International Journal Of Engineering Research And Development
e- ISSN: 2278-067X, p-ISSN: 2278-800X, www.ijerd.com
Volume 14, Issue 10 (October 2018), PP.28-38

Modelling of Volumes and Kots of Keban, Sürgüand Sultan Suyu Dams with Copula Method

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ABSTRACT: *The application of copula in hydrology has been rapidly increasing recently. Aim of this article is that to determine the marginal distribution of water content of the dams in the close coordinates (figure 1) and obtain the relationship between them. For this, we select suitable copula function for data set which is daily volume and kot of Keban, Sürgü and Sultan Suyu dams between 2012-2017 years. For this, we employ a twostep copula model to examine the dependence structure of daily data set. Firstly, we construct the marginal distributions using this data set. To explain dependence structures of the data set, we calculate Kendall Tau and Spearman Rho values which are nonparametric. Based on this method, parameters of copula are obtained. With the help of nonparametric estimation of copula parameters, Kolmogorov Smirnov test and Cramer Von Mises which are goodness of fit test, Maximum likelihood method, Akaike information Criteria and Bayes information criteria, we find the optimal copula family for dependence structure of this data set.*
KEYWORDS: *Dams Copula functions· Akaike information criteria · Schwartz information criteria · Maximum likelihood method· Marginal Modelling.*

Date of Submission: 16-01-2019 Date of acceptance: 28-01-2019

I. INTRODUCTION

Today, increasing population, industrialization and urbanization are increasing the need for water from day to day. Water is becoming a strategic resource as it is added to factors such as pollution and global climate change. Therefore, countries are in an effort to develop the right policies and strategies to avoid future water problems. Dams that freshwater resources are a strategic importance in this context. Dams are especially designed to answer needs such as energy production, irrigation, industry. The dams are constructed for the following purposes; Cities need drinking and running water, industrial water supply, irrigation water, hydroelectric power generation, production of aquaculture products. By the time, increase of population and developing industry had boom more water requirement on the Anatolia and the world. Dams are obtaining great benefits for irrigation, flood control and domestic and industrial use of water. Until today, Markov model are used mostly in hydrology. The copula theory is relatively new to hydrology and hydro climatology but has already established itself to be highly potential in frequency analysis, multivariate modeling, simulation and prediction.Development of joint distribution between multiple variables is the key to analyze utilizing the potential of copulas. The chapter starts with the mathematical theory of copulas and gradually move on to the application.Copula approach is flexible than the one based on parametric bivariate distribution function because it follows the separate modelling of the marginal behavior. Copulas contemporaneously give information on average dependence and upper- and lower tail dependence.[1] have been thoroughly reviewed in copulas. Since then the application on copula theory in finance and economics has grown tremendously. Moreover, practical applications of this modeling approach are found in fields such as hydrology. [2] indicated that using copulas might properly simplify the calculations and might even yield analytical statements for the isolines of the return periods, both in the unconditional and in the conditional case and showed how a new probability distribution might be united with the return period of specific incidents. [3] submitted the modeling of multivariate extreme values using copulas and performed he methodology on two different problems in hydrology; the first application related with the combined risk in the structure of frequency analysis, the second application relates to the joint modeling of peak flows and volumes.[4] presented a Gaussian and a non-Gaussian which two theoretical copula-based models and used Bootstrap-based statistical tests using stochastic simulation of the multivariate distributions to investigate the appropriateness of the models. [5] submitted an introduction to inference for copula models, based on rank methods and used realistic application of the methodology to hydrological data.[6] disputed the bivariate modeling of extreme tails of correlated hydrological random variables, used a copula approach and model the dependence structure independently of the marginal distributions and performed results from the classical extreme value theory to select marginal distributions for extremes of high thresholds, considered six copula families to capture the dependence structure of these extremes.[7]studiedat deriving trivariate rainfall frequency distributions using the Gumbel–Hougaard copula which does not assume the rainfall variables to be independent or normal or have the same type of marginal distributions.[8] investigated a non-Archimedean copula from the Plackett family that is based on the theory of constant cross-product ratio, indicated that the Plackett family not only apply well at the bivariate level, but also allows a trivariate stochastic analysis where the lower-level dependencies between variables could be fully preserved while allowing for specificity at the trivariate level as well, proposed that while the constant cross-product ratio theory was conventionally applied to discrete type random variables and said suitableto continuous random variables.[9] used toa bivariate copula model to obtain the bivariate joint distribution of flood peak and volume, in order to know the probability of occurrence of a given inflow hydrograph. [10] investigated 58 flood events at the Litija gauging station on the Sava River in Slovenia, selected based on annual maximum discharge values, considered suitable hydrograph volumes and durations andperformed different bivariate copulas from three families and compared using different statistical, graphical and upper tail dependence tests.In this study, we evaluated dependence these hydrology variables revealed that volume and kot of Sürgü- Sultan Suyudam's pairs have strong significant dependence and volumes and kots of Keban-Sürgü and Keban-SulanSuyudam's pairs have weak significant dependence. Hence these pairs were used for modelling dependence by employing five types of Copulas; Clayton, Frank, Gumbel, Joe and Gaussian copula. For the marginal modelling, the results of probability distributions fitting to these hydrology variables indicated that the volume and kot time series Weibull, Logistic and Gamma distribution.

