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TheGeometrical solution , of theRegular n-Polygons The UnsolvedAncient Greek Special Problems and Their Nature .

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Markos Georgallides: The ancient - Greek Special Problems, The Regular n-Polygons

ABSTRACT: The Special Problems of E-geometry [47] consist the ,Mould Quantization, of Euclidean Geometry in it , to become →Monad, through mould of Space –Anti-space in itself ,which is the Material Dipole in monad Structure →Linearly, through mould of Parallel Theorem [44- 45],which are the equal distances between points of parallel and line → In Plane , through mould of Squaring the circle [46] , where two equal and perpendicular monads consist a Plane acquiring the common Plane-meter, π,→and in Space (volume) , through mould of the Duplication of the Cube [46], where any two Unequal perpendicular monads acquire the common Space-meter $^3\sqrt{2}$, to be twice each other.[44-47] . Now is added the , Stores of Quantization , which is the Regular-Polygons Mechanism .

The Unification of Space and Energy becomes through [STPL] Geometrical Mould Mechanism , the minimum Energy-Quanta ,In monads → Particles, Anti-particles, Bosons, Gravity –Force, Gravity-Field , Photons, Dark Matter, and Dark-Energy ,consisting the Material Dipoles in inner monad Structures[39-41] .

Euclid's elements consist of assuming a small set of intuitively appealing axioms , proving many other propositions . Because nobody until [9] succeeded to prove the parallel postulate by means of pure geometric logic , many self consistent non-Euclidean geometries have been discovered , based on Definitions , Axioms or Postulates , in order that non of them contradicts any of the other postulates .It was proved in [39] that the only Space-Energy geometry is Euclidean , agreeing with the Physical reality, on ABSegment which is Electromagnetic field of the Quantized on $\overline{AB}$Energy Space Vector , on the contrary to the General relativity of Space-time which is based on the rays of the non-Euclidean geometries. Euclidean geometry elucidated the definitions of geometry-content ,i.e.{[for Point, Segment, Straight Line, Plane , Volume, Space [S] , Anti-space [AS] , Sub-space [SS] , Cave, The Space-Anti-Space Mechanism of the Six-Triple-Points-Line , that produces and transfers Points of Spaces , Anti-Spaces and Sub-Spaces in Gravity field [MFMF] , Particles]} and describes the Space-Energy vacuum beyond Plank´s length level [Gravity`s Length 3,969.10¯62 m] , reaching the absolute Point≡

$L_v = e^{i.\left(\frac{N\pi}{2}\right) b=10^{-N=-\infty}} = 0$ m , which is nothing and the Absolute Primary Neutral space PNS .[43-46] .

In Mechanics , the Gravity-cave Energy Volume quantity [wr] is doubled and is Quantized inPlanck`s-cave Space quantity (h/2π) = The Spin = 2.[wr]³ →i.e. Energy Space quantity,wr ,is Quantized , doubled, and becomes the Space quantityh/πfollowing Euclidean Space-mould of Duplication of the cube, in Sphere volume V=(4π/3).[wr]³ following theSquaring of the circle,π,and in Sub-Space-Sphere volume $^3\sqrt{2}$, and theTrisectingof the angle .

Keywords : The Unsolved ancient-Greek Problems , The Nature of the Special E-Problems.
The solution of All Odd- Regular - Polygons , The Stores of Quantization .

I. CONTENT

Preface :

This article is the completion of the prior [44] and [45-47] .With pure Geometrical logic is presented the Algebraicand Geometric Solution , and the Construction of all the n-Regular Polygons of this very interested problem. A new method for the Alternate Interior angles,The Geometrical - Inversion , is presented as this issues forRight - Angles .In article [62B] is presented the new Geometrical Proof . Thenew article is based on the Geometrical logic with a short procession in Mechanics , without any presuppositionto geometric knowledge on coupler points .

The concept of , The Relation , Mould , of Angles and Lengths ,is even today the main problemin science, Mechanics and Physics .Althoughthe Mould existed in the Theory of Logarithm and in the Theory of Means thisNew Geometrical-Method is the Master key of Geometry and in Algebra andconsequently to the Relation between Geometry and Nature , for theirin between applications .The New Regular Polygons Mechanism , exhibits The How and Where Work (Energy → Kinetic or Dynamic) produced from any Removal , is Stored .The Programming of the Methods is very simple and very interesting for Computer-Programmers . In the next article [64] is prepared the Unification of Energy-monads , The Spin of Black Holes ,with Geometry-Monads , in Black Matter ,through the Material - Geometry – monadsand the Geometrical Inversion.

1.. Definition ofQuantization.

Quantization is the concept (the Process) that any, Physical Quantity → [PQ] of the objective reality (Matter, Energy or Both) is mapping the Continuous Analogous,the points, to only certain Discrete values. Quantization of Energy is done in Space-tanks, on the materialpoints, tiny volumes and on points consisting the Equilibrium,all the Opposite Twin,of Space Anti-space. [61] In Geometry [PQ] are the Points,the nothing , only , transformed into Segments , Lines , Surfaces , Volumes and to any other Coordinate System such as (x,y,z) , (i,j,k) and which are all quantized . Quantization of E-geometry is the way of Points to become as → (Segments, Anti-segments =Monads = Anti-monads),(Segments,Parallel-segments = Equal monads), (Equal Segments and Perpendicular-segments = Plane Vectors), (Un-equal Segments twice – Perpendicular -segments = The Space Vectors = Quaternion) .[46]

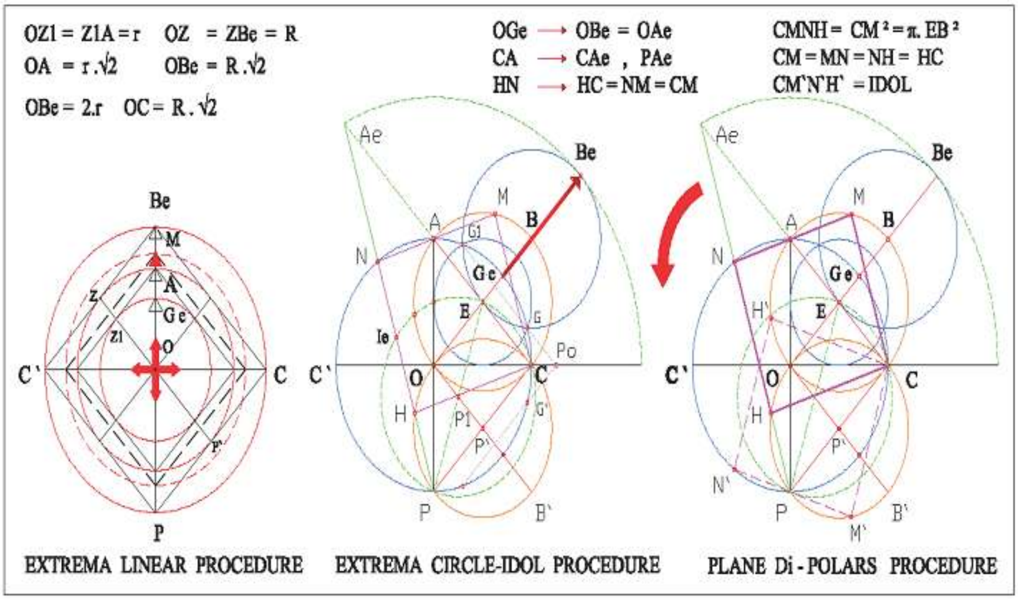
In Philosophy [PQ] are the concepts of Matter and of Spirit or Materialismand Idealism.
a).. Anaximander , claimed that non of the elements could be, Arche and proposed , apeiron , an infinitive substance from which all things are born and to which all will return. b).. Archimedes ,is very clear regarding the definitions, that they say nothing as to whether the things defined exist or not , but they only require to be understood . Existence is only postulated in the case where [PQ] are the Points to Segments (magnitudes = quantization process). In geometry we assume Point , Segment , Line , Surface and Volume , without proving their existence , and the existence of everything else has to be proved .

The Euclid`s similar figures correspond to Eudoxus` theory of proportion .
c).. Zenon, claimed that ,Belief in the existence of many things rather than , only one thing , leads to absurd conclusions and for , Point and its constituents will be without magnitude . Considering Points in space are a distinct place even if there are an infinity of points , defines the Presented in [44] idea of Material Point .
d).. Materialism or and Physicalism , is a form of philosophical monism and holds that matter (without defining what this substance is) is the fundamental substance in nature and that all phenomena , including mental phenomes and consciousness , are identical with material interactions by incorporating notions of Physics such as spacetime , physical energies and forces , dark matter and so on .
e).. Idealism , such as those of Hegel , ipso facto , is an argument against materialism
(the mind-independent properties can in turn be reduced to the subjective percepts) as such the existence of matter can only be assumed from the apparent (perceived) stability of perceptions with no evidence in direct experience .
Matter and Energy are necessary to explain the physical world but incapable of explaining mind and so results , dualism .The Reason determined in itself and its relation to the world creates the very old question as, what is the ultimate purpose of the world?.
f).. Hegel`s conceive for mind , Idea , defines that , mind is Arche and it is retuned to [PQ] the subjective percepts , while Materialism holds just the opposite .
In Physics [PQ] are The , Electrical charge , Energy , Light , Angular momentum , Matter which are all quantized on the microscopic level . They do not seem quantized in the macroscopic scale because the size of the steps between each possible value is so small.
a)..De Broglie found that ,light and matter at subatomic level display characteristics of both waves and particles which move at specific speeds called Energy-levels .
b)..Max Planck found that , Energy and frequency of the Electromagnetic radiation is quantized asthe relation E = h.f .

In Mechanics , Kinematics describes the motion while, Dynamics causes the motion.
c)..Bohr modelfor Electrons in free-Atoms is the Scaled Energy levels , for Standing-Waves is the constancy of Angular momentum , for Centripetal-Force in electron orbit , is the constancy of Electric Potential , for the Electron orbit radii , is the Energy level structure with the Associated electron wavelengths.
d).. Hesiod Hypothesis [PQ] is Chaos, i.e.the Primary Point from which is quantized to Primary Anti-Point . [From Chaos came forth Erebus ,the Space Anti-space, and Black Night ,The [STPL] Mechanism , but of Night were born Aether ,The rest Gravity dipole Field connected by the Gravity Force, and Day , Particles Anti-particles, whom she conceived and Bare ,TheEquilibrium of Particles Anti-particles , in Spaces Anti-spaces , from union in love with Erebus] . [43-46]
e).. Markos model for Physical Quantity → [PQ] is the Energy-Monad produced from Chaos ,which is the Zero-point $0 = \varnothing = \{\oplus + \ominus\}$= The Material-point = The Quantum = The Positive Space and the Negative Anti-Space , between Opposites = The equilibrium of opposite directions →← [58-61]
In article is shown the How and Where this Physical Quantity is stored .

The Special Greek Problems .
1.. The Squaring of the Circle .
The Plane Procedure Method . [45-46]

The property ,of Resemblance Ratio to be equal to 2 on a Square , is transferred simultaneously by the equality of the two areas, when square is equal to the circle,where that square is twice of the inscribed.
This property becomes from the linear expansion in three spaces of the inscribed (O, OG_e) to the circumscribed (O ,OM) circle , in a circle (O, OA) as in . F.1-(1) .
1..The Extrema method of Squaring the circle F.1



(1) (2) (3)

F.1 → The steps for Squaring any circle[O,OA] or (E,EA = EC = EO) ondiameterCA through the –

The Expanding of the Inscribed circleO,OG_e→ to the circle O,OA and to thecircumscribed O,OM and the Four Polar O, A, C, P, Procedure method :

In (1) isExpanding Inscribed circle O,OG_e→ to circle O, OA and to circumscribed O,OM .
In (2) The Inscribed square CBAO is Expanding to squareCMNHandto circumscribedCAC`P
In (3) The Inscribed square CBAO and itsIdol CB`PO , Rotatethrough the pole C , Expand through Pole O on OB line , and Translate through pole P on PN chord . Extrema Edgepoint B_e of circle O,OB_e Rotate to $A_e$point , forming extrema square CMNH=NH² = π.EA² .

The Plane Procedure method :

It isconsisted of twoequal and perpendicular vectors CA , CP , the Mechanism , whereCA = CP and CA⊥CP , such , sothat the Workproducediszero and thisbecauseeach area iszero, withthe three conjugate Poles A , C , P related to central O , withthe three Pole-lines CA ,CP ,AP and the three perpendicular Anti -Pole-lines OB , OB`, OC , and isConverting the Rectilinear motion in (1) , on the Mechanism , to Four - Polar Expanding rotationalmotion.

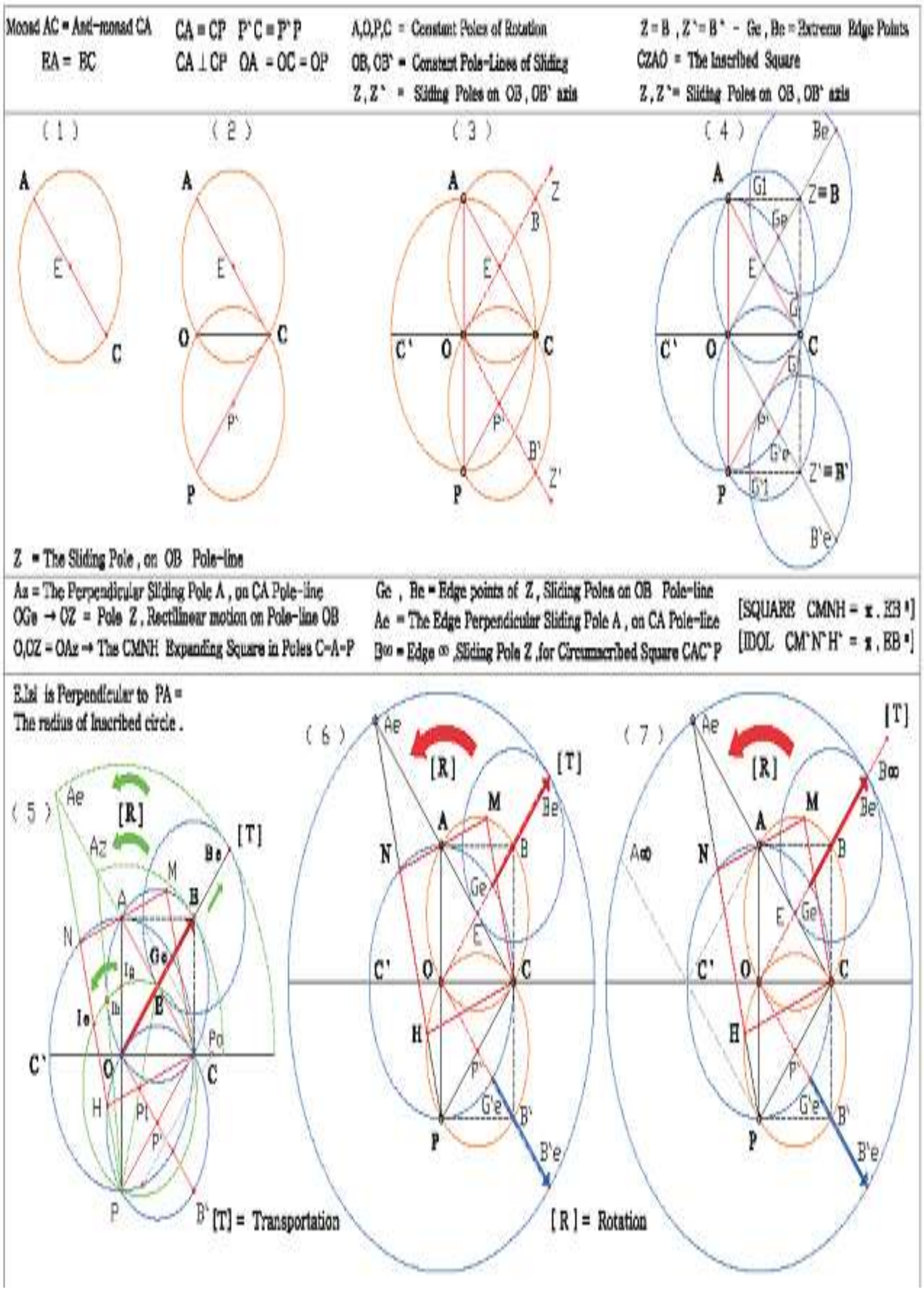
The formulated Five Conjugate circleswith diameters → CA = OB , CP = OB` , $EB_e$= OB , PC= OB` , $P_oG_1 = P_oG`_1$= CA and also the circumscribed circle on them ← define A System of infinite Changable Squares from→ the Inscribed CBAO to → CMNH and to → the Circumscribed CAC`P , through the Four - Poles of rotation .

The Geometrical construction : F.2

1.. Let E be the center , and CA is the diameter of any circle (E, EA = EC) .
2.. Draw CP = CA perpendicular at point C and also the equaldiameter circle (P`, P`C = P`O).

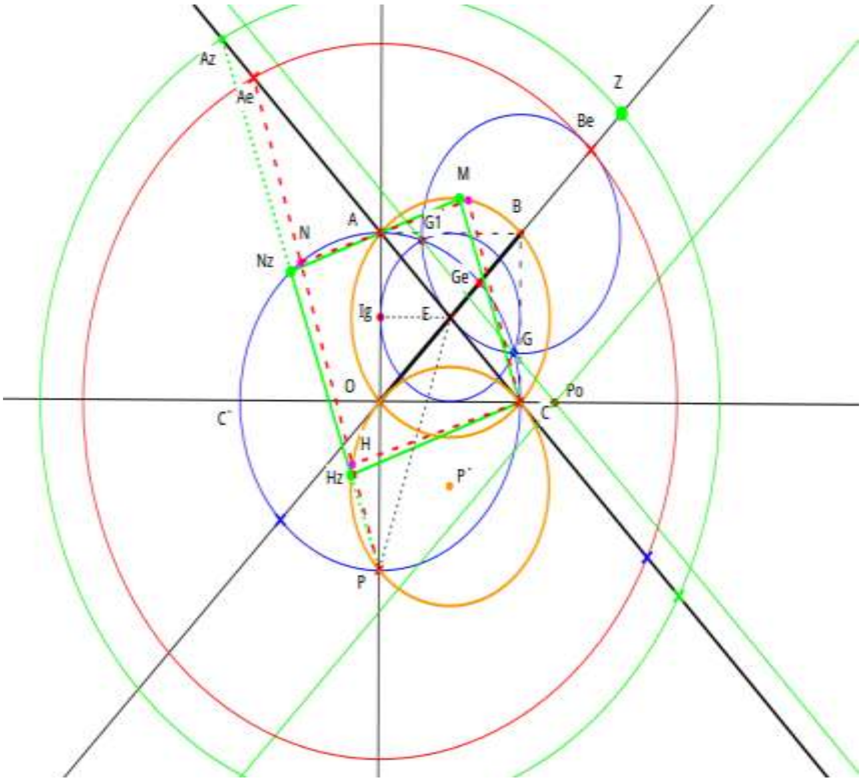
3..Frommid-point O of hypotynuse AP as center , Draw the circle (O, OA = OP = OC) and complete squares , OCBA , OCB`P .
On perpendicular diameters OB , OB` and from points B , B` draw thecircles, (B, BE = Be) , (B`, B`P`) intersecting (O, OA) = (O, OP) circle at double points [G ,$G_1$] , [G`, $G`_1$] respectively, and OB , OB` produced at points B_e, $B`_e$, respectively.
4.. Draw on the symmetrical to OC axis , lines GG_1 and G$G`_1$intersecting OC axis at point $P_o$. 5.. Draw the edge circle ( O, $OB_e$ ) intersecting CA produced at point Ae and draw $PA_e$ line intersecting the circles , (O, OA) , (P`, P`P) at points N-H ,respectively.
6.. Draw line NA produced intersecting the circle (E , EA) at point M and draw Segments
CM , CH and completequatrilateralCMNH ,callingit theSpace = the System . Draw line CM ` and line M `P produced intersecting circle (O,OA) at point N` and line AN ` intersecting circle (E, EA) at point H`, and completequatrilateral CM`N`H`, callingit heAnti-space = Idol = Anti – System .P_1

7.. Draw the circle (P_1, P_1E) of diameter PE intersecting OA at pointI_g,and (E,EA) circle at pointI_b
A.. Show that quadrilaterals CMNH , CM`N`H` are Squares.
B.. Show that it is an Extrema Mechanism , on Four Poles where , The Two dimensional Space (the Plane) is Quantized to a System of infinite Squares → CBAO → CMNH → CAC`P,
and toCMNH square of side CM = HN , where holds CM ² = CH ² = π. EA² = π . EO²
C.. Show that , in circle (E , EA = EC = EO = EB) the Inscribed square CBAO , the square CMNH whichisequal to the circle , and the Circumscribed square CAC`P , Obey , Rotation of Squares through pole P , Translation of circle (E , EO)on OB Diagonal ,and Expansion in CA Segment.



F.2 → The steps for Squaring the circle (E, EA = EC) on diameterCA through Plane Procedure Mechanism

1..Draw on any Orthogonal - System OA ⊥ OC , the circle (O , OA = OC) suchthatintersects the system at points P , C` respectively .
2.. Draw (E , EA = EC) circle on CA hypotynousa , intersecting OE line at point B , and from point B draw the circle (B ,BE = BB_e) and draw on CP hypotynousa circle (P`, P `C = P`P)
3.. Draw circle (O , OB_e) intersecting CA line produced at points at point A_e, and Draw A_eP intersecting (O , OA) circle at point N , and (P` , P ` P) circle at point H .
4.. Draw NA produced at point M on (E , EA) circle , and joinchord MC on circle .
5.. Square CMNH isequal to the circle (E , EA) and issues → $\pi . CE^2 = CM . CH$



F.2-A A Presentation of the Quadrature Method on Dr. Geo-Machine Macro - constructions .

The Inscribed Square CBAO , with Pole-line AOP , rotatesthrough Pole P , to the→ Circle-SquareCMNH with Pole-line NHP ,and to the→ Circumscribed SquareCAC`P , with Pole-lineC`PP ≡ C`P ,of the circleE,EO= EC.
The limiting Position of circle (E , EB)to (B , BE =BB_e)definesB_e point , and OB_e=OA_e radius , such that CMNH Square be equal to π . OA ² .
The Initial relation Position CE² = EB.EO = EO² = $\frac{(CA)^2}{4}$ becomes →$\frac{(CN)^2}{4}$= π .$\frac{(CA)^2}{4}$,
for all Squares $C\ M_z N_z H_z$ on circles of Expanding radius OG_e to OB , to OB_e and to OZ .
This hasa Special-reason for square CE² to becomeequal tonumber π.

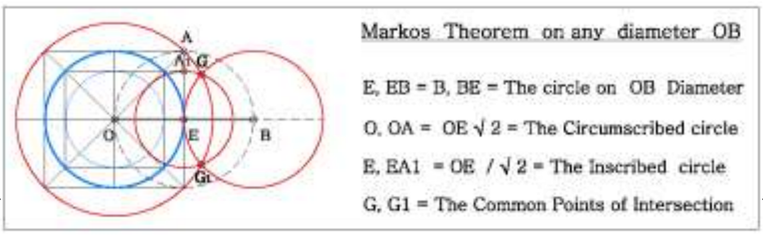
II. ANALYSIS:

In (1) -F.2 , RadiusEA = EC and the unique circle (E , EA) of Segment AC , where AC , CA is The monad the Anti-monad.

In (2) - F.2 , Since circles (E , EA) , (P`, P`P) are symmetrical to OC axis (line) then are equal (conjugate) and since they are Perpendicular so , → No work is executed for any motion ← .

In (3) Points A ,C, P and O are the constant Poles of Rotation , and OB ,OB` ,OC–C A,CP, AP the Six , Pole and Anti – Pole, lines , of sliding points Z , Z`, andA_Z,$A`_Z$, while CA , CP are the constant Pole –lines {PA ,P$A_e$ ,P$A_Z$ , PC`}, of Rotation through pole P . In (4) Circles (E, EO) , (P`, P`O) on diameters OB, OB` follow, My Theorem of the three circles on any Diameters on a circle ,where the pair of points G , $G_1$and G`,$G`_1$consist a Fix and Constant system of lines G$G_1$and G`$G`_1$ . When Points Z,Z` coincide with the Fix points B,B` and thus forming the inscribed Square CBAO or CZAO , ( this is because pointZ is at point A ). The PA , Pole-line , rotates through pole P where $G_e$ , $B_e$,are the Edge points of the sliding poles on thisRectilinear-Rotating System . In (5) When point Point Z≡B , Z`≡B`on lines OB,OB`, then points A_z , $A`_z$ , are the Sliding points while CA,CP , are the constant Pole–lines {PA, P$A_z$, P$A_e$, PC`}, of Rotation through poleP.Sliding points Z, Z`, $A_z$ , $A`_z$, are forming Squares CMNH , CM`N`H` , and this as in Proof [A-B]below ,where PN , AN` are thePole-lines rotating through poles P, A , and diamesus HM passes through O .The circles (E, EO),(P`, P`O) on diameters OB , OB`, blue color, follow also , my Theorem of the Diameters on a circle which follows.

In (6) , Sliding poles Z,Z` being at Edge point $G_e$≡ Z formulatesCBAO Inscribed square, at Edge point $B_e$, $B_e$≡ Zformulates CMNH equal square to that of circleand , at Edge point B∞ , formulates CAC`P square , which is the Circumscribed square.
In (7) , are holding→ CBAO the Inscribed square , CMNH , The equal to the (E, EO = P`O) Circle - square, and CAC`P the Circumscribed square .



F.3. → Markos Theorem , on any OB diameter on a circle .
Theorem : [F.1-(2)] , F.3
On eachdiameterOEB of any circle (E , E B) we draw,
1.. the circumscribed circle(O , OA = OE .√2) at the edge point O as center ,
2.. the inscribed circle(E ,OE/√ 2 = OA/2 = EG)at the mid-point E as center ,
3.. the circle(B , BE = B ,B_e) = (E , EO) at the edge point B as center ,

Then the threecirclespassthrough the common points G ,G_1, and the symmetrical to OB point G_1forming an axis perpendicular to OB, which has the Properties of the circles , wherethe tangent frompoint B to the circle (O , OA = OC) is constant and equal to 2.EB ^{2}, and has to do with ,Resemblance Ratio equal to 2 . Circle issquared on thisGeometric Procedure by Rotation ,Expantion and Translation.
The Common-Proofs [A-B-C] :
In F.1-(2) , F.2-(5) ,
Angle < CHP = 90° becauseis inscribed on the diameter CP of the circle (P′, P′P) .
The supplementary angle < CHN =180 – 90 = 90° . Angle < PNA = PNM = 90° becauseis inscribed on the diameter AP of the circle (O , OA) and Angle < CMA = 90 ° becauseis inscribed on the diameter
CA of the circle (E , EA = EC) .
The upperthree angles of the quadrilateral CHMN are of a sum of 90+90+90 = 270 , and from the total of 360° , the angle < MCH = 360 – 270 = 90° ,Therefore shape CMNH is rightangledand exists CM ⊥ CH .
Sincealso CM ⊥CH and CA ⊥ CP therefore angle < MCA = HCP .
The rightangled triangles CAM , CPH are equalbecause have hypotynousa CA = CP and also angles < CMA=CHP = 90° , < MCA = HCP , thereforeside CH = CM , and Because CH = CM , the rechtangleCMNH is Square . The same for Square CM`N`H` . (ο.ε.δ),(q.e.d) .
This is the General proof of the squares on this Mechanism without any assumptions .
From the equal triangles COH,CBM angle < CHO = CHM = 45∘ because lie on CO chord ,
and so points H,O,M lie on line HM i.e.
On CA line , Any segment PA → PA_z→ PA_e → PC` = CA , drawn from Pole , P,beginning from A to ∞ , is intersecting the circumscribed (O,OA) circle , and the circle (P`, P`P = P`C = EO = EC) at the points N,H, andFormulates SquaresCBAO, CMNH, C$M_zN_zH_z$, CAC`P respectively , which are ,
TheInscribed , In-between, Circumscribed Squares ,of circle(O,OE) = (E,EO = EB) = (P, P`O).
Since angles < CA_zP , HCP have their sides CA_z⊥ CP , A_zP⊥CH_z perpendicular each other , then are equal so angle < PA_zC = PCH_z, and so pointA_z , is common to circle O,OZ , Pole-line CA , and Pole-axis PN , where the perpendicular to CM .
Since PE isdiameter on (P_1,P_1P) circle , therefore triangle E.I_g.P is right-angled and segment ,EI_g, perpendicular to OA and equal to OE/√2 = OA/2 , the radius of the Inscribed circle . Sincealso point ,I_g, lies on PA , therefore moves on (P_1,P_1 P) circle and point A on CA Pole-line , and sopoint B is on the same circle as A_z , while point B moves on circle E,EB .
B.. Proof (1) : F.2-(5), F.2-A
(1)AnyPoint Z , whichmoves on diameter OB produced , BeginningfromEdge-pointG_e of the first circle , Passing from center B of the second circle , Passing fromEdge-pointB_e of the third circle , and Ending to infinite ∞ , → Creates onthe threecircles(O,OA) , (E,EO) , (B,BE) , withtheircenters on the diameter OB , the Changeable moving Squares
a)..The Inscribed CBAO , when point Z ≡ $G_e$and center point O ,
b)..The In-between C$M_zN_zH_z$when point Z ≡ Band center point E ,
c)..The Extrema CMNH , when point Z ≡ B_e and center point B ,
d)..The Circumscribed CAC`P . when point Z ≡ B_∞ and center point ∞ ,
(2). Through the four constant Poles A,C,P – O of the Plane Procedure Mechanism , Squares Rotate through P , the Sides and Diamesus Slide on OB as Squares , Anti-Squares.Point Zmovingfrom Edge points G_e(forming Inscribed square CBAO) , to in-between points G_e -B_e(formingsquares C$M_zN_zH_z$) , to Extrema point B_e (formingsquareCMNHequal to the circle) ,and toB_e - ∞ .
(3). Point I_g,belongs to the Inscribed circle (E,EO) and isRotating ,expanding, Inscribed Edgepoind on (P_1,P_1P) circle to I_g, I_b,$I_e$and to → P point . The othertwo , Sliding , Edgemoving points B,A slide on OB , CA , Pole-linesrespectively .In Initial square COAB and rightangled triangle COB the side CE squaredis CE ² = EB.EO = [√2CB/2] . [√2CB/2] = CB² /2 . In Edge square CMNH and rightangled triangle CHM the side CN/2 squaredis$CE_e{}^2$ = E_eM.E_eH. = [√2CM/2] . [√2CM/2] = CM² /2. In Infinite square CAC`P and rightangled triangle CPA the side CC`/2 =COsquaredisCO^2 = OA.OP = [√2CA/2] . [√2CA/2] = CA² /2 .Fromabove relations and sinceCE=OE ,CE_e= (HM/2),CO=CC`/2then ,
OE ² = CB²/2 = 2.CE² / 2 = [2/2] . CE² = k . CE² , where k = [2/2] = 1
$CE_e{}^2$ = CM² /2 = k.(CB² /2) wherek = CM²/ CB² = CM²/ 2CE²
CO^2 = CA² /2 = 2 . [CB² /2] = 2.CE² =k .CE² , where k = [2/2/2] = 2
A– Proof (2) : F.2-(5),F.2-A

Since BC ⊥ CO , the tangent from point B to the circle (O , OA) isequal to :
BC^2 = BO^2– OC^2 = (2. EB)^2– (EB .√ 2)^2 = 2. EB^2 = (2.EB).EB = (2.BG) . BG and since 2.BG = BG_1 thenBC² = BG . BG_1 , where point G_1 lies on the circumscribed circle , and thismeansthatBG producedintersects circle (O, OA) at a point G_1twice as muchas BG . Since E is the mid-point of BO
and also G midpointof BG_1, so EGis the diamesus of the twosides BO,BG_1 of the triangle BOG_1 and equal to 1/2 of radius OG_1 = OC , the base , and since the radius of the inscribed circle ishalf(½)
of the circumscribed radius then thecircle (E , EB/√2 = OA/2) passes through point G .Because BC is perpendicular to the radius OC of the circumscribed circle , so BCis tangent and equal to
BC ^{2} = 2 . EB ^{2}, i.e. the above relation .
Proofs F.(2) : (5-6) :

Following again prior A-B common proof ,

Angle < CHP = 90° becauseis inscribed on the diameter CP of the circle (P′,P′P) . The supplementary angle < CHN =180 – 90 = 90° . Angle < PNA = PNM = 90° becauseis inscribed on the diameter AP of the circle (O , OA) and Angle < CMA = 90 ° becauseis inscribed on the diameter CA of the circle (E , EA = EC) . The upperthree angles of the quadrilateral CHMN are of a sum of 90+90+90 = 270 , and from the total of 360° , the angle <MCH =360–270 = 90°, thereforeshape CMNH is rightangled and exists CM ⊥ CH .

Sincealso CM ⊥ CH and CA ⊥ CP therefore angle < MCA = HCP .

The rightangled triangles CAM , CPH are equalbecause have hypotynousa CA = CP and also angles < CMA=CHP = 90°, <MCA= HCP and side CH = CM therefore , rechtangle CMNH is Square onCA,CP Mechanism , through the three constant Poles C,A,P of rotation . The same for squareCM\`N\`H\` .From the equal triangles COH , CBM angle < CHO = CHM = 45∘ then points H,O,M lie on line HM .i.e. Diagonal HM of squares CMNH on Mechanism passes through central Pole O.

The twoequaland perpendicular vectors CA , CP , whichis the Plane Mechanism , of theseChangableSquares through the two constant Poles C, P of rotation , isconvertingthe Circular motion to Four-Polar Rotational motion , and aslinear motionthroughpoints O,A .

Transferring the above property to [F.2 –(5)] then when point Z moves on OB line → Point A_Z moves on CA and → PA_ZSegment rotates through point P,defining on circle (P_1,P_1 P =P_1 E) ,

the Idol ,[the points $I_Z$on circles O,OA = The Circumscribed P\`P\`O = The Circle] ,and points H,N such that shapes → CHNM are all Squares between the Inscribed and Circumscribed circle . i.e.

Archimedes trial , The Central – Expansion of the Inscribed to the Circumscribed circle,
is altered to the equivalent as , Polar and Axial motion on this Plane Mechanism .

The areas of above circles are→

Area of Inscribed $= \frac{1}{2}\pi.OE^2 = \frac{1}{2}\ \pi.\frac{CB^2}{2} = \ \pi.\frac{CB^2}{4} = [\frac{k\pi}{4}]\ .CB^2$

Area of Circle $= 1\ \pi.OE^2 = 1\ \pi.\frac{CM^2}{2} = k\pi.\frac{CB^2}{4} = [\frac{k\pi}{4}]\ .CB^2$

Area of Circumscribed $= 2\ \pi.OE^2 = 2\ \pi.\frac{CA^2}{2} = 2\ k\pi.\frac{CB^2}{4} = [\frac{k\pi}{2}]\ .CB^2$

and those of corresponding squares , then one square of Plane Mechanism is equal to the circle ,
but which one ??.

→That square whichisformed in Extrema Case of The Plane Mechanism :

The radius of the inscribed circle is AB/2 and equal to the perpendicular distance between center E and OA , so any circle of EP diameter passes through the edge-point (I_g) , and point (I_b) is the Edge common point of the two circles .G_e,

The Common Edge –Point of the three circles is (I_e) belongs to the Edge point Be of circle (B,BE = BB_e) , so exists ,

Case : [1] [2] [3] [4]
PointZ at →G_e B $B_e B_\infty$
Point A at → A A(I) $A_e A_\infty$
PointIg at → $I_g I_Z = I_b I_e$ P
↓ ↓ ↓ ↓
Square CBAO , $CM_i N_i H_i$,CMNH , CAC\`P

i.e. Square CMNH of case [3] is equal to the circle , andCM ² = CH ² = π . EA² = π . EO ²

On the threeCircles(E,EO), (P_1,P_1,P) , (O , OZ) and Lines OB,CAexists → F.2 - (5)

a)..Circle (O,OZ = OG_e)is Expanding to → (O,OZ = OB_e) Circumscribed circle , for the Inscribed CBAO square ,

b).. PointA , to → (A-A_Z) is The Expanding Pole-line A-$A_Z$for the In-between $CM_Z N_Z H_Z$square ,

c).. Circle (P_1,$P_1$$I_g$) is Expanding to → (P_1,$P_1 I_b$) Inscribed circle (E,E.I_g) to I_b and $I_e$point.

d).. Circle (O,OB →OB_∞, Pole-lines (A –AA_e→A_∞) and(P –PI_e= PP → P) , for CAC\`P square, Point N on (O,OA) , belongs to Circumscribed circle Point$I_e$, on circle withdiameter ,PE , belongs to the Inscribed circle (E ,$EI_g$ = EG) Point H , on (P\`,P\`O) , belongs to the Circle.

i.e. It wasfound a Mechanism where the Linearly Expanding Squares → CBAO – CMNH – CAC\`P , and circles → (P_1,P_1E) – (B, BE) – (O,OA),which are between the Inscribed and Circumscribed ones , arePolarly–Expanded as Four – Polar Squares .

The problemis in two dimensions determining an edge square between the inscribed and the circumscribed circle. A quick measure for radius r = 2694 m givessideof square 4775 m
and π = 3,1416048 → 11/10/2015

The Segments CM = CM \`, is the Plane Procedure Quantization of radius
EC = EO= CP\` in Euclidean Geometry , through this Mould , the Mechanism.

The Plane Procedure Method is called so , because it is in two dimensions → CA ⊥CP , as this happens also in , Cube mould , for the three dimensions of the spaces ,which is a Geometrical
machine for constructing Squares and Anti -Squares and that one equal to the circle .

This is the Plane Quantization of , E-Geometry, i.e.The Area of square CMNH is equal to that of one of the five conjugate circles , or CM ² = π . CE ² ,and System with numberπtobe a constant .

III. Remarks

Since Monads AC = ds = 0 → ∞ are simultaneously (actual infinity) and (potential infinity) in Complex number form , this defines that the infinity exists also between all points which are not coinciding , and ds comprises any two edge points with imaginary part , for where this property differs between the infinite points between edges .This property of monads shows the link between Space and Energy which Energy is between the points and Space on points. In plane and on solids , energy is spread as the Electromagnetic field in surface .The position and the distance of points , can be calculated between the points and so to perform independent Operations (Divergence, Gradient , Curl , Laplacian) on points .

This is the Vector relation of Monads , ds = CA ,(or , as Complex Numbers in their general formw = a + b. i= discrete and continuous) , and which is the Dual Nature of Segments = monads in Plane,tobe discrete and continuous). Their monad –meter in Plane ,and in two dimensions is CM , the analogous length ,in the above Mechanism of the Squaring the circle with monad the diameter of the circle . Monad isds = CA= OB , the diameter of the circle (E,EA) with CBAO Square , on the Expanding by Transportation and Rotation Mechanismwhich is → {Circumscribed circle (O,OA) – Inscribed circle (E , EG = EI_g) - Circle (B,BE) }←In extended moving System → {OB Pole-line – CA Pole-line – Circle $(P_1, P_1\text{B}=P_1.I_g)$} ,andis quantizedto CMNH square.

The Plane Ratio square of Segments –CE , CM- is constant and Linear , and for any Segment CN / 2on circle in Square CMNH exists another one CE such that ,

→ $EC^2 / (CN/2)^2 = k$ = constant←

i.e. the Square Analogy of the Heights in any rectangle triangleCOB is linear to Extrema Semi-segments (CN/2) or to (CA/2) , or the mapping of the continuous analog segment CE to the discrete segment (CN/2) .

