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Mechanical Behavior of Composite Tapered Lamina

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Abstract:- Composite lamina is widely used in a variety of applications. The laminas used are of different shapes. Based on the shape of the laminas its mechanical behavior also changes. Laminas that are tapered along a single direction are one of the most commonly used.We can analyze these laminas by considering them as a set of regular laminas of varying lengths arranged one above the other. This paper gives an alternative set of formulae that can be used to determine the mechanical behavior of such laminas. The formulae speak about the manner in which the tapered lamina deforms along each axis. A Compliance matrix will be formulated specifically for the analysis of composite tapered laminas. For this paper the data obtained from the derived formulae is verified against the data obtained from already existing FEA software like ABAQUS.

Keywords:–Lamina/Ply; Strength; Mechanical Properties; Analytical Modeling; Composite; Stiffness Matrix; Compliance Matrix.

Annotations –
E_x **– Modulus of Elasticity in the fiber (x) direction**
E_y **– Modulus of Elasticity in the transverse (y) direction**
E_z **– Modulus of Elasticity in the out of plane (z) direction**
v_{xy} **– Poison's ratio relating the longitudinal strain in x direction to the transverse strain in y direction**
v_{xz}**– Poison's ratio relating the longitudinal strain in x direction to the transverse strain in z direction**
G_{xy} **– Shear modulus along the xy plane**

I. INTRODUCTION

A unidirectional tapered Composite lamina is one in which the thickness of the lamina is varying linearly from one end to the other. The fiber is laid along the direction of taper. This will develop an orthotropic property in the lamina. When the lamina is loaded it will undergo deformations.

The coordinate system considered and the geometry of the tapered lamina is shown in Figure 1. The x direction refers to the direction in which the fiber is laid. The y direction is perpendicular to the x direction that is, it lies in the transverse direction. The dimension along this direction remains constant throughout. The z direction is perpendicular to the xy plane.It is referred to as the out of plane direction.

The length of the composite lamina is measured along the x direction and is taken as 'l'. The width of the tapered lamina is measured along the y direction and is taken as 'b'. The thickness of the tapered lamina is measured along the z direction and it is considered as 't_1' on one side and 't_2' on the opposite side. Also 't_1' is greater than 't_2'.

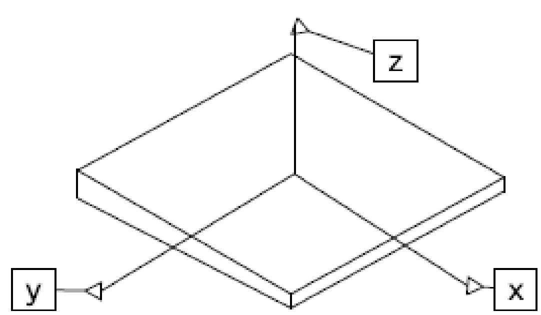

Figure 1 Isometric view of the Tapered Lamina

Figure 2 Side view of Tapered Lamina

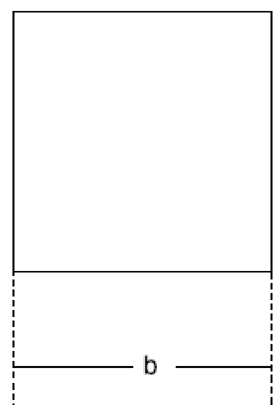

Figure 3 Top View of the Tapered Lamina

A Lamina generally has two of its dimensions much greater than the third dimension. Hence a state of plane stress can be considered.

THEORY

The composite can experience three different loading conditions under the plane stress condition. The composite can be loaded along the xdirection or loaded along the y direction or the lamina can undergo a shear loading on the xy plane. Each of these cases will be considered individually and the deformations developed will be determined. The normal loads considered are to be tensile in nature. But the deformations under compressive loading can also be determined as long as the lamina does not buckle.
The forces applied are such that the lamina will experience only elastic behavior.

A. Load along X direction

When the composite lamina is loaded along the X direction, three deformations result. The first one will be along the loading direction itself called dx. The remaining two deformations are due to the poisson's effect. The loading condition is shown in Figure 4.

