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Wavelet–Galerkin Technique for Neumann-Helmholtz-Poisson Boundary Value problems

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Abstract:- This paper is concerned with Wavelet-Galerkin technique to solve Neumann Helmholtz and Neumann Poisson boundary value problems. In compare, to classical finite difference and finite element, Wavelet-Galerkin technique has very important advantages. In this paper, we have made an attempt to develop a technique for Wavelet-Galerkin solution of Neumann Helmholtz boundary value problem in one dimension and Neumann Poisson problem in two dimensions in parallel to the work of J. Besora [6], Mishra etl. [15]. The Taylors approach have been used to include Neumann condition in Wavelet-Galerkin setup for $\boldsymbol{y}'' + \boldsymbol{\alpha u} = \boldsymbol{f}$. The test examples show that some value of $\boldsymbol{\alpha}$ the result match with the exact solution. Neumann Poisson BVP results show that the given technique is not fit to the solution for some parameter.

Keywords:- Boundary Value Problems, Wavelets, Scaling Function, Connection Coefficients, Wavelet Coefficient ,Wavelet- Galerkin Method.

I. INTRODUCTION

A. History of Wavelets

Wavelet is an important area of mathematics and nowadays it becomes an important tool for applications in many areas of science and engineering.

The name wavelet or ondelette was coined the French researchers Morlet, Arens, Fourgeau and Giard [16]. The existence of wavelet like functions has been known since early part of the century. The concepts of wavelets were provided by Stromberg [18], Meyer [14], Mallat [11]-[13], Daubechies [7], [8], Battle [4], Belkin [5] and Lemarie [10]. Since then, wavelet research in mathematics has grown explosively.

Family of a orthonormal based of compactly supported wavelets for the space of square-integral function $L^2(\mathbb{R})$ was constructed by Daubecheis in 1988 (see [8]). That wavelets as bases in a Galerkin method to solve Neumann mixed boundary value problem needs a computational domain of sample shape. The wavelet based approximations of partial differential equations are more attracting and attention, since the fact that orthogonality of compactly supported wavelets. Since Multiresolution Analysis based Fast wavelet transform algorithm gained momentum to make attraction of wavelet approximations of ODE's and PDE's. Wavelet-Galerkin technique is frequently used nowadays, and its numerical solutions of partial differential equations have been developed by several researchers.

Several researchers used wavelet-Galerkin method in these days by taking Daubechies wavelet as bases in a Galerkin method to solve BVP. The contribution in this area is due to the remarkable work by Latto et al. [9], Xu et al. [24],[25], Williams et al. [19]-[23], Mishra et al [15] , Jordi Besora [6] and Amartunga et al. [1]-[3]. The problems with periodic boundary conditions or periodic distribution have been dealt successfully. However, there is problem in dealing with some boundary conditions, D. Patel and Abeyratne [17] have deal wavelet Galerkin technique for Nuemann and mixed BVP.

In this paper, we have proposed an effective method for solving partial differential equation and examines a family of wavelet-Galerkin approximation to the Neumann and mixed boundary value problem, using compactly supported wavelets as basis functions introduced by Daubechies [7], [8]. We have used Taylor's approach to deal with Neumann and mixed Boundary conditions.

II. WAVELETS

Wavelets are an orthonormal basis functions in $L^2(\mathbb{R})$ which have compactly support, continuity properties, a complete basis that is it can be easily generated by simple recurrence relation and very good convergence properties. In this paper only the Daubechies compactly supported wavelets are used. It was introduced by Ingrid Daubechies in 1988 using a finite set of nonzero $\{a_k\}_{k=0}^{N-1}$ scaling coefficients, with

$$\sum_{k=0}^{N-1} a_k = 2$$

where N denotes the order, or genus of the Daubechies wavelet. $supp(\phi) = [0, N-1]$ and $(N/2-1)$ vanishing wavelets moments.