The Physical notion of Quadrature :

The exact Numeric Magnitude of number ,π, may be found only by numeric calculations.[44] All magnitudes existon the <Plane Formation Mechanism of the first dimentional unit AB >as geometricalelementsconsisting , the Steady Formulation , (The Plane System of the Isosceles Right-angle triangle ACPwith the threeCircles on the sides) and the movingand Changeable Formulation of the twin , System-Image , (This Plane Perpendicular System of Squares , Anti-squares issuchthat, the Workproduced in a betweenclosed area to beequal to zero) .

Starting from this logic of correlation upon Unit , we can control Resemblance Ratio and construct all Regular Polygons on the unit Circle as this is shown in the case of squares .

On this System of these three circles F.3 (The Plane ProcedureMechanism which is a Constant System) is created also, a continues and , a not continues Symmetrical Formation , the changeable System of the Regular Polygons , and the Image (Changeable System of Regular anti-Polygons) theIdol ,as much this in Space and also in Time , and was proved that in this Constant System ,the Rectilinear motion ofthe Changeable Formation is Transformed into a twin and Symmetrically axial-centrifugal Pole rotation (this is the motion on System) .

The conservation of the Total Impulse and Momentum , as well as the conservation of the Total Energy in this Constant System with all properties included , exists in this Empty Space of the un-dimensional point Units of mechanism. All the forgoing referred can be shown (maybe presented) with a Ruler and a Compass , or can be seen , live , on any Personal Computer .The method is presented on Dr.Geo machine . The theorem of Hermit-Lindeman that number , pi , is not algebraic , is based on the theory of Constructible numbers and number fields (on number analysis) and not on the <Euclidean Geometrical origin-Logic onunit elements basis >The mathematical reasoning (the Method) is based on the restrictions imposed to seek the solution <i.e. with a ruler and a compass > . By extending Euclid logic of Units on the Unit circle to unknown and now proved Geometrical unit elements ,thus the settled age-old question for the unsolved problems is now approached and continuously standing solved . All Mathematical interpretation and the relative Philosophical reflections based on the theory of the non-solvability must properly revised .

Application in Physics :

From math theory of Elasticity , Cauchy equations of Stresses in three dimensions are ,

$\frac{\partial \sigma x}{\partial x}+\frac{\partial \tau yx}{\partial y}+\frac{\partial \tau zx}{\partial z}$ + X = 0 $\frac{\partial \tau xy}{\partial x}+\frac{\partial \sigma y}{\partial y}+\frac{\partial \tau xy}{\partial z}$ +Y =0 $\frac{\partial \tau xz}{\partial x}+\frac{\partial \tau yz}{\partial y}+\frac{\partial \sigma z}{\partial z}$ + Z = 0 where are ,

σx ,σy ,σz =Principal stresses in x,y,z axis ,τxy,τxz,τyz= shear-stresses in xy,xz,yzPlane,

X,Y,Z = The components of external forces and of Strain ,$\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} = 0$, $\frac{\partial}{\partial x}\frac{\partial v}{\partial y} =0$,$\frac{\partial}{\partial x}\frac{\partial w}{\partial z} = 0$

where u = u (y,z) → are Deformation components , the displacements , in y,z axis .

v = c x z = the Rotation on z , axis

w = - c x y Anti-rotation in y axis .

Applying above equations on an orthogonal section of a solid , then exist the differential equations of equilibrium , and for the boundary conditions is found that , the Stress function is satisfying equations ,

$\frac{\partial \tau yx}{\partial y}+\frac{\partial \tau zx}{\partial z} = \frac{\partial \gamma yx}{\partial y}+\frac{\partial \gamma zx}{\partial z} = \frac{\partial^2 u}{\partial y^2}+\frac{\partial}{\partial x}\frac{\partial v}{\partial y}+\frac{\partial^2 u}{\partial z^2}+\frac{\partial}{\partial x}\frac{\partial w}{\partial z} = 0$ ……….. (1)

and the boundary conditions on solid`s surface , $\frac{\partial u}{\partial y}$ dz - $\frac{\partial u}{\partial z}$.dy+y.dy+z.dz = 0 ……..(2)

where ,γxy ,γxz,γyz= the slip components where is , γxy = $\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}$.

Equations show that the resultant shear-stress at the boundary is directed along the tangent to the boundary and that , the Stress function u =u(yz) must be constant along the boundary of the cross section . i.e. each cross section on x, axis is rotated as a disk in its plane , from which points follow relation u = u(yz) and since stress function are constant , then from equation (2) y.dy + z.dz = 0 or $y^2 + z^2$ = constant , meaning that , a Cross-section under Stress stays Plane only in circle circumference , or a Plane Space , under Energy Stress , remains Flat only when the Plane becomes a circle , i.e .follows the Plane Mould which is the squaring of the circle.

The same is seen in Laplace`s equation$\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial y^2} \equiv \nabla^2 u = 0$ which is termed a harmonic function.

Placing $\nabla^2 u = 0$ in both parts of the equation of the circle , becomes Identity and $\nabla^2 u.(y^2+z^2) = \nabla^2 u.(c)$,

or any Monad = Quaternion , consisted of the real part the Plane Space , and under Energy Stress the imaginary part , remains in Flat only when the Plane becomes a circle , i.e. theEnergy-Space discrete continuum follows extrema E-geometry Mould ,π, which is the squaring of the circle.

If Potential Energy is zero then vector $\bar{\tau}$ is on the surface indicating the conjugate function. [49]. In Electricity , when an electric current flows through a conductor , then a transverse circular Electromagnetic field is produced around itself following the vector – cross-product Plane mould ,π. Because , the nth- degree - equations are the vertices of the n-polygon in circle so , π , is their mould .

2.. The Duplication of the Cube ,

Or the Problem of the two Mean Proportionals , The Delian Problem.

The Extrema method for the

Duplication of the cube ? [44-45]

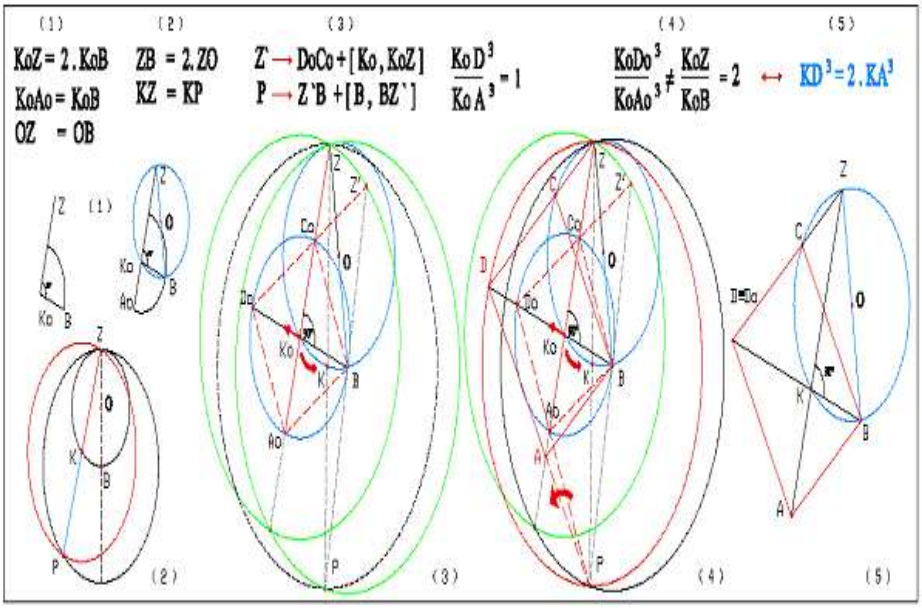
This problem is in three dimensions as this first was set by Archytas proposed by determining a certain point as the intersection of three surfaces , a right cone , a cylinder,a toreor anchoring with inner diameter nil. Because of the three master-meters where there is holdingthe Ratio of two or three geometrical magnitudes , is such that they have a linear relation (continuous analogy) in all Spaces,

the solution of this problem , as well as that of squaring the circle , is linearly transformed .

The solution is based on the known two locusof a linear motion of a point .

The geometrical construction Step – By – Step in F-4 :

The Presentation of the method on Dr-Geo machine for macro constructions in F.4-A.

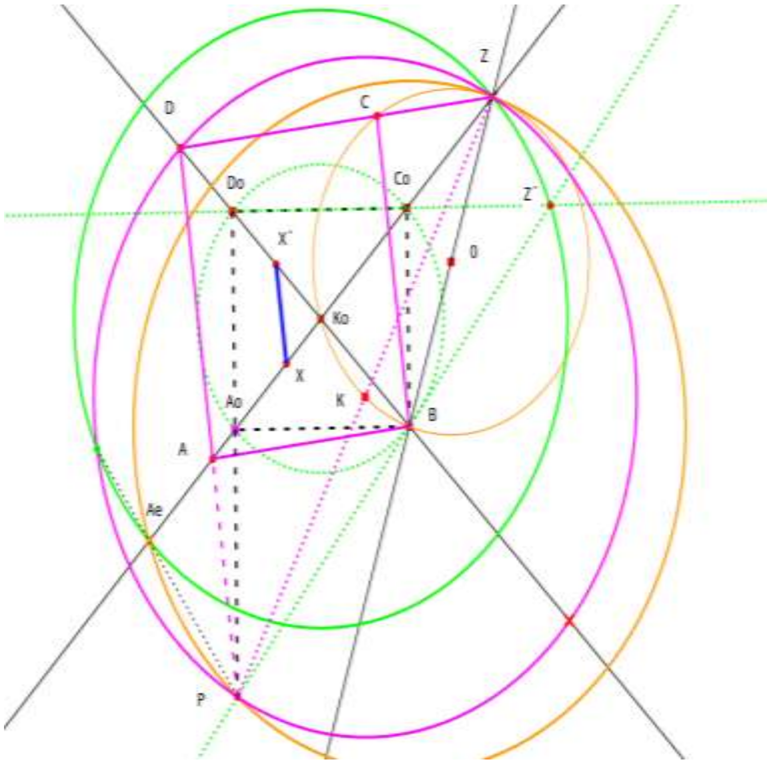


(1) (2) (3) (4) (5)

F.4.. → The Mechanical Extrema Constant Poles Z , K , P of rotationin any circumcircle of triangle ZKoB

1.. Draw on any Orthogonal - System K_oZ ⊥ K_oB , Segment K_oZ = 2.K_oB and on BZ as hypotynousa the circle (O , OB = OZ) .
2.. Draw on K_oZ produced K_oA_o = K_oB and form the square $B\ C_oD_oA_o$, . 3.. Draw the circles (K_o ,K_oZ) , (B , BZ) which are intersected at points Z , A_e , and D_oC_o produced at point Z ` , and $D_oA_o$produced at point P .
4.. Draw on ZP as diameter the circle (K , KZ = KP) intersecting$K_oD_o$produced at point D and join DZ , DP intersecting the circle (O ,OZ) and line $K_oA_o$produced at point A . 5..On Rectangle BCDA , the Cube of Segment K_oD is twice the Cube of Segment KoA and , existsK_oD ³ = 2. K_oA ³

F4-A. → A Presentation of the Dublication Method on Dr.Geo - Machine Macro - constructions
$B\ C_oD_oA_o$, Is the initial Basic Quadrilateral ,square , on K_oZ , K_oB Extrema-lines mechanism.
BCDA is the In-between Quadrilateral , on (K,KZ) Extrema-circle , and on K_oZ-K_oB Extrema lines of common poles Z , P , mechanism . The Initial Quadrilateral B$C_oD_oA_o$, with Pole- lines D_oA_oP - D_oC_oZ `, rotatesthrough Pole P and the moveable Pole Z ` on Z`Z arc , to the → Extreme Quadrilateral BCDA through Pole-lines DAP - DCZ with pointD_o , sliding on BK_oD_oPole-line .
The Final Position of the Rotation – Translation isQuadrilateralBCDA whereK_oD ³ = 2. K_oA ³



2.1. The Processus of The Duplication of Cube : $F.4$, $F4-A1$..Draw Line segment K_oZ tobe perpendicular to its half segment K_oB or as K_oZ = 2.K_oB ⊥K_oB
and the circle (O, BZ/2) of diameter BZ . Line -segment ZK_o produced to K_oA_o= K_oB (or and $K_oX_o \neq K_o$B) is forming the Isosceles right-angled triangle A_oK_oB.

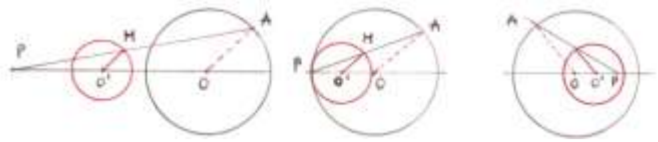
2.. Draw segments BC_o , A_oD_o equal to BA_o and be perpendicular to A_oB such that points C_o ,D_o meet the circle (K_o,K_oB) in points C_o ,D_o,respectively, and thus forming the inscribed square $B\ C_oD_oA_o$. Draw circle (K_o,K_oZ) intersecting line $D_oC_o$produced at point Z ` and draw the circle
(B , BZ) intersecting diameter Z `B , produced at point P (the constant Pole) .

3.. Draw line ZP intersecting (O, OZ) circle at point K , and draw the circle (K , KZ) intersecting line BD_o produced at point D .
Draw line DZ intersecting (O, OZ) circle at point C and Complete Rectangle CBAD on the diamesus BD .
Show that this is an Extrema Mechanism on where ,
The Three dimensional Space KoA→ is Quantized toK_oD as → K_oD ³ = 2. K_oA³ .
Analysis :
In (1) - F.4 , K_oZ = 2.K_oB and K_oA_o= K_oB , K_oB ⊥K_oZ and K_oZ / K_oB = 2.
In (2) Circle (B, BZ) with radius twice of circle (O, OZ) isthe extremacase where circles with radius KZ = KP are formulated and are the locus of all moving circles on arc BK as in F4-(2) , F.5
In (3) Inscribed square $B\ C_oD_oA_o$. passes through middle point of K_oZ so C_oK_o = C_oZ and since angle < ZC_oO = 90▫ , then segment OC_o // BK_o and BK_o = 2.OC_o .
Since radius OB of circle (O,OB = OZ) is ½ of radius OZ of circle (B,BZ =2.BO) then ,D, is isExtrema case where circle(O,OZ) is the locus of the centersof all circles (K_o, K_oZ) , (B, BZ) moving on arc ,K_oB, as this was proved in F.5.

All circles centered on this locus are common to circle (K_o, K_oZ) and (B,BZ) separately. The only case of being together is the common point of these circles which is their common point P , where then → centered circle exists on the Extrema edge , ZP diameter.
In (4) , F4-(4) Initial square A_oBC_oD_o , Expands and Rotates through point B , while segment $D_oC_o$limits to DC , where extrema point Z ` moves to Z . Simultaneously , the circle of radius $K_o$Z moves to circle of radius BZ on the locus of ½ chord $K_o$B . Since angle < Z `D_oA_oP is always 90 º so , exists on the diameter Z`P of circle (B, BZ `) and is the limit point of chord D_oA_o of the rotated square B$C_oD_oA_o$, and not surpassing the common point Z .
Rectangle B$A_oD_oC_o$ in angle < PD_oZ ` is expanded to Rectangle BADC in angle < PDZ by existing on the two limit circles (B,BZ`= BP) and (K_o, K_oZ) and point D_o by sliding to D .
On arc K_oB of these limits is centered circle on ZP diameter , i.e. Extrema happens to →
the common Pole of rotation through a constant circle centered on $\boldsymbol{K_o}$B arc , and since point Do is the intersection of circle (K_o ,K_oB = K_oD_o) which limit to D , therefore the intersection of the common circle (K , KZ = KP) and line K_oD_o denotes that extrema point , where the expanding line D_oC_oZ` with leverarm D_oA_oP is rotating through Pole P, and limits to line DCZ ,and Point P is the common Pole of all circles on arc ,$\boldsymbol{K_o}$B,for the Expanding and simultaneously Rotating Rectangles.
In (5) rectangle BCDA formulates the two right-angled perpendicular triangles ADZ , ADB which solve the problem. Segments $\boldsymbol{K_o}$D , $\boldsymbol{K_oA_o}$= $\boldsymbol{K_o}$B are the two Quantized magnitudes in Space (volume) such that Euclidean Geometry Quantization becomes through the Mould of Doubling of the Cube .
[This is the Space Quantization of E-Geometry i.e. The cube of Segment $\boldsymbol{K_o}$D is the double magnitude of $\boldsymbol{K_o}$A cube , or monad $\boldsymbol{K_o}$D³ = 2 times the monad $\boldsymbol{K_o}$A³] .About Poles in [5] .

The Proof :   F.4. (3)-(4)-(5) .
1.. Since K_oZ = 2.K_oB then (K_oZ / K_oB) = 2 , and since angle < ZK_oB = 90^o then BZ is the diameter of circle (O,OZ) and angle <ZK_oB =90^o on diameterZB
2.. Since angle < Z$K_o A_o$ = 180^o and angle< ZK_oB = 90^o therefore angle   < B$K_o A_o$= 90^o also .
3.. Since B$K_o \perp$ZK_o then K_o is the midpoint of chord on circle (K_o,K_oB) which passes through Rectangle (square) $B\, A_o D_o C_o$. Since angle< ZDP = 90^o (because exists on diameter ZP) and since also angle <BCZ =90^o (because exists on diameter ZB) therefore triangle BCD is right-angled
and BD is the diameter .
Since Expanding Rectangles $B\, A_o D_o C_o$, BADC rotate through Pole ,P, then points A_o,A
lie on circles with BD_o , BD diameter , therefore point D is common to BD_o line and (K,KZ = KP) circle , and BCDA is Rectangle . F.4-(2)   i.e. Rectangle BCDA possess A$K_o \perp$ BD and DCZ a line passing through point Z .
4.. From right angle triangles ADZ , ADB we have ,
On triangle Δ ADZ   →   KD ² = KA . KZ   … (a)
On triangle Δ ADB   →   KA ² = KD . KB   … (b)
and by division (a) / (b) then   →
KD² = KA.KZ KD²   KA.KZ KD³ KZ
----------------- = |------|=|---------| or|-----|=|-----|= 2
KA²=KD.KB   KA² KD.KBKA³ KB   (o.ε.δ),(q.e.d)
i.e. →$\boldsymbol{K_o}$D ³ = 2 . $\boldsymbol{K_o}$A ³ ,which is the Duplication of the Cube .
In terms of Mechanics , Spaces Mould happen through , Mould of Doubling the Cube , where for
any monad ds =$\boldsymbol{K_o}$A analogous to$\boldsymbol{K_o A_o}$, the Volume or The cube of segment $\boldsymbol{K_o}$D is double the volume of $\boldsymbol{K_o}$A cube ,or monad K D³ =2.$\boldsymbol{K_o}$A³.This is one of the basic Geometrical Euclidean Geometry Moulds , which create the METERS of monads →where Linear is the Segment M$\boldsymbol{A_1}$, Planeis the square CMNH equal to the circle and in Space, is volume $\boldsymbol{K_o}$D³ = K D³in all Spaces , Anti-spaces and Sub -spaces of monads = Segments ← i.e
The Expanding square $\boldsymbol{B\, A_o D_o C_o}$ is Quantized to BADC Rectangle by Translation
to pointZ `, and by Rotation , through pointP(the Pole of rotation) to point Z.

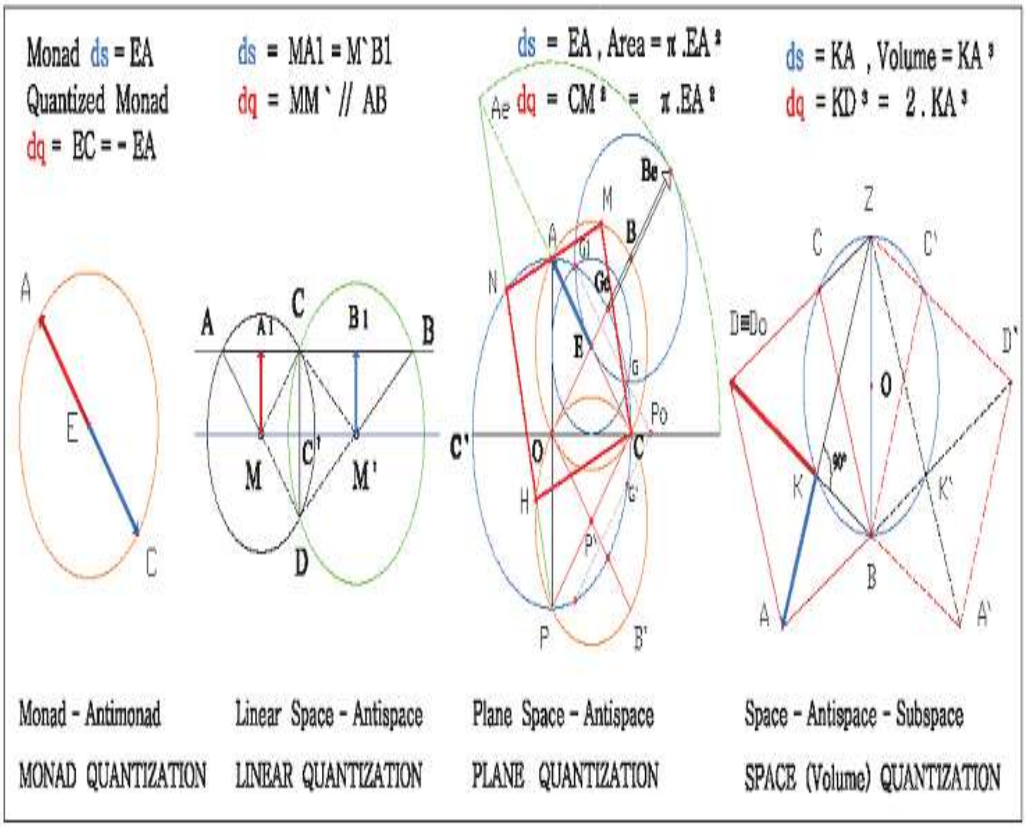
The Constructing relation between segments K_oX , K_oA is → (K_oX)² = (K_oA)².(XX_1 /AD)
such that X X_1// AD , as in Fig.6(4), F7.(3). All comments are left to the readers, 30/8/2015.


(1)   (2)
(3)
F.5. → For any point A on , and POut-On-Incircle [O,OA] and O`P = O`O,exists O`M=OA / 2 .[16]

2.2The Quantization of E-Geometry,{Points,Segments, Lines, Planes, and the Volumes} ,
to its mouldsF-6 .
Quantization of E-geometry is the Way of Points to become asa → (Segments , Anti-segments = Monads = Anti-monads) , (Segments , Parallel-segments = Equal monads) , (Equal Segments and Perpendicular-segments = Plane Vectors) , (Non-equal Segments and twice-Perpendicular-segments = The Space Vectors = Quaternion) , by defining the mould of quantization .

The three Ways of quantizationare→ for Monads= The Material points , the Mouldis the Cycloidal Curl Electromagnetic field,for Linesthe Mould isthat of Parallel Theorem with the
least constant distance ,for Plane the Mould is the Squaring of the circle,π,and ,for Spaceis
the Mould of the Duplication of cube$\sqrt[3]{2}$ .All methods in, F-6below .
In [61] The Glue-Bond pair of opposites [⊖⊕] ,creates rotation with angular velocity w = v/r ,
and velocity v = w.r = $\frac{2\pi}{T}$ = 2πr.f = [$\frac{\sigma}{2}$].(1+$\sqrt{5}$) , frequency f =$\frac{(1+\sqrt{5}\,]).\sigma}{4\pi r}$, Period T = $\frac{4\pi r}{\sigma(1+\sqrt{5})}$

where ± σ are the two Centripetal F_p and Centrifugal F_f forces .
Odd and Even number of opposites , on a Regular Polygon , defines the Quality of Energy- monad.



(1) (2) (3) (4)

F.6. → Quantization for Point E , for Linear ds = MA_1, for Plane ,π, Space (volume) $\sqrt[3]{2}$.

Moulds for E-geometry Quantization are ,ofmonad EA to Anti-monad EC – ofAB line to Parallel lineMM`- ofAE Radius to the CM side of Square of KASegment to KD
Cube Segment .

The numeric METERS of Quantization of anymaterialmonadds = AB are as → In any point A , happens through Mould in itself (The material point as a → ± dipole) in [43] In monad ds = AC , happens through Mould in itself for two points (The material dipole in inner monad Structure as the Electromagnetic Cycloidal field which equilibrium in dipole by the Anti-Cycloidal field as in [43]).
For monad ds = EA the quantized and Anti-monad is dq = EC = ± EA

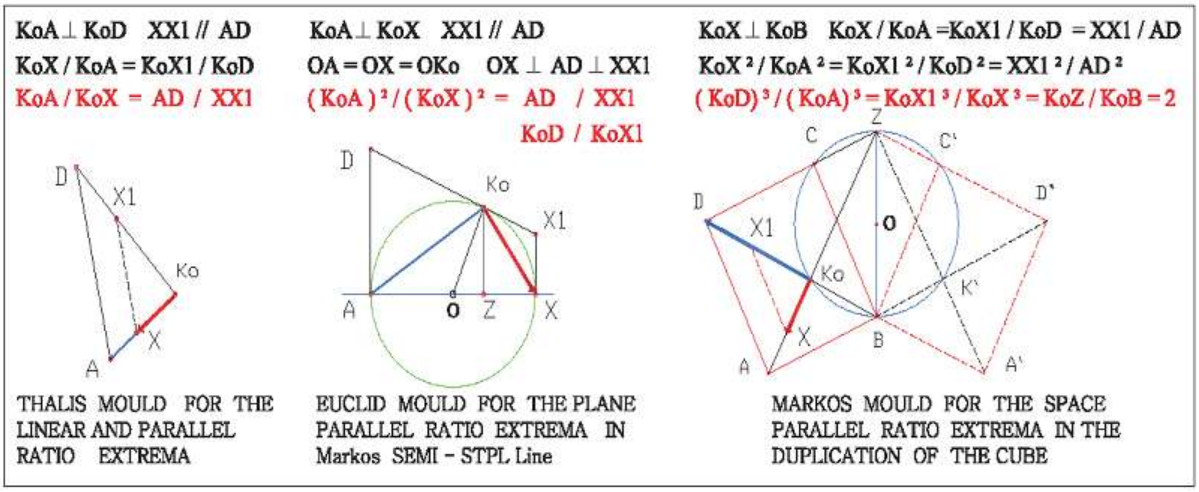
Remark 1: The two opposite signs of monads EA , EC represent the two Symmetrical equilibrium monads of Space-Antispace , the Geometrical dipoleAC on points A,C whichconsistspaceAC as in F6 - (1) Linearly , happens through Mould of Parallel Theorem , where for any point M not on ds = ± AB , the Segment $\boldsymbol{MA_1}$=Segment $\boldsymbol{M`B_1}$= Constant . F6 - (1-2)

Remark 2 : The two opposite signs of monads represent the two Symmetrical monads in the Geometrical machine of the equal and Parallel monads [MM`//AB where$MA_1 \perp$ AB ,
M`$B_1 \perp$ AB and $MA_1$ = M`B_1]which are → The MonadMA_1– AntimonadM`$B_1$ , or → The InnermonadMA1 Structure –The Inner Anti monad structure M`B_1 = - MA_1 = Idle , and
{ The Space = line AB , Anti-space = the Parallel line MM` = constant }.
The Parallel Axiomis no-more Axiombecausethis has been proved as a Theorem [9-32-38-44].
Plainly , happens through Mould of Squaring of the circle , where for any monad ds = CA = CP , the Area of square CMNH is equal to that of one of the five conjugate circles and π = constant , or as $CM^2 = \pi . CE^2$.
On monad ds = EA = EC , the Area = $\pi.EC^2$ and the quantized Anti-monad dq = $CM^2 = \pm \pi. EC^2$and thisbecause are perpendicular and produceZeroWork.F6-(3)

Remark 3 :
The two opposite signs represent the two Symmetrical squares in Geometrical machine of the equal and perpendicular monads as , [CA⊥ CP , and CA = CP] ,whichare → The Square CMNH – Antisquare CM`N`H`, or → The Space – Idol= Anti-Space . In Mechanicsthispropety of monads isveryuseful in Work area , wheretwo perpendicular vectorsproduceZeroWork .{Space = square CMNH ,Anti-space = Anti-square CM`N`H`}. In three dimensional Space , happens through Mould Doubling of the Cube , where for any monad ds = KA , the Volume or , The cube of a segment KD is the double the volume of KA cube , or monad $KD^3 = 2.KA^3$.
On monad ds = KA the Volume = KA^3 and the quantized Anti-monad ,dq = $KD^3 = \pm 2. KA^3$ .F6-(4) Remark 4 :
The two opposite signs represent the two Symmetrical Volumes in Geometrical machine oftriangles [Δ ADZ ⊥Δ ADB] ,whichare → The cube of a segment KD is the double the volume of KA cube – The Anti-cube of a segment K`D` is the double the Anti-volume of K`A` cube , Monad ds = KA , the Volume = KA^3 and the quantized Anti-monaddq = $KD^3 = \pm 2. KA^3$.
{The Space = the cube KA^3 ,TheAnti-Space = the Anti- CubeKD^3 }.
In Mechanicsthisproperty of Material monads isveryuseful in the Interactions ofthe ElectromagneticSystemswhereWork of two perpendicular vectorsis Zero .
{Space =Volume of KA , Anti-space=Anti–Volume of KD, and this in applied to Dark-matter, Dark - Energy in Physics}. [43]

Radiation of Energy is enclosed in a cavity of the tiny energy volume λ ,(which isthe cycloidal wavelength of monad) with perfect and absolute reflecting boundaries wherethis cavity may become infinite in every direction and thus getting in maxima cases (the edge limits) the properties of radiation in free space .

. (1) (2) (3)



F.7.→ The Thales ,Euclid ,Markos Mould , for the Linear – Plane - Space , Extrema Ratio Meters

The electromagnetic vibrations in this volume is analogous to vibrations of an Elastic body (Photo-elastic stresses in an elastic material [18]) in this tiny volume , and thus Fringes are a superposition of these standing (stationary) vibrations .[41]

Above are analytically shown , the Moulds (The three basic Geometrical Machines) of Euclidean Geometry which create the METERS of monads i.e. Linearly is the Segment $M\boldsymbol{A_1}$, In Plane the square CMNH , and in Space is volume KD^3 in all Spaces , Anti-spaces and Sub-spaces .
This is the Euclidean Geometry Quantization in points to its constituents , i.e. the
1.. METER of Point A is the Material Point A , the ,
2.. METER of line is the discrete Segment ds = AB = monad = constant , the
3.. METER of Plane is that of circle ,number π , on Segment = monad , which is the Square equal to the area of the circle , and the
4.. METER of Volume is that of Cube $\sqrt[3]{2}$, of any Segment = monad , which is the Double

Cube of Segment and Thus is the measuring of the Spaces , Anti-spaces and Sub-spaces in this cosmos .

5.. In Physics , METER of Mass is the Reaction of Matter , anything material , against Motion , the contrast Inertia of matter againstkinetic effects , and it is a number only without any other Physical meaning . [39-40]

The meter of mass during a Parallel -Translation is a constant magnitude for every Body ,while for Moment of Inertia during a Rotational - motion is not , except it is referred to the same axis of the Body . markos 11/9/2015 .

2.3 The Three Master - Meters in One , for E-geometry Quantization , F-7

Master - meter is the linear relation of the Ratio , (continuous analogy) of geometrical magnitudes , of all paces and Anti-spaces in any monad .This is so because of the , extrema - ratio - meters . Saying master-meters , we mean That the Ratio of two or three geometrical magnitudes , is such that they have a linear relation (continuous analogy) in all Spaces , in one in two in three dimensions, as this happens to the Compatible Coordinate Systems as these are the Rectangular [x,y,z] , [i,j,k] , the Cylindrical and Spherical -Polar . The position and the distance of points can be then calculated between the points , and thus to perform independent Operations (Divergence , Gradient , Curl , Laplacian) on points only .This property issues on material points and monads .

This is permitted because , Space is quaternion and is composed of Stationary quantities , the position $\bar{r}(t)$ and the kinematic quantities , the velocity $\rightarrow$ $\bar{v}$ = dr/dt and acceleration $\rightarrow$ $\bar{a}$ = d$\bar{v}$/dt = d²r/dt² .

Kinematic quantities are also the tiny Energy volume caves (cycloid is length ,λ, the Space of velocity $\bar{v}$, and $\bar{a}$consist in gravity`s fieldthe infinite Energy dipole Tanks in where energy is conserved) . In this way all operations on edge points are possible and applicable . Remarks :

In F7-(1) ,The Linear Ratio , for Vectors , begins from the same Common point K_0 , of the two concurring and Non-equal , Concentrical and Co-parallel Direction monads K_0X –K_0A and becomes K_0X_1- K_0D .

In F7-(2) ,The Linear Ratio , for Plane , begins from the same Common point K_0, of the two Non-equal , Concentrical and Co-perpendicular Direction monads.

Proof :

Segment K_0A⊥K_0X because triangle AK_0X is rightangled triangle and K_0Z⊥AX . Radius OK_0=OA=OX . Since DA , X_1X are also perpendicular to AX , therefore K_0Z//X_1X //DA. According to Thales theorem ratio (ZA/ZX) = (K_0D/K_0X_1) and since tangentDA = DK_0 and X_1K_0=X_1X then AZ/ZX = DA/XX_1 . From Pythagorean theorem (Lemma 6)→ K_OA²/K_OX²= (AZ/ZX) = (DA/XX_1) = (K_OD/ K_OX_1) i.e. The ratio of the two squares K_OA² , K_OX² are proportional to line segmentsK_OD, K_OX_1) . (ο.ε.δ).

In F7-(3) ,The Linear Ratio , for Volume , begins from the same Common point K_O , of the two Non-equal , Concentrical and Co-perpendicular Direction monads.

In (1) → Segment K_OA ⊥ K_OD , Ratio K_OX / K_OA = K_OX_1 / K_OD , and Linearly (in one dimension) the Ratio of K_OA / K_OX= AD / $X X_1$, i.e. in Thales linear mould [$X X_1$ //AD] , Linear Ratio of Segments $\boldsymbol{X X_1}$, AD is , constant and Linear , and it is the Master key Analogy of the two Segments, monads .

In (2) → Segment K_O A⊥ K_O X, O K_O =OA= OX and since $O X_1$, OD are diameters of the two circles then K_OD = AD, K_OX_1 = $X X_1$,and Linearly (in one dimension) the Ratio of K_OA / K_OX = AD / $X X_1$, in Plane (in two dimensions) the Ratio [K_OA] ² / [K_OX] ² = AD / $X X_1$, i.e.

in Euclid`s Plane mould [K_O A⊥ K_O X] ,

The Plane Ratio square of Segments – $\boldsymbol{K_O}$ A , $\boldsymbol{K_O}$ X- is constant and Linear , and for any Segment $\boldsymbol{K_O}$ Xon circle(O,O $\boldsymbol{K_O}$) exists another one $\boldsymbol{K_O}$ A such that ,

→ $\boldsymbol{K_O}$ A ² / $\boldsymbol{K_O}$ X ²=AD /$\boldsymbol{X X_1}$= $\boldsymbol{K_O}$ D/ $\boldsymbol{K_O X_1}$←

i.e. the Square Analogyof the sides in any rectangle triangleA $\boldsymbol{K_O}$ X is linear to Extrema Semi-segments AD , $\boldsymbol{X X_1}$or to $\boldsymbol{K_O}$ D, $\boldsymbol{K_O X_1}$monads , or the mapping of the continuous analog segment $\boldsymbol{K_O}$ X to the discrete segment $\boldsymbol{K_O}$ A .

In (3) → Segment K_OB⊥ K_OX , O K_O = OB = OZ and since $X X_1$// AD , then K_OA / K_OD= K_OX/ K_OX_1 = AD / $X X_1$, and Linearly (in one dimension) the Ratio of K_OA / K_OX= AD / $X X_1$and in Space (Volume) (in three dimensions) the Ratio [K_O A] ³ / [K_O D] ³ = [K_OX / K_OX_1] ³ = ½ .

i.e. in Euclid`s Plane mould [K_OA // K_OX , K_OD // K_OX_1] , Volume Ratio of volume Segments – $\boldsymbol{K_O}$ A , $\boldsymbol{K_O}$ D-, is constant and Linear , and for any Segment $\boldsymbol{K_O}$ Xexistsanother one $\boldsymbol{K_O X_1}$ such that → ($\boldsymbol{K_O X_1}$) ³ / ($\boldsymbol{K_O}$ X) ³ = 2 ← i.e. the Duplication of the cube.

In F-7 , The three dimensional Space [K_OA⊥ K_OD⊥ K_OX…] , where $X X_1$// AD , The two dimensional Space [K_OA⊥ K_OX] , where $X X_1$// AD , The one dimensional Space [$X X_1$ // AD] , where$X X_1$// AD , is constant and Linearly Quantized in each dimension.

i.e. All dimensions of Monads coexist linearly in Segments –monads and separately (they are the units of the three dimensional axisx,y,z - i ,j, k -) and consequently in all Volumes , Planes, Lines , Segments , and Points of Euclidean geometry, which are all the one point only and which is nothing. For more in [49-51] . 25/9/2015

At the beginning of the article it was referred to Geometers scarcity from which instigated to republish this article and to locate the weakness of prooving these Axioms which created the Non-Euclid geometries and which deviated GR in Space-time confinement. Now is more referred ,

a). There is not any Paradoxes of the infinite because is clearly defined what is a Point and what is a Segment.

b).The Algebra of constructible numbers and number Fields is an Absurd theorybased on groundless Axioms as the fields are , and with directed non-Euclid orientations which must be properly revised.

c).The Algebra of Transcental numbers has been devised to postpone the Pure geometrical thought , which is the base of all sciences , by changing the base-field of the geometrical solutions to Algebra as base.The Pythagorians discovered the existence of the incommensurable of the diagonal of a square in relation to its side without giving up the base of it , which is the geometrical logic.

d).All theories concerning the Unsolvability of the Special Greek problems are based on Cantor`s shady proof, <that the totality of Allalgebraic numbers is denumerable> and not edifyed on the geometrical basic logic which is the foundations of all Algebra .

The problem of Doubling the cube F.4-A , as that of the Trisection of any angle F.11-A , is a Kinematic Mechanical problem with moveable Poles , and could not be seen differently, while Quadrature F.2-Awith constant Poles of rotation and the proposed Geometrical solutions are all
clearly exposed to the critic of the readers .
All trials for Squaring the circle are shown in [44] and the set questions will be answerd on the Changeable System of the two Expanding squares ,Translation [T] and Rotation [R] . The solution
of Squaring the circle using the Plane Procedure methodisnowpresented in F.1,2 , and consists an , Overthrow , to all existingtheories in Geometry , Physics and Philosophy .
e).Geometryis the base of all sciences and itis the reflective logicfrom the objective reality and whichis nature .