Figure 4 Load in the x direction

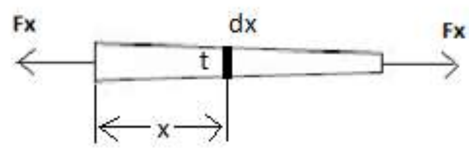

Figure 5 Considered Element

Figure 5 shows the element considered for analysis.
F_x is the load along the x direction, t is the thickness of a section at a distance x from the thicker end.
Consider the relation

$$Modulus\ of\ elasticity\ = \frac{Stress}{Strain} \qquad (1)$$

$$\text{Stress} = \frac{F_x}{b*t} \qquad (2)$$

$$\text{Strain} = \frac{d\Delta x}{dx} \qquad (3)$$

Substituting in the Stress-Strain relation (1) and rearranging we get

$$d\Delta x = \frac{F_x}{E_x*b} * \frac{dx}{t} \qquad (4)$$

Where t = $t_1 - k * x$

$$k = \frac{t_1 - t_2}{l} \qquad (5)$$

So,

$$d\Delta x = \frac{F_x}{E_x*b} * \frac{dx}{t_1 - k*x} \qquad (6)$$

Integrating (6) with respect to x from 0 to l, we get the net deformation in x direction as

$$\Delta x = \frac{F_x*l}{E_x*b*(t_1-t_2)} * \ln\frac{t_1}{t_2} \qquad (7)$$

For deformation in y due to the loading in x direction we have to consider the Poisson's ratio,

$$v_{xy} = -\frac{transverse\ \ strain\ (y)}{longitudinal\ \ strain\ (x)} \qquad (8)$$

$$v_{xy} = -\frac{d\Delta y/_{dy}}{d\Delta x/_{dx}} \qquad (9)$$

Rearranging and substituting the value of $d\Delta x/_{dx}$ in (9),

$$\frac{d\Delta x}{dx} = \frac{F_x}{E_x*b*(t_1-k*x)} \qquad (10)$$

$$\frac{d\Delta y}{dy} = \frac{-v_{xy}*F_x}{E_x*b*t} \qquad (11)$$

Integrating (11) with respect to y from 0 to b we get,

$$\Delta y = \frac{-v_{xy}*F_x}{E_x*t} \qquad (12)$$

The equation (12) shows that the deformation in the y direction is dependent on the cross section at which the deformation is measured. When x = l, the deformation is maximum. Hence loading in the x direction introduces a taper along the y direction as well.

For deformation in z due to the loading in x direction we have to consider the Poisson's ratio,

$$v_{xz} = -\frac{transverse \quad strain\ (z)}{longitudinal \quad strain\ (x)} \tag{13}$$

$$v_{xz} = -\frac{d\Delta z/_{dz}}{d\Delta x/_{dx}} \tag{14}$$

Rearranging and substituting the value of $d\Delta x/_{dx}$ at a section x in (14),

$$\frac{d\Delta x}{dx} = \frac{F_x}{E_x * b * (t_1 - k * x)} \tag{15}$$

$$\frac{d\Delta z}{dz} = \frac{-v_{xz} * F_x}{E_x * b * t} \tag{16}$$

Integrating (16) with respect to z we get,

$$\Delta z = \frac{-v_{xz} * F_x}{E_x * b} * \ln\frac{t_1}{t} \tag{17}$$

The equation (17) shows that the deformation in the z direction is dependent on the cross section at which the deformation is measured. When x = l, the deformation is minimum. Hence loading in the x direction increases the taper along the z direction.

B. Load along y direction

When the composite lamina is loaded along the y direction, three deformations result. They are similar to the previous loading condition. But here the deformations in x and z will result due to Poisson's effect and the deformation in y direction will be due to the load acting in the y direction. Figure 6 shows the loading condition considered.

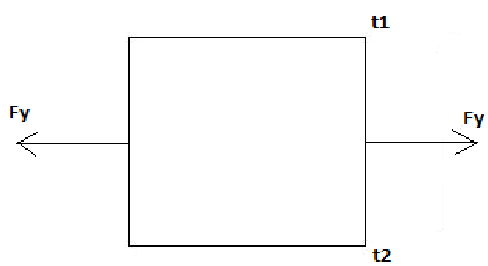

Figure 6 Load in the y direction

Now F_y is the load acting in the y direction. All other geometric parameters are the same.

