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A Detailed Comparative Study between Reduced Order Cumming Observer & Reduced Order Das &Ghoshal Observer

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Abstract:- In this article a detailed comparative study between two well known observer design methodologies namely, reduced order Cumming observer & reduced order Das &Ghoshal observer has been presented. The necessary equations & conditions corresponding to these two types of observers are discussed in brief. Thereafter with the help of a structure wise comparison the similarities & dissimilarities between the above mentioned methods are explained in details. Finally a performance wise comparison between these two is shown using proper numerical example & illustrations in open loop as well as closed loop.

Keywords:- Luenberger Observer, Full & Reduced Order Observer, Cumming Observer, Generalized Matrix Inverse, Das & Ghoshal Observer, State Feedback Control.

I. INTRODUCTION

Observing the states of a system always has been a topic of utmost importance in modern control theory problems. To date various observer design methodologies have been proposed by several authors for different types of systems. Luenberger was the first to develop an elementary construction method for designing observer or state reconstructor. He came up with the design procedures for both full order observer where all system states are estimated & reduced order observer where only the immeasurable (i.e. states not accessible for direct measurement) states are reconstructed [1, 2]. In 1969 S.D.G Cumming proposed a new & simple approach to the design problem of observer of reduced dynamics [3]. But this proposal was not entirely different from Luenberger's approach in the sense that it also presupposes certain observer structure. In [4] N. Munro constructed a reduced order observer for LTI systems using an alternative canonical form to Luenberger's approach. This algorithm can be extended to estimate the entire state vector in an online computer aided design environment and involves complex co-ordinate transformations. In 1981 another novel approach was proposed by Das &Ghoshal for designing reduced order observer using the concept of generalized matrix inverse [5]. The key difference between reduced order Cumming observer & reduced order Das &Ghoshal observer (DGO) is that the later does not presume any particular observer structure, hence does not need to satisfy the constraints associated with Luenberger & Cumming observer. A detailed comparison between DGO and Luenberger reduced order observer is already presented in [6]. The study carried out there, shows that both the methods are more or less same as far as the structures and performances are concerned. Although, DGO has a few advantages over the Luenberger reduced order observer in certain cases.

In this article first both the methods are explained briefly & then a thorough comparison between reduced order Cumming observer & reduced order Das &Ghoshal observer is presented on the basis of structure of observers & their performances in both open loop & closed loop using proper illustrative & numerical example.

The following notations will be used in this paper- R represents the real numbers field, m x n denotes the dimension of a matrix with m rows & n columns. A^g denotes the Moore-Penrose generalized matrix inverse of matrix A. A^T is the transpose of A & I represents the identity matrix of appropriate dimension. R(X) represents the rank of any matrix X.

II. MATHEMATICAL PRELIMINARIES

Cumming's method was simple & straight forward & needs no special mathematical tools. But the concept of generalized matrix inverse has been used in Das &Ghoshal observer to derive the observer dynamics of the system. Generalized matrix inverse is discussed here in brief-

The equation $Ax = y$ (1) has been taken where $A \in R^{m\times n}$ is a matrix; y is a given $m\times 1$ vector, x is an unknown vector $n\times 1$. If matrix $A^g \in R^{m\times n}$ exists that satisfies the four conditions below,

$$(AA^g) = (AA^g)^T$$
$$(A^g A) = (A^g A)^T$$
$$AA^g A = A$$
$$A^g AA^g = A^g$$

Then the matrix A^g is called the Moore-Penrose generalized matrix inverse of A & A^g is unique for A.Now equation (1) is consistent if & only if $AA^g y = y$ …….(2)& then the general solution is given by $x = A^g y + (I - A^g A)v$ ……….(3) where I = Identity matrix of proper dimension v = An $(n \times 1)$ arbitrary vector.

Lemma used by Das & Ghoshal:

For an $m \times n$ matrix C& $n \times k$ matrix L, if the linear space spanned by columns of L is equal to the linear space spanned by the columns of $(I - C^g C)$ then

$LL^g = I - C^g C$ (4)

III. BRIEF CONSTRUCTION PROCEDURE

Consider an LTI system described by $\dot{x} = Ax + Bu, x_0 = x(0)$ (5) $y = Cx$ (6)

Where x is $n \times 1$ unknown state vector to be estimated, x_0 is the initial condition of x. u is a $b \times 1$ input vector, y is $(m \times 1)$ output vector. A, B, C are known matrices of proper dimensions. We assume that the pair {A, C} is completely observable which implies that the simultaneous solutions for equation (1) & (2) is unique when x_0, u, y are given.