The Physical notion of Duplication :

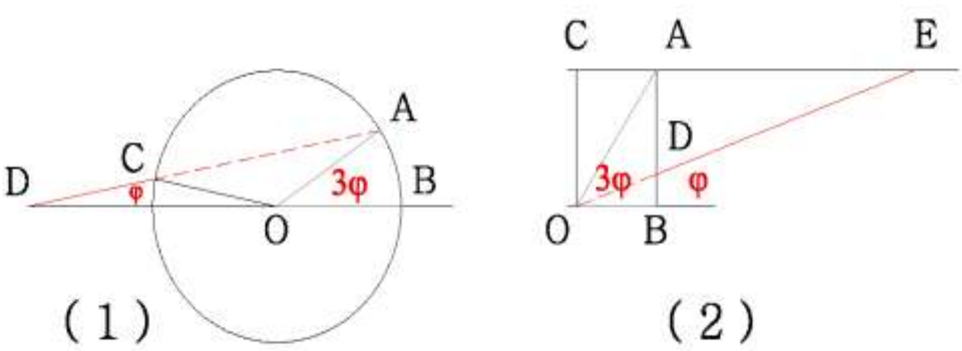
This problem follows , The three dimensional dialectic logic of ancient Greek, Αναξίμανδρος, [« τόμήΟν, Ονγίγνεσθαι » The Non-existentExists when is done , ‘ The Non - existent becomes and never is] , where the geometrical magnitudes, have a linear relation(the continuous analogy on Segments) in all Spaces as ,in one in two in three dimensions, as this happens to the Compatible Coordinate Systems .
The Structure of Euclidean geometry is such [8] that it is a Compact Logic whereNon-Existent is found everywhere , andExistence , monads ,is found and is done everywhere . In Euclidean geometry points do not exist , but their position and correlation is doing geometry. The universe cannot be created , because it is continuously becoming and never is . [9] According to Euclidean geometry ,and since the position of points (empty Space) creates the geometry and Spaces , Zenon Paradox is the first concept of Quantization . [15] In terms of Mechanics , Spaces Mould happen through ,Mould of Doubling the Cube ,where for any monad ds =KoAand analogous to KoD, the Volume or The cube of segment KoD is the doublethe volume ofKoA cube , or monad $KoD^3 = 2.KoA^3$.This is one of the basic Geometrical Euclidean Geometry Moulds , which create the METERS of monads which →Linear is the Segment ds = MA1 , Plane is ,π, the square CMNH equal to the circle , and in Space is $\sqrt[3]{2}$ volume KoD^3, in all Spaces , Anti-spaces and Sub -spaces of monads ← i.e. The Expanding square BAoDoCo is Quantized to BADCRectangle by Translation to point Z `, and by Rotation through point P , (the Pole of rotation) .The Constructing relation between any segments KoX , KoA is →
$(KoX)^3 = (KoA)^3 .(XX1 /AD)$ as in F.7

Application in Physics :

The Electromagnetic waves are able to transmit Energy through a vacuum (empty space) by storing their energy vector in an Standing Transverse Electromagnetic dipole wave , and so considered completely particle like , and in the transverse interference pattern tobe considered as completely wave , so the Same Quantity of Energy is as ,
Energy$I_d = \frac{\rho\pi^2 c^3}{2\lambda^2}[\varepsilon E^2+\mu H^2]$ in volume $V=[\frac{4(w^2 r^2)^3}{3\pi}]$ having mass → ParticleEnergy
$I_d = (\frac{\rho.c}{2}).(wA_o)^2$ in Interference pattern as→ Wave
This is the Wave-Particle duality unifying the classical Electromagnetic field and the quantum particle of light .Angular momentum of particles is → Spin $=\frac{E}{w} = [\pm\bar{v}.s^2]/w = (r.s^2) = w^2 r^3 = [wr]^3$ and ,
as Spin = $\frac{h}{\pi} = 2.[wr]^3$, or Energy Space quantity wr ,is doubled and becomes the Space quantity$\frac{h}{\pi}$
The above relation of Spin shows the deep relation between Mechanics and E-geometry , where in the tiny Gravity-caveof $r = 10^{-62}$ m , the Energy -Volume-quantity [wr] in cave , is doubled and is Quantized in Planck`s-cave Space quantity as , $(\frac{h}{\pi})$ = Spin = $2.[wr]^3$ in $r = 10^{-35}$ m i .e.
Energy Space quantity ,wr , is Quantized , and becomes the New Space quantity ,$h/\pi = 2.[wr]^3$,doubled , following the Euclidean Space-mould of Duplication of the cube by changing frequency,
in tiny Sphere volume $V = (4\pi/3).[wr/2]^3$. Also , Since $w = E / [h/2\pi] = m.c^2/[h/2\pi] = 2\pi.mc^2/h = 2r.s^2$
$= 2.r^3.w^2$, then mass $m = \frac{(wr)^3}{c^2} = \frac{2}{c^2}(wr)^3$, isDoubled as above with Space-mould and , is what is called conversion factor mass ,m, and it is an index of the energychanges .
All Energy magnitudes from , 0 → ∞ , deposit in the same Space ,resonance, by changing frequency

3.. The Trisection of Any Angle .

Because of the three master-meters , where is holding the Ratio of two or three geometrical magnitudes , is such that they have a linear relation (a continuous analogy) in all Spaces ,
the solution of this problem , as well as of those before , is linearly transformed .
The present method is a Plane method , i.e. straight lines and circles , as the others and is not required the use of conics or some other equivalent . Archimedes and Pappus proposals are both instinctively right .



F.8. → (1) Archimedes ,(2) Pappus Method

The Present method :

is based on the Extrema geometrical analysis of the mechanical motion of shapes related to a system of poles of rotation . The classical solutions by means of conics ,or reduction to a ,νεύσις, is a part of Extrema ethod . This method changes the Idle between the edge cases and Rotates it through constant points , The Poles ,Fig.11 . The basic triangle AOD_1 is such that angle OD_1A=30▫ and rotating through pole O .

The three edge positions are ,

a). Angle AOB = 90▫ when OD_1≡ - OE and then point D_1 is at point E on OB axis ,

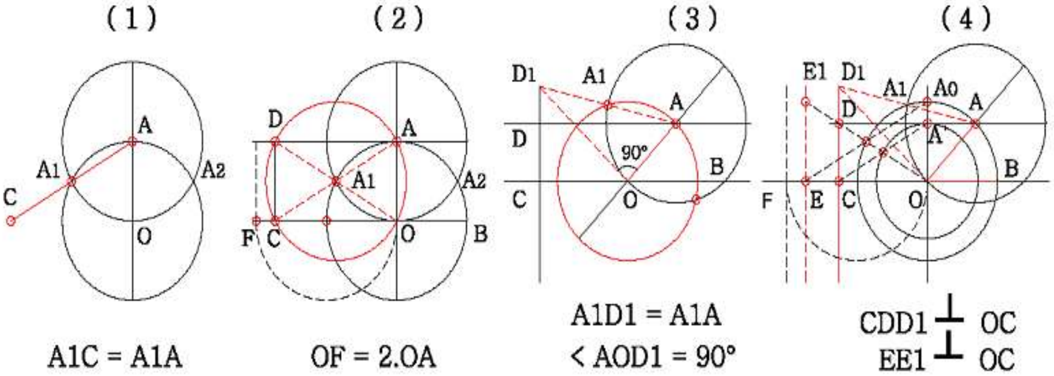
b). Angle AOB = 0 - 90▫ when OD_1= OE and then point D_1 is perpendicular to OB axis ,

c). Angle AOB = 0 when OA ≡ O and then point D_1 is perpendicular to OB axis.

This moving geometrical mechanism acquires common circles and constant common poles of rotation which are defined with initial ones .

This geometrical motion happens between the Extrema cases referred above..

The steps of the basic Rotating Triangle AOD_1 between the extrema cases AOB=180▫, AOB= 0



F.9.→ The proposed Contemporary Trisection method .

We extend Archimedes method as follows :

a . F9.-(2) . Given an angle < AOB =AOC= 90▫

1.. Draw circle (A , AO = OA) with its center at the vertex A intersecting circle (O , OA = AO) at the points A_1 ,A_2 respectively .

2.. Produce line AA_1 at C so that A_1C = A_1A = AO and draw AD // OB .

3.. Draw CD perpendicular to AD and complete rectangle AOCD .

4.. Point F is such that OF = 2 . OA

b . F9.(3-4) . Given an angle < AOB <90▫

1.. Draw AD parallel to OB .

2.. Draw circle (A , AO = OA) with its center at the vertex A intersecting circle (O ,OA = AO) at the points A_1 , A_2 .

3.. Produce line AA_1 at D_1 so that A_1D_1= A_1A = OA .

4.. Point F is such that OF = 2 .OA = 2.OA_o

5.. Draw CD perpendicular to AD and complete rectangle A΄OCD .

6.. Draw A_o E Parallel to A΄ C at point E (or sliding E on OC) .

7.. Draw A_oE΄ parallel to OB and complete rectangle A_oOEE_1.

8.. In F10 -(1-2-3) ,Draw AF intersecting circle (O,OA) at point F_1 and insert after F_1 and on AF segment F_1F_2 equal to OA → F_1F_2 = OA.

9.. Draw AE intersecting circle (O , OA) at point $E_1$and insert after E_1on AE segment E_1E_2equal to OA → E_1E_2= OA = F_1F_2.

To show that :

a). For all angles equal to 90▫ Points C and E are at a constant distance OC = OA . √ 3 and OE = OA_O. √ 3 , from vertices O , and also A΄C //A_OE .

b). The geometrical locus of points C , E is the perpendicular CD , EE_1line on OB.

c). All equal circles with their center at the vertices O , A and radius OA = AO have the same geometrical locus EE_1⊥ OE for all points A on AD , or All radius of equal circles drawn at the points of intersection with its Centers at the vertices O , A and radius OA = AO lie on CD , E E_1perpendicular lines .

d). Angle <D_1OA is always equal to 90▫ and angle AOB is created by rotation of the right-angled triangle AOD_1 through vertex O .

e). Angle < AOB is created in two ways , by constructing circle (O , OA = OA_O) and by sliding , of point A_1 on line A_1 D Parallel to OB from point A_1 , to A .

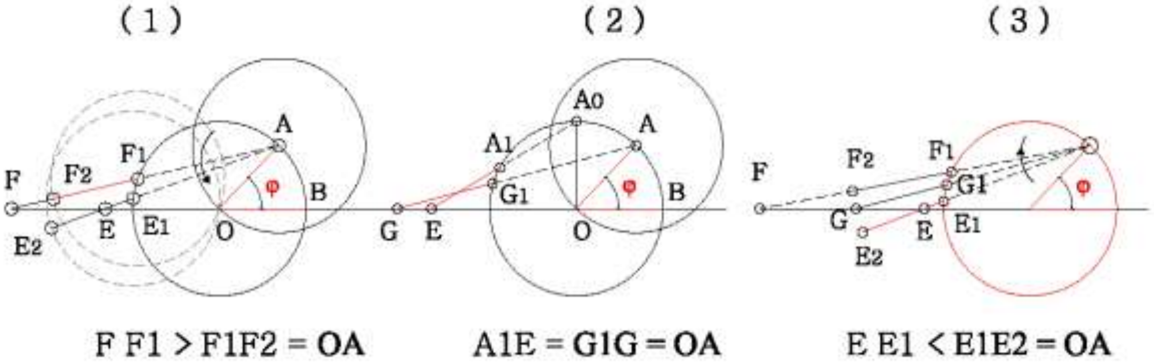
f). Angle < AOB is created in two ways , either by constructing circle (O , OA = OA_O) and by sliding , of point A΄ on line A΄ D Parallel to OB from point A΄ , to A , or on OA circle .

g). The rotation of lines AE , AF (minimum and maximum edge positions)on circle (O,OA =OA_O) from point E to point F which lines intersect circle (O , OA) at the edge pointsE_1, F_1 respectively , fixes a point G on line EF and a point G_1common to line AG and to the circle (O , OA) such that GG_1= OA .

Proof :

a) .. F.9 .(1 - 2 - 4)

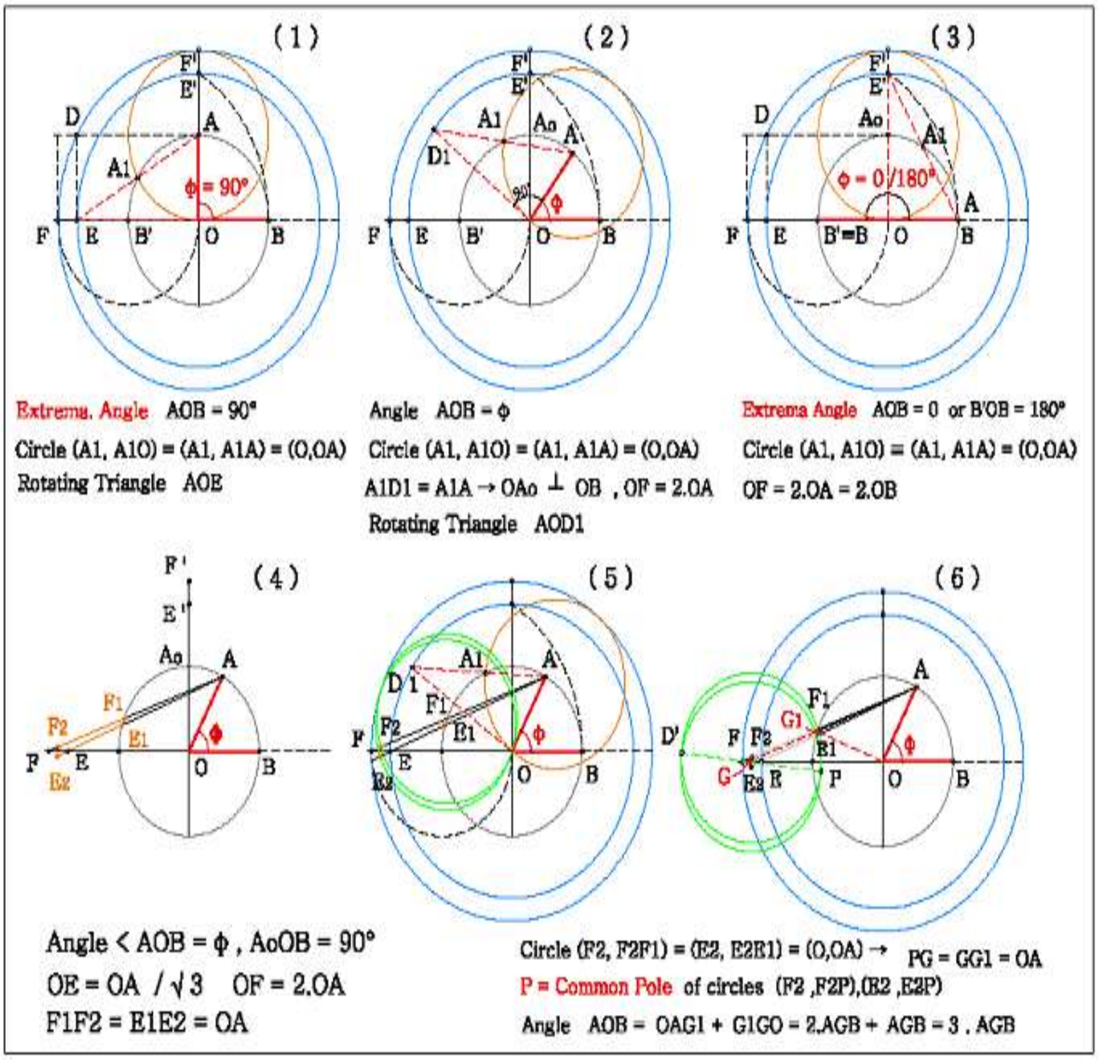
Let OA be one-dimensional Unit perpendicular to OB such that angle < AOB = AOC = 90▫
Draw the equal circles (O,OA) , (A , AO) and let points A_1 , A_2 be the points of intersection . Produce AA_1 to C on OB axis such that A_1C = A A_1.
Since triangle AOA_1has all sides equal to OA (AA_1= AO = OA_1) then it is an equilateral triangle and angle <A_1AO = 60 ▫
Since Angle < CAO = 60 ▫ and AC = 2. OA then triangle ACO is right-angled and since angle
< AOC = 90▫ , so the angle ACO = 30 ▫.
Complete rectangle AOCD , and angle < ADO = 180 – 90 – 60 = 30 ▫ = ACO = 90 ▫ / 3 = 30 ▫
From Pythagoras theorem AC² = AO² + OC² or OC ² = 4.OA² - OA² = 3. OA² and
OC = OA . √ 3 .
For OA = OA_O then A_OE = 2. OA_O and OE = O$\boldsymbol{A_O}$. √ 3 .
Since OC / OE = OA / O$\boldsymbol{A_O}$ → then line CA′ is parallel to E$\boldsymbol{A_O}$.
b) .. F.9.(3 - 4)
Triangle OAA_1 is isosceles , therefore angle <A_1AO = 60 ▫ . Since A_1D_1= A_1O , triangle
D_1A_1O is isosceles and since angle < OA_1A = 60 ▫ , therefore angle < OD_1A = 30 ▫ or , Since
A_1A = $A_1D_1$and angle <A_1AO = 60 ▫ then triangle AO$D_1$is also right-angle triangle and angles <D_1OA = 90 ▫ , < OD_1A = 30 ▫.
Since circle of diameter D_1A passes through point O and also through the foot of the perpendicular from point D_1 to AD , and since also ODA = ODA′ = 30 ▫ , then this foot
point coincides with point D , therefore the locus of point C is the perpendicular CD_1 on OC . For AA_1>A_1D_1, then D_1 is on the perpendicular D_1 E on OC.
Since the Parallel from point $A_1$to OA passes through the middle of OD_1 , and in case where isAOB = AOC = 90 ▫ through the middle of AD , then the circle with diameter D_1A passes through point D which is the base point of the perpendicular , i.e.
The geometrical locus of points C , or E , is CD and E$\boldsymbol{E_1}$,the perpendiculars on OB .
c) .. F.9.(3 - 4)
Since A_1A = $A_1D_1$and angle <A_1AO = 60▫ then triangle AOD_1 is a right-angle triangle and
angle <$\boldsymbol{D_1}$OA = 90 ▫ . Since angle <A$\boldsymbol{D_1}$O is always equal to 30 ▫ and angle <$\boldsymbol{D_1}$OA
is always equal to 90▫ , therefore angle <AOB is created by the rotation of the right -angled triangle A-O-$\boldsymbol{D_1}$ through vertex O .
Since the tangent through $A_O$on to circle (O , OA′) lies on the circle of half radius OA ,
then this is perpendicular to OA and equal to A′A . (F.8)



F.10. → The three cases of the Sliding segment OA = $\boldsymbol{F_1F_2}$= $\boldsymbol{E_1E_2}$between a line OB and a
circle (O,OA) between the Maxima - Edge cases $\boldsymbol{F_1}$F , $\boldsymbol{E_1}$EorbetweenF,Epoints .
On AF, AE lines of F.10 exists :
FF_1>OA GG_1= OA ,A_1E = O$A_O$$E\,E_1$<OA
F_2F_1= OA A_1E =OA_O , EA_1= OA E_1E_2=OA
d) .. F.9-(4) - (F.10 - F.11)
Let point G be sliding on OB between points E and F where lines AE , AG , AF intersect circle (O, OA) at the points E_1, G_1,F_1respectively where then exists $F\,F_1$> OA , $G\,G_1$=OA , $E\,E_1$< OA .
Points E , F are the limiting points of rotation of lines AE , AF (because then for angle < AOB = 90 ▫ → A_1C = A_1A = OA , A_1A_O = A_1E = OA_O and for angle < AOB = 0▫ → OF = 2 .OA) . Exists also E_1E_2 = OA ,F_2F_1 = OA and point G1 common to circle (O , OA) and on line AG such that $G\,G_1$= OA .
AE Oscillating to AF passes through AG so that $G\,G_1$= OA and point G on sector EF.
When point G_1 of line AG is moving (rotated) on circle ($\boldsymbol{E_2}$, $\boldsymbol{E_2E_1}$= OA) and Point $G_1$of $\boldsymbol{G_1}$G is stretched on circle (O , OA), then G_1G ≠ OA .
A position of point $\boldsymbol{G_1}$is such that , when $\boldsymbol{G\,G_1}$= OA point G lies on line EF.
When point G_1 of line AG is moving (rotated) on circle ($\boldsymbol{F_2}$, $\boldsymbol{F_2F_1}$= OA) and point $G_1$of $\boldsymbol{G_1G}$is stretched on circle (O, OA)then length G_1G ≠ OA .
A position of point $\boldsymbol{G_1}$ is such that , when$\boldsymbol{G\,G_1}$= OA point G lies on line EF without stretching .
For both opposite motions there is only one position where point G lies on line OB and is not needed point $G_1$of GA to be stretched on circle (O , OA) .
This position happens at the common point, P,of the two circles which is their point of
intersection . At this point , P,exists onlyrotation and is not needed $\boldsymbol{G_1}$ of GA to be
stretched on circle (O, OA)so that point Gto lie on line EF.
This means that point P lies on the circle (G , $\boldsymbol{G\,G_1}$= OA) , or GP = OA .
Point G_1 in angle < BOA is verged through two different and opposite motions , i.e.1.. From point A′ to point $A_O$where is done a parallel translation of CA′ to the new position EA_O ,this is for all angles equal to 90 ▫ , and from this positionto the new position EA by

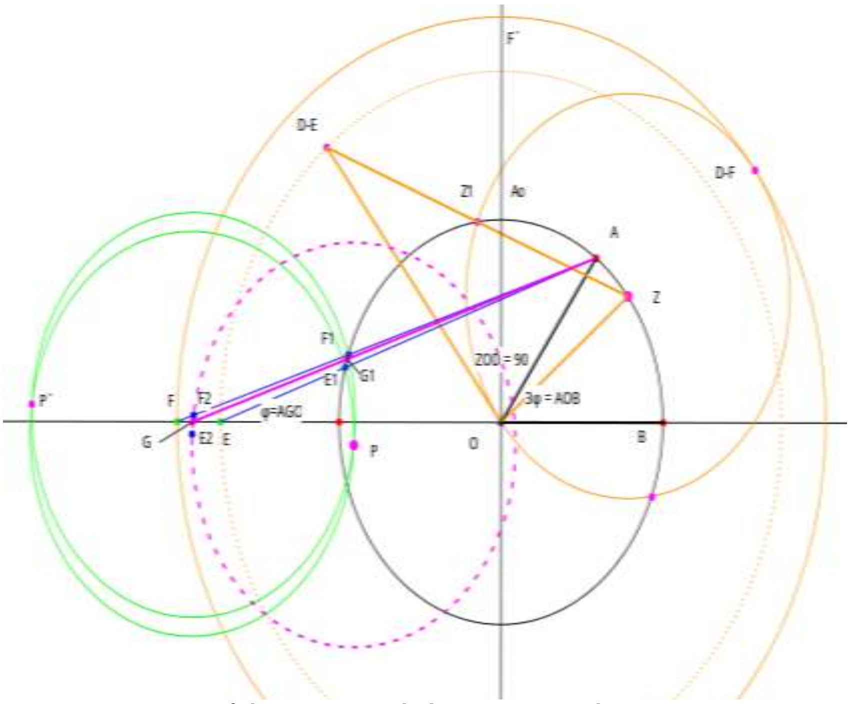
rotating E A_O to the new position EA having always the distance $E_1 E_2$= OA . This motion is taking place on a circle of center E_1 and radius $E_1 E_2$.
2.. From point F , where OF = 2. OA , is done a parallel translation of A΄F to FA_O,and from this position to the new position FA by rotating F A_O to FA having always the distance $F_1 F_2$= OA

The two motions coexist , limit , again on a pointP which is the point of intersection of the circles (E_2 , E_2E_1 = OA) and (F_2, F_2F_1 = OA) .
f) ..(F.9 .3 - 4) - (F.10 -3)
Remarks – Conclusions :
1.. Point $E_1$is common of line AE and circle (O, OA) and point E_2 is on line AE such that E_1E_2= OA and exists E E_1<E_2E_1. LengthE_1E_2= OA is stretched ,moveson EA so that point E_2 is on EF . Circle (E , E E_1<E_2E_1 = OA) cuts circle (E_2 , E_2E_1= OA) at point E_1. There is a point G_1 on circle (O ,OA) such that G_1G = OA , where point G is on EF , and is not needed $\boldsymbol{G_1}$G to be stretched on GA where then , circle (G , $G\,G_1$= OA) cuts circle (E_2 , E_2E_1= OA) at a point P .
2.. Point $F_1$is common of line AF and circle (O,OA) and point F_2 is on line AF such that $F_1 F_2$= OA and exists F F_1>F_2F_1.Segment $F_1 F_2$= OA is stretched ,moveson FA so that point F_2 is on FE . Circle (F , F F_1>F_2F_1= OA) cuts circle (F_2, F_2F_1= OA) at point F_1 .
There is a point G_1 on circle (O,OA) such that G_1G = OA ,where point G is on FE ,and is not needed$\boldsymbol{G_1}$G to be stretchedon OB where thencircle (G ,$G\,G_1$= OA) cuts circle (F_2 , F_2F_1 = OA)at a point P .
3..When point G is at such position on EFthat $\boldsymbol{G\,G_1}$= OA , then point G must be at A COMMON , to the three lines $\boldsymbol{E\,E_1}$, $\boldsymbol{G\,G_1}$, $\boldsymbol{F\,F_1}$, and also to the three circles ($\boldsymbol{E_2}$, $\boldsymbol{E_2\,E_1}$= OA), (G , $\boldsymbol{G\,G_1}$= OA),($\boldsymbol{F_2}$, $\boldsymbol{F_2\,F_1}$=OA)
This is possible at the common point , P , of Intersection of circle(E_2 , E_2E_1= OA)and (F_2, F_2F_1 = OA)and since $\boldsymbol{G\,G_1}$is equal to OA without $\boldsymbol{G\,G_1}$ be stretched on GA ,
then also GP = OA .
4.. In additional , for point G_1:
a.. Point G_1 , from point E_1, moving on circle (E_2 , $E_2\,E_1$ = OA) formulates Segment A E_1E such that E_1E = G_1G < OA , for G moving on line GA.
There is a point on circle (E_2 , $E_2\,E_1$ = OA) such that $G\,G_1$ = OA .
b.. Point G_1,from pointF_1, moving on circle (F_2, $F_2\,F_1$= OA) formulates AF_1F such that F_1F = $G\,G_1$> OA , for G moving on line GA .
There is a point on circle (F_2, $F_2\,F_1$= OA) such that $G\,G_1$= OA .
c.. Since for both Opposite motions there is a point on the two circles that makes $G\,G_1$=OA
then point say P , is common to the two circles .
d.. Since for both motions at point P exists$G\,G_1$= OA then circle (G , $G\,G_1$= OA) passesthrough point P , and since point P is common to the three circles , then fixing point P asthe common to the two circles (E_2 , $E_2\,E_1$ = OA) ,(F_2, $F_2\,F_1$= OA) ,then point G is found as the point of intersection of circle (P,PG= OA) and line EF. This means that the common point P of the three circles is constant to point P of the three circles and is constant to this motion.e.. Since , happens alsothe motion of a constant Segment on a line and a circle , then it is Extrema Method of the moving Segment as stated . The method may be used for part or Blocked figureseither sliding or rotating . Inour case , the Initial triangle forming 1/3 angle is formulating in all cases the common pole ,P, of the three circles .
From all above the geometrical trisection of any angle is as follows ,



F. 11 → The extrema Geometrical method of the Trisection of any angle < AOB

In F.11-(1) Basic triangle AO D_1= OAE defines point E such that angle < AEO = 30 ▫ = AOB/3 .
In F.11-(2)Basic triangle AO D_1defines D_1 point such that angle A D_1O = 30 ▫= AOB/3 .
In F.11-(3) Basic triangle AO D_1 defines E` point such that angle AE`O = 30 ▫ , and it is the Extrema Case for angles AOB = 0 ▫, B`OB = 180▫
In F.11-(4) The two Edge cases (1),(3) issue for any angle AOB= φ▫ where F_1F_2= OA<F_1F ,E_1E_2= OA<E_1E
In F.11-(5) The two circles with centers F_1 , E_1correspond to Edge cases (1),(3) issuing for any angle AOB = φ▫
In F.11-(6) The three circles [F_2, F_2F_1= OA] , [E_2, E_2E_1= OA] ,[G,GG_1= OA = GP] corresponding to Edge cases (1) , (3) define the common axis PP` of all movablepoles and point ,P, of thisrotational system , such that GG_1= OA is stretched on (O,OA) circle and OB line , of any angle AOB = φ ▫ .



F.11-A. → Presentation of the Trisection Method on Dr. Geo - Machine Macro –constructions .

In F.11-AFromInitial position of triangle AOB , where angle< AOB = 90° and Segment A_1C = OA ,to the Final position of triangle , where angle< AOB = BOB = 0°and AOB = B`OB = 180 °, throughtheExtrema position between edge- cases of triangle ZOD where AOB = φ ° and GG_1=GP = OA.3.1.The steps of Trisection of any angle < AOB = 90 ° → 0 ° F.11-[1-6]

1.. Draw circles (O , OA = OB) , (A , AO) , intersected at $A_1 \equiv Z_1$ point .

2.. Draw OA_O⊥ OB where point A_O is on the circle (O ,OA) and on a general circle (Z , D-E = 2. OA) . The circle (O , OD-E) intersects line OB at the Edge point E .

3.. Fix Edge point F on line OB such that → OF = 2. OA

4.. Draw lines AF , AE intersecting circle (O,OA) at points F_1 ,E_1respectively .

5.. On lines F_1A , E_1A fix points F_2 , E_2such that F_1F_2 = OA and E_1E_2= OA .

6.. Draw circles (F_2, F_2F_1 = OA) , (E_2 , E_2 E_1 = OA) and fix point P as their common point of intersection .

7.. Draw circle (P , PG = OA) intersecting line OB at point G and draw line GA intersecting circle (O ,OA) at point G_1, Then Segment $\boldsymbol{G\,G_1}$= OA , and angle < AOB = 3. AGB .

Proof :

1. Since point P is common to circles (F_2, F_2F_1= OA) , (E_2 , E_2 E_1 = OA) ,then PG = PF_2= PE_2 = OA and line AG between AE , AF intersects circle (O,OA) at the point G_1such that $G\,G_1$ = OA .(F10.1 -2) - (F.11-5)

2. Since point G_1 is on the circle (O , OA) and since $G\,G_1$= OA then triangle G G_1O is isosceles and angle < AGO = G_1OG .

3. The external angle of triangle Δ= GG_1O is < AG_1O = AGO + G_1OG = 2. AGO

4. The external angle of triangle GOA is angle < AOB = AGO + OAG = 3.AGO .

Therefore angle < AGB = (1 / 3) . (AOB) (ο.ε.δ.)

A General Analysis :

Since angle < D_1OA is always equal to 90° then angle AOB is created by rotation of the right-angled triangle AOD_1 through vertex O . The circle (A , AO = A_1O) and triangleAO D_1consists the geometricalMechanism which creates the maxima at positions from , AOE , to A_OOE and to BOF` triangles, on (O,OE = √3.OA) , (O,OF = 2.OA) circles. F.11-(5) In (1) Angle AOB = 90° , AE = 2.OA = OF, and point A_1common to circles (O , OA) , (A, AO) define point E on OB line such that A_1E = OA . This happens for the extrema angle AOB = 90° .

In (2) Angle is , 0 < AOB < 90° , AD_1 = 2.OA and point A_1 common to circles (O,OA) , (A,AO)defines point D_1 on (O,OE =√3.OA) circle such that A_1D_1 = OAand on (O,OF =2.OA) circle at any point D_f. In (3) Angle <AOB = 0 or B`OB=180° , AE = 2.OA = BB` and point A_1 common to (O , OA) ,

(A, AO) circles define point E on $O\,A_O$ line such thatE ≡ E`, where then point D ≡ F` .This happens for the extrema angle <AOB = 0 or90°. In (4-5) where angle is , 0 < AOB < 90° , and Segments F_1F_2= E_1E_2 = OA the equal circles (F_2, F_2F_1 = OA) , (E_2 , E_2 E_1 = OA)define the common point P .

Since this geometrical formulation exists on Extrema edge angles , 0 and 90° ,then this point is constant to this formulation , and this point as center of a radius OA circle defines the extrema geometrical locus on it. All Poles are movable except the common Pole line PP` representing the Extrema case of this changeable system .

In (6) Since angle AOB is , $0 \rightarrow 90^\circ$, and pointP is constant,and this because extrema circle (P,PG=OA)whereG on OBline , then is defining (G, $G\,G_1$) circle on GA segment such that point G_1, tobe the common point of segment AG and to circles(O, OA),(G,$G\,G_1$) .

The Physical notion of the Trisection :

This problem follows the two dimensional logic, where , the geometrical magnitudes and their unique circle, have a linear relation (continuous analogy) in all Spaces as , in one in two in three dimensions, and as this happens to Compatible Coordinate Systems , happens also in Circle-arcs. The Compact-Logic-Space-Layer exists in Units , (The case of 90 ▫ angle) ,where then we may find a new machine that produces the 1/3 of angles as in F.11. [11] Since angles can be produced from any monad OB ,and this because monad can formulate a circle ofradius OB , and any point A on circle can then formulate angle <AOB , therefore the logic of continuous analogy of monads in all spaces issues also and on OA radius equal to OB.

Application in Physics :

According to math theory of Elasticity , the total work on free edges where there is no shear becomes from Principal stresses only and work is W = $\frac{\sigma^2}{2E}+\frac{\tau^2}{2G}$and the analogous Energy in monads W=$\frac{1}{2}$[εE²+μH²] spread as the First Harmonic and equal to outer Spin$\bar{S}$ = E / w = 2πr.c .

Equation of Planck`s Energy E = h.f = (h/λ).c is equal to the Isochromatic pattern fringe-order in monad as→ σ1-σ2=(a/d).N=(a/d)nf1=(8πr²/3).n.f1.where n = the order of isochromatic , a number , f1= the frequency of Fundamental-Harmonic .

Since total Energy in cave (wr)² is dependent on frequency only , and stored in the Fundamental and the first Six Harmonics , so the summations bands of these Seven Isochromatic Quantized interference fringe-order-patterns , is total energy E in the same cave (wr)² as ,

E =Spin.w =$\bar{S}$.w = (h/2π).2πf = $[\frac{8\pi r^2 f1}{3}].[\frac{n(n+1)}{2}]$ =$[\frac{4\pi r^2 f1}{3}]$ n.(n+1) ……..(a)

When stress (σ1-σ2) go up then , n = order fringe defining Energy goes up also ,and the colors cycle through a more or less repeating pattern and the Intensity of the colors diminishes .Since phase φ = kx-wt = Spatial and Time Oscillation dependence ,

For n = 1 , Energy in the First Harmonicis , E =2πr.c = $[\frac{4\pi r^2}{3}]$.f1.[1], and for n = 2 Energy in the First and Second Isochromatic Harmonicis , E = $[\frac{4\pi r^2}{3}]$.f1.[3]in threes, and φ is trisectedwith Energy-Bunched variation f2 , i.e.

Energy stored in a homogeneous resonance , is spread in the First of Seven-Harmonics beginning from the Fundamental and after the filling with frequency f1 , follows the Second-Harmonic .

In Second-Harmonic energy as frequency is doubled and this because of sufficient keeping homogeneously in Spatial dependence Quantity kx =(2π/λ).x which is in threes , meaning that , → Dipole–energy is Spatially-trisected in Space -Quantity Quanta the Spin=h/2π as the angle φ , of phase φ=kx-wt=(2π/λ).x , and Bisected by the Energy-Quantity Quanta as in an RLC circuit. [49] .

The Physical notion of the Regular Polygons :

According to Archimedes , Geometric means , speaking of numbers , whether solid or square , observes that , Between Plane One - mean suffices , but to connect two solids Two – means are necessary . This denotes that between two square numbers there is one mean proportional number and between two cubes there are two means proportional numbers .

It was proved that Odd numbers become from any two consequent Even numbers , so the sum of two irrationals may be either rational or irrational .

The Cattle – Problem of Archimedes may be further analysed reaching to equations of any degree.

It was shown in pages 43 – 49 that , all n-Regular Polygons End to equations of n-degree Segment , by finding a suitable value of the Segment , x , That is we have in the general case to solve one or two equations of the form :

A .$R^0.x^n$- B .$R^2.\,x^{n-2}$ + C .R^{n-6}. x³ – D .R^{n-4}. x ² + E .$R^{n-2}.\,x^1$ – F .$R^n.\,x^0$ = 0
for The Even Polygons , and

A .$R^2.\,x^{n-2}$- B .$R^{n-2}.\,x^{n-3}$ + C .$R^{2(n-4)}$. x ³ - D .$R^{2(n-3)}$. x ² + E .$R^{2(n-2)}.\,x^1$ – F .$R^{2(n-1)}.\,x^0$= 0
for The Odd Polygons , where A , B , C , D are constants .

The Presented Geometrical method is the solution of the above equation in the general case . Because , the nth- degree - equations are the vertices of the n-polygon in circle so number , π , is their mould . In Mechanics , by Scanning any Chord $K\,K_1$ to chord $K\,K_2$ of the circle ,then the Work (Energy as → Kinetic or Dynamic) producedfrom any Removal , is Stored . in the Inverted triangles OO_kK_2 ,$K_2P_kP_a$ as in page 60 .

4. The Parallel Postulate, is not an Axiom ,is a Theorem.

The Parallel Postulate. F.13

General : Axiom or Postulate is a statement checked if it is true and is ascertained with logic (the experiences of nature as objective reality).

Theorem or Proposition is a non-main statement requiring a proof based on earlier determined logical properties.

Definition is an initial notion without any sensible definition given to other notions.

Definitions, Propositions or Postulates created Euclidgeometry using the geometrical logic which is that of nature, the logic of the objective reality.

Using the same elements it is possible to create many other geometries but the true uniting element is the before refereed.

4.1.The First Definitions (Dn) =(D), of Terms in Geometrybut the true uniting ,

D1: A point is that which has no part (Position).

D2: A line is a breathless length (for straight line, the whole is equal to the parts) .

D3: The extremities of lines are points (equation).

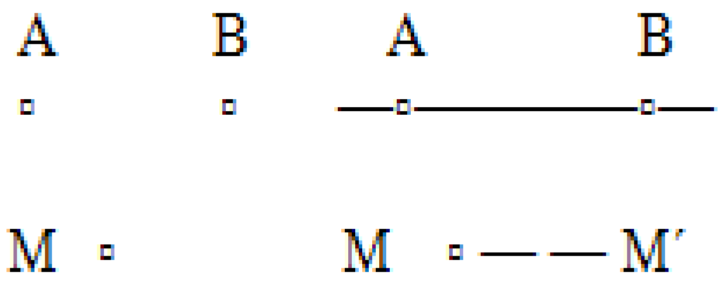
D4: A straight line lies equally with respect to the points on itself (identity).

D: A midpoint C divides a segment AB (of a straight line) in two. CA = CB any point C divides all straight lines through this in two.

D: A straight line AB divides all planes through this in two.

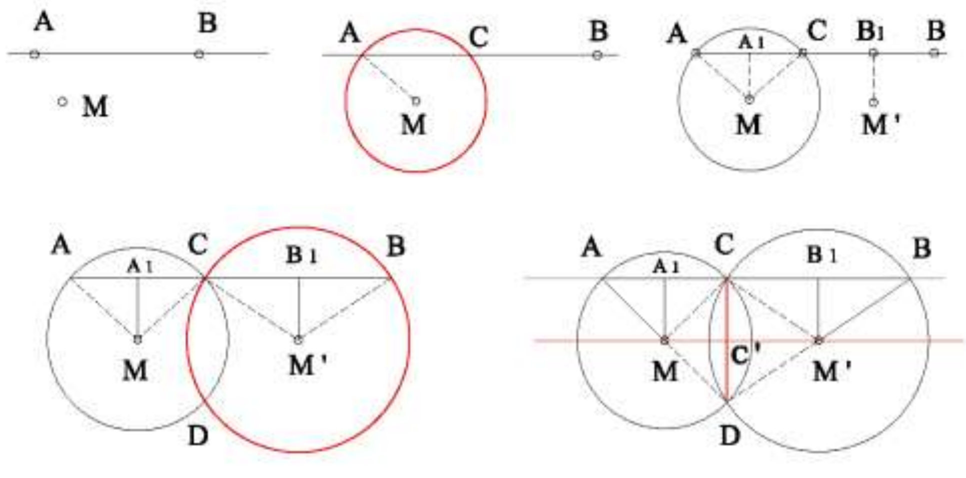
D: A plane ABC divides all spaces through this
in two .

