to solve the mathematical model and provide the report new and recycled spare parts (NSP/RSP).
The formalism of Alami has unfortunately no memory effect. This is a tool for instantaneous resolution and considers that we do sequentially the calculation of the spare part required in the sandstone and at random of the SP market.

In the same area, [44] note that although several fuzzy inventory models have been presented in literature, little has been done on addressing the issue of lead-time reduction in fuzzy environments. For this, he purpose to recast mixture inventory model involving variable lead-time with backorders and lost sales by introducing the fuzziness of lead-time demand, the average demand per year, and the backorder rate of the demand during the stock-out period. The aim of their work is to provide an alternative approach of modeling uncertainty that may appeal in real situations, while they do not attempt to establish the superiority of proposing new models to reduce more inventory cost than before. Moreover, in addition to the centroid method that is often used for defuzzification, a new ranking method for fuzzy numbers namely the signed distance was employed to solve their problems.
[45] Addressed the problem of the demand for spare parts which can sometimes be classified into critical and non-critical demand, depending on the criticality of the equipment for production. To manipulate this problem in spare parts inventory control, authors proposed a (r, r ,Q) inventory model for spare parts where a part of the stock is reserved for critical demand. They proposed to determine both of critical level which is the reorder point r and the reorder quantity Q. To illustrate the results of their method, they present numerical examples of spare parts semiconductor equipment of Taiwan manufacturer.
We will spend with the next section to the BN approach for sustainable decision and uncertainties in SP inventory control.

5. Using BN for sustainable decision and uncertainties

a. Graphical modeling

A “BN” is an appropriate graphical method for modeling of causal processes and probability- based knowledge representation under uncertainty. A “BN” is a directed acyclic graph whose nodes represent random variables and links define probabilistic dependences between variables. These relationships are quantified by associating a conditional probability table with each node, given any possible configuration of values for its parents. Bayesian networks have the ability of capturing both qualitative knowledge (through their network structure), and quantitative knowledge (through their parameters).

While expert knowledge from practitioners is mostly qualitative, it can be used directly for building the structure of a Bayesian network. In addition, data mining algorithms can encode both qualitative and quantitative knowledge and encode both forms simultaneously in a Bayesian network [46].

The static BN can be extended to a DBN model by introducing relevant temporal dependencies that capture the dynamic behaviors of the system at different times. Two types of dependencies can be distinguished in a DBN: contemporaneous dependencies and non-contemporaneous dependencies. Contemporaneous dependencies refer to arcs between nodes that represent variables within the same time period. Non-contemporaneous dependencies refer to arcs between nodes that represent variables at different times [47].

The advantage of DBN over Markov chains is that a DBN is a stochastic transition model factored over a number of random variables, over which a set of conditional dependency assumptions is defined.

Time invariance ensures that the dependency model of the variables is the same at any point in time. While a DBN can in general represent semi-Markovian stochastic processes of order k-1, providing the modeling for k time slices, the term DBN is usually adopted when only two time slices are considered in order to model the system temporal evolution. That's why such models are also called Two time Bayesian Networks (2-TBN) or 2-time- slice temporal Bayesian networks [48].

Each time-slice contains a set of (time-indexed) random variables, some of which are typically not observable. When a first order Markov process assumption holds the future slice at time (t +1), it is conditionally independent of the past ones given the present slice at time t. In this case, it is sufficient to represent two consecutive time slices called the anterior and the ulterior layer to represent the network. However, to specify the entire network and to correctly model the system next parameters have to be provided:

- The prior probabilities for root variables at t=0;
- The intra-slice conditional dependency model, together with the corresponding conditional probabilities;
- The inter-slice conditional dependency model and the transition model, which explicit the temporal probabilistic dependencies between variables.