A. *Reduced Order Cumming Observer* [3]

Reduced order Cumming observer is governed by the following equations & conditions.

In this method the following relations

$$\begin{bmatrix} y \\ \cdots \\ z \end{bmatrix} = \begin{bmatrix} C \\ \cdots \\ H \end{bmatrix} x \qquad (7)$$

Is considered such that $\begin{bmatrix} C \\ \cdots \\ H \end{bmatrix}^{-1}$ is non-singular.

Hence the state vector x can be found as

$$x = \begin{bmatrix} C \\ H \end{bmatrix}^{-1} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} E & F \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} \qquad (8)$$

Therefore differentiating eqn.(7) & using eqn.(8) the output equation & dynamic equation of the observer are obtained as -

$\dot{z} = HAFz + HAEy + HBu$ (9)

$\dot{y} = CAFz + CAEy + CBu$ 10)

The observer equation is given by –

$$\dot{\hat{z}} = (HAF - KCAF)\hat{z} + (HAE - KCAE)y + (HB - KCB)u + k\dot{y} \qquad (11)$$

$$\dot{\hat{q}} = (HAF - KCAF)\hat{q} + (HAFK + HAE - KCAFK - KCAE)y + (HB - KCB)u \qquad (12)$$

Where $\hat{q} = \hat{z} - Ky$ (13)

Also $\hat{x} = F\hat{q} + (E + FK)y$ (14)

B. Reduced Order Das & Ghoshal Observer [5]

Reduced order Das &Ghoshal observer is governed by the following equations & conditions

$x = C^g y + Lh$ (15)

$\dot{h} = L^g ALh + L^g AC^g y + L^g Bu$ (16)

$\dot{y} = CALh + CAC^g y + CBu$ (17)

$$\dot{\hat{h}} = (L^g AL - KCAL)\hat{h} + (L^g AC^g - KCAC^g)y + (L^g B - KCB)u + M\dot{y} \quad (18)$$

$$\dot{\hat{w}} = (L^g AL - KCAL)\hat{w} + \{(L^g AC^g - KCAC^g) + (L^g AL - KCAL)M\}y + (L^g B - KCB)u \quad (19)$$

Where $\hat{w} = \hat{h} - My$ (20)

Also $\hat{x} = L\hat{w} + (C^g + LK)y$ (21)

IV. NUMERICAL EXAMPLE

In this section both Cumming & Das &Ghoshal observers are implemented to estimate the immeasurable states for the case of longitudinal motion of Charlie Aircraft (Lungu 08) [7] in both open loop & closed loop (using state feedback control). The system is governed by the state-space representation as shown below-

$$x = [\Delta u \quad \Delta\alpha \quad \Delta\theta \quad \Delta q]$$

$$A = \begin{bmatrix} -.007 & .12 & -9.81 & 0 \\ -0.128 & -0.54 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ .065 & 0.96 & 0 & -.99 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -.04 \\ 0 \\ -12.5 \end{bmatrix}; \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

u is the aircraft longitudinal velocity; α-the aircraft attack angle; θ-the aircraft pitch angle; q-the aircraft pitch angular rate, while Δ is associated with the perturbation of the variables from their nominal values. The input signal is chosen to be a step signal. During state feedback control the control law used is given by $u = r - G\hat{x}$. The open loop responses of the plant for both Cumming Observer & Das & Ghoshal Observer are shown below.