4.2. Common Notions (Cn) = (CN)
Cn1: Things which equal the same thing also equal one another.
Cn2: If equals are added to equals, then the wholes are equal.
Cn3: If equals are subtracted from equals, then the remainders are equal.
Cn4: Things which coincide with one another, equal one another.
Cn5: The whole is greater than the part.
4.3. The Five Postulates (Pn) = (P)for Construction
P1.. To draw a straight line from any point A to any other point B .
P2.. To produce a finite straight line AB continuously in a straight line.
P3.. To describe a circle with any center and distance. P1, P2 are unique.
P4.. That, all right angles are equal to each other.
P5.. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, if produced indefinitely, meet on that side on which are the angles less than the two right angles, or (for three points on a plane) . Three points consist a Plane .
P5a. The same is plane's postulate which states that, from any point M, not on a straight line AB, only one line MM´ can be drawn parallel to AB.
Since a straight line passes through two points only and because point M is the third , then the parallel postulate it is valid on a plane (three points only).
AB is a straight line through points A, B , AB is also the measurable line segment of line AB , and M any other point . When MA+MB> AB , then point M is not on line AB .(differently if MA+MB = AB , then this answers the question of why any line contains at least two points) ,
i.e. for any point M on line AB where is holding
MA+MB = AB , meaning that line segments MA,MB coincide on AB , is thus proved from the other axioms and so D2 is not an axiom . →
To prove that , one and only one line MM´ can be drawn parallel to AB.



F.12.□ In three points (in a Plane).

4.4. The Process in order to prove the above Axiom isnecessary to show: F.13 ,
a..The parallel to AB is the locus of all points at a constant distancehfrom the line AB, and for point M is MA_1,
b..The locus of all these points is a straight line.



F.13.→ The Parallel Method

Step 1
Draw the circle (M, MA) be joined meeting line AB in C. Since MA = MC, point M is on mid- perpendicular of AC. Let A_1 be the midpoint

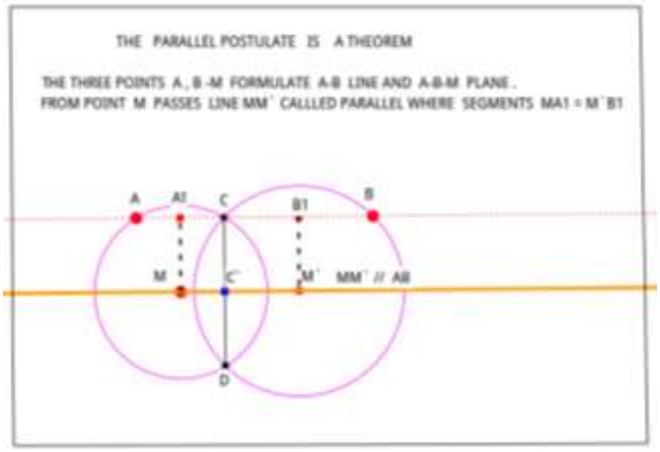
of AC, (it is A_1A+A_1C = AC because$A_1$is on the straight line AC). Triangles MAA_1, MCA_1 are equal because the three sides are equal, therefore angle <MA_1A = MA_1C (CN1) and since the sum of the two angles < MA_1A+MA_1C = 180°
(CN2, 6D) then angle < MA_1A = MA_1C = 90 °.(P4) so, MA_1 is the minimum fixed distance hof point M to AC.

Step 2

Let B_1 be the midpoint of CB,(it is B_1C+ B_1B = CB because $B_1$is on the straight line CB) and Draw B_1M´ = h equal to A_1M on the mid-perpendicular from point$B_1$to CB.Draw the circle (M´, M´B =M´C) intersecting the circle (M, MA = MC) at point D.(P3)
Since M´C = M´B, point M´ lies on mid-perpendicular of CB. (CN1)Since M´C = M´D, point M´ lies on mid-perpendicular of CD. (CN1) Since MC = MD, point M lies onmid-perpendicular of CD. (CN1) Because points M and M´ lie on the same mid-perpendicular (This mid-perpendicular is drawn from point C´ to CD and it is the midpoint of CD) and because only one line MM´ passes through points M, M ´then line MM´ coincides with this mid-perpendicular (CN4) .

Step 3

Draw the perpendicular of CD at point C´. (P3, P1) a..Because MA_1⊥ AC and also MC´ ⊥ CD then angle <A_1MC´ = A_1CC´. (Cn 2,Cn3,E.I.15)Because M´B_1⊥ CB and also M´C´⊥ CD then angle <B_1M´C´ = B_1CC´. (Cn2, Cn3, E.I.15)b..The sum of angles A_1CC´ + B_1CC´ = 180▫ = A_1MC´ + B_1M´C´. (6.D), and since Point C´ lies on straight line MM´, therefore the sum of angles in shapeA_1B_1M´M are < MA_1B_1+ A_1B_1M ´ + [B_1M´ M + M´MA_1] = 90▫ + 90▫ + 180▫ = 360▫ (Cn2) , i.e. The sum of angles in a Quadrilateral is 360▫ and in Rectangle all equal to 90 ▫. (m)
c..The right-angled triangles MA_1B_1, M´B_1A_1 are equal because A_1M = B_1M´ and A_1B_1common, therefore side A_1M´ = B_1M (Cn1). TrianglesA_1MM´,B_1M´M are equal because have the three sidesequal each other, therefore angle <A_1MM´ = B_1M´M, and since their sum is 180▫ as before (6D), soangle <A_1MM´ = B_1M´M = 90▫ (Cn2). d.. Since angle<A_1MM´ = A_1CC´ and also angle<B_1M´M = B_1CC´ (P4), therefore the threequadrilaterals A_1CC´M, B_1CC´M´, A_1B_1M´M are Rectangles (CN3).From the above three rectangles and because all points (M, M´ and C´) equidistant from AB, thismeans that C´C is also the minimum equal distance of point C´ to line AB or, h = MA_1 = M´B_1 =
CD / 2 = C´C (Cn1) Namely, line MM´ is perpendicular to segment CD at point C´ and this line coincides with the mid-perpendicular of CD at this point C´ and points M, M´ ,C´ are on line MM´. Point C´ equidistant ,h, from line AB, as it is for points M ,M´, so the locus of the three points is the straight line MM´, so the two demands are satisfied, (h = C´C = MA_1 = M´B_1 and also C´C ⊥ AB, MA_1⊥ AB, M´B_1⊥ AB). (ο.ε.δ.) –(q.e.d)
e.. The right-angle triangles A_1CM, MCC´ are equal because side MA_1= C´C and MC common so angle<A_1CM = C´MC, and the Sum of angles C´MC + MCB_1 = A_1CM + MCB_1= 180▫F.13-A. → Presentation of the Parallel Method on Dr. Geo - Machine Macro – Constructions . a.. The three Points A , B , M consist a Plane and so this Proved Theorem exist only in plane .b..Points A , B consist a Line and thisbecauseexistspostulateP1 . c.. Point M is not on A B line and thisbecausewhen segment MA+MB > AB then point M is not on line ABaccording to Markosdefinition .
d.. When Point M is on AB line , and thisbecausesegment MA+ MB = AB then point M beingon line ABis an Extrema case , and thenformulates infinite Parallel linescoincidingwith AB line in the Infinite (∞) Planes . All for the extrema Geometry cases in [44-46].



4.5The Succession of Proofs:
1.. Draw the circle (M, MA) be joined meeting line AB in C and let A_1 , B_1 be the midpoint of CA, CB.
2.. On mid-perpendicular B1M´ find point M´ such that M´B1 = MA_1, and draw the circle (M´, M´B = M´C) intersecting the circle (M, MA = MC) at point D.
3.. Draw mid-perpendicular of CD at point C´.
4..To show that line MM´ is a straight line passing through point C ´and it is such that MA_1 = M´B_1= C´C = h , i.e. a constant distance , h , from line AB or, also The Sum of angles C´MC + MCB_1= A_1CM + MCB_1 = 180 ▫
Proofed Succession
1..The mid-perpendicular of CD passes through points M, M ´.
2..Angle <A_1MC´ = A_1MM´ = A_1CC´, Angle <B_1M´C´ = B_1M´M = B_1CC´ <A_1MC´ =A_1CC´ because their sides are perpendicular among them i.e.
MA_1⊥CA,MC´⊥CC´.
a..In case <A_1MM´ +A_1CC´=180▫ and B_1M´M +B_1CC´ = 180▫ then <A_1MM´ = 180▫ - A_1CC´, B_1M´M = 180▫ - B_1CC´, and by summation <A_1MM´ + B_1M´M = 360▫ - A_1CC´- B_1CC´ or Sum of angles <A_1MM´ + B_1M´M = 360 - (A_1CC´ + B_1CC´) = 360 -180▫ = 180▫
3..The sum of angles A_1MM´ + B_1M´M = 180▫ because the equal sum of angles A_1CC´ + B_1CC´ = 180▫, so the sum of angles in quadrilateral MA_1B_1M´ is equal to 360▫.
4..The right-angled triangles MA_1B_1, M´B_1A_1 are equal, so diagonal MB_1= M´A_1 and since triangles A_1MM´, B_1M´M are equal, then angle A_1MM´ = B_1M´M and since their sum is 180 ▫,

therefore angle <A_1MM′ = MM′B_1 = M′B_1A_1 =B_1A_1M = 90 ▫
5..Since angle A_1CC′ = B_1CC′ = 90▫, then quadrilaterals A_1CC′M, B_1CC′M′ are rectangles and for
the three rectangles MA_1CC′, CB_1M′C′, MA_1B_1M′ exists MA_1 = M′B_1 = C′C
6..The right-angled triangles MCA_1, MCC′ are equal, so angle <A_1CM = C′MC and since the sum of
angles <A_1CM + MCB_1 = 180 ▫ then also C′MC + MCB_1 = 180 ▫ →
which is the second to show, as this problem has been set at first by Euclid.
All above is a Proof of the Parallel postulate due to the fact that the parallel postulate is dependent of the other four axioms (now is proved as a theorem from the other four). Since line segment AB is common to ∞ Planes and only one Plane is passing through point M (Plane ABM from the three points A, B, M, then the Parallel Postulate is valid for all Spaces which have this common Plane , as Spherical, n-dimensional geometry Spaces. It was proved that it is a necessary logical consequence of the others axioms, agree also with the Properties of physical objects, d + 0 =
d , d * 0 = 0 , now is possible to decide through mathematical reasoning , that the geometry of the physical universe is Euclidean . Since the essential difference between Euclidean geometry and the two non-Euclidean geometries , Spherical and hyperbolic geometry , is the nature of parallel line, i.e. the parallel postulate so ,
<<The consistent System of the – Non-Euclidean geometry - have to decide the direction of the existing mathematical logic >>.

The above consistency proof is applicable to any line Segment AB on line AB,(segment AB is the first dimensional unit, as AB = 0 → ∞), from any point M not on line AB, [MA + MB > AB for three points only which consist the Plane. For any point M between points A, B is holding MA+MB = AB i.e. from two points M, A or M, B passes the only one line AB. A line is also continuous (P1) with points and discontinuous with segment AB [14] ,which is the metric defined by non- Euclidean geometries ,and it is the answer to the cry about the < crisis in the foundations of Euclid geometry > ***A Line Contains at Least Two Points, is Not an Axiom Because is Proved as Theorem***
Definition D2 states that for any point M on line AB is holding MA+MB = AB which is equal to <segment MA + segment MB is equal to segment AB > i.e. the two lines MA,MB coincide on line AB and thus this postulate is proved also from the other axioms, thus D2 is not an axiom, which form a system self consistent with its intrinsic real-world meaning. F.12-13.

IV. Conclusions

Parallel line.
A line (two points only) is not a great circle(more than three points being in circle`s Plane) so
anything built on this logic is a mislead false.
The fact that the sum of angles on any triangle is 180▫ is springing for the first time, in article
(Rational Figured numbers or Figures) [9] .
This admission of two or more than two parallel lines, instead of one of Euclid’s, does not proof the truth of the admission. The same to Euclid’s also, until the present proved method. Euclidean geometry does not distinguish, Space from time because time exists only in its deviation -Plank's length level-,neither Space from Energy - because Energy exists as quanta on any first dimensional Unit AB , which as above connects the only two fundamental elements of Universe, that of points or Sector = Segment = Monad = Quaternion , and that of Energy. [23]-[39].

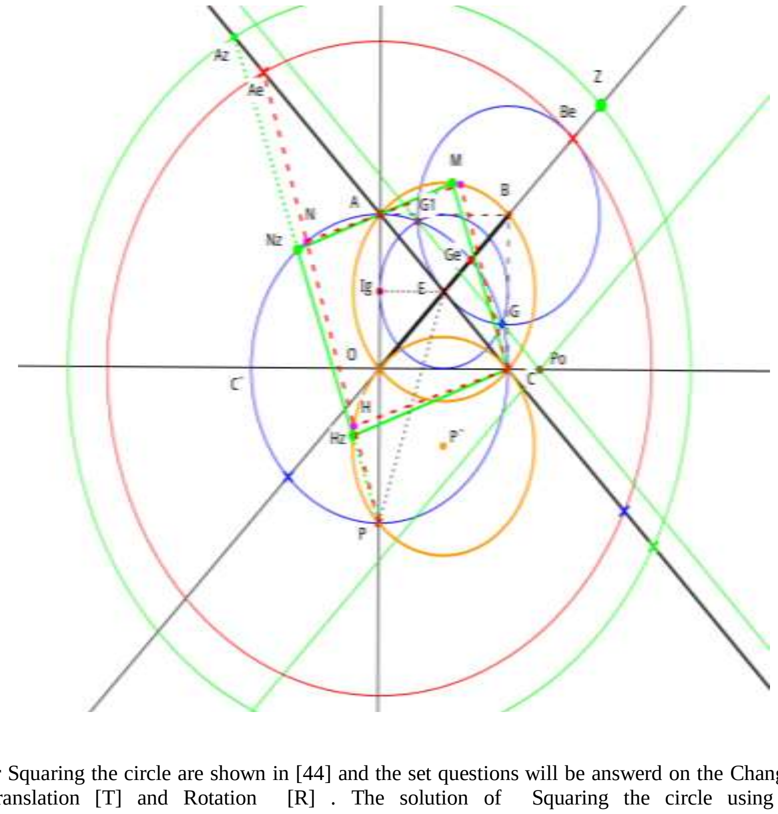
The proposed Method in articles, based on the prior four axioms only, proofs, (not using any other admission but a pure geometric logic under the restrictions imposed to seek the solution) that, through point M on any Plane ABM (three points only that are not coinciding and which consist the Plane) , passes only one line of which all points equidistant from AB as point M , i.e. the right is to Euclid Geometry. The what is needed for conceiving the alterations from Points which are nothing , to segments , i.e. quantization of points as , thediscreteting = monads = quaternion , to lines , plane and volume , is the acquiring and having Extrema knowledge . In Euclidean geometry the inner transformations exist as pure Points , segments, lines , Planes , Volumes, etc. as the Absolute geometry is (The Continuity of Points) , automatically transformed through the three basic Moulds (the three Master moulds and Linear transformations exist as one Quantization) to Relative external transformations , which exist as the , material, Physical world of matter and energy (Discrete of Monads) . [43-44]

The new Perception connecting the RelativisticTime and Einstein`s Energy -is Now Refining Time and Dark –matter Force - clearly proves That Big-Bang have Never been existed .
In [17-45-46] is shown the most important Extrema Geometrical Mechanism in this Cosmoswhich is that of STPL lines , that produces and composite , All the opposite space Points from Spaces to Anti-Spaces and to Sub-Spaces as this is in a Common Circle , this is the Sub-Space , to lines into a Cylinder.This extrema mould isa Transformation , i.e. a Geometrical Quantization Mechanism , →
for the Quantization of Euclidean geometry, points ,
to the Physical world , to Physics , and is based onthe following geometrical logic , Since Primary point ,A, is nothing and without direction and it is the only Space , and this point to exist , to be , at any other point ,B, which is not coinciding with point ,A, then on this couple exists the Principle of Virtual Displacements W = $\int_A^B P.ds$ = 0 or [ds.(P_A+P_B) = 0] , i.e. for any ds > 0 Impulse P = (P_A+P_B) = 0 and Work [ds. (P_A+P_B) = 0] , Therefore ,Each Unit AB = ds > 0 , exists by this Inner Impulse (P) where $P_A + P_B = 0$.
The Position and Dimension of all Points which are connected across the Universe and that of Spaces , exists , because of this equilibrium Static Inner Impulse and thus show the Energy-Space continuum . Applying the above logic on any monad = quaternion(s + $\bar{v}$.∇i) ,where , s = the real part and ($\bar{v}$.∇i) the imaginary part of quaternion so ,
Thrust of two equal and opposite quaternion is the , Action of these quaternions which is ,
(s +$\bar{v}$.∇i) . (s + $\bar{v}$.∇i) = [s+$\bar{v}$.∇i] ² = s²+|$\bar{v}$|².∇i² + 2|s|x|$\bar{v}$|.∇i =s² - |$\bar{v}$|² +2|s|x|$\bar{w}.\bar{r}$ |.∇i =
[s²] – [|$\bar{v}$|²] + [2$\bar{w}$.|s| |$\bar{r}$|.∇i] where,
[+s²] → s² = (w.r)² , → is the real part of the new quaternion which is , the positive Scalar product , of Space from the same scalar product ,s,s with ½ ,3/2,,, spin and this because of ,w,and which represents the massive , Space , part of quaternion → monad .
[-s²] → - |$\bar{v}$|² = - |$\bar{w}.\bar{r}$|² = - [|$\bar{w}$|.|$\bar{r}$|]² = - (w.r)² → is the always , the negative Scalar product , of Anti-space from the dot product of ,$\bar{w}$,$\bar{r}$ vectors , with -½ ,-3/2, spin and this because of , - w , and which represents the massive , Anti-Space , part of quaternion→ monad .
[∇i] → 2.|s| x | $\bar{w}.\bar{r}$ |.∇i =2|wr|.|(wr)|.∇i = 2.(w.r)² → is avector of , the velocity vector product , from the crossproduct of $\bar{w}$,$\bar{r}$ vectors with double angular velocity termgiving 1,3,5, spin and this because of , ± w, in inner structure of monads , and represents the , Energy Quanta , of the Unification of the Space and Anti-Space through the Energy (Work) part of quaternion .
A wider analysis is given in articles [40-43] .

When a point ,A, is quantized to point ,B, then becomes the line segment AB = vector AB = quaternion [AB] → monad , and is the closed system ,A B, and since also from the law of conservation of energy , it is the first law of thermodynamics , which states that the energy of a closed system remains constant , therefore neither increases nor decreases without interference from outside , and so the total amount of energy in this closed system , AB , in existence has always been the same , Then the Forms that this energy takes are constantly changing , i.e.
The conservation of energy is realized when stored in monads andfollowing the physical laws in E-geometry where then are Material → Points , monads , etc ← This is the unification of this Physical world of , what is callled matter and Energy , and that of Euclidean Geometry which are , Points , Segments , Planes and Volumes . For more in [48] .
The three Moulds (i.e. The three Geometrical Mechanism) of Euclidean Geometry which create the ETERS of monads and which are , Linear for a perpendicular Segment , Plane for the Square equal to the circle on Segment ,Space for the Double Volume of initial volume of the Segment , (the volume of the sphere is related to Plane which is related to line and which is related to segment) , Exist on Segments in Spaces , Anti-spaces and Sub-spaces . This is the Euclidean Geometry Quantization to its constituents (i.e. Geometry in its moulds) .
The analogous happens when E-Geometry is Quantized to Space and Energy monads [48].
METER of Points A is the Point A , the
METER of line is the Segment ds = AB = monad = constant and equal to monad , or to the perpendicular distance of this segment to the set of two parallel lines between points A,B , the METER of Plane is that of circle on Segment = monad and which is that Square equal to the circle ,number ,π, the METER of Volume ,$\sqrt[3]{2}$, is that of Cube , on Segment = monad which is equal to the Double Cube of the Segment and Measures all the Spaces , the Anti-spaces and the Subspaces in this cosmos .
Generally is more referred ,
a). There is not any Paradoxes of the infinite because is clearly defined what is a Point a cave and what is a Segment . b).The Algebra of constructible numbers and number Fields is an Absurd theory, based on groundless Axioms as the fields are , and with direction the non-Euclid orientations purposes which must be properly revised .

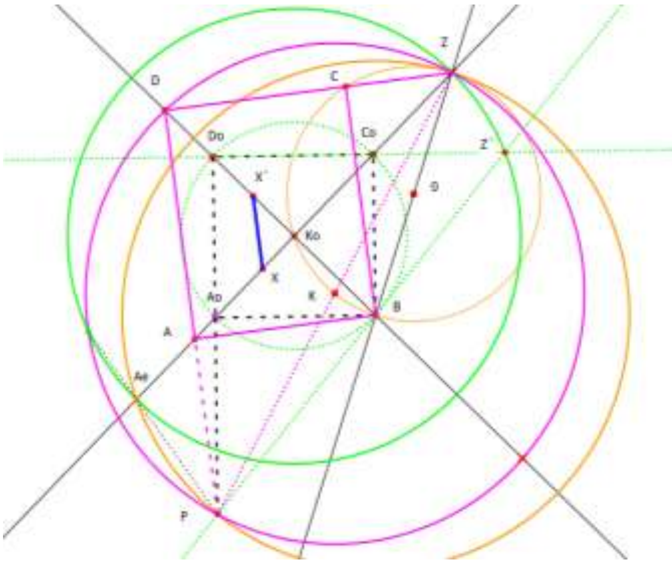
c).The Algebra of Transcental numbers has been devised to postpone the Pure geometrical thought , which is the base of all sciences , by changing the base-field of solutions to Algebra as base . Pythagorians discovered the existence of the incommensurable of the diagonal of a square in relation to its side without giving up the base , which is the geometrical logic.

d). All theories concerning the Unsolvability of the Special Greek problems are based on Cantor`s shady proof , <that the totality of All algebraic numbers is denumerable > and not edifyed on the geometrical basic logic which is the foundations of all Algebra . The problem of Doubling the cube F.4-A , as that of the Trisection of any angle F.11-A, is
a Mechanical problem and could not be seen differently and the proposed Geometrical solutions is clearly exposed to the critic of all readers .



All trials for Squaring the circle are shown in [44] and the set questions will be answerd on the Changeable System of the two Expanding squares ,Translation [T] and Rotation [R] . The solution of Squaring the circle using the Plane Procedure methodisnowpresented in F-1,2 , and consists an , Overthrow , to all existingtheories in Geometry ,Physics and Philosophy . e).Geometryis the base of all sciences and itis the reflective logicfrom the objective reality ,
whichis nature , to ourmind.

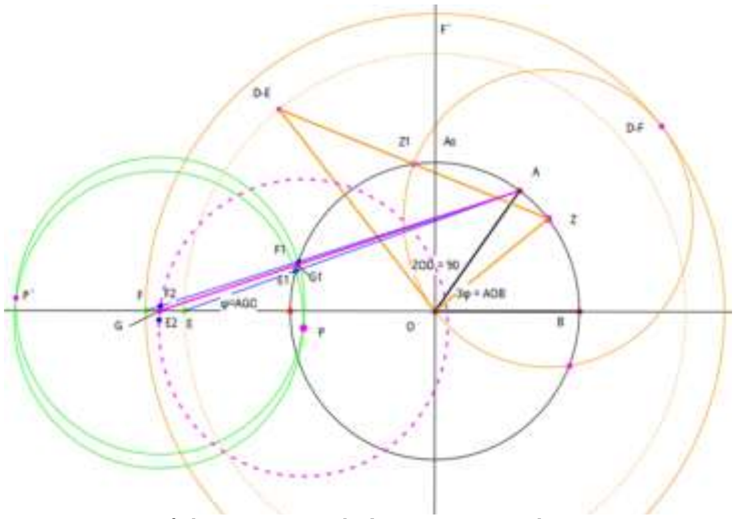
F.2-A →A Presentation of the Quadrature Method on Dr. Geo-Machine Macro - constructions .
The Inscribed Square CBAO , withPole-line AOP , rotatesthrough Pole P , to the→ Circle-SquareCMNH withPole-line NHP , and to the→ Circumscribed SquareCAC`P , withPole-line C`PP = C`P , of the circle E , EO = ECand at position Be , A_eNHP Pole-line formulates square CMNH = π . EO ² whichis the Squaring of the circle . Number $\pi = \frac{CM^2}{EO^2}$ as in [Fig.2-A]



F.4-A. → A Presentation of the Dublication Method on Dr.Geo - Machine Macro - constructions

BCDA Is the In-between Quadrilateral , on (K,KZ) Extrema-circle , and on K_OZ-K_OB Extrema – lines of common poles Z , P , mechanism . The Initial Quadrilateral B$C_O D_O A_O$, withPole-lines $D_O A_O$P , $D_O C_O$Z` , rotatesthrough Pole P and the moveablePole Z ` on Z`Zarc, to the → Extreme Quadrilateral BCDAthroughPole-lines DAP - DCZ with point Do , sliding on B$K_O D_O$Pole-line, and then at point D , KD ³ = 2.KoA ³ whichisthe Dublication of the Cube .
For any initial segment K_OX issues (K_OX`) ³ = 2 .($K_O$X) ³ where $K_O$X` =K_OD.($\frac{KoX}{KoA}$)and

$\sqrt[3]{2}$= ($\frac{KoD}{KoA}$). ($\frac{KoX}{KoX`}$ ) = [$\frac{KoD}{KoA}$] ² = $\frac{KoD^2}{KoA^2}$→ as in [Fig4-A] ,and since ($\frac{KoD}{KoA}$ ) = ($\frac{KoX}{KoX`}$)

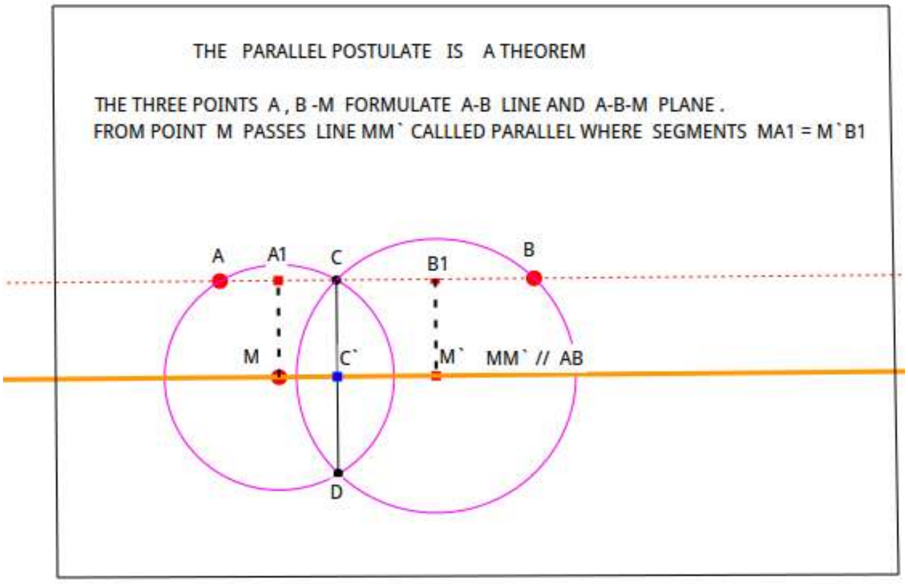


F.11-A. → Presentation of the Trisection Method on Dr. Geo - Machine Macro – constructions .
From Initial position of triangle AOC , where angle AOB = 90▫ and SegmentA_1C = OA ,to

the Final position of triangle , where angle AOB = BOB = 0▫and AOB = B`OB = 180 ▫, throughtheExtrema position between edge- cases of triangle ZOD where AOB = φ ▫ and at common point P ,
PG = OA = GP = GG_1= G_1O and at point G ,thenG_1G = G_1O = OA whichis the Trisection of angle AOB , and Angle <AGB = ($\frac{1}{3}$). AOB .

The Presentation of the Parallel Method .

a.. The three Points A , B , M consist a Plane and so this Theorem exist only in plane .
b.. Points A , B consist a Line and thisbecauseexistspostulate[P1] .
c.. Point M is not on A B line and thisbecausewhen segment MA+MB > AB then point M is not on line ABand MA_1 = M`B_1 .
d.. When Point M is on A B line , and thisbecausesegment MA+ MB = AB then point M being on line ABis an Extrema case , and thenformulates infinite Parallel linescoincidingwith AB line in the Infinite (∞) Planes through AB.



F.13-A. → the Parallel Geo - –

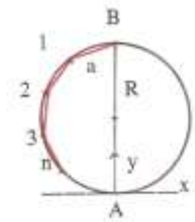
Presentation of Method on Dr. Machine Macro Constructions

V. THE REGULAR POLYGONS :

5.1. The Algebraic Solution:
It has been proved by De Moivre’s , that the n-th roots on the unit circle AB are represented by the vertices of the Regular n-sided Polygon inscribed in the circle . It has been proved that the Resemblance Ratio of Areas , of the circumscribed to the inscribed squares (Regular quadrilateral) which is equal to 2 , leads to the squaring of the circle. It has been also proved that , Projecting the vertices of the Regular n-Polygon on any tangent of the circle , then the Sum of the heights yn is equal to n ∗ R .
This is a linear relation between Heights , h , and the radius of the circle , the monad . This property on the circle yields to the Geometrical construction (As Resemblance Ratio of Areas is now controlled) , and the Algebraic measuring of the Regular Polygons as follows :
when : R = The radius of the circle , with a random diameter AB .
a = The side of the Regular n -Polygon inscribed in the circle
n = Number of sides , a , of the n -Polygon , then exists :

$$n . R = 2 . R + 2 . y1 + 2 . y2 + 2 . y3 + \dots\dots\dots 2 . \boldsymbol{y_n} \qquad \dots\dots\dots\dots (n)$$

the heights y_n are as follows :



y_B= [2 . R]

y_1 = [4.R ² - a ²] / (2 . R)

y_2 = [4.R^4 – 4.R^2.a^2 + a^4] / (2.R^3)
/

y_3 = $\dfrac{[\, 8.R^6 - 10.R^4.a^2 + 6 . R^2.a^4 - a^6] - a^2.\sqrt{64.R^8 - 96.R^6.a^2 + 52.R^4.a^4 - 12.R^2.a^6 + a^8}}{2.R^5}$

y_n= [……………..] / 2.R^n

THE ALGEBRAIC EQUATIONS OF THE REGULAR n - POLYGONS

(a) REGULAR TRIANGLE :

The Equation of the vertices of the Regular Triangle is :

$$3.R = 2.R + \left[\frac{4.R^2 - a^2}{R} \right] \quad >>> \quad R^2 = 4 . R^2 - a^2 \quad >>> \quad a^2 = 3. R^2$$

The side $a_3 = R .\sqrt{3}$……….(1).

(b) REGULAR QUADRILATERAL (SQUARE) :

The Equation of the vertices of the Regular Square gives :

$$4.R = 2.R + \left[\frac{4.R^2 - a^2}{R} \right] \quad >>> \quad a^2 = 2 . R^2$$

The side $a_4 = R .\sqrt{2}$……….(2)

(c) REGULAR PENTAGON :

The Equation of the vertices of the Regular Pentagon is :

$$\frac{5.R = 2.R + \left[\frac{4.R^2 - a^2}{R} \right] + [\, 4.R^4 - 4.R^2 . a^2 + a^4 . \,]}{R^3} \quad >>> a^4 - 5. R^2 . a^2 + 5. R^4 = 0$$

Solving the equation gives :

$$R^2 = \frac{5.R^2 - \sqrt{25. R^4 - 20.R^4}}{2} = \frac{5.R^2 - R^2 .\sqrt{5}}{2} = [\, \{ 5.R^2 - R^2.\sqrt{5} \} / 2] = \frac{R^2}{2} (5 - \sqrt{5})$$

$$a^2 = \left\{ \frac{R^2}{4} \right\} . [\, 10 - 2\sqrt{5} \,] \quad >>> \quad \text{The side} \quad a5 = \left|\frac{R}{2}\right| \sqrt{10 - 2.\sqrt{5}}$$

…..(3)

(d) REGULAR HEXAGON :

The Equation of the vertices of the Regular Hexagon is :

$$6.R = 2.R + \left[\frac{4.R^2 - a^2}{R} \right] + \left[\frac{4.R^4 - 4.R.^2 .a^2 + a^4}{R^3} \right] \quad >> a^4 - 5. R^2.a^2 + 4. R^4 = 0$$

Solving the equation gives :

$$a^2 = \frac{5. R^2 - \sqrt{25.R^4 - 16. R^4}}{2} = \frac{[\, 5 - 3 \,]}{2} . R^2 = R^2 \qquad \text{The side} \quad a6 = R$$

…..(4)

(e) REGULAR HEPTAGON :

The Equation of the vertices of the Regular Xeptagon is :

$$\frac{7. R = 2. R}{R} + \left[\frac{4. R^2 - a^2}{R^3} \right] + \left[\frac{4. R^4 - 4. R^2 . a^2 + a^4}{2. R^5} \right] + [\, 8. R^6 - 10. R^4 . a^2 + 6. R^2 . a^4 a - a^6] \; -$$

$$- \left[\frac{a^2}{2.R^5} \right] . \sqrt{64 . R^8 - 96.R^6 . a^2 + 52 .R^4 .a^4 - 12. R^2 .a^6 + a^8}$$

Rearranging the terms and solving the equation in the quantity a , obtaining :

$$R^2 .a^{10} - 13 .R^4 . a^8 + 63 . R^6 .a^6 - 140 . R^8 .R^4 + 140 , R^{10} .a^2 - 49 . R^{12} = 0 \quad \text{for } a^2 = x$$

$$x^5 - 13 . R^2 . x^4 + 63 .R^4 . x^3 - 140 . R^6 . x^2 + 140 . R^8 . x^1 - 49 . R^{10} = 0 \quad ………..(7)$$

Solving the 5nth degree equation the Real roots are the following two :

$x_1 = R^2 . [3 - \sqrt{2}]$, $x_2 = R^2 . [3 + \sqrt{2}]$ which satisfy equation (7)

Having the two roots , the Sum of roots be equal to 13 , their combination taken 2 ,3, 4 at time be equal to 63 , - 140 , 140 , the product of roots be equal to – 49 , then equation (7) is reduced to the third degree equation as :

$z^3 - 7. z^2 + 14. z - 7 = 0$ …..(7a)

by setting $\psi = z - (-7/3)$ into (7a) , then gives $\psi^3 + \rho. \psi + q = 0$ .... (7b) where ,

$\rho = 14 - (-7)^2 / 3 = 14 - 49/3 = - 7 / 3$ $\rho^2 = 49 / 9$ $\rho^3 = - 343 / 27$
$q = 2. (-7)^3 / 27 + 14 .(-7) / 3 - 7 = 7 / 27$ $q^2 = 49 / 729$

Substituting ρ , q then $\psi^3 - (7 / 3) . \psi + (7 / 27) = 0$ … (7b)

The solution of this third degree equation (7b) is as follows : $\rho = -7 / 3$
$q = 7 / 27$

Discriminant $D = q^2 / 4 + \rho^3 / 27 = (49 / 729 .4) - (343 / 27.27) = - [49 / 108]$ 0

$D = - 49 /108 = i^2 (3.21^2 / 4 .27^2) = i^2 (21 .\sqrt{3}/ 2 .27)^2 = i^2 (21.\sqrt{3}/ 54)^2$

$D = [7 . \sqrt{3}/ 18]^2 . i^2$ also $\sqrt[2]{D} = \left| \frac{7 .\sqrt{3}}{18} \right| . i$

Therefore the equation has three real roots :

Substituting $\psi = w - \rho / 3.w = w + 7 / 9.w$ $>\psi^2 = w^2 + 49 / 81 .w^2 + 14 / 9$
$\psi^3 = w^3 + 343/729w^3 + 49 /27w+7w / 3$
to (7b) then becomes $w^3 + 343 /729 w^3 + 7 / 27 = 0$
and for $z = w^3$ $z + 343 /729 z + 7 / 27 = 0$

$z^2 + 7. z / 27 + 343 / 729 = 0$…(7c)

The Determinant D 0 therefore the two quadratic complex roots are as follows :

$Z1 = [-7/27 - \sqrt{49/27.27 - 4. 343/729}] / 2 = [-7/27 - \sqrt{49/27.27.4 - 49 .7. 4/27.27.4}] / 2$

$= [-7/27 - \sqrt{(49 -49.28)/ 27.27.4}] / 2 = [- 7 - 7 .\sqrt{- 27}] / 27 . 2$

$\frac{=}{2}$ [$- 7 - 21 .\sqrt{-3}$] $/ 3^3 . 2$ $= [- 7] . (1 - 3 . i .\sqrt{3}) / 27 = (-7 / 54).[1 - 3.i.\sqrt{3}]$
$Z2 = [-7/2 . (1 - 3 . i\sqrt{3}] / 27$ $= (- 7 / 54) . [1 + 3.i.\sqrt{3}]$

The Process is beginning from the last denoting quantities to the first ones :

Root $W1,2 = \sqrt[3]{\frac{\sqrt{3}}{2}} = \frac{1}{3} \sqrt[3]{\frac{-7 \quad 21 . i .\sqrt{3}}{2}} = \frac{1}{3} \sqrt[3]{- (7) . [1 \quad 3.i .\sqrt{3}]}$ …...(1)

Root $\psi = W + 7 / 9.W = \frac{1}{3} . \sqrt[3]{\frac{-7 \quad 21. i. \sqrt{3}}{2}} + \frac{7}{3 . \sqrt[3]{\frac{-7 \quad 21 i. \sqrt{3}}{2}}}$…….(2)

Root $X = \psi - \rho / 3 = \psi + 7 / 3 = \frac{7}{3} + \frac{1}{3} . \sqrt[3]{\frac{-7 \quad 21 .i .\sqrt{3}}{2}} + \frac{7}{3 . \sqrt[3]{\frac{-7 \quad 21 .i .\sqrt{3}}{2}}}$

| -------------------- 3 / 2 --------------------- |
1 | 7. / -7 21 . i .$\sqrt{3}$ + / - 7 21 . i .$\sqrt{3}$ + 7 | ……… (3)

X = --- | √ 2 √ 2 | . R^2
3 | ————— |
| ³ √ - 7 ± 21 . i . $\sqrt{3}$ |

The root a7 of equation (7) equal to the side of the regular Heptagon is $a_7=\sqrt{X}$

² / | ------------------ 3/2 ------------------- |
a7 = / 1 | ³ / -7 21. i .$\sqrt{3}$+ / -7 21 . i .$\sqrt{3}$+ 7 |
/ --- | 7. √ 2 √ 2 | . R …….. (4)
/ 3 | ————— |
/ | ³ / -7 21 . i . $\sqrt{3}$ |
√ | √ |

Instead of substituting ψ = w - ρ / 3.w into (7.b) , is substituted ψ = u + v and then gives the equation of second degree as $z^2 + 7.z / 27 + 343 / 729 = 0$ which has the two complex roots as follows :

$$z_{1,2} = \frac{7}{54} . [-1 \pm 3 . i . \sqrt{3}] = \frac{1}{27} . [(-7 \pm 21 . i . \sqrt{3}) / 2]$$ and the side a_7 is as :

² ---
a 7 = / ³ ------ + 7 / ³ ------ / + _7_ and by substituting Z1 , Z2 into (7b)
√√z1 / √z2 / 3 becomes the same formula as in (4) .