Some other rules must be fulfilled for a correct DBN:

- Nodes from the anterior layer must contain only variables having influence on the same variable or on another variable at the ulterior level.
- The inter-slice edges connecting a variable in the anterior layer to the same variable in the ulterior layer are called temporal arcs;

b. Bayesian decision in inventory control

For several years, maintenance inventory management is no longer limited to the classical single vision of the old calculation methods such as GMAO and EOQ [49]. Furthermore, the most commonly used methods to rationalize decisions on managing maintenance inventory are essentially quantitative methods type RCM (Reliability Centered Maintenance: Reliability Based Maintenance) [50] who use not to theoretical models [51]. However, the application of these methods leads to trees or large networks as soon as the complexity of the processes studied is increasing especially with the integration of uncertain variables in the model.

So, to ensure requirements optimization in maintenance services in a company, it is necessary to improve overall performance of management system, its ergonomics and required resources. The aim, still make a profit by optimizing the use of spare parts in maintenance services. The choice of design and maintenance has more than ever, impacts on the overall cost, availability, security and processes quality. About us, the objective is to provide for the operational phase, the availability of spare parts in maintenance services and determine which cost alternatives combination provides the best compromise between: NSP and RSP.

In the field of dependability, recent studies based on industrial processes modeling from the theory of BN have been developed. The articles of [43] demonstrate the relevance of the BN using in the case of industrial processes in the presence of uncertainties and take inventory of published works on the matter.

By addition to the BN a specific nodes such as utility nodes and decision nodes, we can associate costs to decisions by integrating the uncertain variables. This modeling, described by Jensen [52], corresponds to the concepts of Influence Diagram (ID).

Influence Diagrams are graphical models. Considering several scenarios (decisions), an ID evaluates the impact of decisions on costs and variables (certain and uncertain) representing the process. The work of [53] presents influence diagrams in conjunction with decision trees. These two methods are comparable on calculation point of view, but the representation by ID is more compact and easier to apply to decision problems of large size.

Most commonly methods used to rationalize decisions on inventory management maintenance are essentially quantitative and of the type RCM (Reliability Based Maintenance) [54]. However, the application of these methods results in trees or large networks when the complexity of the processes increases with the integration of uncertain variables in the model. Thus, we can notice that all those who have addressed the SP problem have used different models but not yet the DBN resolution.

To ensure maintenance demands in an enterprise, it is necessary to improve overall performance management, its ergonomics and resources. The aim is to make profit by optimizing the use of spare parts in maintenance services. In our work the goal is to provide the availability of spare parts (new and recycled spare

parts) in maintenance services and to determine the impact of integration recycled spare parts on the reliability of the system using a Bayesian approach.
In the field of dependability, recent studies based on modeling of industrial processes using the theory of BN have been developed by [43]. His work demonstrates the relevance of BN as a tool for modeling industrial processes functioning in the presence of uncertainties.
We move in the next section to the Bayesian modeling of recycled spare parts replenishment.

III. BAYESIAN MODELING OF RECYCLED SPARE PARTS REPLENISHMENT

In real maintenance systems, the demand of spare parts is variable due to the changes in the production volume, rate of machine failures and workers availability. In the same time, the inventory volume cannot be set for a planning period, due to the delivery delays or lead times in delivery.
A common characteristic of dynamic inventory models with stochastic demand is the assumption that the demand and the costs distributions are known with certainty. In real systems these are parameters with a level of uncertainty provided by the dynamics of the production and market environment.
To manage this change in behavioral model of spare parts supply management, we present in this part a modeling approach for supply planning of NSP and RSP depending on the level of initial stock and uncertain parameters such as the lead time, demand, the safety threshold and the market price.
The objective is to determine the optimal combination of the stock for both types of parts for a single product range in the context explained above.

To achieve our objective, i.e. finding the optimal solution in terms of supply and the choice between new and recycled parts, we will combine our model with different input variables. We chose the following variables that have the most influence on our decision to purchase:

- Lead time « L (t) »: in spare parts inventory management, it may be a difference between the forecast lead time and real lead time. For this, an input factor remains uncertain variable to manage. Indeed, in our model, the variation of lead time in receiving necessary spare parts for the maintenance department plays a vital role in the determination of our decision.

- Demand « D (t) »: demands for spare parts arrived from the maintenance department are the key factor for our decision. According to the machine state, the choice of the maintenance manager in terms of the part nature can differentiate. Demands vary between new and recycled spare parts.