A wavelet system consist of a mother scaling function $\phi(x)$ and a mother wavelet function $\psi(x)$. The scaling relation is defined as

$$\phi(x) = \sum_{k=0}^{N-1} a_k \phi(2x-k) = \phi(x) = \sum_{k=0}^{N-1} a_k \phi(2x)$$

The scaling function will also hold for $\varphi(2x)$, and, by induction for all $\varphi(2^j x)$, so we can write all the dialation and translation of φ as

$$\phi_{j,k}(x) = 2^{j/2}\phi(2^j x - k)$$

The wavelet function is defined in terms of the scaling function as

$$\psi(x) := \sum_{k=-1}^{N-2} (-1)^k a_{k+1} \phi(2x+k)$$

Alternatively, the scaling functions are sometimes define as

$$\phi(x) := \sqrt{2} \sum_{k=0}^{N-1} h_k \phi(2x-k)$$

This simply means that $a_k = \sqrt{2} h_k$, with the condition

$$\sum_{k=0}^{N-1} h_k = \sqrt{2}$$

Wavelets have so many properties. *V* be a set of all scaling functions $\{\phi(x)\}$ and *W* be the set of all wavelet functions $\{\psi(x)\}$.

1. $\{\phi_{j,k}\}_{j\geq 0, k\in Z}$ is an orthonormal basis for $L^2(\mathbb{R})$.
2. $V_{j+1} = V_j \oplus W_j$.
3. $L^2(\mathbb{R}) = clos_{L^2}\left(V_0 \oplus_{j=0}^{\infty} W_j\right)$.
4. $\{\phi_{0,k}, \psi_{j,k}\}_{j\geq 0, k\in Z}$ is an orthonormal basis for $L^2(\mathbb{R})$.
5. $\int_{-\infty}^{\infty} \phi(x) dx = 1$
6. $\sum_{k\in Z} \phi_{0,k} = 1$.
7. $\int_{-\infty}^{\infty} \phi(x) x^k dx = 0 : \quad k = 0, \dots, \frac{N}{2} - 1$.
8. $\{x^k\}_{k=0}^{\frac{N}{2}-1} \in V_N$

The mulltiresolution analysis nested sequence

$$V_0 \subset V_1 \subset \cdots \subset\in L^2(\mathbb{R})$$

Satisfying the following properties:

1. $\bigcap_{j\in Z} V_j = \{0\}$
2. $clos_{L^2}\left(\bigcup_{j\in Z} V_j\right) = L^2(\mathbb{R})$
3. $f(x) \in V_j \iff f(2x) \in V_{j+1}; \quad \forall j \in \mathbb{Z}$
4. $\exists \phi V_0$ such that $\{\phi_{0,k}(x) = \phi(x-k)\}_{k\in Z}$ forms a Riesz basis for V_0

The wavelet expansion of a function $f(x) \in L^2(\mathbb{R})$ is of the form

$$f(x) = \sum_{l\in Z} c_0 \phi_{0,l}(x) + \sum_{j=0}^{\infty} \sum_{k\in Z} c_{jk} \psi_{j,k}(x)$$

in other words

$$f(x) \in \underbrace{\{V_0 \oplus W_0 \oplus W_1 \oplus W_2 \oplus \cdots \oplus W_\infty\}}_{L^2(\mathbb{R})}$$

Since $V_{j+1} = V_j \oplus W_j$ one can also write

$$f(x) \in \underbrace{\{V_j \oplus W_j \oplus W_{j+1} \oplus W_{j+2} \oplus \cdots \oplus W_\infty\}}_{L^2(\mathbb{R})}$$

and still have an exact representation of $f(x)$. If this representation is truncated at level *j*, approximation of $f(x)$ at resolution *j* (see Mallat [11]-[13] for details).

$$\tilde{f}(x) = \sum_{l \in Z} c_{jl}\, \phi_{j,l}(x) \tag{1}$$

To solve PDEs we need an expansion for $f(x)$, but also for the derivatives of $f(x)$. Daubechies showed that $\exists \lambda > 0$ such that a wavelet of genus N has $\lambda(\frac{N}{2} - 1)$ continuous derivatives; for small N, $\lambda \geq 0.55$. The notation for derivation of the function:

$$\phi_l^d := \frac{\partial^d \phi_l}{\partial x^d}$$

Taking d derivatives of equation (1)

$$f^d(x) = \sum_{l \in Z} c_l \phi_l^d(x)$$

Approximate $\phi_l^d(x)$as

$$\phi_l^d(x) = \sum_m \lambda_m\, \phi_m(x)$$

Where

$$\lambda_m = \int_{-\infty}^{\infty} \phi_l^d(x)\, \phi_m(x) dx$$

which is so called 2-term connection coefficient, when functions are used as basis functions for the Galerkin method.