$$stress = \frac{F_y}{\left(\frac{1}{2}\right) * l * (t_1 + t_2)} \tag{18}$$

$$strain = \frac{\Delta y}{b} \tag{19}$$

From the stress strain relation (1),

$$\Delta y = \frac{2 * F_y * b}{E_y * l * (t_1 + t_2)} \tag{20}$$

For deformation in x due to the loading in y direction from the Poisson's ratio,

$$v_{yx} = -\frac{transverse \quad strain\ (x)}{longitudinal \quad strain\ (y)} \tag{21}$$

$$v_{yx} = -\frac{d\Delta x/_{dx}}{d\Delta y/_{dy}} \tag{22}$$

$$d\Delta y/_{dy} = \Delta y/_{b}$$

This is because the total strain will be the same throughout for this loading condition.

Rearranging and substituting the value of $\Delta y/b$ in (22),

$$\frac{d\Delta x}{dx} = \frac{-2 * v_{yx} * F_y}{E_y * l * (t_1 + t_2)} \tag{23}$$

Integrating (23) with respect to x from 0 to a section at a distance x from the thicker end,

$$\Delta x = \frac{-v_{yx} * F_y * 2 * x}{E_y * l * (t_1 + t_2)} \tag{24}$$

This shows that the deformation in x varies linearly with the taper.

For deformation in z due to the loading in y direction from the Poisson's ratio,

$$v_{yz} = -\frac{transverse \quad strain\ (z)}{longitudinal \quad strain\ (y)} \qquad (25)$$

$$v_{yz} = -\frac{d\Delta z / dz}{d\Delta y / dy} \qquad (26)$$

Again,

$$d\Delta y / dy = \Delta y / b$$

Substituting and rearranging (26) we get,

$$\frac{d\Delta z}{dz} = \frac{-2 * v_{yz} * F_y}{E_y * l * (t_1 + t_2)} \qquad (27)$$

Integrating (27) with respect to dz from 0 to the thickness t at a section we get,

$$\Delta z = \frac{-v_{yz} * 2 * F_y * t}{E_y * l * (t_1 + t_2)} \qquad (28)$$

The deformation in z direction depends on the thickness of the section at which the deformation is measured.

C. SHEAR DEFORMATION

When the composite lamina is loaded by shear it will undergo a deformation as shown in the Figure 8.

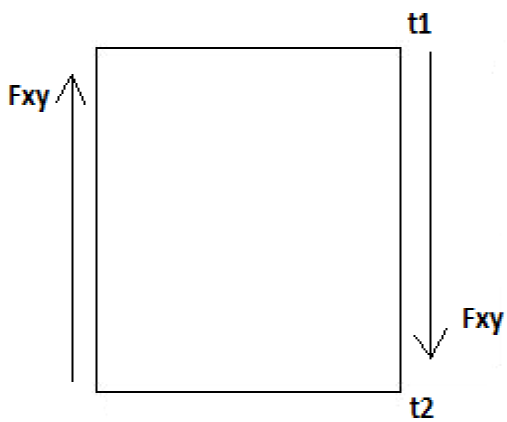

Figure 7 Shear Loading

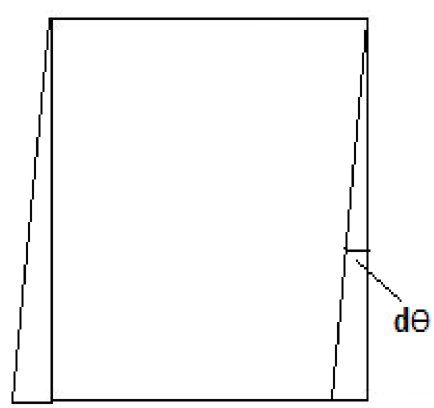

Figure 8 Shear Deformation

The shear load is acting on the xy plane. Its magnitude is taken as F_{xy}.

$$stress = \frac{F_{xy}}{\left(\frac{1}{2}\right)*l*(t_1+t_2)} \quad (29)$$

$$strain = \frac{\Delta l}{l}$$

But,

$$\frac{\Delta l}{l} = \Delta\theta$$

From the stress strain relation (1),

$$\Delta\theta = \frac{2*F_{xy}}{G_{xy}*l*(t_1+t_2)} \quad (30)$$

The change in angle can be considered as the displacement of the face on which the shear load is acting.