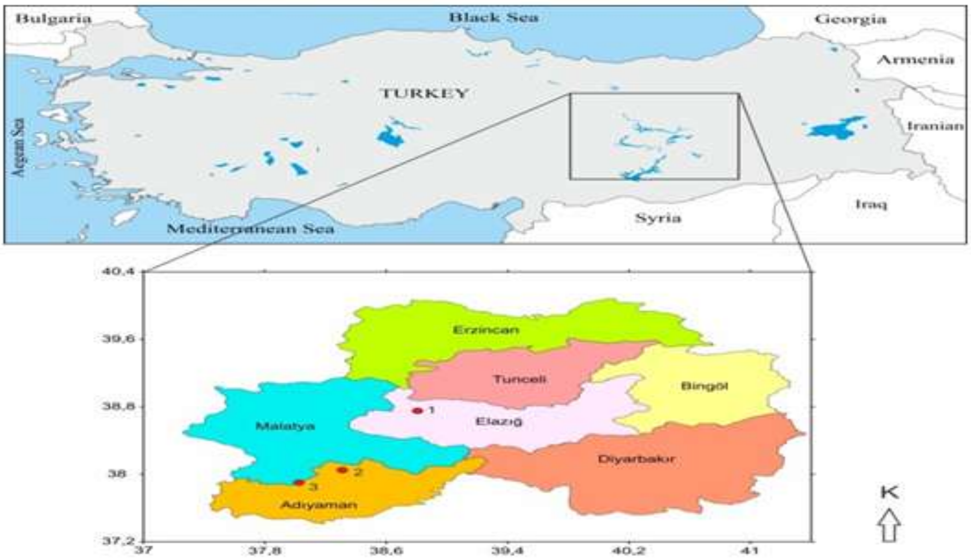

Figure1. Location of Keban, Sürgü and Sultan Suyu dams

II. METERİAL AND METHOD

2.1. Copula Functions

The copula function is proposed to measure dependence of multivariate variables. Based on the famous Sklar's theorem (Sklar 1959), copulas allow to put in place the fruitful idea of splitting the specification of a multivariate model into two parts: the marginal distributions on one side, the dependence structure (copula) on the other part. Let X and Y be random variables with continuous distribution functions F_X and F_Y, which are uniformly distributed on the interval $[0,1]$. Then, there is a copula such that for all $x, y \in R$,

$$F_{XY}\left(X,Y\right) = C(F_X\left(X\right), F_Y\left(Y\right). \qquad (1)$$

The copula C for $\left(X,Y\right)$ is the joint distribution function for the pair $F_X\left(X\right)$, $F_Y\left(Y\right)$ provided F_X and F_Y continuous.