- Parts purchasing cost « CR » et « CN »: according to market price fluctuations, the parts purchase cost varies and still uncertain. In order to minimize the total supply cost, we must take into account this fluctuation in our decision. For this, the part purchasing cost is a major factor in our decisions.

These three key factors constitute the parent nodes (input nodes) for the decision node. Depending on the variability of these factors, the optimal decision will be taken by the node "SP". This decision will be: purchase only new spare part, purchase only recycled spare parts or not to purchase spare parts.
The figure below summarizes the variables influencing the decision.

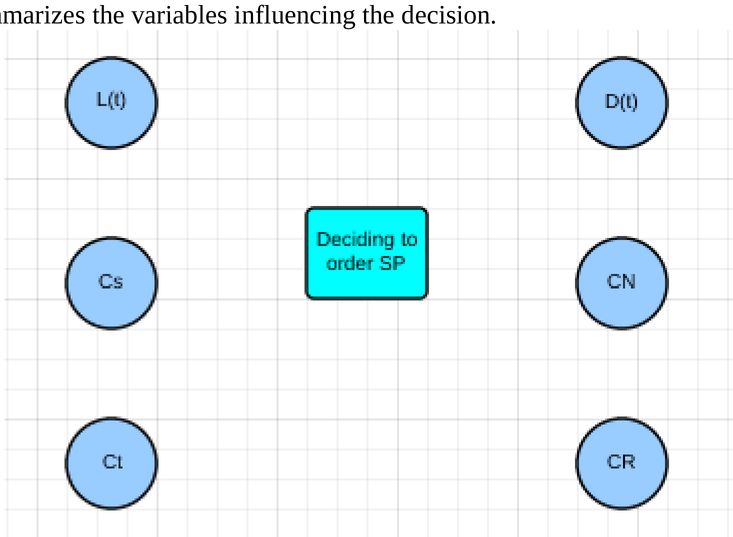

Figure 2: performance indicators of the model

1. Causal structure

Taking into account the behavioral model described above and the sequential nature of the decision that will change the inventory levels and the different costs: the purchasing parts costs, the total storage cost and the total cost of ownership, the causal structure chosen is:

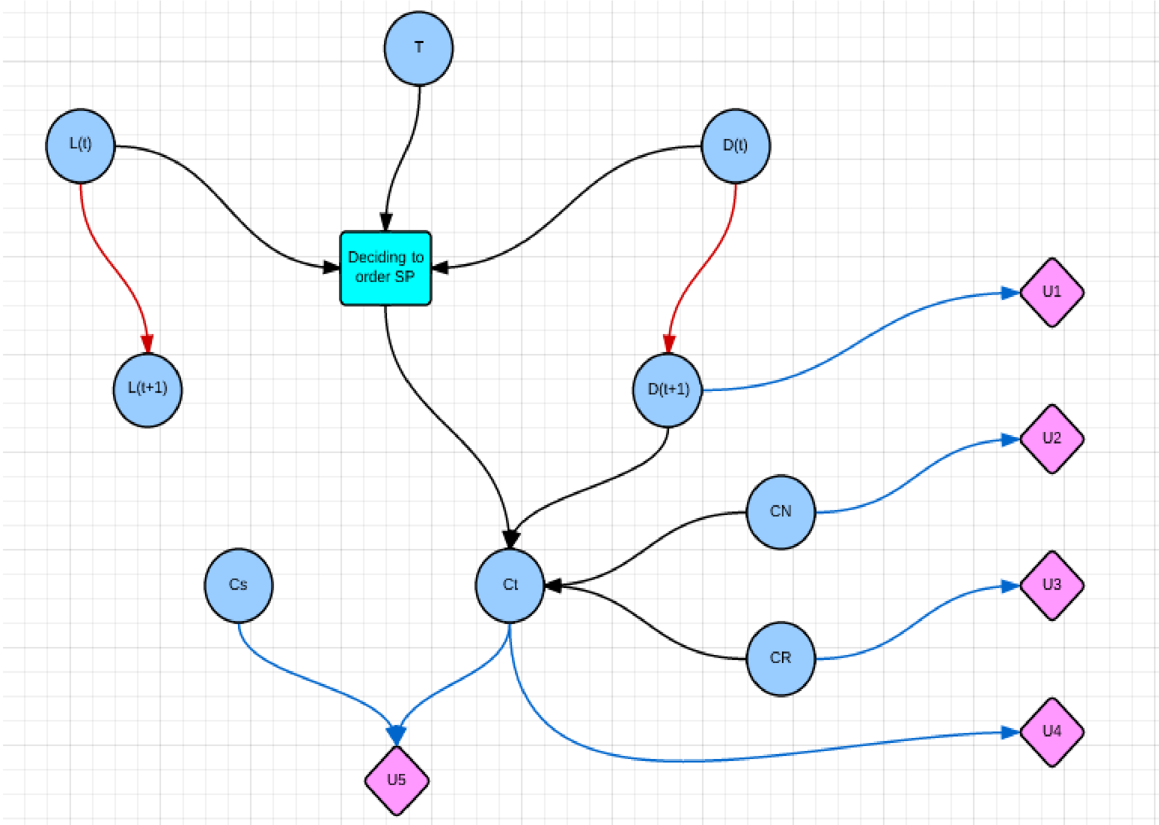

Figure 3: The DBN for recycled spare parts inventory control

As we mentioned in the previous paragraph, a Bayesian network is a graph (consisting of nodes and arcs), associated with a set of probability table of nodes (TPC), so named because there are one TPC for each node in the graph.

The nodes represent discrete random variables. In our case, we introduced nine causality nodes representing:

T: The time variable represented by 20 unities
L (t): Lead time at the first slice of time t
L (t+1): Lead time at the second slice of time t
Cs: Inventory cost
CN: New spare parts acquisition cost
CR: Recycled spare parts acquisition cost
Ct: Total acquisition cost
D (t): demand at (t) time slice
D (t+1): demand at (t+1) time slice

- A decision node "SP": this node can decide whether to purchase new spare parts, recycled or not to purchase SP as required in the maintenance service which is represented by the node causal D (t) . An arc from node D (t) at decision node "SP" models the relationship between these two nodes. Indeed, the demand from the maintenance department affects directly the decision of the spare part nature that has to purchase.

- 5 utilities nodes representing:

U1: represents a utility node that allows evaluating the demand cost at (t +1).
U2: represents a utility node that allows evaluating the total cost of purchasing NSP.
U3: represents a utility node that allows evaluating the total cost of purchasing RSP.
U4: represents a utility node that allows evaluating the total cost of spare parts procurement.
U5: represents a utility node that allows evaluating the total cost of ownership (TCO). It is the sum of the storage cost (or cost of stock maintenance per day per item) and the total cost of spare parts procurement.

Through this modeling, we want to find the best combination (compromise) possible between purchasing NSP and RSP which generates the lowest cost. For this, we took into account the following variables as variables affecting the purchase decision:

- The Demand "D (t)": According to the machinery condition and other factors (priming a new production unit with high volume), the maintenance service request the manager to provide the necessary spare parts (new or recycled). This demand varies from one period to another, for this, we add another node for period 2 which called D (t +1). A temporal arc connects two nodes to show the temporal relationship following the nomenclature of a DBN.

- The Lead time "L (t)": the time between the launch date of the order and the date of receipt of this latter is the lead time. The decision of purchase spare parts or not is directly influenced by the lead time. For this, this node is directly linked by an arc of "cause to effect" to decision node. On the other hand, the lead time also varies depending on the period, for that, we find a temporal arc between the node lead time at time t "L (t)" and the lead time node to the next period (t +1) "L (t +1)".

- Time: it is simply a temporal iteration count that allows upgrading the various states and performance indicators.

The total cost of purchasing spare parts will be calculated based on the result of the decision taken by the node deciding to order "SP", the amount of spare parts needed for maintenance service at the period (t +1) and unit costs of purchasing new parts "CN" and recycled parts "CR".