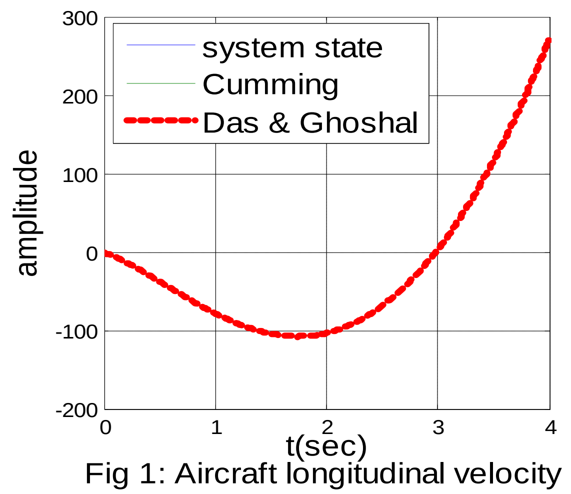

Fig 1: Aircraft longitudinal velocity

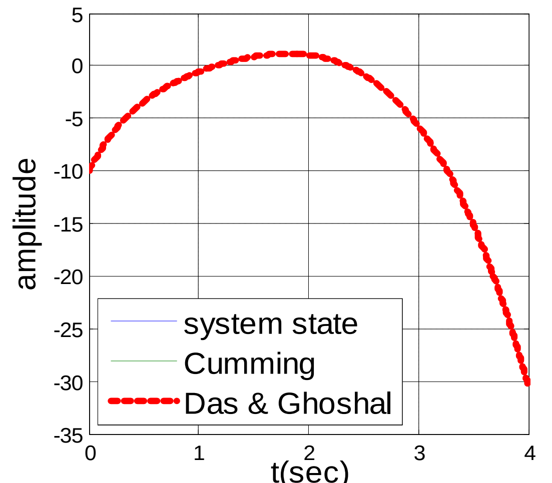

Fig 2: Angle of attack

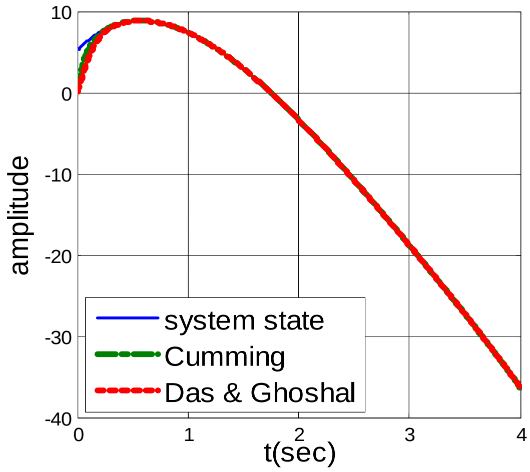

Fig 3:Pitch angle

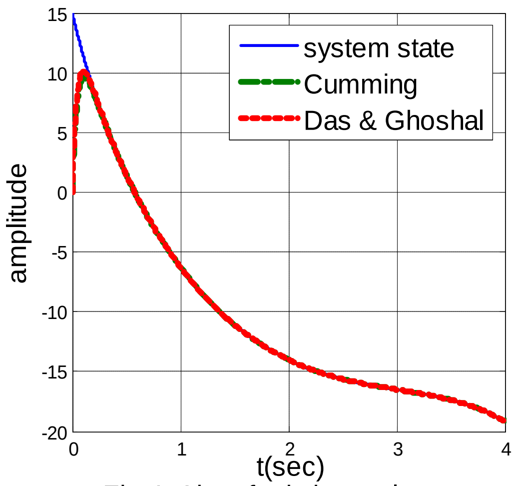

Fig 4: Aircraft pitch angular rate

A closed loop performance of the plant with these two different observers has also been shown below with the help of MATLAB simulations where the two unavailable states x_3 & x_4 are illustrated. We have placed the close loop poles at the same location for both the observers & state feedback gain matrix G is determined accordingly using Ackermann's method. Where, the state feedback gain matrix is chosen by pole placement & is found to be $[6.0076 \quad -28.0996 \quad 14.687 \quad -1.547]$

It is observed that while estimating the unavailable states i.e. pitch angle & aircraft pitch angular rate in closed loop, the performances are almost same for both types of observers as it can be seen from the simulation responses.