It is easy to see that $\sqrt[3]{- (7/2) . [1 – 3 . i .\sqrt{3}]} * \sqrt[3]{- (7/2) . [1 + 3 . i . \sqrt{3}]} = 7$
√√
Analytically is :

x = __ R^2 . [0,753 020 375 967 025 701 777] >> x^2 = 0 , 56704
a 7 = $\sqrt{x}$ = R . [0,867 767 453 193 664 601 …]

By using the formula of the real root of equation (7a) then :

$a.x^3 + b.x^2 + c.x + d = 0$ >>> for a = 1 , b = -7 , c = 14 , d = -7 then $x^3 - 7. x^2 + 14. x – 7 = 0$

$$x = -\frac{b}{3} - \frac{2^{1/3} .(- b^2 + 3.c)}{3[-2b^3+9bc-27d+\sqrt{4(-b^2+3c)^3+(-2b^3+9bc–27d)^2}]^{1/3}} + \frac{[-2b^3+9bc-27d+\sqrt{4(-b^2+3c)^3+(-2b^3+9bc-27d)^2}]^{1/3}}{3 \cdot 2^{1/3}} .$$

Substituting the coefficients to the upper equation becomes :

$-b^2 +3.c = - (-7)^2 + 3.14 = - 49 + 42 = -7$ $-2.b^3 + 9.b.c -27.d = -2.(-7)^3 + 9.(-7).14 - 27.(-7) =$
686 -882 + 189 = -7
$4.(-b^2+3.c)^3 = 4 (-7)^3 = - 1372$ $(-2.b^3 +9.b.c -27.d)^2 = (-7)^2 = 49$ $32^{1/3} = \sqrt[3]{8.4} = 2 . \sqrt[3]{4}$

$$X = \frac{7}{3} - \frac{\sqrt[3]{2} .(-7)}{3 .\sqrt[3]{-7 + 21. i .\sqrt{3}}} + \frac{\sqrt[3]{-7 + 21. i .\sqrt{3}}}{2 .\sqrt[3]{4}}$$ and

$$\boldsymbol{a_7} = \sqrt{X} = \sqrt{\frac{7}{3} + \frac{7 . \sqrt[3]{2}}{3 .\sqrt[3]{-7 + 21.i.\sqrt{3}}} + \frac{\sqrt[3]{-7 + 21 . i . \sqrt{3}}}{2 . \sqrt[3]{4}}}$$ The Side of the Regular Heptagon (4.a)

Further Analysis to the Reader

(f) REGULAR OCTAGON :

The equation of vertices of the Regular Octagon is

$$8.R = 2.R + \frac{(a^2)}{R} + \frac{(4.R^2.a^2 - a^4)}{R^3} + \frac{10.R^4.a^2 - 6.R^2.a^4 + a^6 + a^2.\sqrt{64.R^8 - 96.R^6a^2 + 52.R^4.a^4 - 12.R^2.a^6 + a^8}}{2.R^5}$$

Rearranging the terms and solving the equation in the quantity a , is a 10th degree equation , and by reduction ($x = a^2$) is find the 5th degree equation as follows :

$$a^{10} - 13.R^2.a^8 + 62.R^4.a^6 - 132.R^6.a^4 + 120.R^8.a^2 - 36.R^{10} = 0$$

$$x^5 - 13.R^2.x^4 + 62.R^4.x^3 - 132.R^6.x^2 + 120.R^8.x^1 - 36.R^{10} = 0 \quad \text{.... (a)}$$

Solving the 5th degree equation is find the known algebraic root of Octagon of side a as :

The roots are >>>>>>>>>> $x1 = R^2.[2 - \sqrt{2}]$, $x2 = R^2.[3 - \sqrt{3}]$

$a8 = \sqrt{x} = R.\sqrt{2 - \sqrt{2}}$........ (b)

Verification :

$x = a^2 = R^2(2 - \sqrt{2})$ $x^2 = R^4.(6 - 4\sqrt{2})$ $x^3 = R^6.(20 - 14.\sqrt{2})$

$x^4 = R^8.(68 - 48\sqrt{2})$ $x^5 = R^{10}.(232 - 164\sqrt{2})$ ....(c)

by substitution (c) in (a) becomes :

$R^{10}.[232 - 164.\sqrt{2}] = R^{10}.[232 - 164.\sqrt{2}\ 2]$

$-R^{10}.[884 - 624.\sqrt{2}] = R^{10}.[-884 + 624.\sqrt{2}]$

$R^{10}.[1240 - 868.\sqrt{2}] = R^{10}.[1240 - 868.\sqrt{2}]$

$-R^{10}.[792 - 528.\sqrt{2}] = R^{10}.[-792 + 528.\sqrt{2}]$

$R^{10}.[240 - 120.\sqrt{2}] = R^{10}.[240 - 120.\sqrt{2}]$

$-R^{10}.[36 \quad] = R^{10}.[-36 \quad]$

$R^{10}.[1712 - 1712 + (1152 - 1152).\sqrt{2}] = 0$

$R^{10}.[0+0] = 0$ therefore Side $\boldsymbol{a}_8 = R.\sqrt{2 - \sqrt{2}}$...... (b)

(g) Conclution :

By summation the heights y on any tangent in a circle ,which hold for every Regular n-sided Polygon inscribed in the circle as the next is :

$n.R = 2.R + 2.\boldsymbol{y_1} + 2.\boldsymbol{y_2} + 2.\boldsymbol{y_3} + \ldots 2.\boldsymbol{y_n}$ ....(n)

the sides$\boldsymbol{a_n}$of all these Regular n-sided Polygonsare Algebraically expressed .

The Geometrical Construction of all Regular Polygons has been proved to be based on the solution of the moving Segment ZD of the figure of page 8 and it is the Master Key of Geometry , because so , the nth degree equations are the vertices of the n-polygon .

In this way , all Regular p - gon are constructible and measureable .

The mathematical reasoning is based on Geometrical logic exclusively alone .

As the Resemblance Ratio of Areas on the 4 - gone is equal to 2 , the problem of squaring the circle has been approached and solved by extending Euclid logic of Units (under the restrictions imposed to seek the solution , with a ruler and a compass ,) on the unit circle AB , to unknown and now the Geometrical elements . (the settled age-old question for all these problems is not valid) .

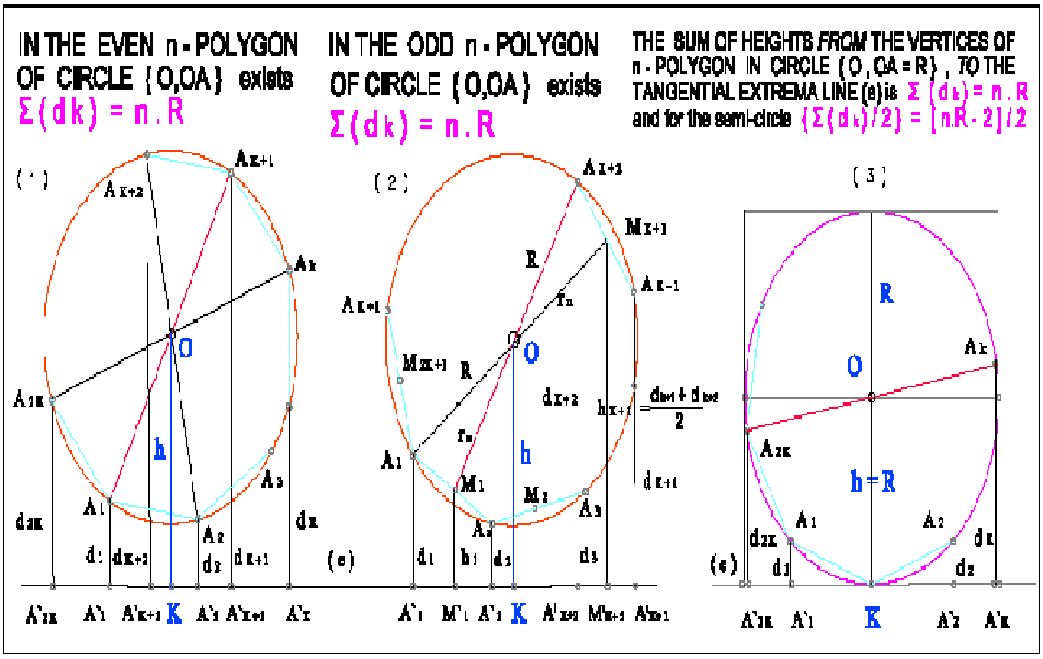
The Regular Heptagon :

According to Heron , the regular Heptagon is equal to six times the equilateral triangle with the same side and is the approximate value of $\sqrt{3}.R/2$

According to Archimedes , given a straight line AB we mark uponit two points C , D such that $AD.CD = DB^2$ and $CB.DB = AC^2$, without giving the way of marking the two points . According to the Contemporary Method , the side of the Regular Heptagon is the root of a third degree equation with three real roots , one of which is that of the regular Heptagon as analytically presented.

5.2. THE GEOMETRICAL SOLUTION :

a.. The Even andOdd n-Polygons :



F.14 $\rightarrow$ A Even and an Odd n-Polygon in circle O ,OA with diameters , $A_k A_{2k}$, passing fromA_{2k} ,
as vertex (apex) of the Polygone , and diameters , $A_{k+2}M_1$perpendicular to sideA_1A_2 .

Let be the n-PolygonA_1 , A_2 , A_3 , A_k , A_{k+1} , A_{k+2} , A_{2k} , in circle (O , OA_1) ,
(e) a straight line not intersecting the circle
d_1 ,d_2 ,d_{2k} ,   The heightsof the vertices to (e) line ,
h_1 ,h_2 ,h_{2k+1} , The heights of the midpoints$M_k M_{k+1}$ of the sides to (e) line and OK = h , The height from the center Oto (e) line .

To proof :
In any n-Polygon , The Sum , Σ=Σ(h) , of the Heights , d_1 ,d_2 ,d_{2k} , of the Vertices A_1 , A_2 , A_3 , A_k ,A_{k+1} , A_{k+2} , A_{2k} , where n = 2k,
from any straight line (e) is equal to

Σ= Σ(h) = n . OK = n . h
Proof   F.14 :
From any vertex A_k , of the n-Polygon draw the diameter (A_k OA_{2k})
a.. When n = 2.k $\rightarrow$ then Vertex A_{2k}belongs to the Polygon
b.. When n = 2.k + 1 $\rightarrow$ then line A_kO , is mid-perpendicular to one of the sides .

Case a.. n = 2.k   F.14 –(1)
Exists $\frac{n}{2}$= $\frac{2k}{2}$ = k , and are the pairs of vertices in opposite diameters as in A_1 ,A_{k+1} , and the , k ,
Trapezium which has bases the heights of the vertices in opposite diametersfrom (e) line , and which
have height OK = h , as Common Height from their Diameter , i.e.

From trapezium A_1 , $A`_1$ , $A_{k+1}$ , $A`_{k+1}$ exists d_1 + d_{k+1} = 2.h   and analogically ,
d_2 + d_{k+2} = 2.h
d_3 + d_{k+3} = 2.h   ………….. d_k + d_{k+1}= 2.h
And by Summation ,

$d_1 + d_2 + \ldots\ d_k + d_{2k}$ = 2.h   or   Σ =(2k) . h = n . h = n . OK………..(1)

Case b.. n = 2.k +1 F.14 –(2)
A_1A_2 ,A_2A_3 , …. $A_{2k+1}A_1$, the sides of the Polygon .
M_1 , M_2 , M_{2k+1} , are the midpoints of sides from line (e)
h_1 , h_2 , …. h_{2k+1} the corresponding heights of midpoints from (e) .
The diameter from vertex A_1 is perpendicular to side $A_{k+1}A_{k+2}$ which has the midpoint M_{k+1} ,
while $A_1M_{k+1} = A_1O + OM_{k+1} = R + r_n$
In trapezium $A_1A`_1M`_{k+1}M_{k+1}$ with Bases $A_1A`_1$ and $M`_{k+1}M_{k+1}$, both perpendicular to (e)
line is parallel to height OK = h and bisects A_1O = R and $OM_{k+1} = r_n$ and from figure , exists

$$OK = h = \frac{R\,h_{k+1} + r_n \cdot d_1}{R+r_n} \qquad \text{……..…(2)}$$

i.e. Height OK is common to all 2k+1 trapezium which are formed as $A_1A`_1M`_{k+1}M_{k+1}$ and OK Height divides also the corresponding to A_1M_{k+1} side with the same analogy as $\frac{R}{r_n}$.
By summation of 2k+1 parts of (2) which are all equal to OK = h , then from the 2k+1 different
Between them trapezium referred exists ,

$$(2k+1).\ h = \frac{R\{h_{k+1}+h_{k+2}+h_{k+1}+h_1+\cdots h_k\} + r_n \cdot \{d_1+d_2+\cdots d_{k+1}+d_{2k+1}\}}{R+r_n} = n\ .\ h = \frac{R.S+r_n\Sigma}{R+r_n} \quad \text{…..(3)}$$

where S = $h_1 + h_2 + \dots\ h_k + d_{2k+1}$. Since h_1 , h_2 , …. h_k , d_{2k+1} are the diameters of trapezium with bases d_1 , d_2 to h_1 , d_2 , d_3 to h_2 and so on and also d_{2k+1} , d_1 to h_{2k+1}then
S = $\frac{d_1+d_2}{2} + \frac{d_2+d_3}{2} + \dots \frac{d_{2k}+d_1}{2} = \frac{2\{d_1+d_2+\cdots d_{2k+1}\}}{2} = d_1 + d_2 + \dots\ d_k + d_{2k}$ = Σ and (3) is
n . h = $\frac{R.S+r_n\Sigma}{R+r_n} = [\frac{R+r_n}{R+r_n}]$. Σ = Σi.e. Σ = n . h for all Even and Odd n-Polygons .

A relation between Heights and the Number of the Regular Polygons .
Case c.. Line (e) is Extrema as Tangential to circle F.14 –(3)
In this case height ,h , is equal to radius R and OK = h = R .
Since the Sum of Heights of the vertices in any n-Polygon is Σ= n . h = n .OK then Σ = n . R This remark helps to construct Geometrically ,i.e. with a Ruler and a Compass , all the Regular n-Polygons because gives the relation of the Apothem , the radius r_n of the inscribed circle which
is related to the Interior anglew = { $\frac{n-2}{n}$}.180^o.
i.e. Angles, w , in a circle of radius , R , define the n-Sides , A_1A_2 , of the Regular Polygonwhich in turn define the Sum , Σ , of their heights equal to Σ = n . R
Since also the relation of radius ,R, between the Circle and ,r, of the Inscribed circle is extended to Heights , this helps Extrema - Method to be applicable on the solution which follows .

b.. The Theory of Means:

It was known from Pappus the how to exhibit in a semicircle all three means , namely ,
The Arithmetic , The Geometric , and The Harmonic mean .

In Fig.15 –(1a) → On the diameter AC of circle (O , OA = OC) , C is any Pont on OC .
Draw BD at right angles to AC meeting the semi-circle in D .
Join OD and draw BE perpendicular to OD .
Show that DE is the Harmonic - Mean between AB , BC

Proof :

For , since ODB is a right – angled triangle , and BE is perpendicular to OD then ,
DE : BD = BD : DO or DE . DO = BD ² = AB . BC
But DO = $\frac{1}{2}$(AB + BC) therefore DE .(AB+BC) = 2 . AB.BC . By rearranging
is AB . (DE – BC) = BC . (AB – DE) or AB : BC = (AB –DE) : (DE –BE) ,
that is , DE is the Harmonic Mean between AB and BC .
In Fig.15 –(1b) → Is given only Segment AB and is defined Harmonic mean AM between AB ,MB
Draw BC at right angles to AB meeting center C of circle (C , CB = AB / 2) .
Join AC intersecting circle (C ,CB) at points D , E where DE = 2.DC = AB .
Draw circle (A , AD) intersecting AB at point M .
Show that AM is the Harmonic - Mean between AB , MB .

The Proof :

For , since ABC is a right – angled triangle , and DE = AB then ,
AB ² = AD . AE = AD . (AD + DE) = AD . (AD + AB) = AD ² + AD . AB therefore ,
AD ² = AB ² - AD . AB = AB . (AB – AD) or AD ² = AB . MB

That is , AM is the Harmonic Mean in AB Segment , or between AB andMB .

6..Markos Theory, on Segmentsand Angles Relation :
The Three Circles Method :
In Fig.15 –(2) → Two Even ,n, and ,n+2, Regular Polygons on the same circle (O , OA) where , n , n +2arethe number of sides differing by an Even number
λ_a = The length of a side of a – [n -Polygon].
λ_b = The length of a side of b – [n+2 Polygon].
r_a = The Apothem (the radius of the inscribed circle of a – Polygon) .
r_b = The Apothem (the radius of the inscribed circle of b – Polygon) .
h_A = The Height of KA_1 side of a – Polygon .
h_B = The Height of KB_1 side of b – Polygon .
Δh = h_A - h_B, the difference of heights .
Δr = r_a - r_b , the difference of apothems .

S = The sum of interior angles equal to (n-2).180° = (n-2).π

$\frac{h_A}{\lambda_a} = \sin\varphi_a$, $\frac{h_B}{\lambda_b} = \sin\varphi_b$, $\frac{h}{\lambda}$ = φ ,

w_a = [$\frac{2}{n}$].180= [$\frac{2}{n}$] π, The Interior angle of the [n - Polygon].

w_b = [$\frac{2}{n+2}$].180 = [$\frac{2}{n+2}$].π , The Interior angle of the [n+2 Polygon].

w_o= An Extrema-angle between w_a,w_b which is related to Heights .

φ_a = [$\frac{n-2}{2.n}$] π, The angle of side λ_a to (e) line for Even , n-Polygon.

φ_b= [$\frac{n}{2(n+2)}$] π , The angle of side λ_b to (e) line for Even , n+2 Polygon.

φ_o= [$\frac{n-1}{2(n+1)}$] π , The angle of side λ_o to (e) line for Odd – Polygon .

Show that , the Extrema-angle ,w_o, and the complementary angle ,φ_o, define the In-between Odd-Regular n-Polygons onthe same circle (O , OA) , byScanning the , Δh ,difference Height, on Circles - Heights-System , and followingtheHarmonic – Mean of Heights .

Proof : Fig.15 –(2 , 3)

a.. Draw on OK circle , the Tangent at point K , and from K any two Chords KA and KB .

From Points A , B draw the Perpendiculars AA`,BB` and the ParallelsAA_1,BB_1 , to Tangent (e).

b.. Draw thecircle of Heights (A_1, A_1B_1)

In right angles triangles KAA` , KBB` , ratios$\frac{AA`}{KA}$= $\frac{h_A}{\lambda_a}$= $\sin\varphi_a$and$\frac{BB`}{KB}$= $\frac{h_B}{\lambda_b}$= $\sin\varphi_b$,

where h_A= λ_a . $\sin\varphi_a$ and h_B= λ_b . $\sin\varphi_b$ and the difference Δh = h_A - h_B, or

Δh = h_A - h_B = $\lambda_a . \sin\varphi_a - \lambda_b . \sin\varphi_b$ ……….(1

Since between the two sequent , n , n+2 , Even – Regular – Polygons exists the Geometric logic of AB

Monads , i.e. In a Segment the whole is equal to the parts , and to the two halves , and for angle φ_a to become φ_b is needed to pass through another one angle φ_o , which is between the two , therefore ,

a.. Between the two sequence Even -Regular-Polygons exists another one Regular-Polygon .

b.. According to Pappus theory of Proportion and Means , between the three terms h , λ , φ

exists one of the three means .

c.. For since the Sum { it is algebraically n + (n+2) = 2n + 2 = 2.(n+1) } must be an Integer which can be divided by 2 .

d.. Between the two Even -Regular-Polygons exists the only one (n+1) Odd-Regular-Polygon .

For the commonly divergence angle , φ , equation (1) becomes h_φ ,

Δh = h_A - h_B = (λ_a- λ_b) . $\sin\varphi$ = $\Delta\lambda$. $\sin\varphi$ ……….. (2)

or , h_A- h_B = (2.r_a. $\sin\varphi$ - 2.r_b. $\sin\varphi$) .$\sin\varphi$ = 2 (r_a- r_b) . $\sin^2\varphi$ i.e.

h_A- h_B = 2 (r_a- r_b) . $\sin^2\varphi$ or $\frac{\boldsymbol{h_A - h_B}}{\boldsymbol{\sin\varphi}} = \frac{\boldsymbol{\sin\varphi}}{\boldsymbol{1/2(r_a - r_b)}}$ ……….. (3)

That is , $\boldsymbol{\sin\varphi}$ = ($\frac{\boldsymbol{h_\varphi}}{\boldsymbol{\lambda_\varphi}}$) , is the Harmonic - Mean between [$\boldsymbol{h_A}$- $\boldsymbol{h_B}$] , [$\frac{\boldsymbol{1}}{\boldsymbol{2(r_a - r_b)}}$]

From (1)Δh = λ_a . $\sin\varphi_a$- λ_b . $\sin\varphi_b$= $\frac{\lambda_a{}^2}{4R^2}$ - $\frac{\lambda_b{}^2}{4R^2}$ =$\frac{1}{4R^2}$($\lambda_a{}^2$ - $\lambda_a{}^2$) or

2.R . Δh = ($\lambda_a{}^2$ - $\lambda_b{}^2$) = [λ_a - λ_b] . [λ_a + λ_b] . ……….. (4)

Show that , the Extrema-angle , w_o, formulates the complementary angle ,φ, defining the In-between Odd - Regular n-Polygons on the same circle (O , OK) , using the Extreme cases of this System { Δh =h_A- h_B = A_1B_1 } , on the Circles of differenceof Height .

Analysis :

1..From above relation of Heights and circle radius for two Sequent – Even - Polygons then ,

Σh_n = n . R = n . OK (a) and Σh_{n+2} = (n+2) . R = (n+2) . OK (b)

By Subtraction (a) , (b)

Σh_{n+2} - Σh_n = (n+2) R – nR = 2.R → constant

By Summation (a) , (b)

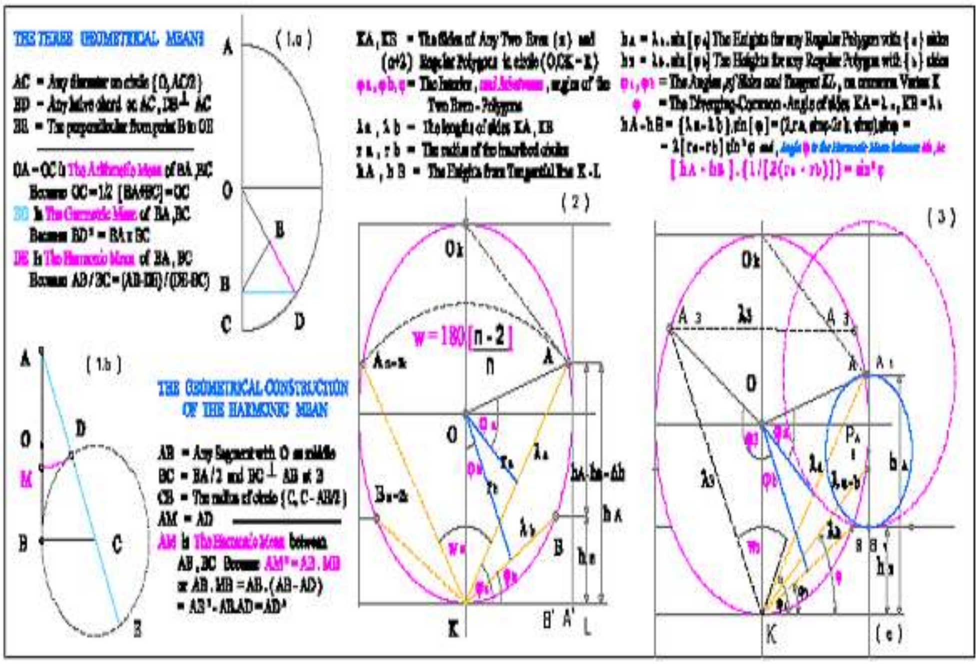
Σh_{n+2} + Σh_n = (n+2) R+nR = (n+1) .2.R→ constant

i.e.in the System of Regular - Polygons the ,Interior angles (w) and Gradient (φ) , Heights (h)and their differences , Δh ,– Summation and Subtraction of Heights are Interconnected and Intertwined at the Common Circle [A , Δh = $\boldsymbol{h_A}$- $\boldsymbol{h_B}$]producing the Common (n+1) ,Odd – Regular - Polygon .

2..In Fig.15- (2-3) → For , KA , KB , chords exists λ_a= 2R.$sin\,\varphi_a$, λ_b = 2R.$sin\,\varphi_b$, and their product [POP] = (λ_a . λ_a) = 4R².[$sin\,\varphi_a$.$sin\,\varphi_b$] …………… (5)
The sum of heights for the n and n+2 Even Regular Polygon isΣh_A=n.R and Σh_B = (n + 2).R
and the In-between Odd Regular Polygon Σh_o = (n + 1). R . The corresponding Interior angles

$w_a = [\frac{2}{n}]\,\pi$ and $\varphi_a = [\frac{n-2}{2.n}]\,\pi$
$w_b = [\frac{2}{n+2}]\,\pi$ and$\varphi_b = [\frac{n}{2.(n+2)}]\,\pi$
$w_o = [\frac{2}{n+1}]\,\pi$ and $\varphi_o = [\frac{n-1}{2.(n+1)}]\,\pi$

The Power of point O to circle of diameter Δhis for λ_o = 2R . $sin\,\varphi_o$, $\lambda`_o$ = 2R . $sin\,\varphi_o$,
[POP] = [λ_o .$\lambda`_o$] = 4R² . $sin^2\,\varphi_o$……….(6) and equal to (5) therefore
$sin\,\varphi_a\,.sin\,\varphi_b = sin^2\,\varphi_o$ or $\frac{sin\,\varphi_a}{sin\,\varphi_o} = \frac{sin\,\varphi_o}{sin\,\varphi_b}$ …………(7)
i.e. Angle $\boldsymbol{\varphi_o}$follows the Harmonic-Mean between angles $\boldsymbol{\varphi_a}$,$\boldsymbol{\varphi_b}$onΔh Difference of Heights.
3..Since Product of magnitudes λ_a . λ_b = constant and also (λ_a - λ_b) . (λ_a+ λ_b) = constant ,
therefore , the Power of any point IN and OUT of the circle of Heights is Constant , meaning
that exists another one Regular – Polygon , between the two Even - Sequence i.e.
The Outer are the two Even-Regular N and N+2 Polygons ,
andThe Inner is the N+1 Odd – Regular Polygon .

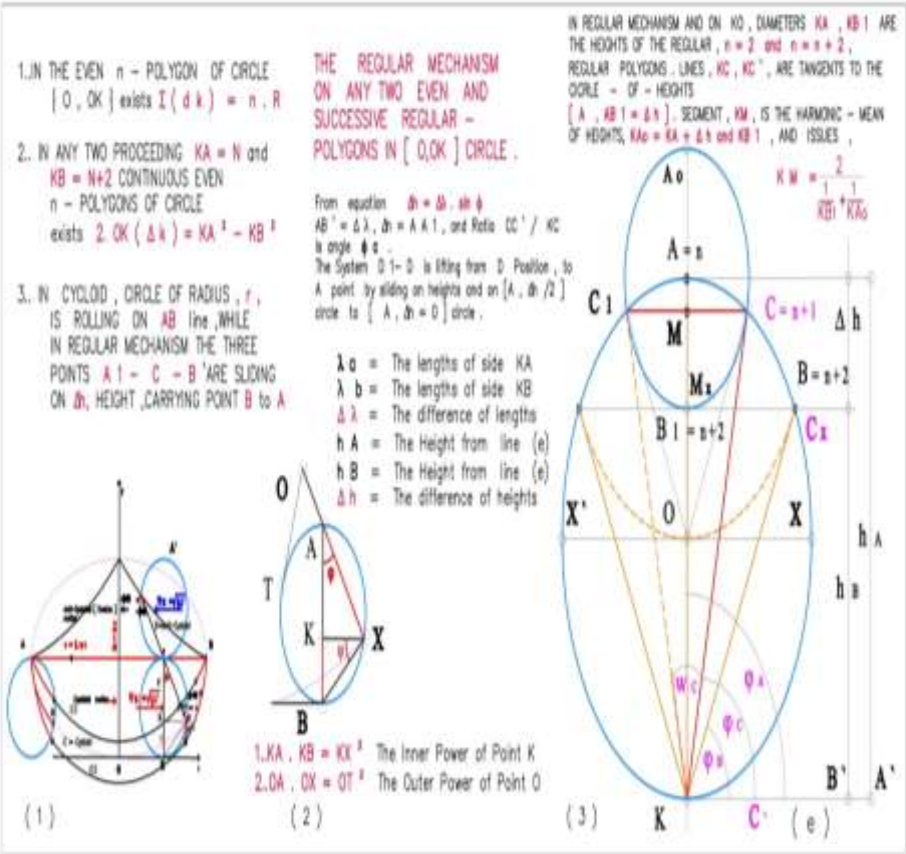


F.15→In (1) are shown the two ways for constructing the three Means on One or TwoSegments .

In (2) is shown the Divergency of Sides to Heights of Two n , and (n+2)Even Polygons .
In (3) is shown the locus of the Two - Circles of Heights (A_1, A_1B_1)andthe parallels to (e) .
to be Extrema case for the two segments KA , and KB .
6.1.Analysis of the Geometrical Construction . Fig.16 - (3)

The construction of all the Even - Regular - Polygons is possible by dividing the circle (O , OK) in 2 , 4 , 6 , 8 , 10 , 12 , 14 …2n parts as $w_a = [\frac{2}{n}]\,\pi$ and $\varphi_a = [\frac{n-2}{2.n}]\,\pi$, n = 1 , 2 , 3 ....
The construction of all the Odd - Regular -Polygons is possible by Applying the Circles on Heights
between the chords of the Even-Sequence of Polygons on [2 , 4] – [4 , 6] – [6 , 8] – [8 ,10] …
[(2n) – (2n+2)] as formulas $w_o = [\frac{2}{n+1}]\,\pi$ and $\varphi_o = [\frac{n-1}{2.(n+1)}]\,\pi$ founded from point$K$. Case A → Digone .

Step 1 :
Draw from point K , of any circle (O , OK) , Tangent (e) at K and Chord KA which is the diameter
(because diameter of the circle is the Side of the Regular - Digone) and any KB , corresponding to the Even (n) and (n+2) Regular Polygon .
Step 2 :
Draw from points A , B , the perpendiculars to (e) and define the difference Δh = h_A- h_B= AB_1 ondiameter KA and Draw circle (A , AB_1) with radius Δh , and line KA intersecting circle at point A_o .
Step 3 :
Draw tangents KC , KC_1 and chord $C\,C_1$ intersecting circle (O , OA) at point C .
Step 4 :
Draw Chord KC which is the Side of the Regular Odd – (n +1) - Regular - Polygon on angle φ_c



F.16→ In (1) is shown the Rolling of a circle on a straight line and forming the Cycloid .

In (2) is shown the Inner - Outer Power of Points , K , O , on circle of AB diameter . In (3) is shown the How and Why KM Segment is the Harmonic-Mean between KA , KB_1 .

Proof :

1.. Because triangle A C K is rightangled then AC is perpendicular to KC therefore Segment

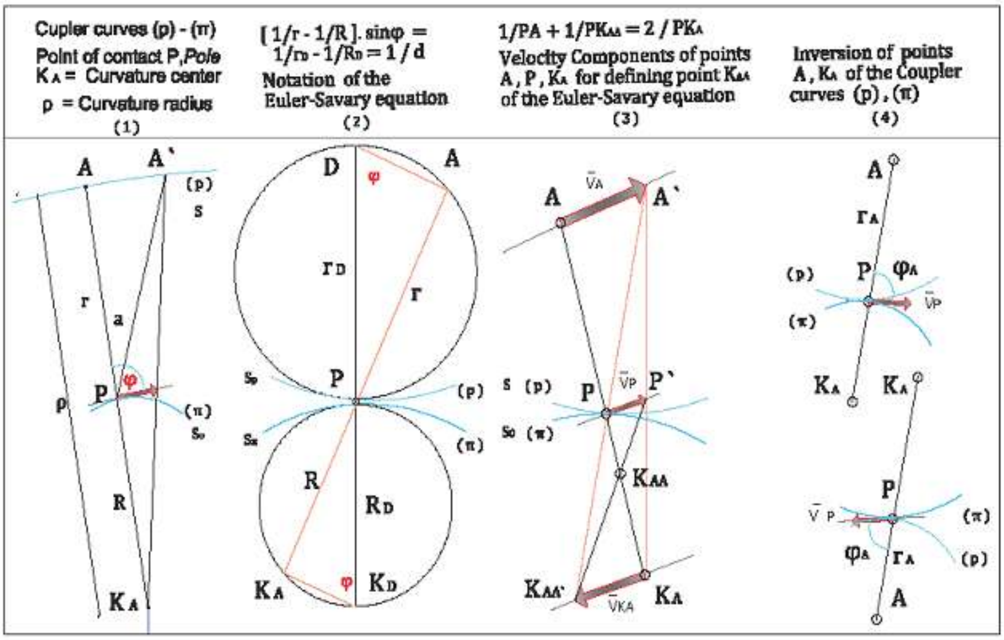
KC is perpendicular to AC and it is Tangential to circle (A , AB_1) .
The same also for KC_1 , which is also tangent tocircle (A , AB_1) .

2.. From relations KA_o = KA + AA_o = KA + AB_1
KB_1 = KA - AB_1 = KA - (KA_o - KA) = 2. KA - KA_o or ,
2 . KA = KA_o + KB_1 = (h_A + Δh) + h_B ……….. (1) therefore
KA = $\frac{h_A + \Delta h + h_B}{2}$ ……….. (2) The Arithmetic – Mean .
3.. From the Power of point K to circle (A , AB_1) exists [KC] ² = [KB_1].[KA_o] therefore
KC = $\sqrt{KB_1 . KA_o}$ = $\sqrt{[h_A + \Delta h] . h_B}$ ….(3) The Geometric – Mean
4.. From the right angled triangle A C M exists KM . KA = KC ² = (KB_1) . (KA_o) or

KM = $\frac{KA_o . KB_1}{KA}$ = [$\frac{KA_o . KB_1}{KA_o + KB_1}$] . 2 = [$\frac{2}{\frac{1}{KA_o} + \frac{1}{KB_1}}$] …….. (4) i.e.
KM is the Harmonic - Mean between KA_o and KB_1 or (h_A+ Δh) , h_B .
For n = 2 , then KA is the Side of the Regular-Digone and equal to the diameter of the circle .
For n = n+2 = 4 , then KB is the Side of the Regular -Pentagon sided on the perpendicular to KA side . Exist h_A = KA , h_B = KO = KB_1 , Δh = AB_1 , and A_3 point coincides with A_2 , and consequence with C point . Parallel line DA_4 coincides with the parallel CC `line and KC is the Side of the n+1 = 3 , Regular – Trigonon KM = KO + $\frac{\Delta h}{2}$=1,5. OK .
Point A is the Vertex and KA is the Side of the Regular Digone .
Point Cis the Vertex and KC is the Side of the Regular Trigon(Triangle).
Point B is the Vertex and KB is the Side of the Regular Tetragon .
In addition , from formula Σ = n . R = 3R = 3.OK , and since every half is $\frac{3}{2}$. OK = 1,5. OK
then Point C is on half Δh , or height h = KO + $\frac{OA}{2}$.
For n = 4 , then KA is the Side of the Regular - Tetragon and equal KX =OK.$\sqrt{2}$ chord .
For n = n+2 = 6 , then KB is the Side of the Regular -Hexagon sided on circle (O , OA) .
For n = n+1 = 5 then it is the side of the Regular-Pentagon .
The How this is Geometrically achieved follows by the following three methods .
a.. The [Antiphon – Archimedes] Ancient Greek–Polygons method .
b.. The[Euler-Savary] Coupler-Curves curvature - centers method .
c.. The [Markos] Geometrical , Three – Circles - Method , in Polygons .
6.2.The Geometrical Construction of ALL Regular Polygons .
Preliminaries : The Coupler Curves .
Geometry :

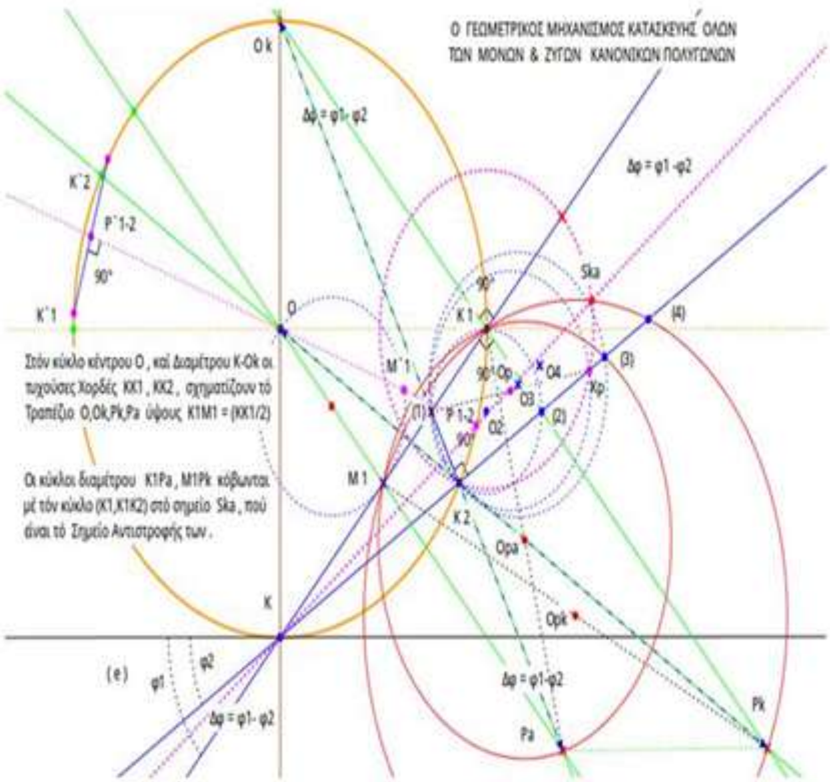
Let A be a point on a Plane System ,S, rolling on the fixed system ,So, as in Fig-17.1
K_A is the center of curvature , the Instaneous center on the fix system .
P is the Instaneous center of curvature on the fix curve So (the pole P),
(p) , (π) are the coupler curves on , S , So
u = The translational velocity of pole P equal to ds/dt = AA`/dt
w = Angular velocity of pole P equal to dr/dt = d(APA`)/dt and for d = u / w then ,
Euler-Savary equation is Ex = [1/r_D - 1/R_D] $sin\,\varphi$ = 1/ d …………………. (a)
When point P lies on the radius of curvature of Polar path (φ = 90) then sinφ =1 and from Fig- 17.2 holds → [1/r_D - 1/R_D] = 1/ d and issues r = r_D . $sin\,\varphi$ and R = R_D.$sin\,\varphi$
i.e. The trajectories of points A on the circumference of circle radius $\boldsymbol{r_D}$, have their center of curvature on circumference of circle of radius $\boldsymbol{R_D}$.
Motion :
The motion of curves (p) , (π) is in Fig -17.3
Let $\overline{v_A}$,$\overline{v_P}$,$\overline{v_{KA}}$, be the velocities of points A , P , K_A to their systems .
For system S the curvature center K_A , the Instaneous center , is found from the intersection of A`P` and AP . For system ,So, the curvature center K_{AA}, the Instaneous center of K_A on fixed system (π) is found from the intersection of P`$K_{AA`}$ and PK_A .
From the above similar triangle K_AAA` , $K_A$PP` exists ,
(K_AA/PA) = (K_AA`/P`A`)=( $K_{AA`}$ A`/P$K_{AA`}$)= K_AK_{AA}/P K_{AA} or { K_AA /PA}={ K_AK_{AA} /K_{AA}P }… (b)
i.e. The Points A ,K_{AA} are harmonically divided by the pointsP, K_A and exists ,
1/PA +1/PK_{AA}= 2/PK_A
Inversing the two Systems by considering fixed system ,So, rolling on ,S, as in Fig-17.4 then ,
Ex = [1/r_A - 1/R_A] sinφ_A = 1/ d and [1/$r_{A`}$ - 1/$R_{A`}$] sin$\varphi`_A$ = 1/ d where in both cases issues ,
(PK_A-PA)/(PK_A.PA)= -(P$K`_A$– PA`)/(P$K_{A`}$. PA`) or Ex =(1/PA –1/PK_A)=(1/P$K_{A`}$ - PA`)= 1 / d … (c)
The Path of the Instaneous-center of curvature , $\boldsymbol{O_A}$, on (k) , (π) coupler envelope curves is proved that , During the rolling of curve (k) of system , S , and the fixed to it envelope (π) , then the Instaneous-center of curvature and those of the constant envelope (π) , coincides to the Instaneous-center of curvature K_A of (k) as in Fig-17.1
The center D , of a Rolling circle (p) on another circle (π) , executes a circular motion with K_D as center which coincides with the center of curvature of the second circle. Because angle
φ =90˚ , then for every point A on (p) exists a center of curvature K_A on AP and C K_p as in Fig-17.2
During the rolling of a circle (p) on (π) line , then the corresponding Instaneous-center of curvature K_A of any point A is the common point of intersection of AP produced and the parallel to DP from point C and the Instaneous-center of curvature K_D for point D is in infinite and KD = ∞ .
The Euler-Savary equation involves the four points A , P , K_A , K_{AA} lying on the path normal.
Equation (b) may be written in the form PA / AK_{AA} = A K_{AA}/ AK_A and is recognized that AK_{AA} is the mean proportional between PA and K_AA .
The Cubic of Stationary curvature :
Euler-Savary formula apply to the analysis of a mechanism in a given position and vicinity .
It gives also the radius of curvature and the center of curvature of a couple-curve. Because couple-curve (Path ↔ Evolute) is the equilibrium

of any moving system , then Complex-plane is involved and the E-S geometrical equations ,
Ex = (1/ PA – 1/ PK_A) i.$e^{i\varphi}$ = h [1/PA – 1/PK_A] = h . ($\frac{d\varphi}{ds}$) and for the homothetic motion
(h= 1) then , Ex = $\frac{1}{PA}$ - $\frac{1}{PK_A}$ = $\frac{1}{P\,K_{AA}}$($\frac{d\varphi}{ds}$) …………. (d)
Equation (d) is that of Rhodonea Hypocycloid curves .
The Inflation circle , ΚύκλοςΚαμπήςκαιΑντίστροφωνΚέντρων, extrema case ,
shows the location of coupler points whose curves havean infinite radius of curvature ,
i.e. on inflection circle lie all centers of curvature of System curves and which , these are rolling
on inflection point on the envelope .(Envelope here are the two or more surfaces in direct contact).The Cubic of Stationary curvature [COSC] indicates the location of coupler points that
willtrace segments of approximate circular arcs . In Geometry , the rolling of a circle , on a circle and or on a line is likewise to Mechanism as , Space Rolling on Anti=space , a Negative particle , Electron , on a Positive particle , Proton , or on many Protons , so the Wheel-Rims represent the , COSC in Mechanics .



