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An analysis of relative Consistancy Coefficients of two- layered blood flow through a narrow vessel

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Abstract:- An attempt has been made to analyse the behavior of relative consistency coefficients of viscosity of two-layered blood flow through a narrow vessel. In the peripheral plasma layer, blood has been taken as Newtonian fluid while in the core layer it has been taken as Bingham Plastic fluid. The determination of relative coefficient ofviscosity has been made by equating the sum ofvolumetric flow rates in two layers to the volumetric flow rate in case the two fluids are replaced by a single newtonian fluid with appropriate viscosity coefficient. The computations of relative coefficient of viscosity are discussed numerically and displayed graphically.

Keywords:-Newtonian fluid, Bingham Plastic fluid, Hematocrit, Effective coefficient of viscosity.

I. INTRODUCTION

Several studies of blood flow through narrow vessels have been carried out by the researchers for quite some time (Fahreaeus and lindquiest, 1931) due to its wide applications in medical science. Blood is actually a complex fluid consisting of a suspension of cells in plasma. There are about 5x 10^9 cells in a mililitre of healthy Human blood of which about 95 percent are real cells called erythrocytes whose main object is to transport oxygen from the lungs to all cells of the body. The percent volume concentration of red blood cells in whole blood is called hematocrit. It is calculated py multiplying the amount of red blood cells by the amount of space occupied by the red blood cells. The coefficient of viscosity ofblood depends on the radius of vessel, hematocrit, shear rates, nature of flow, etc. and the study of the coefficient of viscosity in narrow vessel is of great importance because el vation of blood viscosity has been recognized as serious risk factor in the cardiovascular, hematological, neoplastic and other disorders.

For the reduced diameter of the vessel, blood rheological properties appear more and more important from shear thining and the flow becomes complicated by phase separation in the narrow vessels. Experiments on steady blood flow in narrow vessels exhibit some anomolous feature, e.g. the blunting of velocity profile, the formation of plasma layer and Fahreaeus- Lindqvist effect. Many authors investigated two-layered flow model on the basis of (Segre and Silberg, 1962) pinch effect. A two-layered model with peripheral layer as Newtonian fluid and the core as a couple stress fluid taking into account the size effects and same viscosity for both fluids has been studied by Chaturani and Kalonis, 1976. Bugliarello et al (1965) carried out the measurements in vetro in glass capplillaries with diameters in the range of 40 to 83J.l for a possible estimation of plasma layer thickness by using blood samples with acid-citrate-dextrose and varying hematocrits and shear stress in the range 10-100 dyne/cm^2 • Majumdar et al (1995) analysed a two-layered model consisting of both power- law fluids·. In a recent paper, Sanyal and Pahari (2009) have studied the behavior of viscosity coefficients·of blood by taking the two-layered model consisting of non-newtonian power-law fluid in plasma layer and Herschel-Bulkley fluid in core layer.

In this paper, the natureof.f ll¢fticient of viscosity has been investigated by considering blood as a Newtonian fluid in the penpherai plasma layer while in core-layer as Bingham plastic fluid Mathematical Formulation and Solutmiis

In the present analysis, a laminar, etric, two-layered blood flow through a rigid and narrow vessel with constant pressure gradienthn assumed. The flow field consists of two layers, one
is the peripheral plasma layer and otheris ,cQrelayer (which is suspension of red cells in plasma).
We consider that the peripheral plasma layer behaves like a Newtonian fluid while in core layer behaves as a Bingham Plastic fluid.
The constitutive equations for both Newtonian fluid and Bingham Plastic fluid are expressed as
'= *Jle*
and r=Jle+r_0 (r::::r_0)

e=O(r<o)

where τ_0 is the yield stress and J..1 is the viscosity coefficient.