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The joint probability density of the variables X and Y is obtained from the copula density $(u,v) = \dfrac{\partial^2 C(u,v)}{\partial u \partial v}$, as follows:

$$f_{xy}(x,y) = c(u,v)\, f_x(\mathrm{x})\, f_y(y), \qquad (2)$$

where $f_x(\mathrm{x})$ and $f_y(y)$ are the marginal densities of the random variables X and Y. According to Sklar (1959) an n-dimensional joint distribution can be decomposed into its n-univariate marginal distributions and an n-dimensional copula. In the extension of Sklar's theorem to continuous conditional distributions, Patton (2006) shows that the lower (left) and upper (right) tail dependence of two random variables is given for the copula as:

$$\lambda_l = \lim_{u\to 0} P(\mathrm{F}_X(x) \le u \mid F_Y(x) \le u) = \lim_{u\to 0} C(u,u)/u \ (3)$$

$$\lambda_u = \lim_{u\to 1} P(\mathrm{F}_X(x) > \mathrm{u} \mid F_Y(x) > u) = \lim_{u\to 1} 1 - 2u - C(u,u)/1 - u \ (4)$$

where λ_l and $\lambda_u \in [0,1]$.

2.2. Copula Models

We introduce several copula models in this section; Gumbel copula, Clayton copula, Frank copula Gaussian copula, Student t copula and Joe copula.

Gumbel Copula: This Archimedean copula is definedwith the help of generator function $\phi(t) = (-lnt)^{\theta}$, $\theta \ge 1$;

$$C_\theta(u,v) = \exp\left(-[(-\ln u)^{\theta} + (-\ln v)^{\theta}]^{1/\theta}\right) \ (5)$$

where Θ is the copula parameter restricted to $[1,\infty)$. This copula is asymmetric, with more weight in the right tail. Beside this, it is extreme value copula.

Clayton Copula: This Archimedean copula is defined with the help of generator function $\phi(t) = \dfrac{t^{-\theta} - 1}{\theta}$,

$$C_\theta(u,v) = (u^{-\theta} + v^{-\theta} - 1). \qquad (6)$$

where θ is the copula parameter restricted to $(0,\infty)$. This copula is also asymmetric, but with more weight in the left tail .

Frank Copula: This Archimedean copula is defined with the help of generator function; $\phi(t) = -\ln \dfrac{-e^{-\theta t} - 1}{e^{-\theta} - 1}$;

$$C_\theta(u,v) = -\frac{1}{\theta}\ln\left(1 + \frac{\left(e^{-\theta u} - 1\right)\left(e^{-\theta v} - 1\right)}{\left(e^{-\theta} - 1\right)}\right) \ (7)$$

where θ is the copula parameter restricted to $(-\infty,\infty)$.

Joe Copula: This Archimedean copula is defined with the help of generator function; $\phi(t) = -\ln[1-(1-t)^{\theta}]$

$$C_\theta(u,v) = 1 - [(1-u)^{\theta} + (1-v)^{\theta} - (1-u)^{\theta}(1-v)^{\theta}]^{1/\theta} \ (8)$$

where θ is the copula parameter restricted to $[1,\infty)$.

Gaussian copula: The copula function can be written as;

$$C(u,v;\rho) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(\frac{2\rho rs - r^2 - s^2}{2(1-\rho^2)}\right) drds \ (9)$$

where $u = F_{Y_1}(\mathrm{y}_1)$, $v = F_{Y_2}(\mathrm{y}_2)$ is the inverse of the standard normal distribution and ρ is the general correlation coefficient.