F.17→In (1) A point A on Coupler-curves (p) , (π) define the point of curvature KA , the Instaneous point P , the pole on (π) .
In (2) is the case of point A lying on radius of curvature of polar path (point D) where then the paths of points A in , S , system have the Instaneous center of curvature KA on the fixed system So.
In (3) The Velocity Instaneous center , for curvature point K_A , in S_o system is pointK_{AA} . In (4) The two points A , K_A , of Coupler-curves (p) , (π) , follow the inversed motion
where Poles of rotation , A and K_A , are inverted . Above F.17 is the Master-key for the solution to inscribe in a circle a regular polygon with any given number of sides either Mechanical or Geometrical - Solutions [63].

Η Μέθοδος αφιερώνεται στην Σύγχρονη - Ελλάδα , για να Μη Ξεχνά τους Προγόνους της .



F.17 - ΑΣτον κύκλο(Ο ,ΟΚ)μέ την ευθεία (e) εφαπτομένη στο σημείοΚ , και με διάμετρο

ΚO_K ,Φέρομεν τις τυχουσεςΧορδές KK_1 ,KK_2 και τις αντίστοιχες χορδές O_KK_1 ,O_KK_2 , μέ τις γωνίες <K_1K (ε) = φ_1,<K_2K (ε) = φ_2 και Δφ = φ1 – φ2 .
Από δε του σημείου Ο , Φέρομεν την OM_1 μεσοκάθετο τής χορδής KK_1 . 1.. Προεκτείνομεν την O_kK_1, ώστε να Κόβει την προέκταση της OK_2 στόσημείο P_k , και Φέρομεντον κύκλο (O_{pk} ,$O_{pk}P_k$=$O_{pk}M_1$) κέντρου O_{pk} και διαμέτρου [P_kM_1] .2.. Προεκτείνομεν την O_kK_2, ώστε να Κόβει την προέκταση της OM_1 στόσημείο P_a , καιΦέρομεντον κύκλο (O_{pa} ,$O_{pa}P_a$=$O_{pa}K_1$) κέντρου O_{pa} καίδιαμέτρου [P_aK_1] . 3.. Η ευθείαKK_2 Προεκτεινομένη Κόβει , Την προέκταση της O_KK_1Στο σημείο (2) , Τον κύκλο (K_1 ,K_1K_2) Στο σημείοK_p ,Τον κύκλοδιαμέτρου[P_aK_1] Στο σημείο (3) , και Τον κύκλο διαμέτρου [P_kM_1] Στο σημείο (4) .Ο κύκλος (K_1 ,K_1K_2) κόβει τον κύκλο διαμέτρου [P_kM_1] στο σημείο S_{ka} , η δε Χορδή $K\,S_{ka}$ κόβει τον κύκλο (Ο , ΟΚ) στο σημείο P_{1-2} .

4..□ □ □□□□□ ,□ □

α) Οι κύκλοι (O_{pa} ,$O_{pa}K_1$) , (O_{pk} ,$O_{pk}M_1$) είναι αι ΟρθαίΠροβολαίτου Γεωμετρικού Μηχανισμού {[O_KK_1//OM_1] γωνία <OP_aO_k=$P_aO_kP_k$} του Συστήματος των Απείρων – Αντιθέτων - Κύκλων. β) Η ευθεία [P_{1-2}O]είναι ο Κοινός Ακραίος -Μηχανισμός [$M_1M`_1\perp OM`_1$]Συστήματος Ορθών
και Αντίθετων Προβολών πέριξ ευθείαςδιερχομένης από τού κέντρου Ο .
γ) Στήν περίπτωση όπου οι χορδές KK_1 ,KK_2 ανήκουν σε δύο συνεχόμενα Ζυγά Κανονικά
Πολυγωνα τότε η χορδή [$K\,P_{1-2}$] ανήκει στο ενδιάμεσο ΜονόΚανονικό Πολυγωνο.
ΑΠΟΔΕΙΞΗ :
1.. Τά τρίγωνα KK_1O_k , KK_2O_k , είναι ορθογώνια διότι η υποτείνουσα KO_k , είναι διάμετρος του κύκλου (Ο ,ΟΚ) . Επειδή η γωνία <KK_1O_k = 90ο,άρα και η συμπληρωματική της <KK_1P_k = 90ο.
Το ίδιο και γιά την γωνία <KK_2O_kπού αντιστοιχεί η γωνία <(1)K_2(2) = 90 ,
2.. Επειδή στο τετράπλευρο [(1)K_1(2)K_2] , οι έναντι γωνιαι<(1)K_1(2) = (1)K_2(2) = 90ο , άρα τούτο είναι εγγράψιμο σε κύκλο .
3.. Επειδή η γωνία <(1)K_2K_P = 90 ,άρατά σημεία (1) , K_2 , K_P, είναι εγγράψιμα σε κύκλο . Το ίδιο ισχύει και δια τά σημεία (1)K_2 (3)καιτά (1)K_2(4) .
4.. Η δύναμη των σημείων P_k , P_aστον κύκλο (Ο ,ΟΚ) είναι οι εφαπτομένεςT_{pk} ,T_{pa} τού κύκλου και ίσαιμέ$T^2{}_{pk}$ = $(P_kO)^2$ - (ΟΚ) ²
καί$T^2{}_{pa}$ = $(P_aO)^2$ - (ΟΚ) ² , άρα ισχύει ,
(ΟΚ) ² = $(P_kO)^2$ - $T^2{}_{pk}$ = $(P_aO)^2$ - $T^2{}_{pa}$ ……..(1)
5.. Η δύναμη των σημείων P_k , P_a στον κύκλο (K_1 ,K_1K_2) είναι οι εφαπτομένεςT_{pk1},T_{pa1}τούκύκλου και ίσαιμέ$T^2{}_{pk1}$ =[P_kK_1]² - [K_1K_2]² ,
καί$T^2{}_{pa1}$=[P_aK_1]² - [K_1K_2]² , άρα ισχύει ,
[K_1K_2]² = [P_kK_1]² -$T^2{}_{pk1}$καί [K_1K_2]² = [P_aK_1]² -$T^2{}_{pa1}$, οπότε [P_kK_1]² -$T^2{}_{pk1}$ = [P_aK_1]² -$T^2{}_{pa1}$ ή$T^2{}_{pa1}$-$T^2{}_{pk1}$= [P_aK_1]²- [P_kK_1]² ……..(2)
Επειδή η Χορδή [P_kK_1]του κύκλου διαμέτρου [P_kM_1], είναι ίση με [P_kK_1]² = [P_kM_1]²- [M_1K_1]²η (2) γινεται$T^2{}_{pa1}$ -$T^2{}_{pk1}$ = [P_aK_1] ² - {[P_kM_1]²- [M_1K_1]²}=[P_aK_1] ² - [P_kM_1]² + [M_1K_1]² .... (3)
Δηλαδή η Δύναμη του Συστήματος τωνΔύο Κύκλωνσχετίζεταιμε τις Εντός -Εναλλάξ Διαμέτρους των[P_aK_1] ,[P_kM_1] ,επίτου Ορθογωνίου Τραπεζίου [$P_kK_1M_1P_a$] ύψους K_1M_1 , πού αμφότεραιπροβάλλωνται στο αυτόύψος K_1M_1όπου και ο κύκλος (K_1 ,K_1K_2) .6.. Επειδή το σημείο P_a ευρίσκεται επί της OM_1// O_KK_1,άρα όλοι οι κύκλοι διαμέτρου [P_aM_1]
προβάλλωνται στο σημείο M_1 , και όταν το σημείο $P_a \rightarrow \infty$, είναι στό άπειρο, τότε ο κύκλος (P_a , $P_a\infty$) ταυτίζεται με την χορδή KK_1 .
Επίσης το σημείο P_k ευρίσκεται επί της O_KK_1// OM_1, άρα όλοι οι κύκλοι διαμέτρου [P_kK_1]προβάλλωνται στο σημείο K_1 , και όταν το σημείο $P_k \rightarrow \infty$
είναι στό άπειρο , τότε ο κύκλος (P_k , $P_k\infty$) ταυτίζεται επίσης μέ την χορδή KK_1 .Δηλαδή , Η χορδή KK_1 είναι ο Γεωμετρικός τόπος των

άπειρων κύκλων επί των παραλλήλων OM_1, $O_K K_1$, του Τραπεζίου [Ο$O_k P_k P_a$] με τις χορδές του να κόβωνται επί του κύκλου (K_1 ,$K_1 K_2$) .
7.. Η χορδή KK_1 περιστρεφομένη πέριξ του σημείου Κ , στήν ευθείαKS_{ka} , καθορίζειτο κοινό σημείο S_{ka} τών κύκλων (K_1 ,$K_1 K_2$)καί τού κύκλου της μεγαλυτέρας διαμέτρου [$P_a K_1$] ή [$P_k M_1$], πούείναιο οινόςΓεωμετρικός - ΤόποςδιάβασηςτούΣυστήματος από το Απειρο , ∞ , στην θέση [KK_2] .
Η ευθεία ,P_{1-2}Ο, πού περνά από το σημείο Ο, είναι η Ακραία Κοινή ευθεία Ορθής Προβολής τουΣυστήματος του Τραπεζίου [Ο$O_k P_k P_a$] μεταξύ των Χορδών [KK_1] ,[KK_2].8.. Στήν περίπτωση όπου οι χορδές KK_1 ,KK_2 ανήκουν σε δύο συνεχόμενα Ζυγά Κανονικά Πολύγωνα η χορδή,K P_{1-2} , ανήκει στο ενδιάμεσο ΜονόΚανονικό Πολυγωνο , διότιστο σημείο Αναστροφής
των κύκλων , η Διάμετρος $P_{1-2}O$ γίνεται κάθετος της πλευράς του , Αντιστροφή των γωνιών πέριξ τουάξονος ,P_{1-2}- P `$_{1-2}$.

Ακολουθούν οι διάφορες σκέψεις Προσεγγιστικές και Μη πού έγιναν .

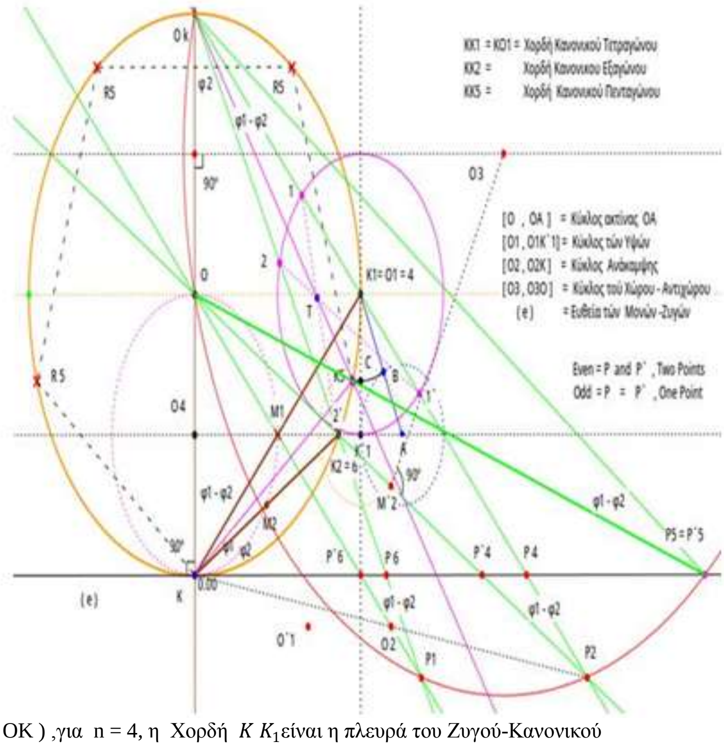
6.3.   Αι Μέθοδοι :
Προκαταρκτικά   :Το Θέμα , F.16(3).
Ο τυχόν κύκλος (Ο , ΟΚ) είναι δυνατόν να χωριστεί σε ,
α..Δύο ίσα μέρη από την διάμετρο ΚΑ [Είναι το Δίπολο ΑΚ] με γωνία<ΑΟΚ = 180 ^{o} . β..Τέσσερα ίσα μέρη από την Διχοτόμο των 180 ^{o}πού είναι η Κάθετη δεύτερη Διάμετρος Χ ` Χ
γ..Οκτώ ίσα μέρη από την Διχοτόμο των τεσσάρων γωνιών πού είναι 90 ^{o} .
δ..Δεκαέξει ίσα μέρη από την Διχοτόμο των Οκτώ γωνιών πού είναι 45^o και ούτω καθ` εξής .
ε.. Ο κύκλος έχων 360 ^{o} = 2π ακτίνια δύναται να χωριστεί σε ,
Τρία ίσα μέρη   360^o / 3   = 120^o πού είναι δυνατό [Το Ισόπλευρο τρίγωνο] , Έξη ίσα μέρη   360^o / 6   = 60^o πού είναι δυνατό με τις διχοτόμους του τριγώνου   [Το Κανονικό Εξάγωνο] ,   Δώδεκα ίσα μέρη   360^o / 12   = 30^o πού είναι δυνατό με τις διχοτόμους του Εξαγώνου
[Το Κανονικό Δωδεκάγωνο] ,και ούτω καθ` εξής σε 15^o , 7,5^o……..

Παρατήρηση .
α…Η σειρά των Ζυγώναριθμών είναι 2 , 4 ,6 , 8 ,10 ,12 ,14 ,16 ,18 ,20 ,…………….
Η σειρά των Μονών αριθμών είναι 1 ,3 ,5 ,7,9 ,11 ,13 ,15 ,17,19 ,21 , ……………
προερχομένη από το ημι-άθροισμα του Προηγούμενου και του Επόμενου Ζυγού αριθμού π.χ.
Ο αριθμός $5 = \frac{4+6}{2} = \frac{10}{2} = 5$. Η Λογική της Πρόσθεσης ισχύει και στην Γεωμετρία αλλά στα δικά της πλαίσια πού είναι η Λογική του Υλικού – Σημείου , δηλαδή το Μηδέν (0 = Τίποτα) Υπάρχει ως άθροισμα του Θετικού + Αρνητικού [ίδε , Υλική Γεωμετρία 58-60-61]
β…Στην άνω παράγραφο 5.5(Casec) απεδείχθη η σχέση (1) Σ(h) = (2k) . h = n .h =n .OK, όπου
Σ = Το άθροισμα των Υψών , των Κορυφών του Κανονικού (n) – Πολυγώνου ,
από των Κορυφών K_n , μέχρι της εφαπτομένης (e) στο σημείο K ,
h = OK , Το ύψος του κέντρου O από την(e) ,
n = O αριθμός των Πλευρών του Κανονικού – Πολυγώνου , …… και πού
Μετατρέπει το Άθροισμα των Υψών από της Εφαπτομένης (e) σε πολλαπλάσιο αριθμό της ακτίνας του κύκλου ,πού σχετίζεται άμεσα με τις γωνίες $\boldsymbol{\varphi_n}$, και τις κορυφές των πλευρών, KK_n.
γ...Εις τυχούσα Χορδή $\boldsymbol{KK_1}$ του κύκλου (O , OK) , η Κεντρική γωνία <KOK_1 , είναι διπλάσια της Εγγεγραμμένης της και η γωνία <$KO_K K_1$= KOM_1. Η Μεσοκάθετος OM_1είναι παράλληλος της Καθέτου $O_K K_1$, άρα τέμνονται στο άπειρο (∞) . Επειδή δε αι δύο Κάθετοι περνούν από τα σημεία O και O_K , αυτά αποτελούν τους Πόλους περιστροφής των .
Εις το Σχήμα F.18 – A, το τυχόν Σημείο $\boldsymbol{K_2}$, επί του κύκλου , σχηματίζει την δεύτερη Χορδή $\boldsymbol{KK_2}$ η δε Κάθετος $O_K K_2$προεκτεινόμενη κόβει την OM_1, παράλληλο της $O_K K_1$, σε ένα σημείο P_1 πού είναι ο Πόλος -Σχηματισμού των δύο Χορδών , ή , γωνιών .
Το γιατί είναι διότι το σημείο P_2κινείται επί της OM_1από το άπειρο μέχρι της διαμέτρουKP_1 .
Επί της διαμέτρουKP_2του κύκλου (O_2 , O_2P_2= O_2K) , καιμε κέντρο το O_2,Σχηματίζονται οι ίδιες γωνίες φ_1 ,φ_2από τις ΧορδέςP_1M_1, P_2K_2 , ώστε η γωνία <$M_1P_1K_2$ = K_1KK_2=OP_1O_k
Δηλαδή , Σε δύο Χορδές , $\boldsymbol{KK_1}$, $\boldsymbol{KK_2}$, κύκλου (O , OK) , κοινής κορυφήςK , η Μεσοκάθετος$\boldsymbol{OM_1}$της πρώτης ,και η Κάθετος$\boldsymbol{O_K K_2}$ της δεύτερης , κόβονται σε ένα σημείο $\boldsymbol{P_1}$πού σχηματίζει τον κύκλο($\boldsymbol{O`_1}$ , $\boldsymbol{O`_1P_1}$)πού είναιοΣυζυγής του Κύκλος, {είναι ο κύκλος των Ίσων-Γωνιών με τονκύκλο (O ,OK)} . Το ίδιο και με τον κύκλο ($\boldsymbol{O_2}$,$\boldsymbol{O_2P_2}$= $\boldsymbol{O_2}$K).
δ...Από την σχέση Σ = (2k) . h = n .h = n .OK , διά n = 2 τότε Σ = 2.h = 2.OK δηλαδή η διάμετρος KO_K . Διά n = 3 τότε Σ = 3.h = 3 .OKκαι n = 4 τότε Σ = 4.h = 4 .OK. Επειδή οι Μονοί αριθμοί είναι ο Αριθμητικός - Μέσος των δύο γειτονικών Ζυγών άρα και το 3.OK είναι (2.OK +4.OK)/2 .
Η διαφορά των υψώνείναιΔh = h_{K1}- h_{K2}=$K_1K`_1$και μεταξύ των παραλλήλων των σημείων , $K_1$ , $K_2$ , και της (e) . O κύκλος ( $K_1$ , $K_1K`_1$) είναι ο Κύκλος των Υψομετρικών-Διαφορών των Χορδών KK_1 , KK_2 , και μεταβάλλεται ανάλογα με το σημείο $K`_1$ ή το ίδιο με το K_2 . Δηλαδή ,
O Κύκλος των Υψομετρικών - Διαφορών ($\boldsymbol{K_1}$, $\boldsymbol{K_1K`_1}$) αλληλοσχετίζεται με τιςΧορδές ,
[$\boldsymbol{KK_1}$,$\boldsymbol{KK_2}$] , [$\boldsymbol{O_K K_1}$,$\boldsymbol{O_K K_2}$] τού κύκλου (O , OK) μέσωτων αντίστοιχων κορυφώνK , $\boldsymbol{O_K}$ και με τον Κύκλο–Ίσων Γωνιών ($\boldsymbol{O`_1}$ ,$\boldsymbol{O`_1P_1}$) μέσω της Μεσοκαθέτου $\boldsymbol{OM_1}$ της πρώτης Χορδής $\boldsymbol{K\,K_1}$,και της Καθέτου$\boldsymbol{O_K K_2}$ της δεύτερης Χορδής $\boldsymbol{KK_2}$.
Αυτός ο Αλληλοσχηματισμός τωνΤεσσάρων κύκλων ,
{ (O,OK) -($\boldsymbol{K_1}$,$\boldsymbol{K_1K`_1}$) -($\boldsymbol{O`_1}$, $\boldsymbol{O`_1P_1}$)- ($\boldsymbol{O_2}$,$\boldsymbol{O_2P_2}$)}
καθέτων προς την εφαπτομένη (e) , επιτρέπει, Στον οποιονδήποτε κύκλο(O , OK), να καθορίσει μέσω τωνΔύο ΧορδώνKK_1 , KK_2 ,και γωνιώνφ_1 , φ_2 , την μεταξύ των κίνηση,ήτοι
Από την σχέση αθροίσματος των Υψών Σ = (2k) . h = n .h = n .OK , προκύπτει ότι το Άθροισμα των Υψών δύο συνεχομένων Κανονικών - Πολυγώνων n , n+2 είναι → $\frac{\Sigma\ 2(h1)}{2} + \frac{\Sigma\ 2(h2)}{2} =$
[$\frac{n_1}{2} + \frac{n_2}{2}$].OK = [$\frac{n_1+n_2}{2}$].OK=n_3 .OK , όπου n_3= [$\frac{n_1+n_2}{2}$] είναι ο Αριθμός των Κορυφών του μεταξύ των δύο Ζυγών n_1 , n_2, Μονού –Αριθμού - Κορυφώντου Κανονικού-Πολυγώνου .
Επί της Υψομετρικής –Διαφοράς Δh =$O_1K`_1$ καθέτου της (e) διατηρούνται οι ιδιότητες Άθροισης .
Από την ταυτόχρονο θέση των γωνιών φ_1 , φ_2 , στους δύο κύκλους ορίζονται και οι χορδές .
ε...Επειδή αιKK_1 , KK_2 , είναι κάθετοι τωνOP_1 ,$O_K P_1$, άρα το σημείο K είναι τοΟρθόκεντρο όλων των καθέτων των τριγώνων από τούτου , καθώς και της κοινής χορδής των δύο κύκλων
(O_2 , O_2P_2) , (O , OK) . Επειδή δε ο Γεωμετρικός -Τόπος των ΧορδώνKK_1 , KK_2 ,του Κοινού Ορθοκέντρου K είναι→για τον κύκλο (O ,OK) το τόξο K_1K_2 ,για τον κύκλο(O_2 ,O_2K=O_2P_2) το τόξο M_1K_2, και για τον κύκλο (O_1 , $O_1P`_1$) το τόξο (1)-(2) με τα σημεία τομής των χορδών, ΑΡΑ τασημεία(1) ,M_1είναι τα Ακραία σημείατων κύκλων τούτωνώστε ναείναι K$M_1 \perp P_1M_1$.

Αι ανωτέρω δύο λογικές καταλήγουν στη Μηχανική και Γεωμετρική λύση πού ακολουθεί .

Η κατά προσέγγιση ΜηχανικήΑπόδειξη :
Εις το σχήμα F. 18- A., έστω κύκλος (O , OK)με την ευθεία (e) εφαπτομένη στο σημείο, K , και την KO_Kδιάμετρο του κύκλου.
Ορίζουμε επί του κύκλου και από της αρχής , K , τις Κορυφές K_1 , K_2να αντιστοιχούν σε άκρα πλευρών Ζυγών - Κανονικών – Πολυγώνων και τις αντίστοιχες γωνίεςτων , φ_1 , φ_2, μεταξύ των πλευρών KK_1 ,KK_2, και της εφαπτομένης (e) .
Φέρομεν από των σημείων K_1 , K_2 , τας παραλλήλους προς την (e) από δε της Κορυφής K_1κάθετο προς την (e) πούνα τέμνειτην παράλληλο από του σημείου K_2 , στο σημείο$K`_1$, και εν συνεχεία φέρομεν την κάθετο $K_1K`_1$ ωςακτίνα τούΚύκλου (K_1 , $K_1K`_1$).
Φέρομεν την$O_K K_1$πούπροεκτεινόμενη τέμνει την OK_2 προεκτεινόμενη(από το σημείο O) στο σημείο P_2από δε τουO_2(μέσουτης διαμέτρου KP_2),φέρομεν τον κύκλο (O_2,O_2 K=O_2P_2).
Προεκτείνομεν τιςπλευρές O_kK_1 , O_kK_2 , ώστε να κόβουν τον κύκλο (O_1 ,$O_1K`_1$) στα σημεία 1 , 1`,και 2 , 2`, αντίστοιχα και εν συνεχεία φέρομεν τις εναλλάξ χορδές 1 - 2` και 2 -1` .

Ορίζουμε το κοινό σημείο , T, των χορδών 1 - 2` και 2 - 1` και προεκτείνουμε την , O_kT,
ώστε να κόβει τον κύκλο (O , OK) στο σημείοK_5 . Ή , με τον Αρμονικό-Μέσο
Φέρομεν από τού σημείου$K`_1$ κάθετο , $K`_1$A =($K`_1 K_1$)/2 καιτον κύκλο ( A,A$K`_1$) ώστε να κόβει την
χορδή O_1A στο σημείο B . Φέρομεν από το K_1τον κύκλο (K_1 ,K_1B) ώστε να κόβει την κάθετο $K_1 K`_1$ στο σημείο , C , από δε του σημείου
C παράλληλο της (e) ώστε να κόβει τον κύκλο (O, OK) στο σημείο K_5 . Η χορδή K$\boldsymbol{K_5}$είναι η πλευρά του Μονού – Κανονικού - Πολυγώνου, διότι ,
Ο κύκλος (O_4,O_4 K = O_4 O) είναι ο κύκλος των μέσον των χορδών KK_1 ,KK_2Άρα και τηςKK_5.
Οι γωνίες <KM_1O_2 =$KM_2O`_1$= 90ο , <KM_1P_1 = KM_1O = 90ο , <KK_2P_1=KK_2O_κ= 90ο ,
Άρα το σημείο K είναι το Ορθόκεντρο των τριγώνωνKOM_2 ,KOP_1, KO_kP_2, KO_kO_1 .
Οι γωνίες <K_1KK_2 ,$K_1O_kK_2$,OP_1O_k , OP_2O_k ,P_2OP_1 είναι ίσες μεταξύ των ,
Διότι Είναια) Εγγεγραμμένες στο ίδιο τόξο , K_1K_2 , τού κύκλου (O, OK) ,
β) Οι πλευρές τωνP_1M_1 , P_1K_2 , κάθετες των KK_1 ,KK_2ευρίσκονται
εντός του κύκλου ($O`_1$ ,$O`_1$ K =$O`_1$ P_1) ,
γ) Εντός εναλλάξ μεταξύ των δύο παραλλήλων ,OP_1, καιO_kP_2
των κύκλων (O_4 , O_4 K =O_4 O) , (O_2 , O_2 K =O_2 P_2) .
Οι Χορδές O_kK_1 , OM_1 είναι κάθετοι της χορδής KK_1 , Άρα είναι παράλληλοι ,
Οι Χορδές O_kK_2 , OM_2 είναι κάθετοι της χορδής KK_2 , Άρα είναι παράλληλοι ,
Ο Γεωμετρικός Τόπος του σημείουK_1, από του Σημείου $\boldsymbol{K_1}$ προς $\boldsymbol{K_2}$, στο κύκλο(O , OK)
είναιτο τόξο K_1K_2 του κύκλου , ενώ επί του κύκλου (O_1 ,$O_1K`_1$ )το τόξο 1 , 2` του κύκλου.
Ο Γεωμετρικός Τόπος του σημείου K_2 , από του Σημείου $\boldsymbol{K_2}$ προς $\boldsymbol{K_1}$, στο κύκλο (O , OK)
είναι το τόξο K_2K_1 του κύκλου , ενώ επί του κύκλου (O_1 ,$O_1K`_1$ ) το τόξο 2 , 1` του κύκλου.
Ο Γεωμετρικός Τόπος από του σημείου , O, των παραλλήλων της Χορδής O_kO_1 , είναι οι Χορδές
OP_1, $O_4O`_1$ , από δε τού πόλου , $O_k$ , η τομή , T , των χορδών 1 , 2` και 2 , 1` αντίστοιχα .
Επειδή δε η γωνία <O_kO_1K= O_kK_2K = 90ο , Άρα η τομή , T , κινείται παράλληλα της O_1K ,
και είναι το κοινό σημείο των δύο Γεωμετρικών Τόπων .
Επειδή τα σημεία $\boldsymbol{K_1}$, $\boldsymbol{K_2}$είναι οι Διαδοχικές Κορυφές των Χορδών - Ζυγών – Κανονικών - Πολυγώνων του κύκλου (O , OK) , και
συνάμα τα σημεία $\boldsymbol{O_1}$, $\boldsymbol{P_2}$,οι αντίστοιχοιΑκραίοι πόλοι
επί των κύκλων(O_1 ,$O_1K`_1$) , (O_2 ,O_2K) , πού ακολουθούν την ΚΟΙΝΗ δέσμευση του σημείου K , να είναι Ορθόκεντρο και Αρχή των Πολυγώνωνκαι το σημείο, T,ο σταθερόςκοινός πόλος του συστήματος , ΑΡΑ η ευθείαO_kT , είναι σταθερά και κόβει τον (O , OK) ,στο σημείο $\boldsymbol{K_5}$ πού είναιη Κορυφή του Ενδιάμεσου Μονού –Κανονικού –Πολυγώνου ?? ,Ή
Επειδή , από την Αρμονική σχέση (1) και (4) ($K_1K`_1$) ² = ($K_1C$). ($K_1C$+ $K_1K`_1$) ορίζεται
το Αρμονικό ύψος $\boldsymbol{K_1C}$ και με την παράλληλο χορδήCK_5 ,το σημείο $\boldsymbol{K_5}$ επί τού κύκλου ,
(O , OK) ώστε να αντιστοιχεί η ανωτέρω Αρμονική σχέση ,ΑΡΑκαι η χορδή $K K_5$ είναι
επίσης του Ενδιάμεσου Μονού –Κανονικού –Πολυγώνου . ο.ε.δ.
Μάρκος ,5/5/2017



F.18 – A→ Στον κύκλο (O , OK) ,για n = 4, η Χορδή $K K_1$είναι η πλευρά του Ζυγού-Κανονικού

Τετραγώνου ενώγια , n =n+2 = 6, η $K K_2$ είναι η πλευρά του Ζυγού - Κανονικού Εξαγώνου η δε Χορδή $\boldsymbol{K K_5}$ του Κανονικού – Μονού - Πενταγώνου. Οκύκλος (O_1 ,$O_1K`_1$ ) είναι ο , κύκλος Καμπής, των Υψομετρικών Διαφορών με Δh= $h_{K1}$-$h_{K2}$=$K_1K`_1$,ο δε κύκλος (O_2, O_2P_2) είναι ο , κύκλος Ανακάμψεως , [Euler-Savary]. Ο κύκλος (O_4 , O_4 K=O_4 O) είναι ο , κύκλος των Μέσων των Χορδών,από του σημείου K .

Οι χορδές 1 , 2` και2 , 1` κόβονται στο σημείο C , πού είναι το Σταθερό σημείο στιςΠεριβάλλουσες επί της παραλλήλου της $K O_1$από του σημείου, C , και με κέντρο Καμπυλότητας το άπειρο , ∞ . Επειδή δε οκύκλος των Υψομετρικών Διαφορών [K_1,$K_1K`_1$]είναικαι

Προβολή τού ΚύκλουΤαχυτήτων[K_1 ,K_1K_2]πού είναι και κύκλος Καμπής , με κοινότο σημείοK_1 κέντρου Καμπυλότητας στο άπειρο ,∞,Άρα όλες οι Γεωμετρικές - Ιδιότητες των δύο Κύκλων είναι Κοινές .

Πρώτη Προσεγγιστική Γεωμετρική Απόδειξη :

Επειδή οι πλευρές P_1O_k ,P_1Oείναι κάθετοι των , KK_2 , KK_1αντίστοιχα , Άραη γωνία <OP_1O_k = K_1KK_2, και επειδή ηP_2O , είναι χορδή μεταξύ των παραλλήλων P_1O , P_2O_k,Άρακαι οι γωνίες <OP_1O_k, OP_2O_k , είναι ίσες , τόσον επί των Σταθερών πόλων , κορυφών ,O , O_k , όσον και των κινουμένωνπόλων , των κορυφών ,P_1 , P_2.

Επειδή οι γωνίες OP_1O_k , OP_2O_k , είναι ίσες Άρα βαίνουν επί κύκλου χορδής OO_k .Επειδή δε επί του ιδίου κύκλουβαίνουν οι πόλοι, O_k , O ,P_1 , P_2 ,Άρα το κέντρο του κύκλου τούτου ευρίσκεται ως τομή της Μεσοκαθέτου των χορδών αυτών,OO_k και OP_2 , και πού είναι το σημείοO_3 .

Το σημείο K , της ευθείας (e) είναι κοινό των Άπειρων (∞) Κανονικών-Πολυγώνων των κύκλων κέντρουO καιμεακτίνα KO = 0 → ∞ , Άρατο Άπειρο - Κανονικό - Πολύγωνο είναι η ευθεία (e) το Κανονικό - Πολύγωνο του κύκλου (O , OK) είναι το ζητούμενο , το δε Μηδενικό – Κανονικό – Πολύγωνο το σημείοK .

Επειδή δε οι κινούμενοι πόλοι P_1 , P_2, των δύο Ζυγών Κανονικών Πολυγώνων , ευρίσκονται επί του κύκλου [O_3 , O_3O] , κύκλος του Αντιχώρου,[12] , Άρα ο ενδιάμεσος Κινούμενος πόλος του Μονού - Κανονικού – Πολυγώνου ,περνά απότο,∞ , πού είναι η τομή της ευθείας (e) και του κύκλου τούτου , πού είναι το κοινό σημείο P_5 . Το ίδιο παρουσιάζεται και με την γωνία των 90 º πού συμβαίνει με δύο κάθετες ευθείες οι οποίες περνούν από το άπειρο . Η χορδή OP_5 αντιστοιχεί στην Ανακαμπτομένη χορδή των κύκλων Ανακάμψεως [O_2 ,O_2P_2] στο άπειρο πού είναι το σημείο P_5. Τα Δύο - ζεύγη τωντομώνP_4 , $P`_4$και$P_6$ , $P`_6$, συγκλίνουν στο Ένα- Ζεύγος με ένα σημείοP_5=$P`_5$, όπου τα δύο σημεία συμπίπτουν .ο.ε.δ.

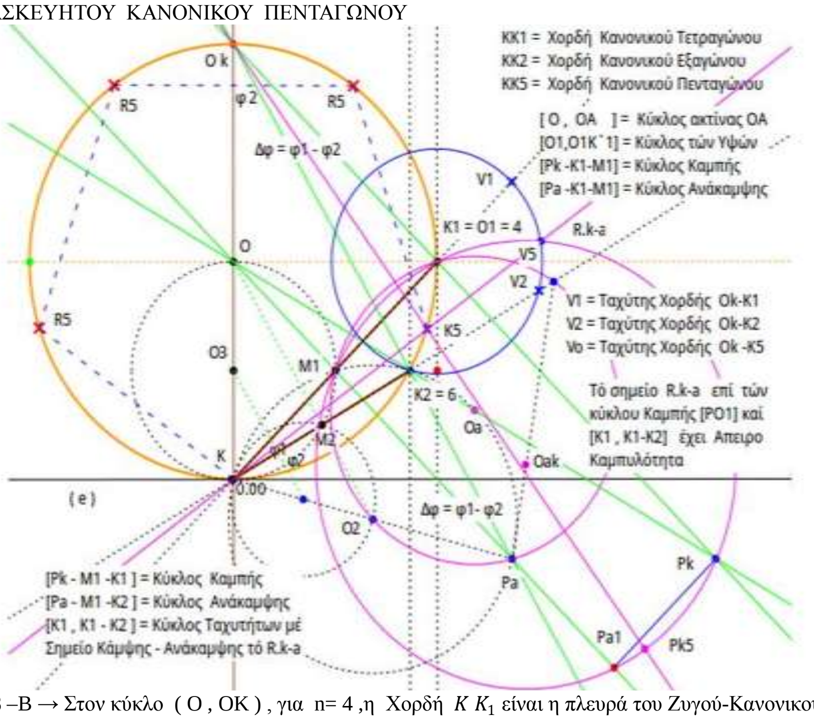
Παρατήρηση .

Η ανωτέρω Γεωμετρική Απόδειξη επιλύει μερικώς το πρόβλημα των Κανονικών – Πολυγώνων παρακάμπτοντας τούς μέχρι σήμερα περιορισμούς στην Αλγεβρική-θεωρία των Πρώτων προς αλλήλους αριθμούς.Στο σχήμαF16.(3) είναιOX ┴ OΑδηλαδή η γωνία<XOK = 90º. ΤυχούσαγωνίαXOC<XOA<90ºισούται με την συμμετρική της X`O$C_1$, εφόσον περάσει από την θέση OAόπουη γωνία <XOA = X`OA = 90ºκαι η πλευρά OCπερνά από το άπειρο .

Στο σχήμα 18-B, λόγω του ότι οι χορδές O_kK_1 , O_kK_2 , είναι κάθετες των KK_1 ,KK_2,άρα και η γωνία <$K_1O_kK_2$ = K_1K K_2. Η αλλαγή της θέσης των καθέτων από του νέου κέντρουO , σχηματίζει την Αντισυμμετρική γωνία OP_aO_kίση με τις άλλες εφόσον περάσει μία κάθετος παράλληλος της KK_2από το άπειρο . Επειδή η Αντισυμμετρική γωνία βαίνει στη χορδή OO_kτων δύο σταθερών κορυφώνσχηματισμού των γωνιών , οι κύκλοι πού περνούν από τα σημεία K ,K_2,P_a ,είναι οι Κύκλοι Ανάκαμψης, λόγω του ότι οι σταθερές περιβάλλουσεςKK_1,KK_2,KK_i όλων των πλευρών αυτού του Συστήματος των γωνιών Ανακάμπτονται στα σημεία συνάντησης τωνμε κοινό το K_1 τού κύκλου, οι δε κύκλοι από τα σημεία K ,K_1, P_k ,είναι οι , Κύκλοι Καμπής, πού αντιστρέφουν τις γωνίες των κύκλων Ανάκαμψησε , Εντός-Εναλλάξ ίσες γωνίες όπως είναι<OP_aO_k=OP_kO_kεπί των παραλλήλωνO_k ,OP_a.