III. WAVELET GALERKIN METHOD

Galerkin Method was introduced by V.I Galerkin. Consider one-dimensional differential equation.

$$Lu(x) = f(x); \qquad 0 \leq x \leq 1; \tag{2}$$

where

$$L = -\frac{d}{dx}\left(a(x)\frac{du}{dx}\right) + b(x)u(x)$$

with boundary conditions, $u(0) = 0$ and $u(1) = 0$.

Where a, b and f are given real valued continuous function on $[0,1]$. We also assume that L is a elliptic differential operator.

Suppose, $\{v_j\}$ is a complete orthonormal basis of $L^2([0,1])$ and every v_j $C^2([0,1])$ such that,

$$v_j(0) = 0 \qquad v_j(1) = 0$$

We can select the finite set Λ of indices j and then consider the subspace S,

$$S = span\{v_j;\ j \in \Lambda\}.$$

Approximate solution u_s can be written in the form,

$$u_s = \sum_{k \in \Lambda} x_k v_k \in S \tag{3}$$

where each x_k is scalar. We may determine x_k by seeing the behaviour of u_s as it look like a true solution on S. i.e.

$$\langle L\, u_s\, , v_j \rangle = \langle f, v_j \rangle \qquad \forall\, j \in \Lambda, \tag{4}$$

such that the boundary conditions $u_s(0) = 0$ and $u_s(1) = 0$ are satisfied. Substituting u_s values into the equation (4),

$$\sum_{k \in \Lambda} \langle L\, v_k\, , v_j \rangle\, x_k = \langle f, v_j \rangle \qquad \forall\, j \in \Lambda \tag{5}$$

Then this equation can be reduced in to the linear system of equation of the form

$$\sum a_{jk}\, x_k = y_i \tag{6}$$

or

$$AX = Y$$

Where $A = [a_{jk}]_{j,k \in \Lambda}$ and $a_{jk} = \langle L\, v_k\, , v_j \rangle$, x denotes the vector $\{x_k\}_{k \in \Lambda}$ and y denotes the vector $\{y_k\}_{k \in \Lambda}$. In the Galerkin method, for each subset Λ, we obtain an approximation $u_s \in$ S by solving linear system (6).

If u_s converges to u then we can find the actual solution.

Our main concern is the method of linear system (6) by choosing a wavelet Galerkin method. The matrix A should have a small condition number to obtain stability of solution and A should sparse to perform calculation fast.

Similarly we can do the same thing in above set up.

Let $\psi_{j,k}(x) = 2^j \psi(2^j x - k)$ (7)

is a basis for $L^2([0,1])$ with boundary conditions

$$\psi_{j,k}(0) = \psi_{j,k}(1) = 0 \qquad \forall\, j, k \in \Lambda \text{ and } \psi_{j,k} \text{ is } C^2.$$

We can replace equations (4) and (5) by

$$u_s = \sum_{j,k\in\Lambda} x_{j,k}\psi_{j,k}$$

and

$$\sum_{j,k\in\Lambda}\langle L\,\psi_{j,k}\,,\psi_{l,m}\,\rangle\, x_{j,k} = \langle f,\psi_{l,m}\rangle \qquad \forall\, l,m\in\Lambda$$

So that *AX=Y*.

Where $A = \left[a_{l,m;j,k}\right]_{(l,m),(j,k)\in\Lambda}$; $\qquad X = \left(x_{j,k}\right)_{(j,k)\in\Lambda} \qquad Y = \left(y_{l,m}\right)_{(l,m)\in\Lambda}$

Then

$$a_{l,m;j,k} = \langle L\,\psi_{j,k}\,,\psi_{l,m}\,\rangle$$
$$y_{l,m} = \langle f,\psi_{l,m}\rangle$$

Where l,m and j,k represent respectively row and column of A.

This is an accurate method to find the solution of partial differential equation.

IV. CONNECTION COEFFICIENT

To find the solution of differential equation using the Wavelet Galerkin technique we have to find the connection coefficients which is also explored in Latto et al.([9]),

$$\Omega_{\ell_1\ell_2}^{\mathrm{d}_1\mathrm{d}_2} = \int_{-\infty}^{\infty}\Phi_{\ell_1}^{\mathrm{d}_1}(\mathrm{x})\,\Phi_{\ell_2}^{\mathrm{d}_2}(\mathrm{x})\mathrm{dx} \tag{8}$$