2. Formalization and causal quantification

To formalize our problem as Bayesian inference, we must define the set of random variables which are included in this problem. These variables have a direct impact on the decision to take and the cost of this decision.

After having defined these random variables, we model them as Bayesian laws. We recall in this paragraph that the key factors in our decision are: the lead time "L (t +1)" and the demand "D (t+1)". In our modeling, we assume that these two variables follow a continuous probability distribution which is the normal distribution.

Recall that in probability theory, a real random variable X follows a normal distribution (Gaussian or, Laplace-Gauss) with mean μ and standard deviation σ strictly positive (then with variance σ2).

This is usually seen as follows:

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

In our model, the two random variables are: D (t +1) and L (t +1).

- For D (t +1): this variable follows a normal distribution with mean μ = the normal law of the previous demand at time t, i.e. μ = D (t), and a standard deviation that can has σ = 1. Indeed, we assume a variation of the demand from one period to another of one unit σ2 = 1.

  Concretely: Pr (D(t+1)/D(t), t)= N (D(t),1)

- For L (t +1): this variable follows a normal distribution with mean μ = the normal lead time to the previous period t, i.e. μ = L (t), and a standard deviation that can has σ = 1. Indeed, we assume a variation of the lead time from one period to another of one unit σ2 = 1.

  Concretely : Pr (L(t+1)/L(t), t)= N (L(t),1)

Afterwards, the decision to purchase new parts, recycled or even not to purchase will be taken through the simulation based on the variation of these two variables over time.

Once our decision is taken, we will calculate spare parts purchasing cost "Ct" depending on the parts nature to purchase (i.e. the decision), the quantity to purchase (i.e. D (t +1)) and NSP purchasing cost "CN" and RSP purchasing cost "CR".

The realization of this process will be conducted through a deterministic equation according to which:

"Ct" =?

[If "SP" = "No" → "Ct" = 0

If "SP"= "NSP" → "Ct"= "CN"*"D (t+1)"

If "SP"= "RSP" → "Ct" = "CR"*"D (t+1)"]

Interpretation: the deterministic function of the total spare parts purchasing cost "Ct" is calculated using the Bayesian law as described above. According to this law, we assume three possible results depending on three decisions that can be taken.

- If the optimal decision is not to purchase spare parts, then the total purchase cost Ct = 0.

- If the optimal decision taken is to purchase only NSP, in this case the total cost is equal to the NSP unit cost multiplied by the amount requested by the maintenance service at the time (T+1).

- If the optimal decision taken is to purchase only RSP, in this case the total cost is equal to the RSP unit cost multiplied by the amount requested at the time (T+1).

Recall that in inventory management, total SP purchasing cost (TCO) calculated according to SP supplying cost "Ct" and the storage cost (or stock maintenance cost per day per item) "Cs". So we have:

$$TCO = Ct + Cs$$

In conclusion, we find that for each decision and each time we have a different TCO allowing an evaluation of the decision total cost based on uncertain variables: the lead time which vary from one supplier to another, demand which is generally uncertain and depends on the machines state to maintain, the purchase price of NSP and RSP which vary according to market fluctuation and the storage cost which varies depending on the number of article stored.

In this way, we modeled the procurement of new and recycled SP according to Bayesian laws and based on several uncertain variables.

In the next section, we will implement this model through a numerical example.

IV. APPLICATION

We consider in a context of SP stocks forecasting on the basis of specific data, consistent but uncertain (see for aspects of imperfect data the work of [55] [56]. The planning horizon is divided into periods and the monitoring is to determine at the end of each period the proportions of the quantities of NSP and RSP to purchase depending on the nature of the spare parts demanded (new or recycled), lead time and purchase price. The objective of the model proposed is to determine a spare parts procurement policy for a single product line and for each period of the planning horizon.

1. Application context

We make the following assumptions:

- The maintenance service accepts introducing RSP in machines to maintain
- The availability is continuous for both types of SP (new and recycled) during the time horizon
- Prices rise and fall depending on the availability.

The objective of planning is to determine the best combination between purchasing New and Recycled SP for each period based on inventory levels while ensuring a minimum acquisition cost.