Έτσι προκύπτει η Ακριβής Γεωμετρική Επίλυσητών Κανονικών - Μονών - Πολυγώνων .

ΗΓΕΩΜΕΤΡΙΚΗ ΚΑΤΑΣΚΕΥΗΤΟΥ ΚΑΝΟΝΙΚΟΥ ΠΕΝΤΑΓΩΝΟΥ



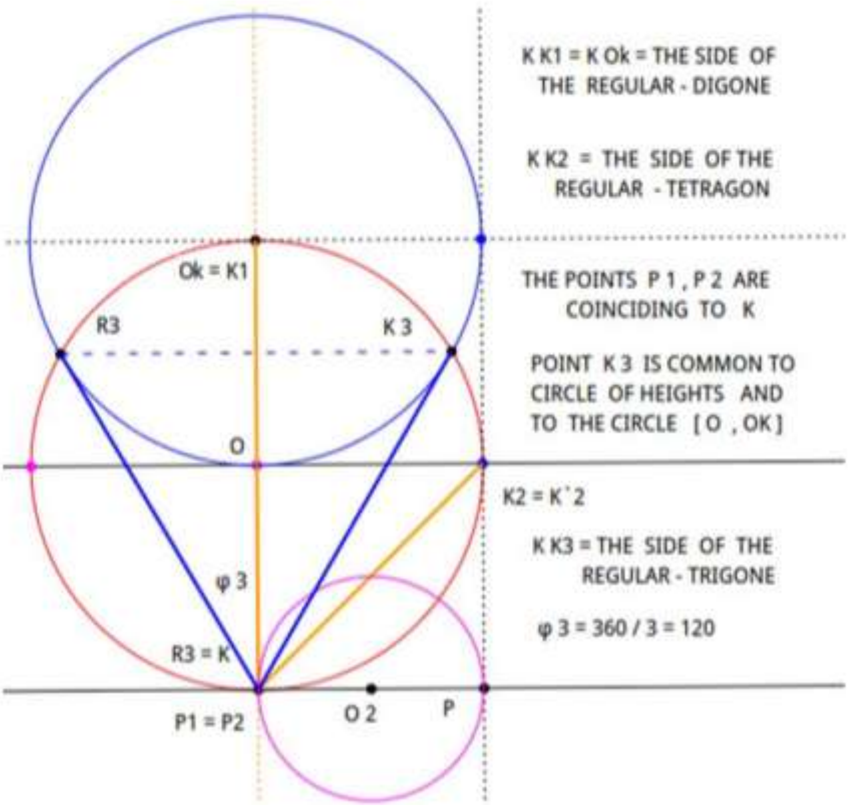
F.18 –B → Στον κύκλο (O , OK) , για n= 4 ,η Χορδή K K_1 είναι η πλευρά του Ζυγού-Κανονικού

Τετραγώνου ενώ για , n = n + 2 = 6 , η K K_2 είναι η πλευρά του Ζυγού - Κανονικού Εξαγώνου η δε Χορδή $\boldsymbol{KK_5}$ του Κανονικού – Μονού - Πενταγώνου. O κύκλος (O_1 ,$O_1K`_1$ ) είναι ένας , κύκλος Καμπής, τωνΥψομετρικών Διαφορών με Δh= $h_{K\,1}$-$h_{K\,2}$ = $K_1K`_1$,δε κύκλος (O_a, O_aP_a) είναι ο , κύκλος Ανακάμψεως , [Euler-Savary] , και ο κύκλος [PO_1, PO_1P_k=PO_1M_1] είναι ο , Κύκλος Καμπής, από του σημείου O_k.Η γωνία <$P_kO_kP_a$ =OP_aO_k είναι μία Περιβάλλουσα επί των κύκλων - Καμπής , η δε γωνία <OP_aO_k ηαντίστοιχη Περιβάλλουσα επί τωνκύκλων Ανάκαμψης .

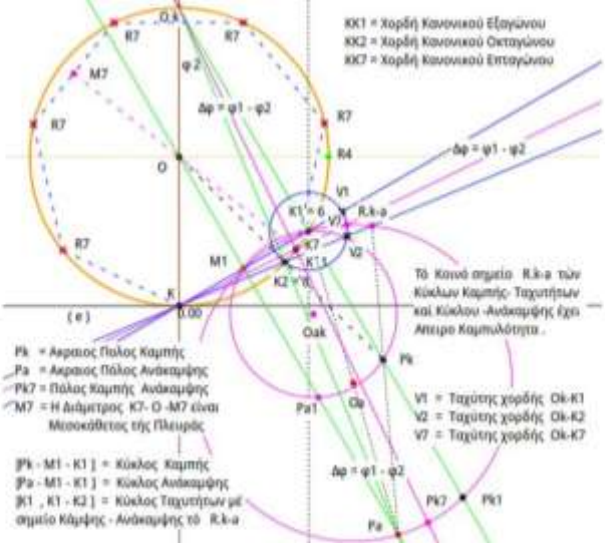
Το εγγεγραμμένο σχήμα$P_kK_1M_1$ P_{a1} ,εντός του Κύκλου Καμπής , είναι ορθογώνιο διότι η γωνία <$P_kK_1M_1$ = $K_1M_1P_{a1}$ = 90 º, Άρακαι η χορδή P_kP_{a1}// K_1M_1 . Επειδή δε η γωνία <OP_kO_k έχει την πλευρά OP_k , μεταξύ των παραλλήλων πλευρώνO_kP_k , OP_a , άρα είναι ίση με την Εντός - Εναλλάξ <$P_kO_kP_a$. Η γωνία <$P_kO_kP_{k5}$ =$OP_\infty O_k$στην θέση O_kP_∞όπου το σημείο P_∞ευρίσκεται επί της παραλλήλου OP_a . Δηλαδή ,F.18 - B ,

Στο σημείο $\boldsymbol{P}_{\infty}$ γίνεται η Αντιστροφή των γωνιών σε Εντός - Εναλλάξ μεταξύ των σημείων $\boldsymbol{P_k}$ του Κύκλου - Καμπής , και$\boldsymbol{P_a}$ του Κύκλου –Ανάκαμψης , αλλά Πού ?? Να δειχθεί ότι ο κύκλος [$\boldsymbol{K_1}$, $\boldsymbol{K_1K_2}$]Ταχυτήτων , διέρχεται διά του Κύκλου-Καμπής .
Φέρομεν τον κύκλο [K_1 , K_1K_2] πού τον ονομάζουμε , Κύκλο - Ταχυτήτων , του σημείου K_1, και
τούτο διότιτο σημείο K_1κινούμενο επί του κύκλου [O ,OK] κατευθύνεται ακαριαία στο σημείοK_2με ταχύτητα το μέγεθος ,K_1K_2 . Από την θεωρία του Κέντρου Καμπυλότητας (Euler–Savary) η ταχύτης V_1 του σημείου K_1, στρεφομένουπέριξ του σημείου O_k είναιίση με$\overline{V_1}$=K_1K_2 καικάθετος της O_kK_1 , του δε σημείουK_2στρεφομένου πέριξ του ιδίου πόλουO_kείναι $\overline{V_2}$ = K_1K_2 και κάθετος τηςO_kK_2 , δηλαδή , Οι τροχιές των σημείων του κύκλου [K_1 , K_1K_2] έχουν τα κέντρα καμπυλότητας των επί του κύκλου διαμέτρου KO_k , η δε κατεύθυνση των ταχυτήτωντων σημείων K_1, K_2 του κύκλου [K_1 , K_1K_2] ευρίσκονται επί των καθέτων χορδών KK_1, KK_2 αντίστοιχα .
Όταν όμως το σημείο$\boldsymbol{K_1}$κινείται επί της χορδής $\boldsymbol{K_1}$K, τότε το Κέντρο καμπυλότητας αρχίζει από το σημείο $\boldsymbol{P_k}$, κινείται επί της $\boldsymbol{O_kP_k}$ και κατευθύνεται προς το άπειρο∞σχηματίζοντας έτσι την Περιβάλλουσα των Κύκλων - Καμπής , όπου και Αντιστρέφεταιη κίνηση προς τα πίσω όπως τούτο
συμβαίνει σε γωνίες 90°μεταξύ δύο καθέτων . Για να φτάσει το σημείο $\boldsymbol{K_1}$ στη θέσητουσημείου $\boldsymbol{M_1}$ από το άπειρο της ευθείας $\boldsymbol{OP_a}$στοσημείο $\boldsymbol{P_a}$, σχηματίζοντας έτσι την Περιβάλλουσα των Κύκλων – Ανάκαμψης περνά και απόένα Κοινό σημείο των δύο κύκλων το , $\boldsymbol{R}_{\boldsymbol{k-a}}$, πούείναι τέτοιοώστεοι
Εντός-Εναλλάξ γωνίες πού είναιίσες ,να είναι και επί των πόλων K ,$\boldsymbol{O_k}$,και πού είναι στην θέση $\boldsymbol{K_5}$.Επειδή η Διάμετρος από τες Κορυφές K_1 , K_2 περνά από Κορυφές των , n ,και ,n+2, Ζυγών
Κανονικών Πολυγώνων , η δε Διάμετρος από την Κορυφή τού $K_{5\,=\,n+1}$περνά από το μέσο$\boldsymbol{M_5}$τήςέναντι Χορδής Άραείναι και Μεσοκάθετος της , Δηλαδή περνά από Σημεία Καμπής σε Σημείο Ανάκαμψης όπως τούτο συμβαίνει και στους τρείς ανάλογους Κύκλους . Ο κύκλος (PO_1 ,PO_1 K_1 = PO_1P_k = PO_1M_1) είναι ο Οριακός–Κύκλος- Καμπής πού περνά από τα σημεία K_1,M_1 , P_k , ο δε κύκλος (O_a, O_aK_1 = O_aP_a = O_aM_1) είναι ο Οριακός -κύκλος -Ανάκαμψης
πού περνά από τα σημεία K_1,M_1 , P_a . Το σημείοK_1με ταχύτητα V_1επί τούκύκλουταχυτήτων κινείται επί του κύκλου ταχυτήτων μέχρι του σημείουK_2και μεταχύτητα $V_1 \rightarrow V_2$. Επειδή η καμπύλη Κίνησης , η Τροχιά , του σημείου K_1είναι η ευθεία ,KK_1μέχρι το Άπειρο , πού είναι και η Σταθεράπεριβάλλουσα , Άρα το σημείο K_1 είναι και το αντίστοιχο κέντρο - καμπυλότητας τηςKK_1, και οι τροχιές των , καθώς επίσηςκαιο κύκλος των ταχυτήτων των ,
έχουν το αντίστοιχο κέντρο καμπυλότητας στο άπειρο.
Το άκρο του ΒέλουςV_1(η αιχμή τουV_1) , διαγράφεικατά την στιγμήν αυτήντροχιά παρουσιάζουσα Καμπή ,Άρα η Αιχμή του $\boldsymbol{V_1}$διέρχεται διά του Κύκλου - Καμπής .ο.ε.δ.Επειδή επί της O_kK_1 άπειροι κύκλοι περνούν από το σημείοK_1 , Άρα είναι οι Οριακοί Κύκλοι – Ανάκαμψης Διαμέτρου τά τμήματα K_1P_k από τού σημείου $K_1 \rightarrow P_k \rightarrow \infty$.
Το ίδιο συμβαίνει και με την Αιχμή του V_2 του σημείου K_2 ,και τους Κύκλους Ανάκαμψης από τόK_2. Επειδή δε ισχύει η σχέση των Υψών , Σ = n .OK , και στα Μονά, n+1, Κανονικά Πολύγωνα η Διάμετρος από την Κορυφή , K , είναι κάθετος της έναντι πλευράς , Άρα πρέπειναυπάρχει ένα τέτοιο Κοινό σημείοκαι στις Περιβάλλουσες , πού είναι πράγματι το σημείο$\boldsymbol{R}_{\boldsymbol{k-a}}$.
Εις την περίπτωση πού , ο Οριακός - Κύκλος - Καμπής (PO_1 , PO_{1-} K_1 =$PO_{1-}P_k$ =$PO_{1-}M_1$)
τέμνειτον άξοναOO_k τότε το σημείο $\boldsymbol{R}_{\boldsymbol{k-a}}$, Αντιστρέφεται και κινείται επί του άξονοςOP_k .

H Γεωμετρικη Κατασκευη Του Κανονικου Τριγωνου



F.19 → Στον κύκλο (O , OK) , για n = 2 , η Χορδή KK_1 , είναι η πλευρά του Ζυγού - Κανονικού Διγώνου ενώ για , n = n+2 = 4 , η $K\,K_2$

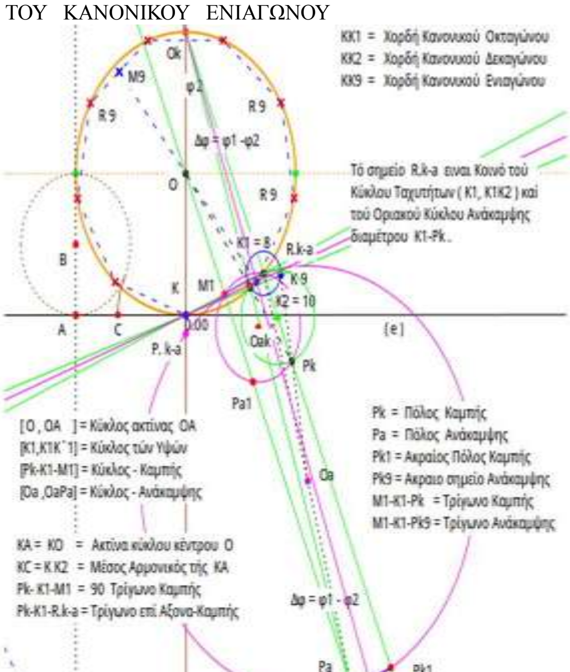


είναι η πλευρά του Ζυγού - Κανονικού – Τετραγώνου , η δε Χορδή $\boldsymbol{K K_3}$ του Κανονικού – Μονού – Τριγώνου.For n = 2 , then $K K_1$ is the Side of the Regular - Digone and equal to 2.OK.. For n = n+2 = 4 , then $K K_2$ is the Side of the Regular – Tetragon and equal to OK.$\sqrt{2}$, the point K_2 on (O , OK) circle . Exist Δh = h_{K1} - h_{K2} = O_k O . The Circle of Heights is (K_1 ,K_1O) .The Coupler - Circle is (O_2 , $O_2 P$) , Points P_1 ,P_2 are the intersections of Sides $K K_1$, $K K_2$ produced . Point K_3 is the intersection of $P_2 O_k$ Segment , and the circle (O , OK) .

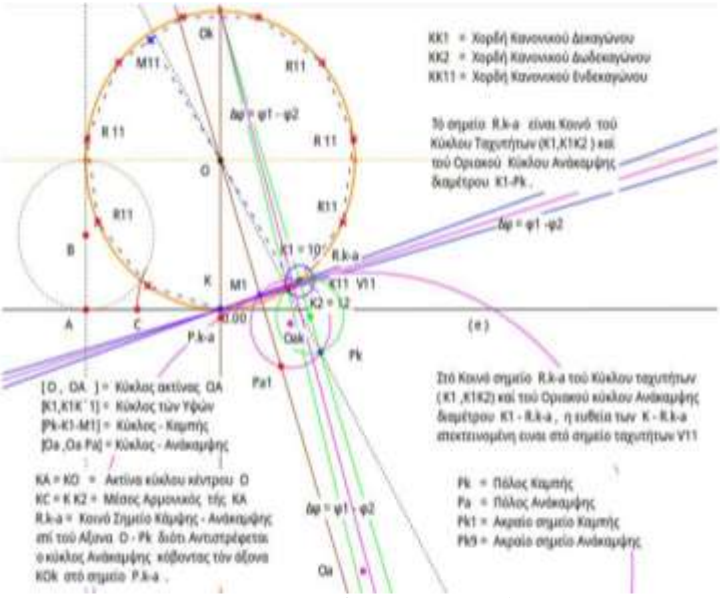
Η ΓΕΩΜΕΤΡΙΚΗ ΚΑΤΑΣΚΕΥΗ ΤΟΥ ΚΑΝΟΝΙΚΟΥ ΕΠΤΑΓΩΝΟΥ

F.20 → Στον κύκλο (O , OK) , για n = 6, η Χορδή $K K_1$ είναι η πλευρά του Ζυγού -Κανονικού Εξαγώνου ενώ για , n =n+2 = 8 , η χορδή $K K_2$είναι η πλευρά του Ζυγού -Κανονικού Οκταγώνου , η δε Χορδή $\boldsymbol{K K_7}$ του Κανονικού – Μονού – Επταγώνου.→
Το εγγεγραμμένο σχήμα $P_{k1}K_1M_1 P_a$, εντός του Κύκλου Ανάκαμψης , είναι ορθογώνιο παραλληλόγραμμο διότι η γωνία <$P_{k1}K_1M_1$ = $K_1M_1P_a$ = 90 º, Άρα και η χορδή $P_{k1}P_a$// K_1M_1 η δε γωνία $P_{k1}P_aP_{k7}$ = $K_1K\ K_2$ διότι έχουν τις πλευρές των παράλληλες μεταξύ των από των σημείων P_{k1} , K . Η γωνία <$P_{k1}P_a\ P_{k7}$ =$P_aP_{k1}P_\infty$ = $K_1K\ K_2$, διότι είναι Εντός - Εναλλάξ στη χορδή $P_{k1}P_a$ επί του Κύκλου Καμπής .Ο κύκλος Ταχυτήτων [K_1 , K_1K_2] απεδείχθη ότι είναι ένας κύκλος Καμπής πού κόβει τον Οριακό κύκλο Ανάκαμψης [O_a ,O_aP_a] στο σημείο R_{k-a} , η δε ευθείαKR_{k-a} κόβει τον κύκλο [O,OK] στο σημείο K_7 πού η χορδήKK_7 είναιη πλευρά του Κανονικού Επταγώνου ΣτόΤραπέζιο $\boldsymbol{OO_kP_kP_a}$η Εντός Εναλλάξ γωνία <$OP_a O_k$= $P_a O_k P_k$=K_1KK_2= $\Delta\varphi_{(\varphi1-\varphi2)}$.
Οι ΔιάμετροιK_1OK \`$_1$ , $K_2OK$ \`$_2$,των Κανονικών , Εξαγώνων – Οκταγώνων διέρχονται από τις έναντι κορυφές τωνK \`$_1$ ,$K$ \`$_2$,ΕΝΩ η ΔιάμετροςK_7OM_7 διέρχεται του μέσου της έναντι Πλευράς και είναι Μεσοκάθετος της . Στο σημείο K_7 γίνεται η Αναστροφή της Διαμέτρου κατά γωνία 90º.

Η ΓΕΩΜΕΤΡΙΚΗ ΚΑΤΑΣΚΕΥΗ ΤΟΥ ΚΑΝΟΝΙΚΟΥ ΕΝΙΑΓΩΝΟΥ



F.21 → Στον κύκλο (O , OK) , για n = 8, η Χορδή $K K_1$ είναι η πλευρά του Ζυγού –Κανονικού



Οκταγώνου ενώ για , n = n + 2 = 10, η χορδή $K\,K_2$ είναι η πλευρά του Ζυγού -Κανονικού Δεκαγώνου , η δε Χορδή $\boldsymbol{K\,K_9}$ του Κανονικού – Μονού - Ενιαγώνου.→

Το εγγεγραμμένο σχήμα $P_{k1}K_1M_1\,P_a$,εντός του Κύκλου Ανάκαμψης , είναι ορθογώνιο παραλληλόγραμμο διότι η γωνία <$P_{k1}K_1M_1$ = $K_1M_1P_a$ = 90 º, Άρα και η χορδή $P_{k1}P_a$// K_1M_1 η δε γωνία<$P_{k1}P_aP_{k9}$ = $K_1K\,K_2$διότι έχουν τις πλευρές των παράλληλες μεταξύ των από των σημείων P_{k1} , K . Η γωνία <$P_{k1}P_a\,P_{k9}$ =$P_aP_{k1}P_\infty$ = $K_1K\,K_2$, διότι είναι Εντός - Εναλλάξ στη χορδή $P_{k1}P_a$επί του Κύκλου Καμπής.Ο κύκλος Ταχυτήτων [K_1 , K_1K_2] απεδείχθηότι είναι ένας Κύκλος - Καμπής πού κόβει τον ΟριακόΚύκλο - Ανάκαμψης , Διαμέτρου K_1P_k στο σημείο R_{k-a}και τούτο , διότι οι κύκλοι Ανάκαμψης Αντιστρέφονται, η δε ευθεία $K\ R_{\cdot k-a}$επεκτεινομένηπερνά από το σημείο V9 , και κόβει τον κύκλο [Ο ,ΟΚ] στο σημείο K_9 η δε
χορδή $K\,K_9$είναι η πλευρά του Κανονικού Ενιαγώνου . Τούτο συμβαίνει στά Πολύγωνα πού ο Κύκλος Καμπής των ή και Ανάκαμψης κόβει τον άξονα O_k-O-K, οπότε η Αντιστροφή γίνεται στον Οριακό κύκλο Ανάκαμψης διαμέτρου K_1P_k .

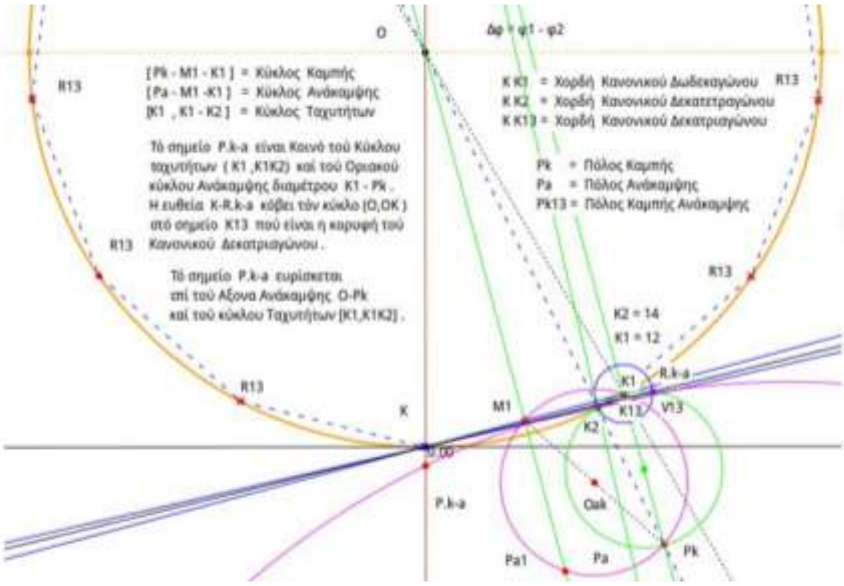
Η ΓΕΩΜΕΤΡΙΚΗ ΚΑΤΑΣΚΕΥΗ ΤΟΥ ΚΑΝΟΝΙΚΟΥ ΕΝΔΕΚΑΓΩΝΟΥ

F.22 → Στον κύκλο (Ο , ΟΚ) , για n = 10, η Χορδή $K\,K_1$ είναι η πλευρά του Ζυγού -Κανονικού Δεκαγώνου , ενώ για , n = n + 2 = 12,η χορδή $K\,K_2$ είναι η πλευρά του Ζυγού -Κανονικού Δωδεκαγώνου η δε Χορδή$\boldsymbol{KK_{11}}$τουΚανονικού– Μονού - Εντεκαγώνου.→

Το εγγεγραμμένο σχήμα $P_{k1}K_1M_1\,P_a$, εντός του Κύκλου Ανάκαμψης , είναι ορθογώνιο παραλληλόγραμμο διότι η γωνία <$P_{k1}K_1M_1$ = $K_1M_1P_a$ = 90 º, Άρα και η χορδή $P_{k1}P_a$// K_1M_1 η δε γωνία<$P_{k1}P_aP_{k11}$ =$K_1K\,K_2$ διότι έχουν τις πλευρές των παράλληλες μεταξύ των από των σημείων P_{k1} , K . Η γωνία <$P_{k1}P_a\,P_{k11}$ =$P_aP_{k1}P_\infty$ = $K_1K\,K_2$, διότι είναι Εντός - Εναλλάξ στη χορδή $P_{k1}P_a$ επί του Κύκλου Καμπής . Ο κύκλος Ταχυτήτων [K_1 , K_1K_2] απεδείχθη ότι είναι ένας κύκλος Καμπής πού κόβει τον ΟριακόΚύκλο - Ανάκαμψης , Διαμέτρου K_1P_k στο σημείο R_{k-a}και τούτο , διότι οι κύκλοι Ανάκαμψης Αντιστρέφονται , η δε ευθεία $K\ -R_{k-a}$, η οποία και περνά από το Οριακό σημείο ταχυτήτωνV11 ,επεκτεινομένηπερνά και από το σημείο V11, όπουκαι κόβει τον κύκλο[Ο ,ΟΚ] στο σημείο K_{11}, η δε χορδή $K\,K_{11}$ είναι η πλευρά του Κανονικού Εντεκαγώνου .

Τούτο συμβαίνει στά Πολύγωνα πού ο Κύκλος Καμπής των ή και Ανάκαμψης κόβει τον άξονα O_k-O-K, οπότε η Αντιστροφή γίνεται στον Οριακό κύκλο Ανάκαμψης διαμέτρου K_1P_k .

Η ΓΕΩΜΕΤΡΙΚΗ ΚΑΤΑΣΚΕΥΗ ΤΟΥ ΚΑΝΟΝΙΚΟΥ ΔΕΚΑΤΡΙΑΓΩΝΟΥ



F.23 → Στον κύκλο (O , OK) , για n = 12, η Χορδή $K\,K_1$ είναι η πλευρά του Ζυγού -Κανονικού

Δωδεκαγώνου , ενώ για , n = n+2 = 14 , η χορδή $K\,K_{13}$ είναι η πλευρά του Ζυγού - Κανονικού Δεκατετραγώνου η δε Χορδή$\boldsymbol{KK_{13}}$του Κανονικού–Μονού -Δεκατριαγώνου.
Το εγγεγραμμένο σχήμα $P_{k1}K_1M_1\,P_a$, εντός του Κύκλου Ανάκαμψης , είναι ορθογώνιο παραλληλόγραμμο διότι η γωνία <$P_{k1}K_1M_1$ = $K_1M_1P_a$ = 90 º, Άρα και η χορδή $P_{k1}P_a$// K_1M_1 η δε γωνία <$P_{k1}P_a\,P_{k13}$ =$K_1K\,K_2$ διότι έχουν τις πλευρές των παράλληλες μεταξύ των από των σημείων P_{k1} , Κ . Η γωνία <$P_{k1}P_a\,P_{k13}$ =$P_aP_{k1}P_\infty$ = $K_1K\,K_2$, διότι είναι Εντός - Εναλλάξ στη χορδή $P_{k1}P_a$ επί του Κύκλου Καμπής . Ο κύκλος Ταχυτήτων [K_1 , K_1K_2]απεδείχθη τι είναι ένας κύκλος Καμπής πού κόβει τονΟριακόΚύκλο - Ανάκαμψης , Διαμέτρου K_1P_k
στο σημείο R_{k-a}και τούτο ,διότι οι κύκλοι Ανάκαμψης Αντιστρέφονται , η δε ευθεία $K - R_{k-a}$επεκτεινομένηπερνά από το σημείο V13, και κόβει τον κύκλο [O , OK] στο σημείοK_{13}η δε χορδή$K\,K_{13}$είναι η πλευρά του Κανονικού Δεκατριαγώνου .Τούτο συμβαίνει στά Πολύγωνα πού ο Κύκλος Καμπής των ή και Ανάκαμψης κόβει τον άξοναO_k-O-K, οπότε η Αντιστροφή γίνεται στον Οριακό κύκλο Ανάκαμψης διαμέτρου K_1P_k .
Η Αναστροφή των κύκλων Καμπής $P_kK_1M_1$ γίνεται διότι η Διάμετρος K_1OM_{13} του Κανονικού Δεκατριαγώνου είναι Μεσοκάθετος της έναντι πλευράς του στο μέσο σημείο M_{13},εν αντιθέσει με την Διάμετρο K_2OM_2 ≡ OK_2→ P_kπού διέρχεται από την ορυφήτου Κανονικού Δεκατετραγώνου.

Η ΓΕΩΜΕΤΡΙΚΗ ΚΑΤΑΣΚΕΥΗ ΟΛΩΝ , ΤΩΝ ΚΑΝΟΝΙΚΩΝ – ΜΟΝΩΝ – ΠΟΛΥΓΩΝΩΝ
ΜΕ ΤΗ ΜΕΘΟΔΟ ΤΩΝ ΤΡΙΩΝ ΚΥΚΛΩΝ .

Η ανωτέρω Γεωμετρική Απόδειξη επιλύει γενικά το πρόβλημα των Κανονικών – Πολυγώνων παρακάμπτοντας τούς μέχρι σήμερα περιορισμούς στην Αλγεβρική-θεωρία των Πρώτων προς αλλήλους αριθμούς . Η Αναστροφή γωνίας πέριξ άξονος OA { σχήμα F16.(3) } είναι ότανσυμβαίνει OX ⊥ OA δηλαδή η γωνία <XOK = X`OK = 90º . Τυχούσα γωνία XOC<XOA< 90º ισούται με την συμμετρική της X `OC_1 , εφόσον περάσει από την θέση OA όπου καίXOA = X `OA = 90º(Αναστροφή)και η πλευρά OCπερνά από το άπειρο ∞ . Στο σχήμα F-20

Το Σύστημα των Κύκλων - Καμπής – Ανάκαμψης σχηματίζεται από τον κύκλο μεγαλυτέρας διαμέτρου του κύκλου , καιείναι το Ορθογώνιο Παραλληλόγραμμο $K_1M_1P_aP_{k1}$ είτε το$K_1M_1P_{a1}P_k$. Ο Οριακός Κύκλος – Καμπής επί του τριγώνου $M_1K_1P_k$έχει την κορυφή P_k επί της O_kP_k , ενώ ο ΟριακόςΚύκλος – Ανάκαμψης επί του τριγώνου $K_1M_1P_a$, έχει την κορυφή P_a επί τηςOM_1 παραλλήλου τηςO_kP_k .Επειδή δε οι χορδές O_kP_k , OP_a είναι κάθετοι της KK_1,άρα είναι και παράλληλοι , και επειδή οι χορδές , OP_k , O_kP_a , είναι μεταξύ των παραλλήλων , άρα και οι Εντός Εναλλάξ γωνίες των <P_aOP_k = OP_kO_k καιη<$P_aO_kP_k$ = OP_aO_k = K_1KK_2 =$\varDelta\varphi = \varphi_1 - \varphi_2$.

Οι χορδές O_kP_k ,OP_a είναι παράλληλοι , ΑΡΑ , το Τετράπλευρο $\boldsymbol{OO_kP_kP_a}$είναι Τραπέζιο μεΎψοςK_1M_1με τα Ανάστροφα τρίγωνα $P_kK_1M_1$,$P_aM_1K_1$.Οι κύκλοι επί των διαμέτρων P_kM_1 , P_aK_1είναι
ο Ακραίος Κύκλος –Καμπής καιΑνάκαμψης αντίστοιχα .

Η Αναστροφή των κύκλων γίνεται διότι η Διάμετρος$\boldsymbol{K_7OM}_7$είναι Μεσοκάθετος της έναντι πλευράς στο μέσο σημείο$\boldsymbol{M_7}$, εν αντιθέσει με την Διάμετρο $\boldsymbol{K_2OM_2}$≡$\boldsymbol{OK_2}$→$\boldsymbol{P_k}$πού διέρχεται από κορυφή . Για να καταστούν οι γωνίες <$P_kO_kP_a$,OP_aO_k, Εντός – Εναλλάξκαι ίσες της <K_1KK_2, πρέπει η ευθεία O_kP_kνα περιστρέφεται πέριξ του Πόλου O_k από το άπειρο (∞) μέχρι τη χορδήO_kP_a. Αυτή η περιστροφική κίνηση της ευθείας είναι Ισοδύναμη με την κίνηση του σημείουK_1προς το σημείοK_2επί του κύκλου [O , OK] , με τα κάτωθι επακόλουθα :

1..Με τηνπεριστροφή της χορδής O_kP_k πέριξ του πόλου O_k, η χορδή O_kK_1 έχει την κάθετο ταχύτητα K_1V_1 επί της επέκτασης της KK_1. Το ίδιο συμβαίνει και διά την χορδή O_kK_2πού έχει την κάθετο ταχύτητα K_2V_2 επί της επέκτασης της KK_2 ,Δηλαδή , έκαστο σημείο K_7μεταξύ των σημείωνK_1, K_2 έχει μίαν κάθετο ταχύτητα , έστω τηνK_7V_7 , επί του κύκλου ταχυτήτων [K_1 , K_1K_2] και με κατεύθυνση την$O\ K_7$, στην εκάστοτε θέση του σημείου . Απεδείχθηπροηγουμένως ότι η Αιχμή του Βέλους V_1,διέρχεται διά Κύκλου Καμπής , (και τούτο διότι όταν το σημείο P_k είναι στο ∞ , τότε ο κύκλος (P_k,P_k∞) προβάλλεται στο σημείοK_1 και γίνεται η εφαπτομένη του σημείου πού είναι η KK_1.

όπως και κάθε άλλου βέλους V_7έχοντας σχέση με την Θέση - Αναστροφής της Διαμέτρου . 2..Με την περιστροφή της χορδής O_kP_k πέριξ του πόλου O_k , Άπειροι Κύκλοι – Καμπής απότα ορθογώνια τρίγωνα $P_kK_1M_7$σχηματίζονται με διάμετρο τηνP_kM_7,(όπου

M_7 είναι η τομή της $O_k K_7$και της$K\, K_1$) , με Οριακό Κύκλο –Καμπής τον επί της διαμέτρου $P_k M_1$, ταυτόχρονα δε, Άπειροι Κύκλοι–Ανάκαμψηςσχηματίζονται από τα ορθογώνια τρίγωνα $P_a\, , M_1\, , M_7$με διάμετρο την$P_a M_7$και με Οριακό Κύκλο Ανάκαμψης τον επί της μεγαλυτέρας διαμέτρου $P_a K_1$ ευρισκόμενο. Η Αναστροφή των κύκλων Καμπής $P_k K_1 M_1$ γίνεται διότι η Διάμετρος $K_1 OM_{n+1}$ του Κανονικού (n+1)Μονού Πολυγώνου είναι Μεσοκάθετος της έναντι πλευράς του , στο μέσο σημείο M_{n+1},εν αντιθέσει με την Διάμετρο $K_2 OM_2 \equiv OK_2 \rightarrow P_k$πού διέρχεται από την κορυφή του Ζυγού-Κανονικού (n) , (n+2)Πολυγώνου .Η κίνηση της Κορυφής ,K, στη θέση O_k,διατηρεί την Χορδή$K_1 K_2$ σταθερή.

3.. Απεδείχθη ότι η εξίσωση , Σ (h) = n .OK , δηλαδή το άθροισμα των Υψών , h , των κορυφών των Κανονικών (n) Πολυγώνων από τυχούσα ευθεία (e) εφαπτομένη σε μίαν κορυφή του , είναι , n , φορές την ακτίνα του κύκλου . Όταν δε , n , n +2 , είναι οι Αριθμοί των Κορυφών δύο διαδοχικών Ζυγών Πολυγώνων , τότε μεταξύ των υπάρχει και το , n +1 , Μονό Πολύγωνο .

Η θέση του Μονού Πολυγώνου είναι κοινή του Κύκλου - Καμπής και του Κύκλου – Ανάκαμψης. Επίσης απεδείχθη ότι , η Αιχμή του Βέλους επί του Κύκλου των Ταχυτήτων $[K_1, K_1 K_2]$ διέρχεται διά της Περιβάλλουσας των Κύκλων-Καμπής, οπότε η τομή τών Οριακών Κύκλων –Ανάκαμψης με ΔιαμέτροτόΤμήμα $K_1 P_k$, καθορίζει το σημείο $R._{k-a}$καί την κατεύθυνση $K_1 V_7$, πού είναι αυτή τούn+1 Μονού – Κανονικού –Πολυγώνου .

Δηλαδή , η ευθεία$K\, V_7$ κόβοντας τον κύκλο [O ,OK] στο σημείο K_7 , καθορίζει την χορδή KK_7 πού είναι η Πλευρά του Ενδιάμεσου Μονού - Πολυγώνου , και Στην περίπτωση όπου ο Κύκλος Καμπής ή και Ανάκαμψης τέμνει τον άξονα O_k-O-K στο σημείοP_{k-a},ή και έχοντας την μεγαλυτέρα διάμετρο τότε το Κοινό σημείο Καμπής ευρίσκεται επί του Οριακού κύκλου Ανάκαμψης διαμέτρου $K_1 P_k$, και του κύκλου των Ταχυτήτων .

ο.ε.δ.Μάρκος 16 /06/2017 .

VITHE GEOMETRICAL CONSTRUCTION OF ALL THE ODD - REGULAR -POLYGONS USING THE THREE CIRCLES METHOD

But Simultaneously , are formulatedInfinite Reflection - Circles circumscribed in the rightangled triangles $P_a M_1 M_7$ with diameter $P_a M_7$, limiting to the Reflection –circle of $P_a K_1$ diameter . Inversionof the circles happens because Diameter$\boldsymbol{K_7 OM_7}$ is Mid-perpendicular to the opposite Side in the middle point $\boldsymbol{M_7}$in contradiction to Diameter$\boldsymbol{K_2 OM_2}$which passes through the verticesof Polygon .

3.. It was proved the equation Σ (h) = n .OK , the Summation of heights h , of the vertices of any (n) Polygon from any (e) line tangential to any vertices , is equal to , n , times the radius OK . When , n , n+2 , are the numbers of the vertices of any two sequent and Even Polygons , then exists the In-between , n+1, Odd -Polygon . The position of this Odd-Polygon is common to the Inflection and Reflection circles . It was proved also , that the edge of arrowV_1passes through the Inflection circle $[K_1, K_1 K_2]$ and through the Envelope of Inflection circles where then , the point of intersection , R.k-a , defines the direction $K_1 V_7$, which belongs to the n+1 Odd – Regular –Polygon . i.e. line KV_7 intersecting the circle [O , OK] at point K_7defines chord $K\, K_7$ which is the Side of the intermediate Odd – Regular – Polygon. i.e.In circle [O , OK] of diameter $K - O_k$, any two chords $K\, K_1, K\, K_2$ and the circle [$K_1\, , K_1 K_2$] , Formulate the Trapezium $\boldsymbol{OO_k P_k P_a}$ and $\boldsymbol{K_1 M_1 P_a P_k}$, such that the two circles on the Diamesus and diameters $M_1 P_k$,$K_1 P_a$, intersect the circle [$K_1\, , K_1 K_2$] at the pointS_{ka} such that , this tobe the common Inversion point of the two Inverted circles . (q.e.d) .

6.3. The Methods :

Preliminaries : The Subject , F.16(3).

Anycircle(O , OK) can be divided into ,

a..Twoequalpartsbythediameter KA [It is the Dipole AK] with angle < AOK = 180 º .

b..FourequalpartsbytheBisector of180º which is the perpendicular and second diameter X `X.

c..EightequalpartsbytheBisector of the four angles which are90 º .

d..Sixteen equalpartsbythe Bisector of the Eight angles which are 45º, and so on .

e..Thecirclehaving 360 º = 2πradians , can be divided into ,

Threeequalpartsas360º / 3 = 120ºand which is possible [The Equilateral triangle] ,

Sixequalpartsas360º / 6 = 60ºand which is possibleby the bisectors of the triangle

[TheRegularHexagon] ,

Twelveequalpartsas360º / 12 = 30ºand which is possibleby the bisectors of the Hexagon

[TheRegularDodecagon] , and so on , to 15º , 7,5º ……...