Assuming that peripheral plasma layer is of thickness 8 and the radius of the vessel is R, the equations governing the motion in the peripheral and core layers, respectively, are

d [p(}

R2 p . 1 8 1,
m --<YJ< = TJ .
R- -
1)

and _i_[c(-:zc)]=R2 (TJ- C)p in c s; TJ s; 1- (2)
dTJ 2 P R

where TJ =P = –(:) the pressure gradient, J..l.p and J..l.c are the coefficent of viscosity

in peripheral and core layers respectively. vzp and vzc are the axial velocity components in these two layers. Without loss of generality, it may be assume that

1
– J..l.p h –h 1- TJ n]
Jlc – 1-Ph(TJ)' (TJ) – m [(1-.%.
J

Where J3 is constant having the value 2.5, n_1 is shape parameter and hm is the maximum hematocrit at the centre of the vessel. The boundary conditions are given by

(i) vzp = 0 at TJ = 1

(ii) dvzc =0 at TJ = 0
dYJ

$$\text{(iii)}\ \ v_{zp} = v_{zc}\ \ \text{at}\ \ \eta = 1 - \frac{\delta}{R}$$

· dvzp dv 0
(iv) f..lp = f..lc d c at l1= 1-R

The last relation (iv) is due to equality of shear stress at the interphase of the two layers. The solution of equation (1) is given by

R2P vzp =-(1-'11²)--A logl1 (3)
4f..lp f..lp

where A is constant of integration to be determined by the condition (i) Solution of equation (2) is given by

$$v_{zc} = -\frac{PR^2}{2\mu_P}\left[(1-L)\left(\frac{\eta^2}{2} - C_P\eta\right) + La^{n_1}\left(\frac{\eta^{n_1+2}}{n_1+2} - \frac{C_P\eta^{n_1+1}}{n_1+1}\right)\right] + D \tag{4}$$

where L = Bhm, a = (1-%f and D is constant of integration.

Usin g the condition (iv), it is obtaihed that A= PR_(g R - 1)(CP +%)
2
2

Thus the solution (3) reduces to

$$V_{zp} = \frac{PR^2}{2}\left(1-\eta^2\right) - \frac{\log\eta}{\mu_P}\left[\frac{PR^2}{2}\left(\frac{\delta}{R}-1\right)\left(C_P + \delta/R\right)\right] \tag{5}$$

The constant D in (4) is given by using the condition (iii) as

$$D = \frac{PR^2}{4\mu_P}\left[1-\left(1-\frac{\delta}{R}\right)^2\right] - \frac{\log\left(1-\delta/R\right)}{\mu_P}\left\{\frac{PR^2}{2}\left(\frac{\delta}{R}-1\right)\left(C_P+\frac{\delta}{R}\right)\right\}$$

1-){1-L L }-cP(1-%)(1-L- JJ

2 PR 2 (6)

2f..lp R 2 n_1 +2 n_1 +1

Thus, (4) and (6) constitute the solution for V*v:,*·
The volumetric flow rate in peripheral plasma layer and core layer are denoted, respectively, by QP and Qc.

Now QP = *f* Vzp 2nr dr
R-1\

= 2nR^2 *f* Vzpll dfJ
t-%.

4 1 1

P[.!_ _.!_(1)(c+)-(-%f +(+%f+2(-1)(c+)

= nR

2f..lp 6 2 R P R 2 3 R P R

$$\left\{\frac{\left(1-\delta/_R\right)^2}{2}\log\left(1-\delta/_R\right)-\frac{\left(1-\delta/_R\right)^2}{4}\right\}\Bigg]\Bigg]$$

Q_c can be calculated as

R-1>
Qc = J Vzc 2nr dr
0

t-%.
= 2nR^2 J Vzc11 d11
0

$$=2\pi R^2\left[\frac{-PR^2}{2\mu_P}\left((1-L)\left(\frac{\left(1-\delta/_R\right)^4}{8}-\frac{Cp\left(1-\delta/_R\right)^3}{3}\right)\right.\right.$$

$$\left.\left.+L\left\{\frac{\left(1-\delta/_R\right)^4}{(n_1+2)(n_1+4)}-\frac{Cp\left(1-\delta/_R\right)^3}{(n_1+1)(n_1+3)}\right\}\right)+\frac{D}{2}\left(1-\delta/_R\right)^2\right]$$