III. DATA SET

Keban, Sürgü and Sultan suyudams data was obtained from Elazığ dams directorateas daily prices between 01.01.2012 – 01.012017. There are 1828 observations in total. Graphical representations of the data employed are shown in figures 2.Table 1 summarizes statistics of series. In table 1,mean values of the data are different from each other and the corresponding standard deviations are fairly different. Skewness of the sultan suyudams volume, sürgü dams volume, sultan suyu dam kot and Sürgü dams kot is negative; Keban dam vol and kot series is positive. This indicate sultan suyudams volume, sürgü dams volume, sultan suyu dam kot and Sürgü dams kot that are skewed left beside this Keban dam vol and kot series are skewed right. The high kurtosis of the Keban dam kot and Sürgü dam Kot reveals that extreme value changes often occur when the tail of series distributions shows fatness. The Jarque-Bera (JB) test shows that the normality of each return series distribution is strongly rejected at 0.05 level, which means all the six index distributions are non-normal.

Table 1. Summary Statistcs

	Kebanvol	Sultan suyuvol	Sürgüvol	Kebankot	Sultan suyu kot	Sürgükot
Mean	23489,68	37112,91	42640,23	835,136	896,6379	1302,731
Median	23468,00	38172,00	43408,00	835,130	897,4250	1304,225
Maximum	33353,00	53995,00	70135,00	938,920	903,0300	1309,620
Minumum	18802,00	16817,00	8088,000	742,690	879,9000	1209,510
Std.Dev.	2660,887	11385,18	19394,63	5,72127	4,921772	6,258238
Skewness	0,165497	-0,18412	-0,104979	0,94109	-0,39988	-2,308559
Kurtosis	2,127441	1,822521	1,635692	97,5944	1,983450	28,69835
Jarque-Bera	66,33482	115,9302	145,1295	681817,3	127,4156	51924,63
Probability	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000

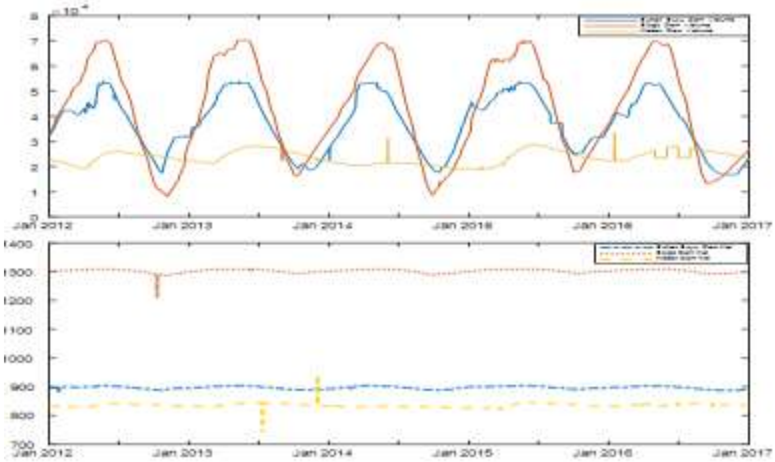


Figure 2. Volume and kot of Sultan suyu, Keban and Sürgübaragecahnge over years respectively

IV. RESULTS

4.1. Fitting marginal distrubutions to volumes and kots of Keban Sultan Suyu and Sürgü Dams

Before evaluating the dependence, marginal distributions are fitted to each of the variables. For volumes and kots of Keban Sultan Suyu and Sürgü dams, we use the most popular distributions namely Logistic, Weibull, Gamma and Exponential. The probability density distribution and parameter estimates are shown in Table 2 and Table 3. In all cases, the estimates are obtained by using the method of maximum likelihood. We select the best distribution based on Akaike information criteria, Bayes information Criteria and graphical indicator. From Table 2, volumes and kots of Sultan suyu and Sürgü dams time series are the best Weibull distribution and volume and kots of Keban dam time series are the best Gamma and Logistic distribution, respectively.Graphical representations of the data employed are shown in figures 3.