II. VERIFICATION

The above derived formulae are verified against the deformations obtained from an FEA tool such as ABAQUS. A 3 dimensional model of the tapered lamina is developed with the following dimensions.

l = 50mm
b = 50mm
t_1 = 6mm
t_2 = 3mm

The material considered is S-Glass epoxy composite with the following properties,

E_x = 53e3 N/mm^2
E_y= 16e3 N/mm^2
v_{xy} = Vxz = Vyz = 0.26
v_{yx} = 0.078
G_{xy} = 6.35e3 N/mm^2

The respective deformations were found from the formula and from the software, for ten different load values in each case. The error is calculated and it is plotted against the load values for each deformation.
All deformation values obtained are in mm.

A. LOAD IN X AXIS

The force acting in the x direction is considered to vary from 600 N to 6000N in steps of 600 N.The figures (Figure 9 to 11) shows the error obtained when compared with ABAQUS results for loading in X direction.
For deformation in x direction,

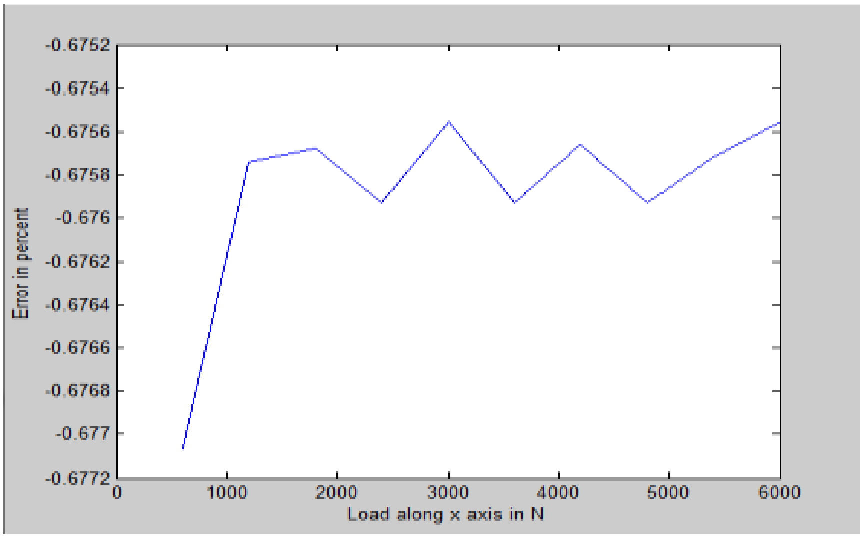

Figure 9 Error vs Load for deformation in x

For deformation in y direction,

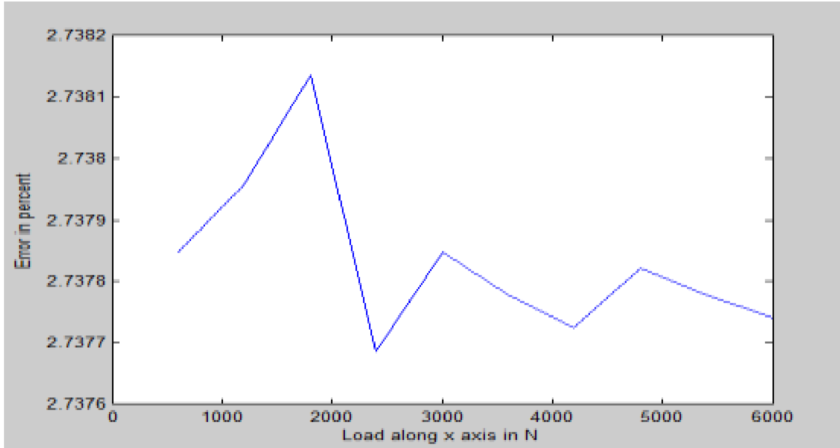

Figure 10 Error vs Load for deformation in y

For deformation in z direction,

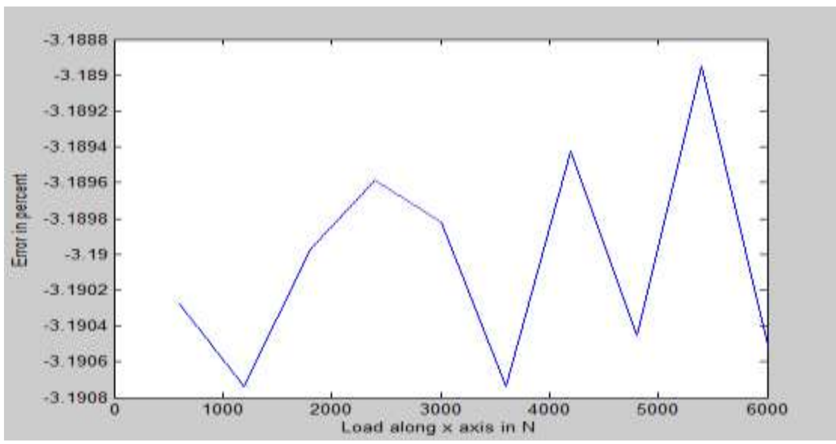

Figure 11 Error vs Load for deformation in z

B. LOAD IN Y AXIS

The force acting in the y direction is considered to vary from 450 N to 4500 N in steps of 450 N.The figures (Figure 12 to 14) shows the error obtained when compared with ABAQUS results for loading in Y direction.
For deformation in y direction,