Thus total flux is
Q = QP + Qc

=7l'R4P[_!.__!.(t-8/)(c +8/) (t-%)2 (1+%)3 *(8/-)(* %)
2 6 2 /R P /R 2 + 3 +^{2} /R l CP + R

$$\left\{\left(\frac{1-\delta/_R}{2}\right)^2\log\left(1-\delta/_R\right)-\frac{\left(1-\delta/_R\right)^2}{4}\right\}\Bigg]\Bigg]$$

+2ffR'[-:'({(1-Lf- r c,(I %l'}

+L{ (t-%_f _ Cp(t-%r }]+n(1_8/)2]
(n_1 +2)(n_1 +4) (n_1 +l){n_1 +3) 2 /R

If the tube was completly filled by a single newtonian fluid, then the volumetric flow rate would be

$$Q_0=\frac{\pi R^4 P}{8\mu_C}$$

in which *Pc* is the effective viscosity coefficient of two fluids, Assuming the fluxes Q and Q_0 to be the same, we find the relative coefficient of viscosity as

4
f.lc =–
s

where S=- (1-%)(c,+%)-(l-:r

+(- r+2(%-1)(c,+%)f-)'log(!-%) (!-)'}

$$-2(1-L)\left(\frac{\left(1-\delta/R\right)^4}{8}-\frac{C_P\left(1-\delta/R\right)^3}{3}\right)-2L\left(\frac{(1-\delta/R)^4}{(n_1+2)(n_1+4)}-\frac{C_P(1-\delta/R)^3}{(n_1+1)(n_1+3)}\right)$$

+(!-?)'[(1-(1-%)')-log(l-%){(-l)(c;+%)}

+(1-%)'{{IL)-n, Jc,(l-%)(1-L-n,L+]]

NUMERICAL RESULTS
For heamatocrit exceeding 5·8 percent, it has been found that the yield stress is given by

τ =A(H-Hm)/100

I

where A=(0.008±0.002)3 dyne/cm^2, His normal heamatocrit and Hm is the heamatocrit

below which there is no yield stress. Taking H as 45 percent and Hm as 5 percent, the yield stress of normal uman blood should be between 0·01 and 0·06 dyne/cm^2• On the basis ofthese results, numerical computations are done with the use of following data and exhibited in figs. 2 – 5 for different values of shape parameter.

h.	**45%**	**30%**	**20%**	**9%**
CP = 2r$_0$ /PR	**0·2**	**0·15**	**0·1**	**0**

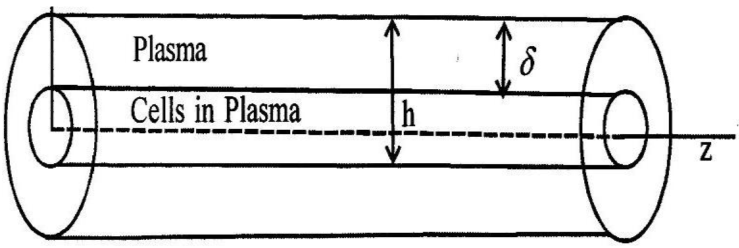


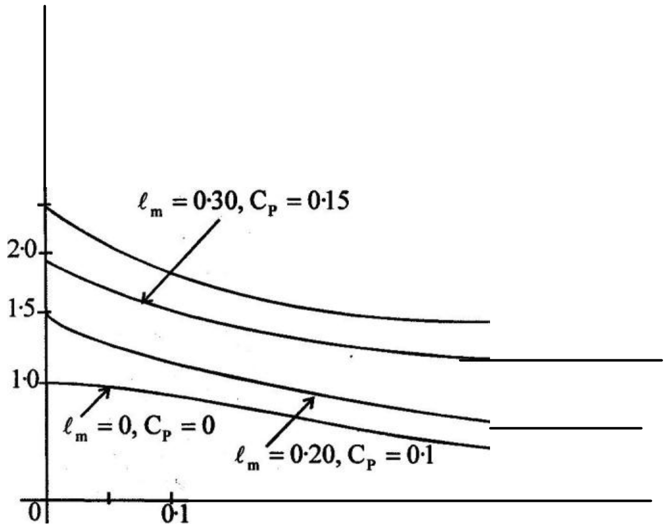
Fig. 1. Geometry of vessel

2·0

0 ()-1 ()-4 0·2 0·3

δ/R →

Fig-2. Variations of relative consistency coefficient of viscosity for different values of hm and