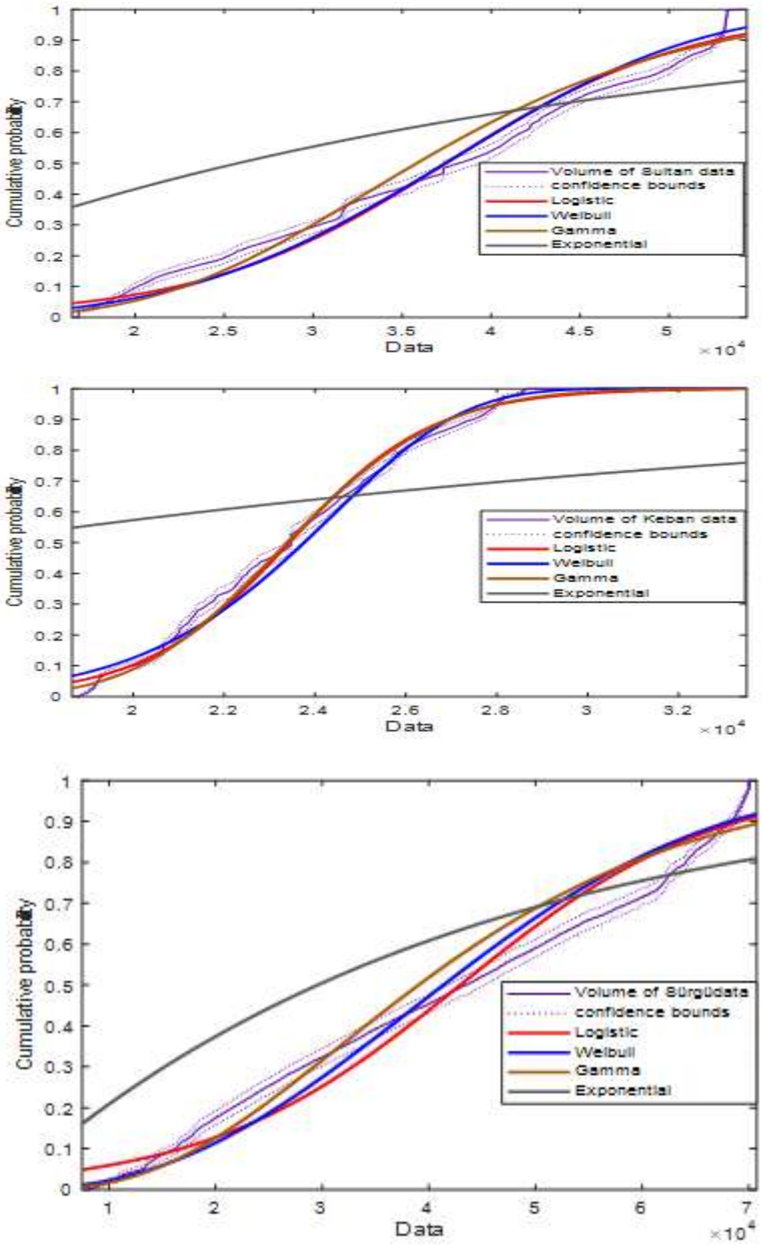
Table 2. Performance evaluation of different probability distrubutions fitted to volumes and kots of Keban Sultan Suyu and Sürgü dams

	Logistic	Weilbull	Gamma	Exponential
Keban vol logl	-4,01202	-4,00774	**-3,99636**	-4,75639
AIC	12,02404	12,01548	**11,99272**	13,51278
BIC	38,06795	38,05939	**38,03663**	39,55669
Sultan suyu vol logl	-7,26302	**-7,22073**	-7,28545	-7,87776
AIC	18,52604	**18,44146**	18,5709	19,75552
BIC	44,56995	**44,48537**	44,61481	45,79943
Sürgü vol logl	-8,70595	**-8,67446**	-8,77453	-9,23547
AIC	21,4119	**21,34892**	21,54906	22,47094
BIC	47,45581	**47,39283**	47,59297	48,51485
Keban kot	**-5555,821**	-7353,89	-5779,79	-14126

