

## **Finite Element Buckling Analysis Of Stiffened Plates**

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**Abstract:-** Structures consisting of thin plates stiffened by a system of ribs have found wide application for aircraft, ships, bridges, and buildings as well as in many other branches of contemporary structural engineering. Buckling analysis can be used to find critical compressive loads at which a structure becomes elastically unstable. In this paper, buckling analysis of stiffened plates was carried out using finite element software NISA. The analytical investigations of stiffened plates are rarely found in literature; hence it is found apt to carry out such investigations to provide design recommendations.

**Keywords:-** buckling , stiffener ,finite element, stiffness, nonlinear analysis.

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### **I. INTRODUCTION**

Stiffened plates are extensively used as structural components in naval and offshore industry. These stiffened elements, representing a relatively small part of the total weight of the structures, substantially influence their strength, stiffness and stability. The plates used in ship structure are of very thin dimension subjected to very high compressive loading include the passenger crew, machines and other combinations of dead load and live load. To increase the stability of the plates and to withstand the stress and deformations developed it is not possible to increase the thickness of the plates.

Instead of increasing the thickness of plates and there by increasing the cost of materials, stiffeners are provided. These stiffener elements presenting relatively small part of the weight of the structure substantially influence the strength of the structure under different loading conditions. It is assumed that stiffener will have the same displacement as that of plating. The stiffeners can be positioned any were within the plate element and need not necessarily be placed on nodal lines as the stiffness matrix of a stiffened plate element is comprised of the contributions of the plate element and that of the stiffener element. The stability of stiffened plate is determined from the arrangement and the loading conditions on the plates. There are three types of stiffening systems,

- 1) Transverse
- 2) Longitudinal
- 3) Combination of both longitudinal and transverse

### **II. REVIEW OF LITERATURE**

Buckling analysis of these plate forms subjected to inplane loads is very relevant in the present scenario. An attempt has been made here to realize the state of art in the analysis and design of stiffened plates. Literature describing finite element analysis of stiffened plates are reviewed and presented below.

Jiang et al carried out finite element analysis for the bending and buckling of unstiffened, sandwich, and hat stiffened orthotropic, rectangular plates. Systematic calculations were performed for deflection; stress and critical buckling load of the plate using first order shell elements and first and second order three-dimensional solid elements. The calculated results are compared with available analytical solution for unstiffened plates. First order shell and second order three-dimensional solid elements are found to model accurately bending and buckling of the plate structures. For thin unstiffened plates one layer of second order three-dimensional solid elements through the thickness is found to give sufficient accuracy.

Masoka et al have developed a simple design equation for predicting the ultimate compressive strength of unstiffened plate with misalignment, initial deflection and welding residual stresses is developed in this study. A non-linear finite element method is used to investigate the ultimate strength of the imperfect plate. The method incorporates both geometric and material nonlinearity. Buckling and plasticity behavior of the plate were expressed using the finite element system. The results from the finite element method and an analytical method using large deflection theory together with rigid plastic theory were compared. Reduction factors of the ultimate strength due to welding residual stresses and initial deflection were generated from the results of the nonlinear finite element method. A new equation for ultimate strength of imperfect plates were developed using these reduction factors. The accuracy of the proposed new equation was confirmed by comparing it with the finite element results.

Rao et al have presented finite element large deflection behavior of stiffened plates using the isoparametric quadrilateral stiffened plate bending element. The evaluation of fundamental equation of the stiffened plates is based on Mindlin's hypothesis. The large deflection equation were based on von Karman's theory. The solution algorithm for the assembled non linear equilibrium equation is based on Newton Raphson iteration technique. Numerical solutions are presented for rectangular plates and skew stiffened plates. Bergain et al have carried out large deflection analysis of plates using the finite element method. Sadek et al have developed A finite element model for the analysis of stiffened laminated plates.

**III. FINITE ELEMENT ANALYSIS**

The finite element method is an approximate technique by which an object is decomposed in to pieces and treated as isolated, interacting sections. Finite element method provides a greater flexibility to model complex geometries than finite difference and finite volume methods do. They also help in solving complex elasticity, structural analysis problem. The advancement in computer technology enables as to solve even larger system of equations, to formulate and assemble the discrete approximation, and to display the results quickly and conveniently. Another important feature is the meshing that is discretization of a continuous domain in to a set of sub-domains. When we use more number of elements, that is as the mesh size becomes coarser we converge more to our accurate result. In this method we apply load in increments to each element. Each element is checked for its stresses and deflections. So we are able to visualize the pattern of behavior for each element. These all helped in finite element method in becoming a powerful tool.

**IV. FINITE ELEMENT PROCEDURE**

Following are the steps involved in finite element analysis for buckling analysis.