Taking derivatives of the scaling function d times, we get

$$\phi^d(x) = 2^d\sum_{k=0}^{L-1} a_k\phi_k^d(2x-k) \tag{9}$$

We can simplify equation (8) then for all $\Omega_{\ell_1\ell_2}^{\mathrm{d}_1\mathrm{d}_2}$ gives a system of linear equation with unknown vector $\Omega^{\mathrm{d}_1\mathrm{d}_2}$

$$T\Omega^{\mathrm{d}_1\mathrm{d}_2} = \frac{1}{2^{d-1}}\Omega^{\mathrm{d}_1\mathrm{d}_2} \tag{10}$$

where $d = \mathrm{d}_1 + \mathrm{d}_2$ and $T = \sum_i a_i\, a_{q-2l+i}$. These are so called scaling equation.

But this is the homogeneous equation and does not have a unique nonzero solution. In order to make the system inhomogeneous, one equation is added and it derived from the moment equation of the scaling function ϕ. This is the normalization equation,

$$d! = (-1)^d\sum_l M_l^d\,\Omega_l^{0,d}$$

Connection coefficient $\Omega_l^{0,d}$ can be obtained very easily using $\Omega_l^{\mathrm{d}_1\mathrm{d}_2}$,

$$\Omega_l^{0,d} = \int\phi^{\mathrm{d}_1}\phi_l^{\mathrm{d}_2}dx$$
$$= [\phi^{\mathrm{d}_1-1}\phi_l^{\mathrm{d}_2}]_{-\infty}^{\infty} - \int_{-\infty}^{\infty}\phi^{\mathrm{d}_1-1}\phi^{\mathrm{d}_2+1}dx$$

As a result of compact support wavelet basis functions exhibit, the above equation becomes

$$\Omega^{\mathrm{d}_1\mathrm{d}_2} = -\int_{-\infty}^{\infty}\phi^{\mathrm{d}_1-1}\phi_l^{\mathrm{d}_2+1}dx \tag{11}$$

After d_1 integration,

$$\Omega_l^{\mathrm{d}_1\mathrm{d}_2} = (-1)^d\int_{-\infty}^{\infty}\phi^{\mathrm{d}_1-1}\phi_l^{\mathrm{d}_2+\mathrm{d}_3}dx = (-1)^d\,\Omega_l^{0,d}$$

The moments M_i^k of ϕ_i are defined as

$$M_i^k = \int_{-\infty}^{\infty} x^k\phi_i(x)dx$$

With $M_0^0 = 1$

Latto et al derives a formula as

$$M_i^m = \frac{1}{2(2^m-1)}\sum_{t=0}^{m}\binom{m}{t} i^{m-t}\sum_{l=0}^{t-1}\binom{t}{l}\sum_{i=0}^{L-1}a_i i^{t-l} \tag{12}$$

Where a_i's are the Daubechies wavelet coefficients. Finally, the system will be

$$\begin{pmatrix} T - \frac{1}{2^{d-1}}I \\ M^d \end{pmatrix}\quad \Omega^{\mathrm{d}_1\mathrm{d}_1} = \begin{pmatrix} 0 \\ \mathrm{d}! \end{pmatrix} \tag{13}$$

Matlab software is used to compute the connection coefficient and moments at different scales. Latto et al [9] computed the coefficients at j=0 and L=6 only. The computation of connection coefficients at different scales have been done by using the program given in Jordi Besora [6]. The scaling function at *j*=0 and *L*=6 connection coefficient prepared by Latto et al [9] is given in Table 1 and Table 2 respectively.

Table I: Scaling function at j=0 and L=6 have been provided by Latto et al. [9]

x	Phi(x)	x	Phi(x)
0.000	0	2.500	-0.014970591
0.125	0.133949835	2.625	-0.03693836
0.250	0.284716624	2.750	-0.040567571
0.375	0.422532739	2.875	0.037620632
0.500	0.605178468	3.000	0.095267546
0.625	0.743571274	3.125	0.062104053
0.750	0.89811305	3.250	0.02994406
0.875	1.090444005	3.375	0.011276602
1.000	1.286335069	3.500	-0.031541303
1.125	1.105172581	3.625	-0.013425276
1.250	0.889916048	3.750	0.003025131
1.375	0.724108826	3.875	-0.002388515
1.500	0.441122481	4.000	0.004234346
1.625	0.30687191	4.125	0.001684683
1.750	0.139418882	4.250	-0.001596798
1.875	-0.125676646	4.375	0.000149435
2.000	-0.385836961	4.500	0.000210945
2.125	-0.302911152	4.625	-7.95485E-05
2.250	-0.202979935	4.750	1.05087E-05
2.375	-0.158067602	4.875	5.23519E-07
		5.000	-3.16007E-20