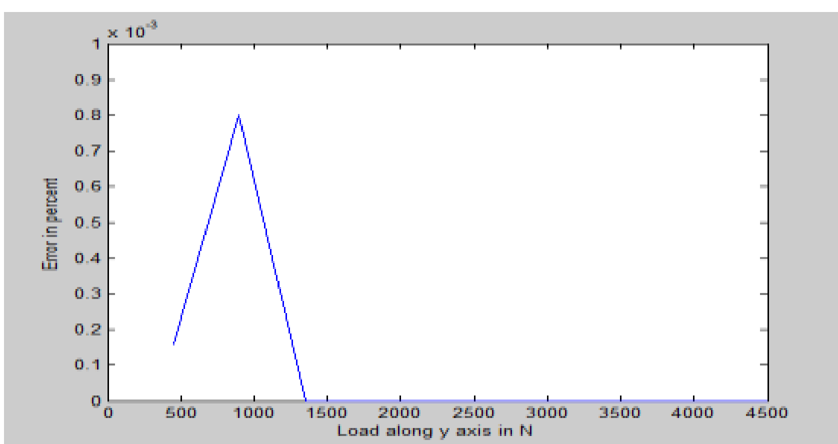

Figure 12 Error vs Load for deformation in y

For deformation in x axis

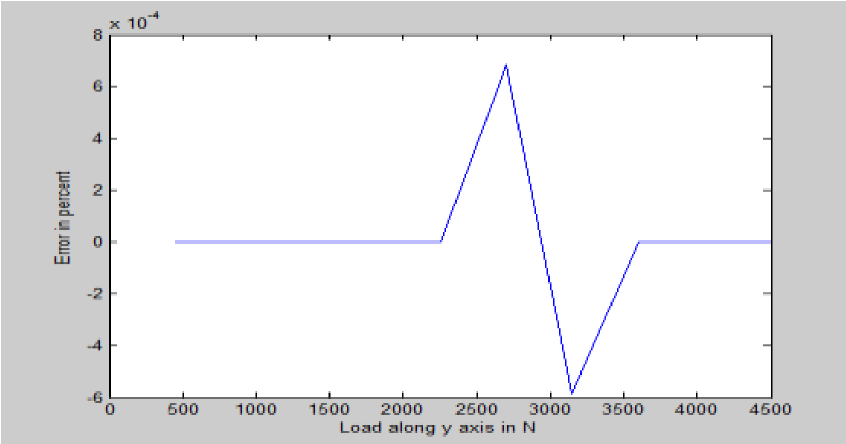

Figure 13 Error vs Load for deformation in x

For deformation in z axis,

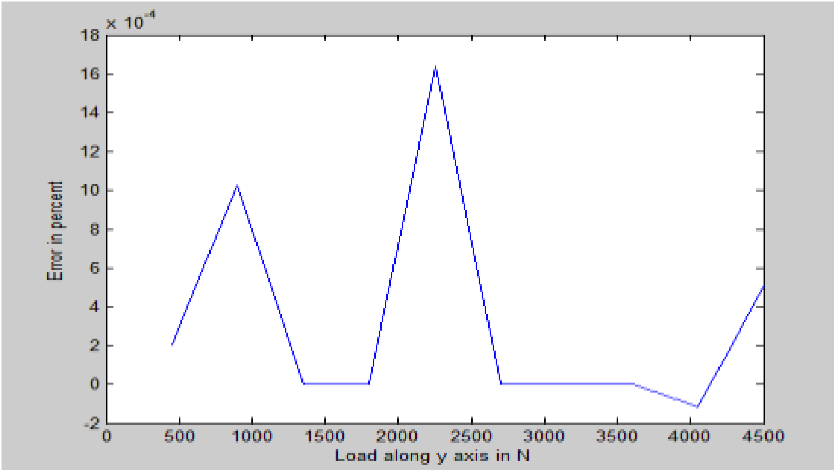

Figure 14 Error vs Load for deformation in z

C. SHEAR LOAD

The shear force acting in the xy plane is considered to vary from 450 N to 4500 N in steps of 450 N.
The Figure 15 shows the variation of the error obtained when the results are compared with ABAQUS results.
Shear Deformations,

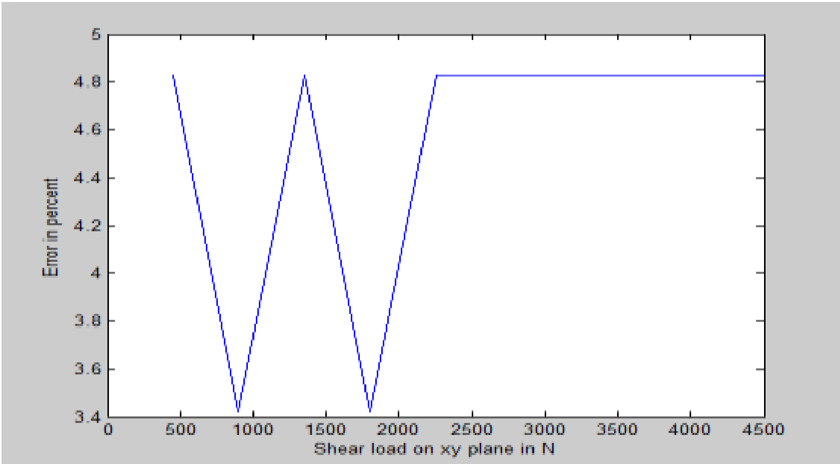

Figure 15 Error vs Load for shear deformation

III. COMPLIANCE MATRIX

The compliance matrix for a tapered composite lamina experiencing in plane stresses will be developed now.

$$\begin{matrix} \Delta x \\ \Delta y \\ \Delta\theta \end{matrix} = \begin{matrix} S11 & S12 & S13 \\ S21 & S22 & S23 \\ S31 & S32 & S33 \end{matrix} * \begin{matrix} F_x \\ F_y \\ F_{xy} \end{matrix}$$

$$S11 = \frac{l}{E_x * b * (t_1 - t_2)} * \ln\frac{t_1}{t_2}$$