**Idealisation:-**

The process of converting the actual 3-D problem in to structure, which is simple and easy to handle, is called idealization. Here the plate element with the stiffeners is analyzed.

**Discretization:**

The plate is cut in to many smaller elements called finite elements and these elements are connected at fixed nodes. The degrees of freedom are chosen and the shape functions are selected.

**Formulation of stiffness matrices:-**

Bifurcation buckling pressure is determined from linear buckling analysis. Linear buckling analysis is performed by constructing linear elastic stiffness matrix signifying the internal strain energy and geometric stiffness matrix representing the work done by the prebuckling stresses on the buckling displacement of the complete structure.

**Element stiffness matrix [K]:-**

The stiffness matrix of an element [K] calculated from the equation,

$$K = \int_v [B]^T [C] [B] dv$$

Where [B] is the strain-displacement matrix.

[C] is constitutive matrix.

**Formulation of geometric stiffness matrix [Kg]:-**

Geometric stiffness matrix can be derived from the expression [16] given below.

$$[K_g] = \int_v d[B_{nl}]^T \{\sigma\} dv$$

Where [B<sub>nl</sub>] is the nonlinear strain-displacement matrix,

{σ} is the compressive stress matrix

**Assembly of global stiffness matrices and application of boundary condition:-**

The global stiffness matrix of the stiffened plate finite element model is obtained by computing the element stiffness matrix of each shell element and assembling them by posting them in appropriate global locations determined by node numbering and connectivity.

**Linear buckling analysis**

During buckling the total stiffness matrix becomes singular or the determinant of the total stiffness matrix vanishes. The eigen value problem of instability is therefore formulated as

$$([K_o] + [K_G]) \{\delta\} = 0 \quad \dots\dots\dots (1.2)$$

$$([K_o] + \lambda_b [K_g]) \{\delta\} = 0 \quad \dots\dots\dots (1.3)$$

The buckling pressure is evaluated for the condition

$$[K_o] + \lambda_b [K_g] = 0 \quad \dots\dots\dots (1.4)$$

where λ<sub>b</sub> is the nondimensional buckling pressure.

The solution procedure adopted for obtaining the linear buckling pressure is the determinant search procedure. A value of λ is assumed and the determinant of the matrix is calculated. The process is repeated by

changing the value of  $\lambda$  until the determinant changes its sign. The value of  $\lambda$  for zero determinant is the buckling pressure.

In the solution, eigen values will be the buckling pressure and eigen vectors will be the buckling mode. Linear prebuckling analysis has the advantage of avoiding a full nonlinear analysis, which may be expensive and time consuming.

## V. FINITE ELEMENT MODELING

Stiffened plates are modeled as discrete stiffener model and super element model and are described subsequently.

In the discrete stiffener model plate is modeled using 3D shell element (NKTP-20), which includes membrane, bending and shear deformation effects. The element has 6 degrees of freedom per node. The stiffeners are modeled as a 3D beam element (NKTP-12).

In the super element model plate as well as stiffener is modeled using 3D shell element (NKTP-20), which includes membrane, bending and shear deformation effects. The element has 6 degrees of freedom per node.

Finite element modeling using DISPLAY III consists of two steps viz., geometric and finite element modeling. The preprocessing phase consists of mesh generation, material definition, constraint definition load definition and model displays. In the solution phase the element matrices, displacements and stress are calculated. In the post processing phase stress contours and displacements are displayed.

## VI. NISA-TOOL FOR ANALYSIS

Several software packages are now a days available for finite element analysis. NISA is a general purpose finite element package for structural analysis. Different types of analysis like static, dynamic and nonlinear analysis can be done by this package.

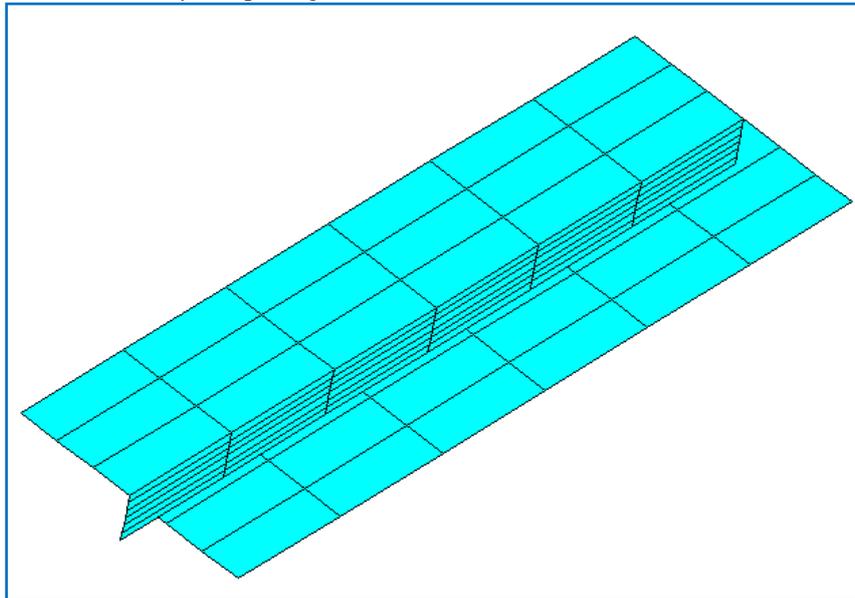


Fig. 1: Finite element model of stiffened plate ( $a/b=3$ , flat-bar stiffener)