Table II: Connection coefficients at *j*=0 and *L*=6 have been provided by Latto et al. [9] using $\boldsymbol{\Omega[n-k]=\int \phi''(x-k)\phi(x-n)dx}$

$\Omega[-3]$	5.357142857141622e-003
$\Omega[-2]$	1.142857142857171e-001
$\Omega[-1]$	-8.761904761905105e-001
$\Omega[0]$	3.390476190476278e+000
$\Omega[1]$	-5.267857142857178e+000
$\Omega[2]$	3.390476190476152e+000
$\Omega[2]$	-8.761904761904543e-001
$\Omega[3]$	1.142857142857135e-001
$\Omega[4]$	5.357142857144167e-003

V. TEST PROBLEMS

Consider

$$\frac{d^2u(x)}{dx^2}+\beta u(x)=f \tag{14}$$

Now we use Wavelet-Galerkin method solution

Here, we consider $L=6\ \ and\ \ j=0$

We can write the solution of the differential equation (14) is,

$$u(x)=\sum_{k=L-1}^{2^j} c_k 2^{\frac{j}{2}}\Phi(2^j x-k), \qquad x\in[0,1]$$

$$=\sum_{k=-5}^{1} c_k\Phi(x-k), \qquad x\in[0,1] \tag{15}$$

Where c_k are the unknown constant coefficients

Substitute (14) in (15) we get

$$\frac{d^2}{dx^2}\sum_{k=-5}^{1} c_k \Phi(x-k) + \beta \sum_{k=-5}^{1} c_k \Phi(x-k) = 0$$

$$\sum_{k=-5}^{1} c_k \phi''(x-k) + \beta \sum_{k=-5}^{1} c_k \phi(x-k) = 0$$

Taking inner product with $\phi(x-k)$
We have

$$\sum_{k=-5}^{1} c_k \int_{\frac{1-L}{2^j}}^{\frac{L-1+2^j}{2^j}} \phi''(x-k)\phi(x-n) + \beta \sum_{k=-5}^{1} c_k \int_{\frac{1-L}{2^j}}^{\frac{L-1+2^j}{2^j}} \phi(x-k)\phi(x-n) = 0$$

$$\Longrightarrow \sum_{k=-5}^{1} c_k \Omega[n-k] + \beta \sum_{k=-5}^{1} c_k \delta_{n,k} = 0 \qquad (16)$$

$n = 1-L, 2-L, \dots 2^j$
i.e; $n = -5, -4, \dots, 0\ 1$
where

$$\Omega[n-k] = \int \phi''(x-k)\phi(x-n)dx$$

$$\delta_{n,k} = \int \phi(x-k)\phi(x-n)dx$$

By using Neumann Boundary conditions
$u(0) = 1;\quad u'(1) = 0$

Considering left and right boundary conditions we can write

$$u(0) = \sum_{k=-5}^{1} c_k \phi(-k) = 1 \qquad (17)$$

$$u'(1) = \sum_{k=-5}^{1} c_k \phi(1-k) = 0 \qquad (18)$$

We use Taylors method to approximate derivative on the right side

$$u(x) = u(a) + hu'(a) + \frac{h^2}{2!} u''(a) + \cdots\cdots$$

which gives the approximation of derivative at the boundary using forward, backward or central differences. Equation (17) and (18) represent the relation of the coefficient c_k.

We can replace first and last equations of (16) using (17) and (18) respectively. Then we can get the following matrix with $L=6$

TC=B

$$T = \begin{bmatrix} 0 & \phi(4) & \phi(3) & \phi(2) & \phi(1) & 0 & 0 \\ \Omega[1] & \Omega[0] & \Omega[-1] & \Omega[-2] & \Omega[-3] & \Omega[-4] & \Omega[-5] \\ \Omega[2] & \Omega[1] & \Omega[0] & \Omega[-1] & \Omega[-2] & \Omega[-3] & \Omega[-4] \\ \Omega[3] & \Omega[2] & \Omega[1] & \Omega[0] & \Omega[-1] & \Omega[-2] & \Omega[-3] \\ \Omega[4] & \Omega[3] & \Omega[2] & \Omega[1] & \Omega[0] & \Omega[-1] & \Omega[-2] \\ \Omega[5] & \Omega[4] & \Omega[3] & \Omega[2] & \Omega[1] & \Omega[0] & \Omega[-1] \\ 0 & 0 & p(1) & p(2) & p(3) & p(4) & 0 \end{bmatrix}$$