For this, the following elements are considered:

- The unit purchase cost of NSP,
- The unit purchase cost of RSP,
- The storage cost.

The approach we propose in this paper allows taking into account uncertainties about the nature of demand for SP, on the fluctuation of parts purchase cost which is based on availability and lead times. These uncertainties are characterized by probability distributions for each period of the planning horizon.

2. Modeling

In this modeling, we will greatly simplify the model since the aim of this paper is to prove that can be modeled by DBN any behavior of procurement policy, if one has the necessary data.

For each variable previously defined, we have modeled according to different possible states:

- For the lead time L (t): it can be modeled with "ok" if our supplier is on schedule for delivery of SP, with Bad in case where the lead time was exceeded by a few days and modeled with over passed in case where the supplier has far exceeded the lead time.

- The demand D (t): we have assumed in this work that after the failure of one or more machines, the technician may request an amount of SP that does not exceed 5 parts per planning period, and therefore we defined five possible states for the demand. The determination of the demand will follow a normal distribution N (μ, σ). The average demand is 3 parts with a standard deviation of 1.

- The decision "SP": it can take three forms: purchase only NSP, purchase only RSP or not to purchase SP. This decision must be optimal in the sense that it should generate the least cost to the maintenance service. It will be taken depending on the demand in period t, the lead time in period t and the time.

- NSP purchase cost "CN": this cost follows the fluctuation of the market. To facilitate calculations, we assumed the existence of five possible costs that we have modeled in five different states. The calculation of this cost follows the normal distribution N (μ, σ). The average unit cost is 3 units with a standard deviation of 1.

- RSP purchase cost “CR”: this cost also follows the fluctuation of the market. As for NSP, we assumed the existence of five possible costs that we have modeled in five different states. The calculation of this cost also follows the normal distribution N (μ, σ). The average unit cost is 3 units with a standard deviation of 1.
- SP purchase cost “Ct”: It is determined through a deterministic equation that takes into account the effect of the four parent nodes which are: the demand in period (t +1) “D (t + 1)”, the NSP purchase cost “CN”, RSP purchase cost “CR” and the decision to supply SP.
- The storage cost “Cs”: this node represents in the area of inventory management, storage cost or stock maintenance cost per day per item. In our case, we determine this cost through a probabilistic equation which follows a normal distribution with a mean of 3 and a standard deviation of 1. To facilitate calculations, it takes five different node costs that vary from 1 to 5.
- The Total Cost of Ownership (TCO): this cost includes the costs above. It evaluates the overall cost of the procurement decision depending on fluctuations in the SP price on the market.

3. Simulation and results

To validate our model by Bayesian networks, we will implement this approach by a numerical example to better understand the operating principle of our model. The principle of work remains the same as explained above and the results are summarized in the following table:

D(t)	Time	Lead time L(t)	No	NSP	RSP
1	**5**	**Overpassed**	**2,513**	0,396	0,901
1	**5**	**Bad**	**2,407**	0,379	0,864
1	**5**	**Ok**	**1,068**	0,168	0,383
1	**6**	**Overpassed**	0,608	**4,826**	0,931
1	**6**	**Bad**	0,583	**4,626**	0,893
1	**6**	**Ok**	0,259	**2,051**	0,396
1	**7**	**Overpassed**	**3,088**	0,624	0
1	**7**	**Bad**	**2,962**	0,599	0
1	**7**	**Ok**	**1,313**	0,266	0
2	**8**	**Overpassed**	0	0,789	**6,979**
2	**8**	**Bad**	0	0,757	**6,695**
2	**8**	**Ok**	0	0,336	**2,968**
2	**9**	**Overpassed**	**3,154**	0	0,402
2	**9**	**Bad**	**3,025**	0	0,385
2	**9**	**Ok**	**1,341**	0	0,171
2	**10**	**Overpassed**	0	**7,17**	0
2	**10**	**Bad**	0	**6,878**	0
2	**10**	**Ok**	0	**3,049**	0
3	**14**	**Overpassed**	**3,489**	0	0
3	**14**	**Bad**	**3,347**	0	0
3	**15**	**Ok**	**1,371**	0,244	0
3	**16**	**Overpassed**	**3,126**	0,578	0
3	**16**	**Bad**	**2,999**	0,554	0
3	**16**	**Ok**	**1,329**	0,246	0
4	**18**	**Overpassed**	0,667	0	**5,617**
4	**18**	**Bad**	0,64	0	**5,389**
4	**18**	**Ok**	0,284	0	**2,389**
4	**19**	**Overpassed**	0,254	1,59	**5,768**