### Finite Element Details

#### a) Applied boundary conditions:

Along the short edge of the plate, we have

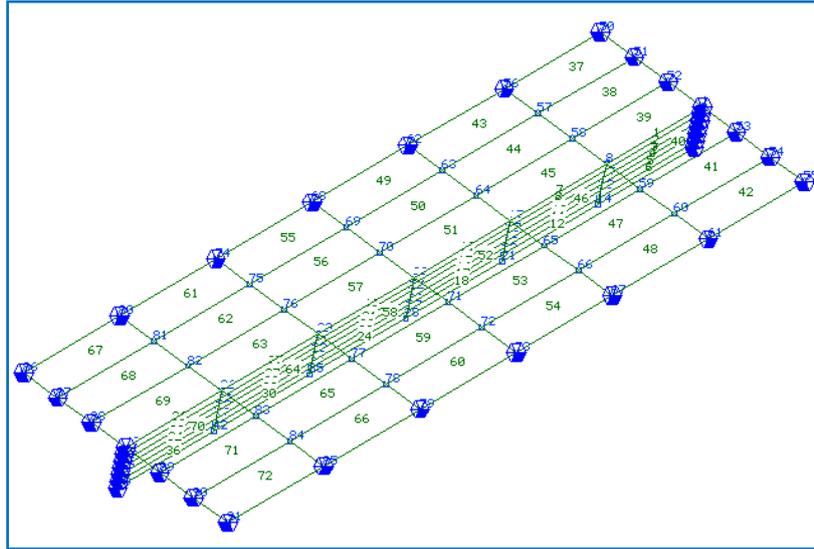
$R_Y = 0$ , that is rotation about Y direction is restricted

Along the long edge of the plate, we have

$U_Y = 0$ , that is deflection in the Y direction is restricted

$R_X = 0$ , that is we restrict rotation about X direction

Along the line of transverse girder,  $U_X, U_Y, U_Z$  is constrained



**Fig. 2 Finite element model of stiffened plate (a/b=3, flat-bar stiffener) with meshing and boundary conditions**

**b) Loads applied**

Here we are going for only one load case that is inplane loading of compressive nature. Inplane load is applied along the width of the plate and stiffener.

**c) Load calculation**

The load is applied as nodal load along the width of the plate as well as stiffener.

The total load on plate is calculated as,

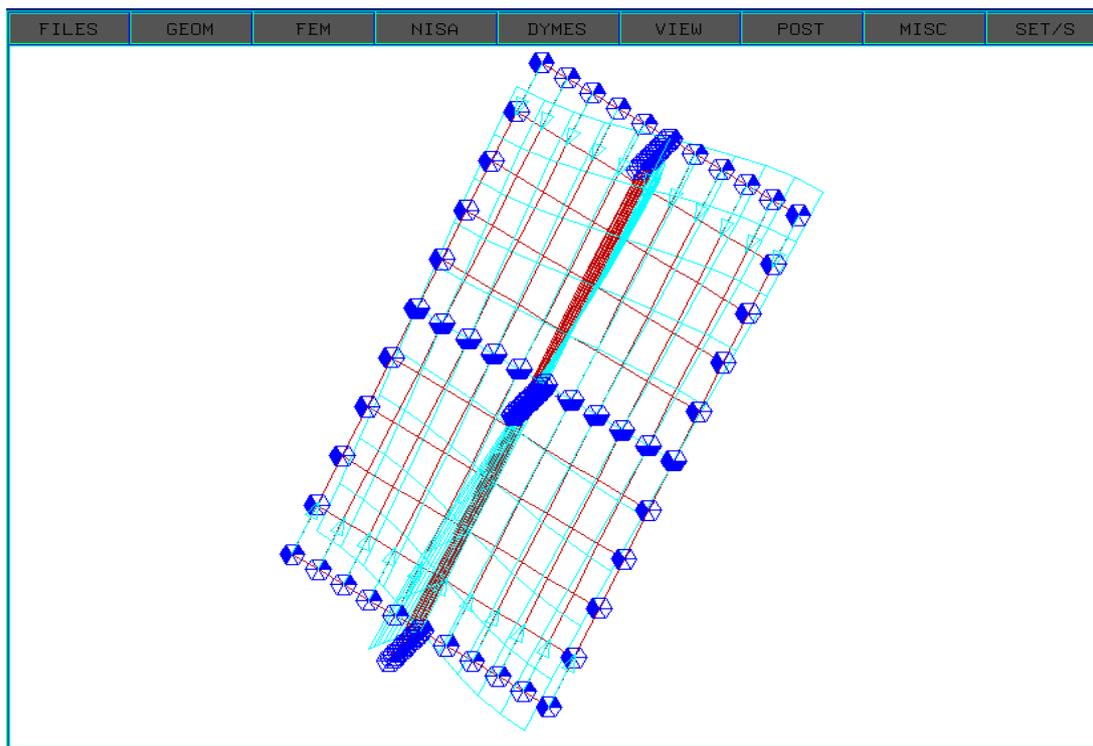
- In plates – yield stress x cross sectional area of plate
- In stiffeners – yield stress x cross sectional area of stiffener