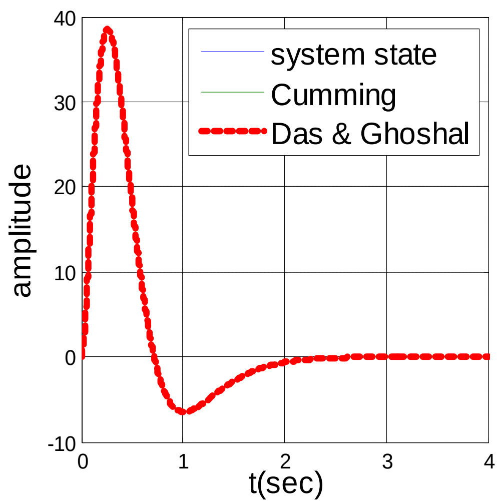

Fig 5: Pitch angle (closed loop)

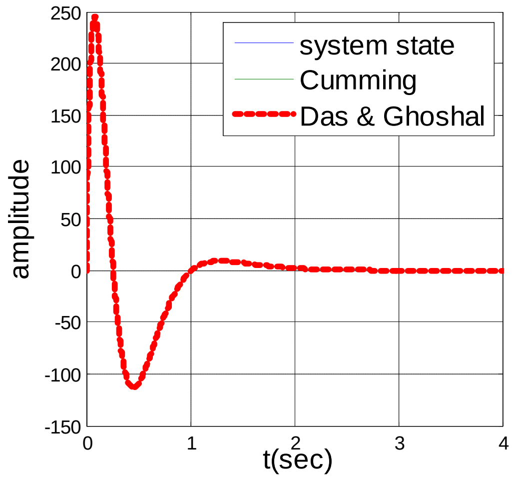

Fig 6: Aircraft pitch angular rate (closed loop)

V. COMPARATIVE STUDY BETWEEN CUMMING OBSERVER & DAS & GHOSHAL OBSERVER

[In the foregoing comparative study CO – Cumming Observer DGO – Das & Ghoshal Observer]

1) In CO system variable can be described as

$$x = \begin{bmatrix} C \\ H \end{bmatrix}^{-1} \begin{bmatrix} y \\ z \end{bmatrix}$$

$$x = \begin{bmatrix} E & F \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix}$$

Here we have to choose H matrix in such a way that the inverse of $\begin{bmatrix} C \\ H \end{bmatrix}^{-1}$ exists.

In DGO $x = C^g y + Lz$

$$x = \begin{bmatrix} C^g & L \end{bmatrix} \begin{bmatrix} y \\ h \end{bmatrix}$$

2) Estimated state variable can be presented as

In CO $\hat{x} = [E + FKF] \begin{bmatrix} y \\ \hat{q} \end{bmatrix}$

In DGO $\hat{x} = [C^g + LKL] \begin{bmatrix} y \\ \hat{w} \end{bmatrix}$

3) In CO $\hat{q} = [-KI] \begin{bmatrix} y \\ \hat{z} \end{bmatrix}$

In DGO $\hat{w} = [-KI] \begin{bmatrix} y \\ \hat{h} \end{bmatrix}$

4) Comparing the observer dynamics of CO & DGO

In CO $\dot{z} = HAFz + HAEy + HBu$

In DGO $\dot{h} = L^g ALh + L^g AC^g y + L^g Bu$

5) In CO it is assumed that $\begin{bmatrix} C \\ H \end{bmatrix}^{-1}$ exists & is found to be $\begin{bmatrix} E & F \end{bmatrix}$ with H, F, L having proper dimensions.

$$\begin{bmatrix} C \\ H \end{bmatrix}^{-1} = \begin{bmatrix} E & F \end{bmatrix}$$

$$\begin{bmatrix} C \\ H \end{bmatrix} \begin{bmatrix} E & F \end{bmatrix} = \begin{bmatrix} I_m & 0_{m\times(n-m)} \\ 0_{(n-m)\times m} & I_{n-m} \end{bmatrix}$$

DGO is based on the concept of generalized matrix inverse, according to which a g-inverse exists for every matrix & is unique.

6) In CO $CE = I_m$

In DGO $CC^g = I_m$

This is not a constraint for DGO. If only C matrix is in the form $[I_m \quad 0]$then this condition holds.