AIC	**11115,64**	14711,78	11563,58	28256
BIC	**11141,69**	14737,82	11589,62	28282,04
Sultan suyu kot logl	-5577,242	**-5435,697**	-5507,89	-14255,9
AIC	11158,48	**10875,39**	11019,78	28515,8
BIC	11184,53	**10901,44**	11045,82	28541,8
Sürgü kot logl	-5906,87	**-5700,657**	-5952.72	-14938,8
AIC	11817,75	**11407,31**	11909,44	29881,6
BIC	11843,78	**11371,3**	11935,48	29907,64

Table 3. Parameters of the probability distrubutions fitted to volumes and kots of Keban Sultan Suyu and Sürgü dams, respectively

	Logistic		**Weilbull**		**Gamma**		**Exponential**
	μ	σ	α	β	α	β	μ
Keban vol	23773,3	1594,07	24991,1	9,78398	78,9997	301,152	23790,9
Sultan suyu vol	41399,5	6186,82	44529,6	4,74919	12,5449	3236,66	40603,6
Sürgü vol	53229,2	9802,32	57128	3,711	6,81489	7550,66	51456,9
Keban kot	835,137	2,88164	838,642	50,2051	835,137	32,6478	835,137
Sultan suyu kot	896,93	2,95467	898,975	223,958	33157,6	0,002704	896,638
Sürgü kot	1303,29	3,50255	1305,48	288,881	43022,5	1423,05	1302,73