The load acting on each node is calculated as: total load/no of elements

As an example consider the following load calculation on a plate of size 2400 x 800 x 10 and stiffener of dimensions 150 x 17

Total load on plate = 313.6 x (800 x 10)

Total load on stiffener = 313.6 x (150x17)



**Fig. 3 Buckled shape for plate with flat bar as stiffener**

## VII. COMPARISON BETWEEN DISCRETE STIFFENER MODELING AND RIGOROUS MODELING

There are two methods for modeling stiffened panels

1. Discrete stiffener modeling and
2. Rigorous modeling

### Discrete stiffener modeling

In discrete stiffener modeling the stiffeners are discretised using 3D beam element and plate is discretised using shell elements. Merging ensures the connection between the plate and stiffeners the nodes is the interface between plate and stiffeners.

### Rigorous modeling

In rigorous modeling the plate as well as the stiffener are discretised using shell elements. In NISA the advantage of using rigorous modeling is that we can visualize the cross section of the stiffener and see the deformation pattern for the web as well as flanges of the stiffener.

## IV. RESULTS AND DISCUSSIONS

Table I shows the load factor obtained for flat bar stiffener using discrete stiffener modeling as well as rigorous modeling.

Table I: Load Factor for Plate with Flat Bar Stiffener Using Discrete Stiffener and Rigorous Modeling

Type	Load factor		%
	discrete stiffener	rigorous modeling	variation
F31015	0.4184	0.366	12.52
F31315	0.6165	0.5564	9.75
F31515	0.7835	0.7152	8.72

From the analysis results given in table I, it can be seen that the result obtained for discrete beam analysis and rigorous analysis are close to each other. The percentage variations are 12.52% , 9.75%, 8.72% respectively for plates having thickness 10,13,15 mm. The results obtained from rigorous analysis fall on the safer side. Hence the rigorous stiffener modeling may be considered as more accurate. Since the % variation is less and the and the memory space required for discrete analysis are minimal discrete stiffener analysis can be effectively used for the analysis of complex structures. If the accuracy level required is much high rigorous stiffener modeling may be adopted.

## REFERENCES

- [1]. Rajagopalan, K., Finite Element Buckling Analysis of Stiffened Cylindrical Shells. Oxford and IBH Publishing Company, New Delhi, India, 1993.
- [2]. Felippa, C.A., Lecture Notes in Nonlinear Finite Element Method, Center for Aerospace Structures, Boulder, Colorado, 1999.
- [3]. Cook, R.D, Malkus, D.S., Pleasha, M.E. Concepts and application of finite element analysis, 3/E, John Wiley and sons, 2000.
- [4]. S.Rajasekaran, Finite Element Analysis in engineering Design, Wheeler Publishing, 1993.
- [5]. C.S Krishnamoorthy, Finite Element Analysis-Theory and Progarmming,Tata.Mc Graw-Hill Publishing Company Ltd.1997
- [6]. Crisfield, M.A., Incremental/Iterative Algorithm that Handles Snap through, Computers and Structures, Vol. 13,pp 55-62, 1981.
- [7]. Hughes, T.J.R., Hinton, E., Finite Element Method for Plate and Shell Structures, Vol. 1: Element Technology; Vol. 2 Formulation and Algorithms, Pineridge, Swansea, 1986.
- [8]. NISA manual, 2003.
- [9]. Bryan,G.H., On the stability of a plane plate under thrust in its own plane, with application to buckling of side of ship, London Mathematical Society Proceedings,22,54-67,1981
- [10]. Troitsky, M. S., Stiffened plates bending, stability and vibrations, Elsevier publishing company-USA, 1976.