Cp(n = 2)

3·0

μ_r

£m = 0·45, Cp =0·20

()-2 ()-3 0·4
%

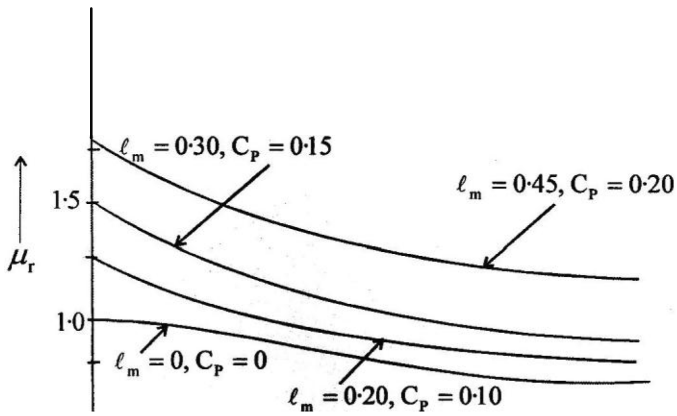


Fig-3. Variations of relative consistency coefficient of viscosity for different values of hm and Cp(n = 4)

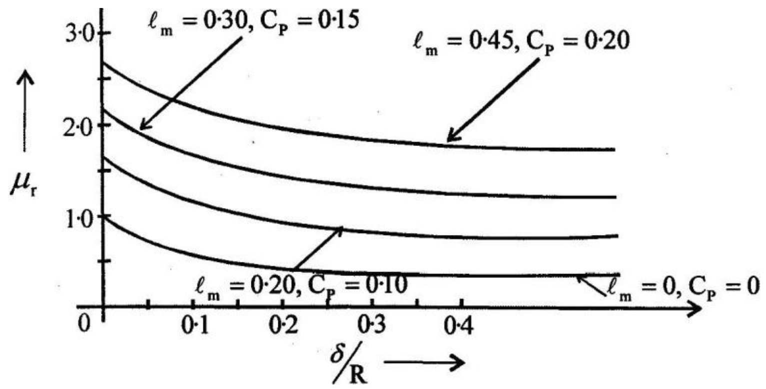


Fig-4. Variations of relative consistency Coefficient of viscosity for different volumes of hm and CP (n=6)

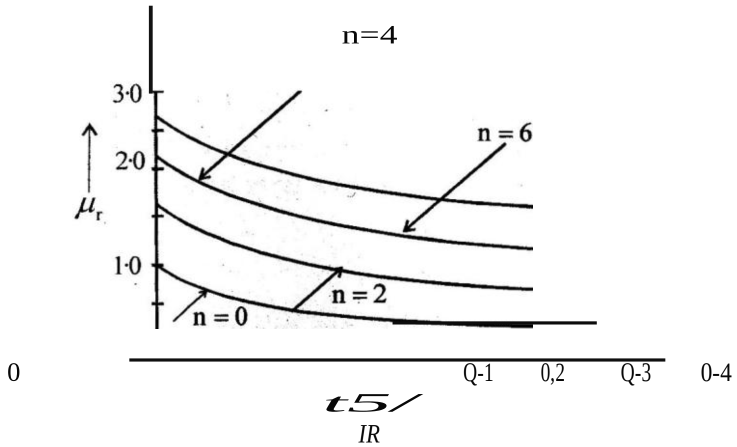


Fig-5. Variations of relatives consistency coefficient of viscosity for different values of shape parameter(hm = 0.45, Cp = 0.20)

CONCLUSIONS

From the figs 2-5, we arrive in the following conclusions:

In the absence of hematocrit and yield stress, the relative consistency coefficient is heady constant in the all cases. For a fixed value of%. and fixed value of shape parameter, the relative consistency coefficient increases with the increase of hematocrit and yield stress. At fixed hematocrit and yield stress, the relative consistency coefficient in reases accordingly at any value of %..Also it is clear that, for any shape parameter, the relative c nsistency coefficient decreases with the increase of %. Thus, the present blood flow model may be useful of better understanding of blood flow through narrow vessels and in clinical applications.

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