4	**20**	**Bad**	0,711	0,364	**5,129**
4	**20**	**Ok**	0,315	0,161	**2,274**
5	**1**	**Overpassed**	0,374	**3,607**	1,517
5	**1**	**Bad**	0,359	**3,46**	1,455
5	**1**	**Ok**	0,159	**1,534**	0,645
5	**2**	**Overpassed**	**2,105**	0,395	0,812
5	**2**	**Bad**	**1,926**	0,378	0,779
5	**2**	**Ok**	**0,88**	0,168	0,345
5	**3**	**Overpassed**	**2,244**	0,526	0,373

Table 2: The determination of the decision for each state

This table shows the results of the simulation according to which, we identify the decision for each period. Indeed, for each D (t), we took the example of different periods and different lead time to have the decision corresponding to each state.

In fact, we can notice that the optimal decision differs from one period to another depending on the demand of spare parts and the lead time required for receipt of order. Take the example of an application for two parts from the maintenance service for the period 8, 9 and 10. We note that, for the period 8, whatever the lead time, over passed, bad or ok, the best solution is to purchase only RSP. However, for the period 9, the best solution is not to supply SP and this whatever the lead time. Finally for the period 10, the optimal solution is to purchase as NSP.

Once we decided what to buy ie new or recycled spare part, we proceed to calculate the cost of this decision based on the fluctuation of the market prices. According to the simulation result, we find that in the case when the maintenance service in the period (t +1) request 2 spare parts and the optimal decision is to purchase new ones (this decision is calculated based on the lead time, demand in period t and the time), the cost of the NSP in this period is 3 units, which we will generate a total SP purchase cost “Ct” of 6 units as shown in the table follows.

D(t+1)	CN	CR	Deciding to order SP	Ct	Cs	Total cost of ownership
0	**1**	**1**	**No**	**0**	**0**	**0**
1	**2**	**3**	**RSP**	**3**	**2**	**5**
1	**2**	**4**	**No**	**0**	**1**	**1**
1	**2**	**4**	**NSP**	**2**	**3**	**5**
1	**2**	**4**	**RSP**	**4**	**3**	**7**
2	**3**	**3**	**No**	**0**	**1**	**1**
2	**3**	**5**	**NSP**	**6**	**2**	**8**
2	**3**	**3**	**RSP**	**6**	**4**	**10**
2	**3**	**4**	**No**	**0**	**1**	**1**
3	**5**	**4**	**NSP**	**15**	**5**	**20**
3	**5**	**4**	**RSP**	**12**	**3**	**15**
3	**5**	**5**	**No**	**0**	**3**	**3**
3	**5**	**5**	**NSP**	**15**	**1**	**16**
3	**5**	**5**	**RSP**	**15**	**2**	**17**
4	**1**	**1**	**No**	**0**	**4**	**4**
4	**1**	**1**	**NSP**	**4**	**4**	**8**
4	**1**	**1**	**RSP**	**4**	**5**	**9**
4	**1**	**2**	**No**	**0**	**4**	**4**

4	**1**	**2**	**NSP**	**4**	**4**	**8**
5	**2**	**5**	**NSP**	**10**	**3**	**13**
5	**2**	**5**	**RSP**	**25**	**5**	**30**
5	**3**	**1**	**No**	**0**	**4**	**4**
5	**3**	**1**	**NSP**	**15**	**4**	**19**
5	**3**	**1**	**RSP**	**5**	**3**	**8**

Table 3: New and recycled spare parts replenishment cost

This table shows different purchase costs as decided by decision node "SP". In the area of inventory management, total cost of supply called "CTO" is constituted by unit purchase cost plus the storage cost. For this, we add to our Bayesian network utility node that evaluates the overall cost.