Remark :

a…Theseries of Even Numbers is 2 , 4 ,6 , 8 ,10 ,12 ,14 ,16 ,18 ,20 ,…………….

Theseries of Odd Numbers is 1 ,3 ,5 ,7,9 ,11 ,13 ,15 ,17,19 ,21 , ……………

Becomingfromthe Arithmetic - mean between two Adjoined - Even numbers , as for example , Numberfive $5 = \frac{4+6}{2} = \frac{10}{2} = 5$. Thelogicofadditionissuesin Geometry in its moulds which is the logic of Material – Point , which is Zero (0 = Nothing) and exists as the Addition of Positive + Negative (→ + ←) . [See , Material Geometry 58 – 60 – 61]

b… In previous paragraph 5.5(Casec) was proved(1) Σ(h) = (2k) . h = n .h = n .OK , where Σ = TheSummationof Heights ,h , of the Vertices (n) – in the Regular Polygon fromthe verticesK_n , projected to tangential(e) at the initial point K ,

h = OK , The height of center , O, measured on (e) tangent ,

n = The number of Sides of the Regular Polygon …… andwhichChangestheSumofheightsfromthe Tangentialline (e) to a Linear and Integer number of the

radius of the circle , and which is directly related to angles,$\boldsymbol{\varphi_n}$,and vertices of sides ,$\boldsymbol{KK_n}$.

c...OnanyChord$\boldsymbol{KK_1}$ofcircle (O ,OK) , the central angle <KOK_1 , is twice the Inscribed and equal to <K$O_K K_1 = KOM_1$. The mid - perpendicular OM_1 , is parallel to the Perpendicular line $O_K K_1$, therefore cut each other to infinite (∞) . BecausethetwoperpendicularspassfromOand$O_K$points , these consist the Poles of their rotation .

InF.18 -A , anyPoint $\boldsymbol{K_2}$ oncircle, formulates the second chord $\boldsymbol{KK_2}$, while the perpendicular $O_K K_2$ projectedcutsO M_1 , theparallelto$O_K K_1$atapointP_1, which is the Pole of rotation of the two chords , or angles , and this because point $P_2$is moving on $OM_1$from infinite to KP_1diameter .

Ondiameter KP_2ofcircle (O_2 , O_2P_2= O_2K) , and centerO_2 , are formulated the same angles φ_1 , $\varphi_2$bychordsP_1M_1 , P_2K_2 , such that angles are equal <$M_1P_1K_2 = K_1$K$K_2 = OP_1O_k$, Thatis , on any two chords $\boldsymbol{KK_1}$, $\boldsymbol{KK_2}$, of circle(O , OK) , with common verticesK , the Mid - Perpendicular$\boldsymbol{OM_1}$of the first , and the Perpendicular$\boldsymbol{O_K K_2}$of the second , cut each other

atapoint$\boldsymbol{P_1}$, which defines its conjugate circle ($\boldsymbol{O`_1}$ , $\boldsymbol{O`_1 P_1}$) , { it is the Circle of equal angles with circle (O ,OK) } . The same happens with circle($\boldsymbol{O_2}$,$\boldsymbol{O_2P_2}$= $\boldsymbol{O_2}$K).

d...From relationΣ = (2k) . h = n .h = n .OK , Forn = 2 thenΣ = 2.h = 2.OKthat is diameter KO_K .

Forn = 3 thenΣ = 3.h = 3 .OKandfor n = 4 thenΣ = 4.h = 4 .OK . BecausetheOdd - numbersare the Arithmetic - mean between two Adjoined - Even numbers so for 3.OK is (2.OK +4.OK)/2 .

ThedifferenceofheightsisΔh = h_{K1}- h_{K2}=$K_1K`_1$and it is between the parallels through points $K_1$ , $K_2$ , and line (e) . Circle ( $K_1$ , $K_1K`_1$) isthecircleofHypsometric differences of the chords KK_1 , KK_2 , andchangesaccordingtopoint$K`_1$or the same with point $K_2$ . That is ,The circle of the Hypsometric differences( $\boldsymbol{K_1}$ , $\boldsymbol{K_1K`_1}$) is correlated with chords [$\boldsymbol{KK_1}$,$\boldsymbol{KK_2}$] ,

[$\boldsymbol{O_K K_1}$,$\boldsymbol{O_K K_2}$] of circle(O , OK) through the corresponding vertices , $\boldsymbol{O_K}$ and with that ofEqualanglescircle($\boldsymbol{O`_1}$ ,$\boldsymbol{O`_1P_1}$) through the mid - perpendicular $\boldsymbol{OM_1}$ of the first chord $\boldsymbol{K K_1}$,and the mid - perpendicular$\boldsymbol{O_K K_2}$of the second chord$\boldsymbol{KK_2}$.

This co relation of this Formation between these four circles ,

{ (O , OK) - ($\boldsymbol{K_1}$, $\boldsymbol{K_1K`_1}$ ) - ( $\boldsymbol{O`_1}$, $\boldsymbol{O`_1P_1}$) - ($\boldsymbol{O_2}$, $\boldsymbol{O_2P_2}$) }

and Perpendiculartoline (e) ,Allowsto Any circle (O , OK) to define their in between motion through the two chords KK_1 , KK_2 , or and anglesφ_1 , φ_2 ,that is , From the relation of Heights Σ (h) = (2k) . h = n .h = n .OK , becomes that the Summation of heights of any two Adjoined – Even egularPolygons,n , n+2 is → $\frac{\Sigma\ 2(h1)}{2} + \frac{\Sigma\ 2(h2)}{2} = [\frac{n_1}{2} + \frac{n_2}{2}]$.OK = $[\frac{n_1+n_2}{2}]$.OK = n_3 .OK , where $n_3 = [\frac{n_1+n_2}{2}]$ is the number of vertices between the two Evenn_1, n_2,

The Odd – Number- Vertices Regular – Polygon .

On the Hypsometric differenceΔh =$O_1K`_1$and on the perpendicular to line (e) are kept all properties f the addition .From the Instaneous position of anglesφ_1,φ_2 , to the two circles the chords are defined. e...BecausechordsKK_1 , KK_2 , areperpendiculartoOP_1 ,$O_K P_1$,lines , ThereforepointK istheOrthocenter ofallperpendicularand rightangledtriangles, as well as their common chord K_1M_1, of the two circles (O_2 , O_2P_2) , (O , OK) . BecausetheGeometriclocusofchordsKK_1 , KK_2 , of the Common Orthocenter K is→ for circle (O ,OK) the arcK_1K_2 , and for circle (O_2 ,O_2K=O_2P_2)

arc$M_1\ K_2$, andforcircle (O_1 , $O_1P`_1$) arc (1)-(2) with the points of the chords intersection , Thereforepoints(1) , $M_1$are limit points of these circles such that exists K$M_1 \perp P_1M_1$.

Theabovelogicsresult to the , Mechanical and Geometrical solution , which follows . The new Mechanical Approach :

InF. 18 - A. is thecircle (O ,OK) with the tangential line (e) at point K , and the diameter KO_K .Define on the circle from vertices , K , The vertices K_1 , K_2 corresponding to the edges of sides oftwo Adjoined Even - Regular Polygons and the corresponding angles φ_1 , φ_2, between sides $K\ K_1$,$K\ K_2$, and the tangent line (e) .

Draw the parallels from vertices K_1 , K_2 , to (e) line and from vertices K_1 perpendicular to (e) , such that cuts the parallel from pointK_2 , at point $K`_1$, and draw the perpendicular $K_1K`_1$as the radius

the circle (K_1 , $K_1K`_1$).

Draw$O_K K_1$ produced which cuts OK_2extended (from point O)at point $P_2$and from point O_2

(the middle of diameter KP_2) draw the circle (O_2 ,O_2 K = O_2P_2).

Extendsides$O_k K_1$, $O_k K_2$, so that they cut circle (O_1 ,$O_1K`_1$) at points 1 ,1`, and 2 , 2`, and draw

chords 1 - 2` και 2 -1` respectively .

Define the common point , T , of chords 1 - 2` και 2 - 1` andproduce , O_kT, such that cuts circle

(O , OK) at point K_5 . OR, withtheHarmonicMean,

Draw from point $K`_1$ the perpendicular , $K`_1$A =($K`_1K_1$)/2 and the circle ( A , A$K`_1$) cutting the chord O_1Aat point B .

DrawfrompointK_1thecircle (K_1 , K_1B) suchthatintersectstheperpendicular$K_1K`_1$ atpoint , C , and from this point C the parallel to (e) so that cuts circle (O, OK) at point K_5 . ThechordK$\boldsymbol{K_5}$is the side of the Regular - Odd- Polygon , and this because Thecircle (O_4 ,O_4 K = O_4 O) is the circle of the middle of chords KK_1 ,KK_2so and for KK_5.

Angles<$KM_1O_2 = KM_2O`_1$ = 90° , <KM_1P_1=KM_1O = 90° , <$K\ K_2P_1$=KK_2O_κ = 90° ,

ThereforepointKis the Orthocenter of the triangles KOM_2 , KOP_1 , KO_kP_2 , $K\ O_kO_1$.

Angles<K_1KK_2 , $K_1O_kK_2$, OP_1O_k , OP_2O_k , P_2O$P_1$are equal between them ,

Becausetheseareα) Inscribed to the same arc, K_1K_2 , of circle (O, OK) ,

β) TheirsidesP_1M_1 , P_1K_2 , and being perpendicular toKK_1 ,KK_2

are in circle ($O`_1$ , $O`_1$ K = $O`_1\ P_1$) ,

γ) Alternate Interior anglesbetween the parallels ,OP_1, andO_kP_2

of the circles (O_4 , O_4 K = O_4 O) , (O_2 , O_2 K = $O_2\ P_2$) .

ChordsO_kK_1 , OM_1areperpendicularto chord KK_1 ,Therefore are parallels , ChordsO_kK_2 , OM_2areperpendicularto chord KK_2 , Therefore are parallels , TheGeometricallocusofpointK_1 , from Point $\boldsymbol{K_1}$to point$\boldsymbol{K_2}$, andon circle (O , OK)

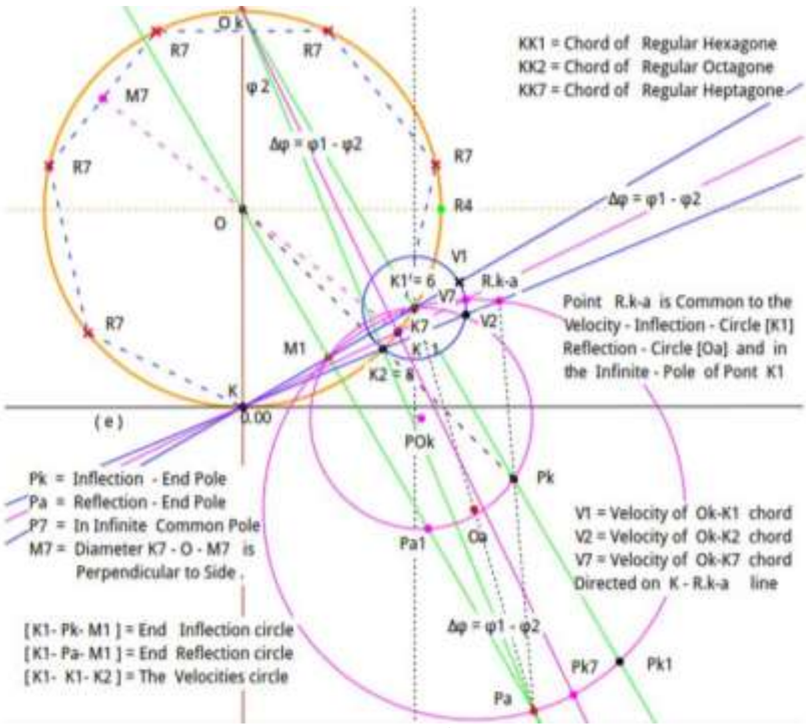
isarcK_1K_2ofthecircle , while on circle (O_1 ,$O_1K`_1$ ) arc 1 , 2` of the circle .TheGeometricallocusofpointK_2 , from Point $\boldsymbol{K_2}$to point$\boldsymbol{K_1}$, andon circle (O , OK)

isarcK_2K_1ofthecircle , while on circle(O_1 ,$O_1K`_1$ ) arc 2 , 1` of the circle .

TheGeometricallocusfrompoint, O,oftheparallels to chord O_kO_1 , are the chords OP_1 , $O_4O`_1$,

andfromPole , O_k , section , T , between chords 1 , 2` and 2 , 1` respectively .
Becauseangle<$O_k O_1 K$ = $O_k K_2 K$ = 90º, Therefore section , T, moves parallel toline O_1K , nditis the common point of the two Geometrical loci .
Becausepoints$\boldsymbol{K_1}$, $\boldsymbol{K_2}$are the two Adjoined - Even Regular Polygonsofcircle (O ,OK) and simultaneouslypoints$\boldsymbol{O_1}$, $\boldsymbol{P_2}$, thecorrespondingextremePolesoncircles(O_1 ,$O_1 K$ `$_1$),(O_2 ,O_2K), following the common joint for point K , to be the Orthocenter andthe Pole of Polygons ,and point, T , the constant and common Pole of the System , Thereforeline$O_k T$, is constant and cutscircle (O ,OK) , atpoint$\boldsymbol{K_5}$ which is the vertices of the intermediate Regular – Odd – Polygon ??
OR , because of the Harmonic relation (1) and (4)as($K_1 K$ `$_1$) ² = ( $K_1 C$). ($K_1 C$+ $K_1 K$ `$_1$) isdefinedtheharmonicheight$\boldsymbol{K_1 C}$and from parallel chord CK_5 ,point $\boldsymbol{K_5}$, on circle (O ,OK) suchthatcorrespondstheabove Harmonic relation , Therefore chord KK_5isalso of the inner and The between Odd –Regular- Polygonq.e.d
Μάρκος , 5/5/2017

ThenewGeometricalApproach :
InF. 18 - A.ofcircle (O ,OK) , sincethe sides$P_1 O_k$, P_1O are perpendicular to K K_2 , K K_1respectivelySoangle<$OP_1 O_k$ = $K_1 K K_2$, and since also $P_2 O$chord is between the parallel lines$P_1 O$, $P_2 O_k$, Thereforeangles<O$P_1 O_k$,O$P_2 O_k$are equal , either on the constant Poles of the vertices O , O_k , or on themovablePolesofverticesP_1 , P_2 . Sinceangles<O$P_1 O_k$, $OP_2 O_k$, are equalSolie on a circle of chord OO_k .Sincealsoexist onthesamecirclethePolesO_k ,O, P_1 ,P_2Thereforelieon a circle of center the intersection of the mid-perpendicular of chords OO_k, OP_2, and is point O_3ThepointKofline (e) iscommontotheinfinite(∞) Regular – Polygons of the circles with centerthepoint ,O, andradiusKO = 0 → ∞ , ThereforetheInfinite Regular Polygon becomes line (e) , the Regular Polygons lie on circle (O , OK) and theZero Regular Polygon is point K .
SincethemovablePoles P_1 , P_2 , ofthetwoAdjoined - EvenRegularPolygonslieoncircle [O_3 , O_3 O]TheAnti-Spacecircle [12] , Sotheinterandmovablepoleof the Odd – Regular – Polygon passes fromtheinfinite, ∞ ,andwhichistheintersectionofline (e) and this circle and it is the common point P_5 . Thesamehappenswithangleof 90 ºwith two lines passing from infinite .
ChordOP_5 correspondstotheReflection chords of the Reflection-circle[O_2 , $O_2 P_2$]with center in infinite and which is in point P $_5$ .Thetwointersectingpairs P_4 , P`$_4$and$P_6$ , $P$`$_6$, converge to theone pairsuchthatP_5 = P `$_5$,where the two points coincide . q.e.d.
Remarks :
InF. 18 – B , chords$O_k K_1$, $O_k K_2$, are perpendicular toKK_1 , KK_2, therefore angle <$K_1 O_k K_2$ =$K_1 K$ K_2 .Chord$O_k K_1$isparalleltoOM_1 , O$P_a$andsincechord$P_a O_k$ is between the two parallels thentheAlternate Interior angles<O$P_a O_k$,$P_a O_k K_1$are equal . In order that point P_k reaches to P_a ,which means from Inflection - Envelope to the Reflection - Envelope , line $O_k P_k$must move from point K_1 to point M_1 perpendicularly .This motion presupposes that the point K_1islying on Inflection circle which happens because the perpendicular velocities of $O_k K_1$ chord are always directed onKK_1chord .i.e. the Velocity - circle [$\boldsymbol{K_1}$,$\boldsymbol{K_1 K_2}$]is an Inflection circle .
Since theEnd –Inflection –Circle passes through$\boldsymbol{K_1}$, $\boldsymbol{P_k}$points , and theEnd –Reflection –Circlepasses through$\boldsymbol{K_1}$, $\boldsymbol{P_a}$points, with point $\boldsymbol{K_1}$ always common , then Passes also through the outerCommon -Inflection - Reflection –Point which lies on the Velocity –circle,where for point $\boldsymbol{K_1}$ the Pole of Rotation is in infinite and the Alternate Interior anglesreversible.
BecausetheDiametersthroughthe vertices K_1 , K_2 pass through the corresponding , n , and , n+2 , Odd – Regular – Polygons , the Diameter through the vertices $K_{7\,=\,n+1}$passes through the center of the Opposite Side , Therefore it is Mid-perpendicular between the Inflation and to the Reflation point .
The Exact Geometrical Solution of the Odd – Regular – Polygons follows :



The Geometrical Construction Of The Regular Heptagon
F.20 - A → In circle(O , OK) For n = 6, then K K_1 is the Side of the Even –Regular - Hexagon
while for n = 8 , then KK_2 is the Side of the Even - Regular –Octagon .
$\boldsymbol{K}$ $\boldsymbol{K_1}$is the Side of the Odd - Regular – Hexagon ,
$\boldsymbol{K}$ $\boldsymbol{K}$ $_{\boldsymbol{2}}$is the Side of the Odd - Regular – Octagon ,

ExistsCircle of Heights Δh = $h_{K\,1}$- $h_{K\,2}$= $K_1K`_1$ and Velocity Inflection circle ΔV=K_1K_2 Straight - Line {O_k ,K_1 , P_k } is parallel to {O , M_1 ,P_a } and the Alternate Interior angles equal ,
< O P_aO_k =$P_kO_kP_a$ = K_1KK_2 . The same for angle < O O_kP_k = P_kOP_aThe Inflection Circle [PO_k ,PO_k- K_1] or the Reflection circle [O_a ,O_a- K_1] cut the Inflection
Velocity - Circle [K_1,ΔV =K_1K_2]at Edge point ,$R_{\cdot k-a}$.
Line K $R_{\cdot k-a}$ intersects the circle (O ,OK) at point K_7which is the vertices of the n+1 =7 Regular Odd Polygon , and which is the Regular –Heptagon .
$\boldsymbol{K\,K_7}$is the Side of the Odd - Regular - Heptagon ,

The Geometrical Proof :

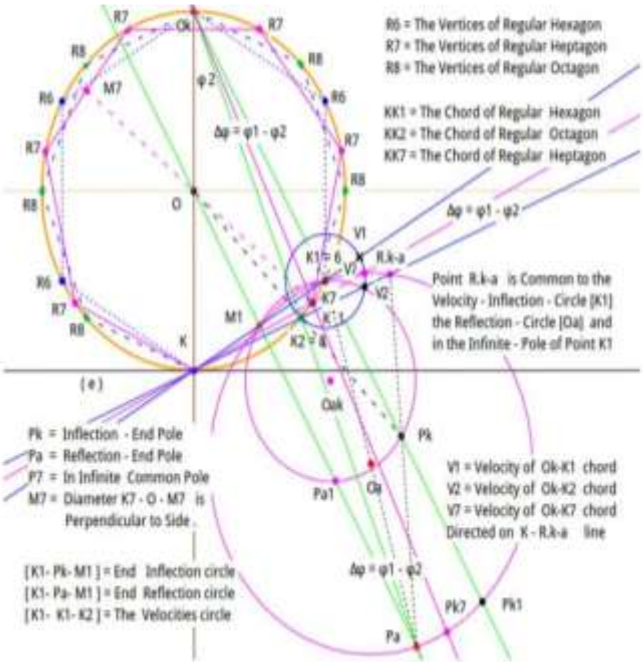
In circle(O ,OK) of F.20-A(B) , the points K_1, $K_2$are the Vertices and$K\,K_1$, $K\,K_2$ are the Sidesof two Adjoined - Even Regular Polygons . Chords O_kK_1 , O_kK_2 are perpendicular to the sides $K\,K_1$, $K\,K_2$ because lie on diameter $K\,O_k$. The mid-perpendicular OM_1 ofKK_1 side , is parallel to O_kK_1 chord because both are perpendicular to $K\,K_1$ side . Line OK_2 produced intersects O_kK_1 line at point $P_k$and since Segment OP_klies between the two parallels , theAlternate - Interior angles <OP_kO_k ,P_kOP_a are equal .
Line O_kK_2 produced intersects OM_1 line at point $P_a$and since Segment O_kP_a lies between the two parallels then the ,Alternate Interior angles<OP_aO_k ,$P_aO_kP_k$ are equal , and since angle <$K_1O_kK_2$ = $K_1K\,K_2$, then also angle< OP_aO_k =$P_aO_kP_k$ = $K_1K\,K_2$.
Segments O_kP_k, $OP_a$are parallel therefore , Quadrilateral $\boldsymbol{OO_kP_kP_a}$is Trapezium of height K_1M_1.
Since the right angle triangles , $P_kK_1M_1$, $P_aM_1K_1$ occupy the common segment K_1M_1=M_1K_1therefore are Inverted(either Inflection or Reflection) Triangles and their Hypotenuses P_aK_1 , P_kM_1 , formulate theReflection [$P_aM_1K_1$] and theInflection [$P_kK_1M_1$] Circles on K_1M_1= M_1K_1common segment .
[This terminology of , Inflection and Reflection circle, becomes from Mechanics] . q.e.dRemark : Trapezium$OP_aP_kO_k$ is a Geometrical mechanism with its Alternate Interior angles equal tothe angle<$K_1K\,K_2$of Sides . When triangle OO_kK_1changes from K_1 to K_2position then ,
the right angled triangles $K\,K_1O_k$,$K\,K_2O_k$ are directed on $K\,K_1$,$K\,K_2$, lines and in the (K_1 ,K_1K_2) circle as K_1V_1 , K_2V_2 , segments , because these lie on perpendicular Segments , while the Inverted (Backing Formation) circles [O_a, O_aK_1= O_aP_a] , [O_{ak} ,$O_{ak}M_1$= $O_{ak}P_k$] are constant for every combination .
The End –Inflection circle is of Diameter $M_1P_k$and is Inverted to (K_1 ,K_1K_2) circle . The End –Reflection circle is of Diameter$K_1P_\infty$and is Inverted to (K_1 ,K_1K_2) circle since the Infinite circles passing Tangentially from K_1 and K_1V_1 .
Inversion of circles happens in infinite through the Trapezium , in where ,
a.. Triangles O_kP_kO, $O_kP_kP_a$ are of equal area , because lie on the common Segment O_kP_k , and the commonheightK_1M_1 . Since triangle $O_kP_kK_2$ is common to both triangles therefore the remaining triangles K_2O_kO , $K_2P_aP_k$ are of equal area , andpoint$\boldsymbol{K_2}$is a constant point to this mechanism .
Since also trianglesK_2O_kO , $K_2P_aP_k$ lie on opposites of line $O_kK_2P_a$ position then are Inverted on this line . (the Alternate Inverted triangles)
The Inversion of the circles happens because Diameter K_7OM_7 is the Mid - perpendicular to the opposite Side of the Odd in the middle point M_7 in contradiction to DiameterK_2OM_2≡$OK_2 \rightarrow P_k$ which passes through the vertices of the Even-Regular-Polygon forming angle <K_1OK_2= 2.K_1KK_2
b.. Because at point $K_1$of chord $O_kK_1 \perp KK_1$, infinite points $P_k$exist on $O_kK_1$for all points K_2≡ K_1 and circle of radius K_1K_2 = 0 , Therefore separately must issue and for chord O_kK_2 . But since is $K_1K_2 \neq 0$ then Chords KK_1 ,KK_7 ,$KK_2$are all projected on the ($\boldsymbol{K_1}$, $\boldsymbol{K_1K_2}$)circle , and Diameter $P_kM_1$is Inverted to Diameter P_aK_1 with their circles .The edges of Segments K_1V_1 , K_2V_2 , are on KK_1 , KK_2 lines , so all triangles of Parallel sides of Trapezium , occupy the point K , as the same Orthocenter for all the Regularly-Revolving triangles KO_kP_k , $KO_kK_{\infty \to 7}$, KO_kP_a ,with the Sides $O_kP_k \rightarrow O_kP_7 \rightarrow O_kP_a$, and the Inverted Circles [O_a ,O_aK_1= O_aP_a] ,[O_{ak} ,$O_{ak}M_1$= $O_{ak}P_k$] . c.. That Inverted circle [O_a , O_aK$_1$= O_aP_a] , [O_{ak} ,$O_{ak}M_1$= $O_{ak}P_k$] with the greater diameter intersecting the circle (K_1 ,K_1K_2) between the pointsV_1 ,V_2 defines the Inverted Position , i.e. that of the Odd-Regular -Polygon .
In case that Inverted circles intersect axisO_kOK, Then The - Inverted - Position is the Common - Point of the circle (K_1 , K_1K_2)and the circle of diameter K_1P_k , and this is because , the Tangential Inflection circle becomes the End Inflection circle on K_1P_k .

In all cases Trapezium [$\boldsymbol{OO_kP_kP_a}$] is the New Regular Polygons Mechanism and exhibits
The How(By Scanning Chord$K\,K_1$ to $K\,K_2$) and Where (In the Inverted triangles OO_kK_2 ,$K_2P_kP_a$)
Work(Energy → Kinetic or Dynamic) produced from any Removal , is Stored .
A wide analysis for the Energy - Storages in [64] .

In F.20-A , For n = 6, then $K\,K_1$ is the Side of the Even - Regular – Hexagon
For n = 8 , then $K\,K_2$is the Side of the Even - Regular – Octagon .
For n = 7 , then $K\,K_7$is the Side of the Even - Regular – Heptagon . q.e.d

THE REGULAR - POLYGONS

In F.19– (Page 69) , Is shown the Geometrical construction of the Regular –Triangle ,
Through the Regular → Digone and Tetragon .
In F.18-B – (Page 67) , Is shown the Geometrical construction of the Regular – Pentagon ,
Through the Regular → Tetragon and Hexagon.
In F.20 – (Page 70) , Is shown the Geometrical construction of the Regular – Heptagon ,
Through the Regular → Hexagon and Octagon.
In F.21 – (Page 71) , Is shown the Geometrical construction of the Regular – Ninegone ,
Through the Regular → Octagon and Decagon.
In F.22 – (Page 72) , Is shown the Geometrical construction of the Regular – Endekagone ,
Through the Regular → Decagon and Dodecagon .



In F.23 – (Page 73) , Is shown the Geometrical construction of the Regular – Dekatriagone ,

Through the Regular → Dodecagon and Dekatriagone .
F.20 - B → In circle(O ,OK)=(O ,OO_k)and[O_a, O_aK_1= O_aP_a] , [PO_k, PO_kM_1= PO_kP_k] , (K_1 ,K_1K_2)
For n = 6 , then $\boldsymbol{K\,K_1}$ is the Side of the Odd - Regular – Hexagon ,
For n = 8 , then $\boldsymbol{K\,K_2}$ is the Side of the Odd - Regular – Octagon ,
For n = 7 , then $\boldsymbol{K\,K_7}$is the Side of the Even - Regular – Heptagon . 5 / 8 / 2017
The Physical notion of the Regular and Not - Polygons:
Segment $\boldsymbol{M_1K_1}$or chord $\boldsymbol{KK_1}$is the locus of the infinite circles on $\boldsymbol{OM_1}$,$\boldsymbol{O_kK_1}$parallels of Trapezium [$\boldsymbol{OO_kP_kP_a}$] which intersect ($\boldsymbol{K_1}$, $\boldsymbol{K_1K_2}$) circle .Chord $\boldsymbol{KK_1}$revolving (Scanning) through point K , to $\boldsymbol{KK_7}$ and to $\boldsymbol{KK_2}$produces , Work , when the Trapezium System passes through infinite.
Since triangles $\boldsymbol{KK_1O_k}$, $\boldsymbol{KK_2O_k}$ are rightangle triangles , then $\boldsymbol{KK_1 \perp O_kK_1}$,$\boldsymbol{KK_2 \perp O_kK_2}$, and for anyremoval of point $\boldsymbol{K_1}$to $\boldsymbol{K_2}$the Work produced is zero .
In all Odd and Even - Regular -Polygons , AND in Any – Non- Regular –Shape , The Area of the Space triangle , $\boldsymbol{K_2O_k}$O , is equal to the Area of the Anti – Space triangle$\boldsymbol{K_2P_kP_a}$.
Generally by Scanning Any Space-Monad $\boldsymbol{KK_1}$to a Space –Monad $\boldsymbol{KK_2}$ of the circle , the Work produced is conserved in the first Space - triangle of the circle , and in theOutside of the Equal area triangle .The area of the first triangle denotesthe, Work Produced[i.e.Energy as Electricity , as Vibrationas Frequency ,as Thermal , as Movement , as anyother Alteratione.t.c] ,while the area of the second triangle denotes the , Work Quantizedinthe Plane – Stores of Anti-Space . [61C]

Epiloque:

In Material Geometry [58-61] ,Zero - point 0 = ∅ ={⊕+⊖}= The Material-point = The Quantum = The Positive Space and the Negative Anti-Space , between Opposites =The equilibrium of opposite →←
Point O , is nothing and maybe anywhere .
Point K , is nothing and maybe anywhere .
Segment$\overline{OK}$, is the Monad OK, ⊕, and maybe on circle [O ,OK] where OK is , the ⊕Space .
Point O_k,is nothing and this is in Opposite Position of point O such that Segment$\overline{OO_k}$≡The Quantum ≡ Anti-Monad ($\boldsymbol{OO_k}$) = - (OK) = ⊖, andOpposite direction (OO_k) → = - (OK) ← is , the Anti-Space .
Any Point K_1 , is nothing also and maybe on circle [O , OK] .
Segment $\overline{KK_1}$ is the monad KK_1 and it is the chord on circle [O, OA] , where $KK_1$is the ⊕Space.
Segment $\overline{O_kK_1}$ is the monad O_kK_1 = is the ⊖Spaceand it is the perpendicular chord on circle [O,OA] , where ,since O_kK_1 is perpendicular to KK_1then No-Work is produced ,therefore the velocities of chords are also perpendicular . Here Velocity is the change of direction of the Space KK_1 and always on K_1O_k.
Any Point K_2 , is nothing and maybe on circle [O , OK] also , and which occupies all above .
Angle <$K_1K\ K_2$is theInbetween-Spaceof chords KK_1 ,KK_2 on triangle $K_1K\ K_2$,the Space triangle , which locus is the constant circle (O , OK) and Triangle $K_1O_kK_2$is , the Anti-Space triangle .
Chord K_1K_2 remains constant during the Removal of point K , the ⊕Space , in order to reach pointO_k the Anti-Space⊖, and this because arc $\widehat{K_1K_2}$ of the circle is constant . Since K_1K_2 Segment is constant therefore point K_2 lies on (K_1 , K_1K_2) circle which we call , Velocity circle .
Conclusion 1 :
On monad [OK] , The Quantum , exists the equilibrium and the oppositeAnti-monad $[OO_k]$ = - [OK] and from points K ,$O_k$are formed Infinite monads either as couple of chords $K\ K_1$,O_kK_1- $K\ K_2$,O_kK_2 , or as the angles <$K_1K\ K_2$, $K_1O_kK_2$which have common their velocity circle (K_1,K_1K_2) .On this velocity circle any motion of Space$K\ K_1$,$K\ K_2$lies on Anti-spaceO_kK_1 , O_kK_2 and the opposite .
This is the equilibrium of , ⊕, Space$K\ K_1$and ,⊖, Anti-space $O_kK_1$in Material Geometry .
It was shown [12]that SpaceK_1O≡ ⊕ is in equilibrium with the Anti-space K_1O_k≡ ⊖ through the area of triangle K_1O_kO , and it is the Work embedded in point K_1 of Space .
The case of the Space K_2O is the same as in K_1Oinfront.
In case of simultaneous SpacesK_1O≡ ⊕≡ K_2Othen lineOK_2produced , intersects O_kK_1≡ ⊖at point $P_k$which is called the Inflection Pole, and this because point $K_2$is Inflected on circle (O , OK) .
Line O_kK_2 produced , intersectsOM_1 line produced ,the parallel to O_kK_1 passes through the center $M_1$of the chord KK_1,at the point P_a, which is called the Reflection Pole , and this because point M_1 is Reflected on triangle K_1OK .
Since lines OP_a , O_kP_k are parallels , and this because are both perpendicular to $K\ K_1$chord , then quadrilateral $OP_aP_kO_k$ is Trapezium , and since Segments OP_k , O_kP_a are between the parallels then , theAlternate Interion angles<OP_aO_k , $P_aO_kP_k$are equal , and both equal to angle <$K_1K\ K_2$ and this because angle <$P_aO_kP_k$≡ $K_2O_kK_1$≡ $K_1K\ K_2$.
The same also for theAlternate Interior angles<$OP_kO_k = P_kOP_a$.

Since triangles P_kO_kO , $P_kO_kP_a$, occupy the common segment P_kO_k and common height K_1M_1 , so are equal , and therefore the Area of trianglesP_kO_kO ,$P_kO_kP_a$equal, and since also triangle $P_kO_kK_2$ is common to them , then the Remaining triangles K_2O_kO , $K_2P_kP_a$ are also equal .
Since the Area [S] of triangle K_2O_kO represents the Work embedded in Point K_2 therefore the Work is conserved in triangle $K_2P_kP_a$ of this trapezium .
It was found that when λ_a= the length of the side of the Regular Polygon and R = OK is the radius of the circle then , the Area S = $\frac{\lambda_a}{4}.\sqrt{4R^2-\lambda^2{}_a}$and Polygon`s Length $\lambda_a=\sqrt{2.R^2\pm\sqrt{R^4-4S^2}}$
A wide analysis for the nature of Polygon`s length $\lambda_a$in [63] .
Conclusion 2 :
Any relative motion of ,Space ≡ ⊕ monad $KK_1$to KK_2 ,it is an alterating Chord - Scanning , and is defined in the Outer Space K_2P_aas the Area of triangle$K_2P_aP_k$, and it is the conserved Work , and equal to K_2O_kO Area , it is the Work .i.e.
The Work produced in any Removal of Space is conserved in the Plane triangle of Anti-Space . This is the Conservationof Work , in Material Geometry , for monads either as Segments or Anglesthrough the Area of the Space triangle ,K_2O_kO , to the Area of the Anti - Space triangle , $K_2P_kP_a$.
The circles of diameters K_1P_a , M_1P_k , are called the , Reflection and the Inflection circle alternatelybecause these lie on common height of Trapezium , the Segment K_1M_1 , and are reflected at point K_1which pass from the removable P_a , P_k, Poles of this Quadrilateral .
On the Anti - space chordK_1O_k, Infinite Inflection circles exist on the diameters K_1P_k , for point $P_k\ \equiv\ K_1\to\infty$ and for $P_k\ \equiv\infty$ then all parallels to K_1O_k , lie on the Space - Chord K_1K_∞ with the Infinite Inflection circles passing from $K_1\equiv V_1$ point . The same also for Anti-Space Chord K_2O_k where velocity at $K_2\equiv V_2$ point .
Since circle (K_1 , K_1K_2) lies on K_1O_kline with center at point K_1 , then is the End-Inflection -circle, and since also K_1P_a diameter is equal to zero , then is also , and the End Reflection circle .
For both Anti - Space - Chords K_1O_k , K_2O_k corresponds the Intermediate - Space - Chord $O_kK_7$onKV_7 line withthe Reflection - Circle of diameter K_1P_a passing from V_7common point , and to the End Inflection Velocity circle (K_1 , K_1K_2) .
Conclusion 3 :
On any Anti - Space - Chord K_1O_k and the corresponding Space - Chord K_1K , the Work done from any Removal is equal to the Area of triangle K_1O_k Oand is spread on line $K_1P_k\to\infty$, and in case of a ,simultaneously , second Anti-Space –Chord K_2O_k , then Work is gothered to$K_2P_aP_k$triangle .
The Reflection circle of diameter K_1P_a intersects the End-Inflection-Velocity Circle of diameter M_1P_k at a point $R_{\cdot k-a}$, between the two points V_1, V_2such that lineKV_7intersecting the circle (O,OA) at point K_7 , and the Work produced is equal to the Area of triangle K_7O_kO , which is conserved .

The above Geometrical Mechanism Constructs , Points K_7 , Chords $K\,K_7$, Triangles K_7O_kO inwhere Work for any Removal is conserved . Since the Area of the triangles can be transformed to Equal Area of any other Shape then thisShapeconsists the Conservation-Work-Stores in Material –Geometry .
In case that Points K_1 , K_2, consist the Vertices of any Two Sequent - Even - Polygons, then $K_7$is the Vertices of The Inbetween - Odd - Regular - Polygon with the Produced and Conserved Work the Area of the TriangleK_7O_kO .
This is the the Quantization of Work in Monads , Either-as , Odd - Regular - Polygons and their Interior Angle , OR – as , ofAny - Shape - Area equal to the Space triangle ,K_2O_kO , and equalalso to the Area of the Anti - Space triangle ,$K_2P_kP_a$.
By Scanning The Space-Monad $K\,K_1$to Space –Monad $K\,K_2$ of the circle , The Work produced is conserved in $[OO_kK_2]$ Space - triangle , and in the equal area triangle $K_2P_kP_a$of theAnti–Space .
The above relation of Work, Quantization in Geometry– Shapes, in Area – Stores of Anti-Space ,
is the Unification of Geometry-monads with the Energy monads (The How in [61] , ≡ where→
The How Energy from Chaos Becomes Discrete Monads).
Conclusion 3 : The Physical meaning .

In article was shown the Geometrical construction of all the - Regular - Polygons in a circle and for Odd , between any two sequent Even Polygons . Any two Chords $K\,K_1$, K_1O_kat the Ends of a diameter are perpendicular each other , and consist the Space and Anti-Space monads respectively and since are Perpendicular each other , these do not produce Work (Stored Work = Area of triangle $O\,K_1O_k$) .
In case of a Removal of any two chords the Work Produced between them is equal to the Central triangle Surface which consists the Quantization of Work in Monads .For , Odd - Regular - Polygons and their Angle , OR - for , Any - Shape of Area equal to the Space triangle ,K_2O_kO , Work Quantization[Energy as Electricity ,as Vibration ,as Frequency ,as Thermal , as Movement , as any Alteration e.t.c] , is equal also to the Area of the Anti - Space triangle $K_2P_kP_a$. It was also proved that , By Scanning Any Space-Monad $K\,K_1$to a Space – Monad $K\,K_2$ of the circle ,
the Work produced is conserved in a Space - triangle in the circle ,The Store , and in one of the equal area triangle outside thecircle , which is the Anti-Space triangle , meaning that ,
The above relation of this Plane Work ,it isThe Quantization ofGeometry –Shapesinto thePlane – Stores of Anti-Space, consists the Unification of the Geometry – monadswith those ofEnergy monads , and which were analyzed and have been fully described .markos 20/8/2017

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