$$S12 = \frac{-2 * v_{yx} * x}{E_y * l * (t_1 + t_2)}$$

S13 = 0

$$S21 = \frac{-v_{xy}}{E_x * (t_1 - k * x)}$$

$$S22 = \frac{2 * b}{E_y * l * (t_1 + t_2)}$$

S23 = 0

S31 = 0

S32 = 0

$$S33 = \frac{2}{G_{xy} * l * (t_1 + t_2)}$$

To determine the deformation in the z direction,

$$\Delta z = \frac{-v_{xz} * F_x * l * t}{E_x * x * b * (t_1 - t_2)} * \ln\frac{t_1}{t} + \frac{-v_{yz} * 2 * F_y * t}{E_y * l * (t_1 + t_2)}$$

The Compliance Matrix will be as shown below,

$$\begin{matrix} S11 & S12 & 0 \\ S21 & S22 & 0 \\ 0 & 0 & S33 \end{matrix}$$

IV. CONCLUSION

The formulae were derived for the composite tapered lamina. Using these formulae the deformations experienced by the lamina was calculated for various load values. These results were compared with the results obtained from the FEA tool ABAQUS and the two results were found to match with minimum error. The compliance matrix obtained can be used to study the behavior of a tapered lamina under plane stress condition with good accuracy.

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