To calculate the CTO, we will consider the previous example where we will supply 2 NSP to 3 units the part. We noted that the purchase cost is 6 units. Let us add now the storage cost; we find that the total cost of ownership is 8 units i.e. the storage cost for 2 NSP is 2 units.

The utility node evaluates for each period the new and recycled SP purchasing cost according to the different uncertain variables affecting the decision of supply.

After a simulation made to determine the CTO depending on uncertain demand, uncertain lead time and uncertain supply costs, there is a fraction of simulation that allows us to show some CTO generated following the simulation and taking into account the uncertainty of the model variables.

		Deciding to order SP		
		No	NSP	RSP
D(t+1)	1			**6**
	2		**8**	
	3			**16**
	4	**4**		
	5			**19**

Table 4: Average total cost of ownership

Table 2 allowed us to generate the above table which gave us the following diagram:

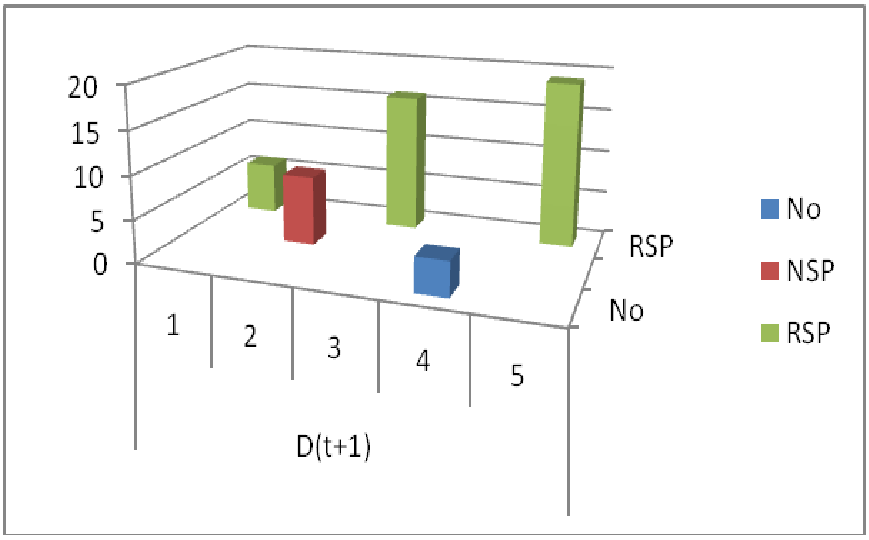

Figure 4: Average total cost of ownership

This diagram takes into account all possible costs of supply for a planning horizon. Indeed, we estimate the average cost of supply for a single decision and this for different periods.

We will take the example of the calculation of average cost of SP purchasing for the case when the demand is one part with the decision to purchase RSP. We find during the planning period different possible costs. For this fraction of simulation, we take the average of two different costs and we find that the mean TCO for purchase a RSP is 6 units such as shows the diagram above.

This histogram becomes a decision aided tool for SP replenishment policies and this for various possible situations in each planning period.

V. CONCLUSION

The purchasing policy of spare parts for maintenance and the reverse inventory chain logistics has been attracting researcher's attention in recent years. In spare parts inventory control the aim is to maintain the inventory at an optimal level and to minimize the purchasing and the inventory costs. This paper deals with integration of the recycled spare parts in the economic models of inventory systems. In order to contribute to a realistic model of the spare parts inventory control, we proposed an approach based on the Dynamic Bayesian Networks formalism. This method allows choosing the best proportion between recycled and new spare parts so as to minimize the costs by taking into account the uncertainties that can occur in the supply chain inventory system. A new method to integrate the uncertainty in the policy of spare parts inventory control was proposed in this paper. The method is based on the Bayesian theory and integers the dynamical time aspect of the inventory supply chain. In the same time, the problem of replacing new spare parts with recycled spare parts was addressed so as to minimize the inventory costs.

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