$$p(1) = \phi(4) - \phi(4-h)$$
$$p(2) = \phi(3) - \phi(3-h)$$
$$p(3) = \phi(2) - \phi(2-h)$$
$$p(4) = \phi(1) - \phi(1-h)$$

$$C = \begin{bmatrix} c_{-5} \\ c_{-4} \\ c_{-3} \\ c_{-2} \\ c_{-1} \\ c_0 \\ c_1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

1. Suppose given Boundary Value Problem is,

$$u_{xx} + u = 2 \tag{19}$$

with boundary conditions $u(0) = 1 \ \ and \ \ u'(1) = 1$

The exact solution is,

$$u(x) = -\cos x + \left(\frac{1 - \sin 1}{\cos 1}\right) \sin x + 2$$

$c_{-5} =$ -2.7724

$c_{-4} =$ -2.8070

$c_{-3} =$ -1.7428

$c_{-2} =$ -0.4842

$c_{-1} =$ 0.7705

$c_0 =$ 1.8628

$c_1 =$ 2.0618

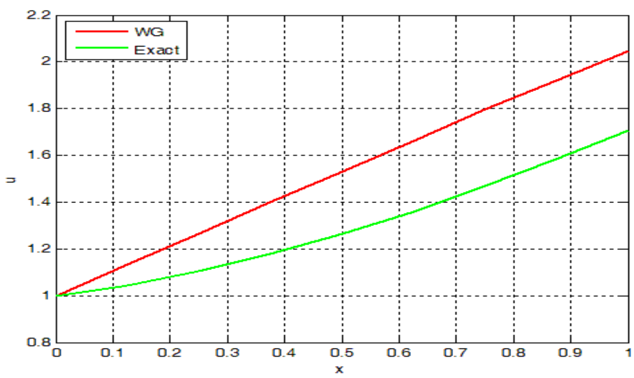

Fig.1: Wavelet-Galerkin Solution for $u_{xx} + u = 2,\ u(0) = 1 \ \ and \ \ u'(1) = 1$ with *L*=6 and *j*=7

$c_{-5} =$ 2.3410

$c_{-4} =$ 2.8697

$c_{-3} =$ 2.1410

$c_{-2} =$ 1.2187

$c_{-1} =$ 0.9752

$c_0 =$ 1.5022

$c_1 =$ 1.7439

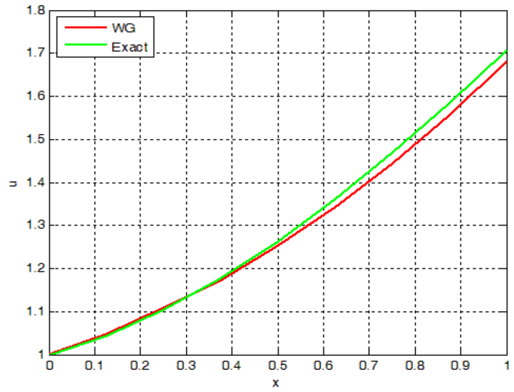

Fig.2: Wavelet-Galerkin Solution for $\boldsymbol{u_{xx} + u = 2,\ u(0) = 1\ \ and\ \ u'(1) = 1}$ **with** ***L*=6 and *j*=0**

$$c_{-5} = \quad 1.8830$$

$$c_{-4} = \quad 2.9599$$

$$c_{-3} = \quad 2.9275$$

$$c_{-2} = \quad 2.0728$$

$$c_{-1} = \quad 1.1728$$

$$c_{0} = \quad 0.9016$$

$$c_{1} = \quad 0.9843$$

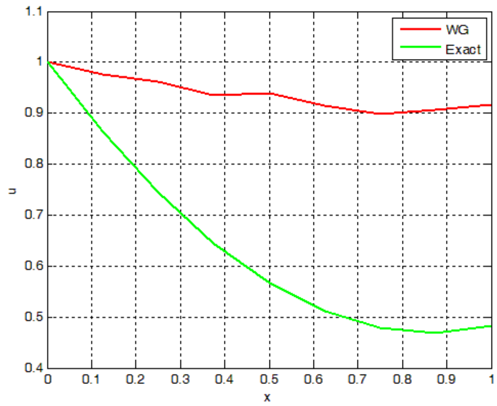

Fig 3 : Wavelet-Galerkin Solution for $\boldsymbol{u_{xx} + u = 2,\ u(0) = 1\ \ and\ \ u'(1) + u(1) = 0}$ **with** ***L*=6 and *j*=0**

It is observe that this kind of Neumann problem the solution of Helmholtz problem only works for value of α=1. For other value of α Wavelets Galerkin techniques to solve the Helmholtz problem (See figure 2 and 3).