7) In CO $HF = I_{n-m}$

In DGO $L^g L = I_{n-m}$

8) In CO $CF = 0_{m\times(n-m)}$

In DGO $CL = 0_{m\times(n-m)}$

9) In CO $HE = 0_{(n-m)\times m}$

In DGO $L^g C^g = 0_{(n-m)\times m}$

This is not a constraint for DGO. If only C matrix is in the form $[I_m \quad 0]$ then this condition holds.

10) In CO the pair (HAF, CAF) should be completely observable.

In DGO the pair $(L^g AL, CAL)$ has to be completely observable.

11) The output equation is

In CO $\dot{y} = CAFz + CAEy + CBu$

In DGO $\dot{y} = CALh + CAC^g y + CBu$

12) The observer equation is given by

In CO $\dot{\hat{z}} = (HAF - KCAF)\hat{z} + (HAE - KCAE)y + (HB - KCB)u + K\dot{y}$

In DGO $\dot{\hat{h}} = (L^g AL - KCAL)\hat{h} + (L^g AC^g - KCAC^g)y + (L^g B - KCB)u + M\dot{y}$

13) The final observer dynamic equation is given by

In CO $\dot{\hat{q}} = (HAF - KCAF)\hat{q} + (HAFK + HAE - KCAFK - KCAE)y + (HB - KCB)u$

In DGO $\dot{\hat{w}} = (L^g AL - KCAL)\hat{w} + (L^g ALK + L^g AC^g - KCALK - KCAC^g)y + (L^g B - KCB)u$

14) Error dynamics equations are given by

In CO $\dot{e} = (HAF - KCAF)e$

In DGO $\dot{e} = (L^g AL - KCAL)e$

15) CO presupposes that the observer structure is basically the same that of the plant.
In DGO no such observer structure is presumed. A generalized approach is used.

16) Observer poles can be chosen arbitrarily (but they should lie in the left half of the s-plane so that a stability matrix is achieved to ensure the asymptotical stability of the observer system).
In CO the stability matrix is
$(HAF - KCAF)$, where we can choose K accordingly.
In DGO the stability matrix is
$(L^g AL - KCAL)$, where we can choose K accordingly.

VI. CONCLUSIONS

In this article a thorough & exhaustive comparative study between reduced order Cumming observer & reduced order Das & Ghoshal observer based on the concept of generalized matrix inverse has been carried out in both open loop and closed loop. The study signifies the fact that basically these two types of observers are almost identical in terms of structural scheme as well as performance. Though it is shown that Cummings observer needs to satisfy some constraints while that is not necessary at all for Das &Ghoshal observer. Finally a numerical example for the case of longitudinal motion of Charlie Aircraft has been selected & both the observer systems are implemented to examine their respective response &performance.

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REFERENCES

[1]. D.G. Luenberger, "Observing the states of a linear system", IEEE Transactions Mil. Electron. Vol. MIL-8, pp. 74-80, April.1964.
[2]. D.G. Luenberger, "An Introduction to Observers", IEEE Transactions on Automatic Control, vol. AC-16, no. 6, pp. 596-602, December. 1971.
[3]. S.D.G. Cumming, "Design of Observers of Reduced Dynamics", Electronics Letters, vol.5, Issue:10, pp. 213-214, 1969 .
[4]. N. Munro, ―Computer-aided-design procedure for reduced-order observers estimate of entire state vector, Proceedings of IEE, Vol. 120, No. 2, pp. 319-324, February 1973.
[5]. G. Das and T.K. Ghosal, "Reduced-order observer construction by generalized matrix inverse", International Journal of Control, vol. 33, no. 2, pp. 371-378, 1981.
[6]. Parijat Bhowmick and Dr.Gourhari Das, "A detail Comparative study between Reduced Order Luenberger and Reduced Order Das &Ghoshal Observer and their Application" Proceedings of the International Conference in Computer, Electronics and Electronics Engineering, pp. 154 – 161, Mumbai, March 2012.
[7]. Lungu, Mihai, and Romulus Lungu. "Full-order observer design for linear systems with unknown inputs", International Journal of Control vol. 85.10, p.p. 1602-1615, 2012.
[8]. F.A.Graybill , Introduction to Matrices with Applications in Statistics ,Belmont ,CA: Wadsworth, 1969
[9]. J.O'Reilly, Observer for Linear Systems, New York, Academic,1983