2. Suppose given boundary value Problem is,

$$u_{xx} + u_{yy} = q \tag{20}$$

with $u(0, y) = 1; \qquad \frac{\partial}{\partial x}(1, y) = 1; \quad 0 \leq y \leq 1$

Let $u = u(x)e^{-\zeta y}$

$$u_{xx} + u_{yy} = (u'' + \zeta^2) = qe^{\zeta y}$$

$$u_{xx} + \zeta^2 u = qe^{\zeta y}; \qquad u(0) = 1; \; u'(1) = 1$$

Put $\zeta^2 = b$ and $qe^{\zeta y} = Q$

$$u_{xx} + bu = Q; \qquad u(0) = 1; \; u'(1) = 1$$

The exact solution is,

$u(x) = (1 - \mathrm{Q})\cos\zeta x + \left(\frac{1}{\zeta}\sec\zeta + (1 - Q)\tan\zeta\right)\sin\zeta x + Q$

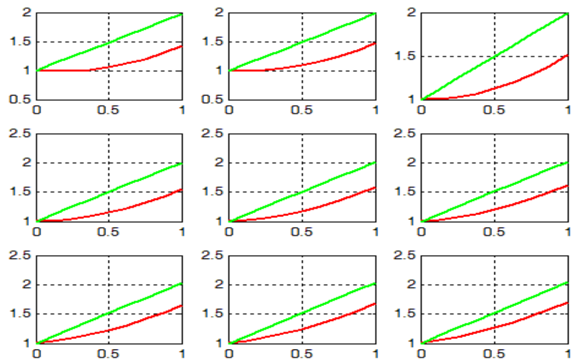

Fig.4: Wavelet-Galerkin Solution for $u_{xx} + \zeta^2 u = 2e^{\zeta y},\ u(0) = 1\ and\ u'(1) = 1$

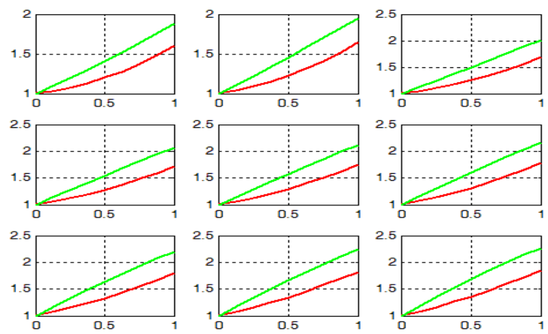

Fig.5: Wavelet Wavelet-Galerkin Solution for $u_{xx} + \zeta^2 u = 2e^{\zeta y},\ u(0) = 1\ and\ u'(1) = 1$

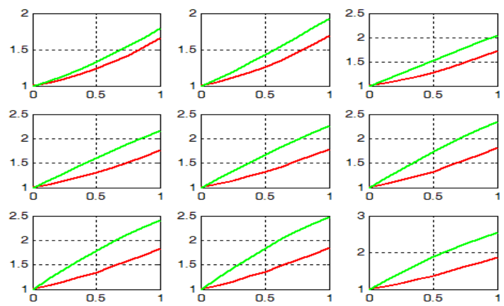

Fig.6: Wavelet-Galerkin Solution for $u_{xx} + \zeta^2 u = 2e^{\zeta y},\ u(0) = 1\ and\ u''(1) = 1$

It is observe that changing value b and q the Wavelet Galerkin techniques gives some different answers. Figure 6 shows that the graph of error verses β. From the figures we conclude that for certain value of β Wavelet Galerkin solution near to the exact solution.

VI. CONCLUSIONS

Wavelet method has shown a very powerful numerical technique for the stable and accurate solution of one dimensional and two dimensional Neumann Helmholtz and Neumann Poisson boundary value problems. The exact solution correlates with numerical solution, using Daubechies wavelets.

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