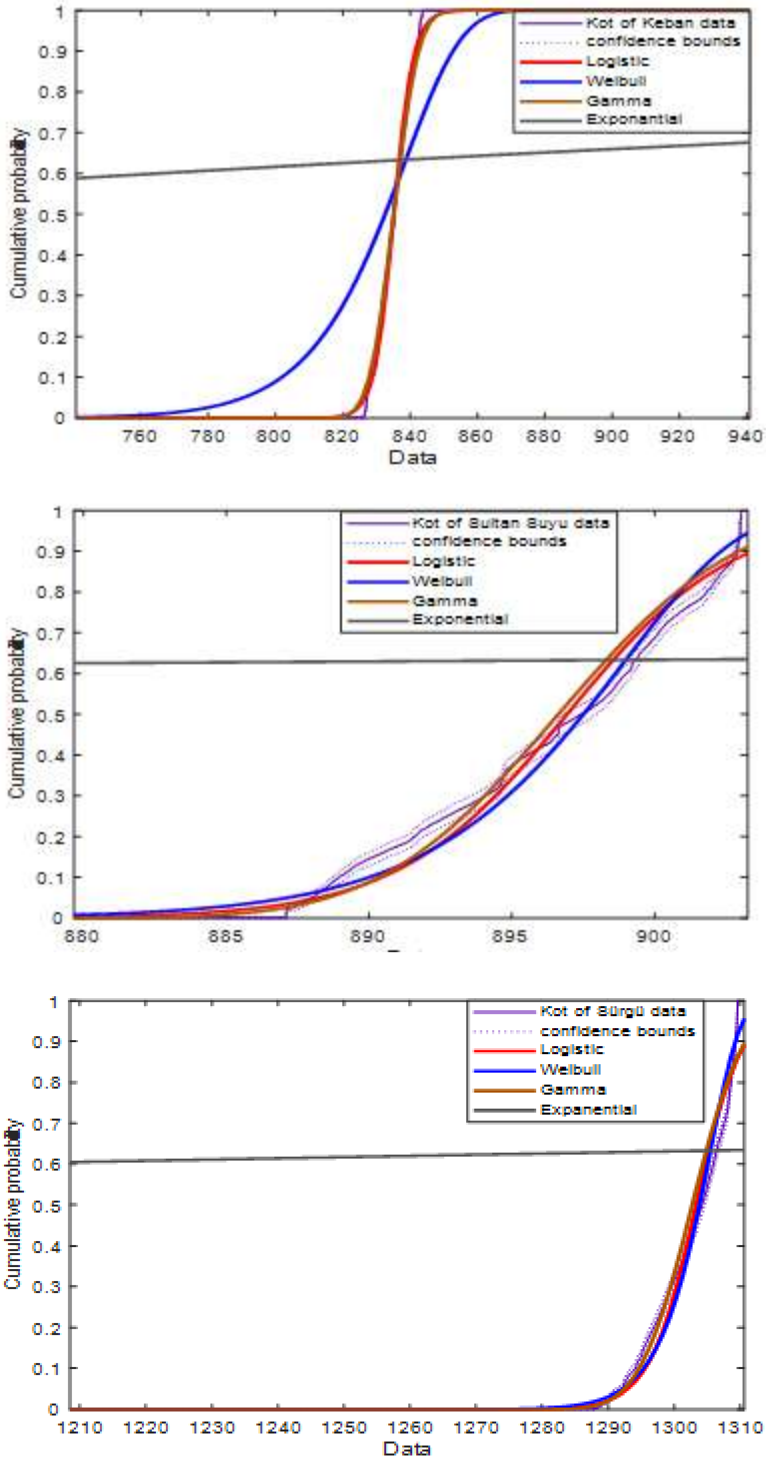


Figure 3. Cumulativedistribution function of Logistic, Weibull, Gamma and Exponential volumes and kots ofKeban Sultan Suyu and Sürgü dams, respectively.

4.2. Copula Modelling

In this study, to model the dependence of volume and kots of Sultan suyu, Sürgü and Keban dams, we use copula method. It is shown that emprical distribution functions are as shown in figure 4, we use Clayton, Gumbel Frank, Joe, Gaussian copula family. We present realtionship between volume of Keban-Sürgü, Keban-Sultan Suyu and Sultan suyu- Sürgü dams and relationship between kot of Keban-Sürgü, Keban-Sultan Suyu and Sultan suyu- Sürgü dams in table 4. Accordingly, it is observed that the relationship between volume of Keban-Sürgü, Keban- Sultan suyu dams are weak in the positive direction and the relationship between volume of Sürgü- Sultan suyu dams is strong positive direction in table 4. Similarly, we observe that the relationship between kot of Keban-Sürgü, Keban- Sultan suyu dams are weak in the positive direction and the relationship between kot of Sürgü- Sultan suyu dams is strong positive direction. From table 4 and table 5, according to the AIC, BIC criteria, Kolmogorov Smirnov and Cramer Von Mises test statistics, it is obtained that as the relationship between volume of Keban-Sürgü, Keban- Sultan suyu dams is modelled by Joe copula and the relationship between volume of Sürgü- Sultan suyu dams is modelled by Frank copula. Similarly, the relationship between kot of Keban-Sürgü, Keban-Sultan suyu dams are modelled by Joe copula and the relationship between kot of Sürgü- Sultan suyu dams is modelling by Frank copula. From Table 5, it is obvious that the Frank copula and Joe copula performs best for the pairs. In table 4, the calculated tail dependence values for the pairs Sultan Suyu-Sürgü volume and Sultan Suyu-Sürgükot when λ_l =0, λ_u = 0, symmetric tail dependency is observed in the tail of these pairs. From table 4, for the Keban-Sürgü, Keban- Sultan suyu volume and Keban-Sürgü, Keban- Sultan suyukot pairs, tail dependency coefficients are λ_u = 0,13 λ_l =0, λ_u = 0,05 λ_l =0 and λ_u = 0,12 λ_l =0, λ_u = 0,05 λ_l =0 respectively. According to these values, Keban-Sürgü volume and Keban-Sürgükot pairs have the highest upper tail dependency than Keban- Sultan suyu volume and Keban-Sultan suyukot pairs.In figure 4,5,6,7,8,9 for each copula, we show scatter graph of the relationship between volume of Keban-Sürgü, Keban- Sultan suyu, Sürgü- Sultan suyu dams and the relationship between kot of Keban-Sürgü, Keban- Sultan suyu, Sürgü- Sultan suyu dams.

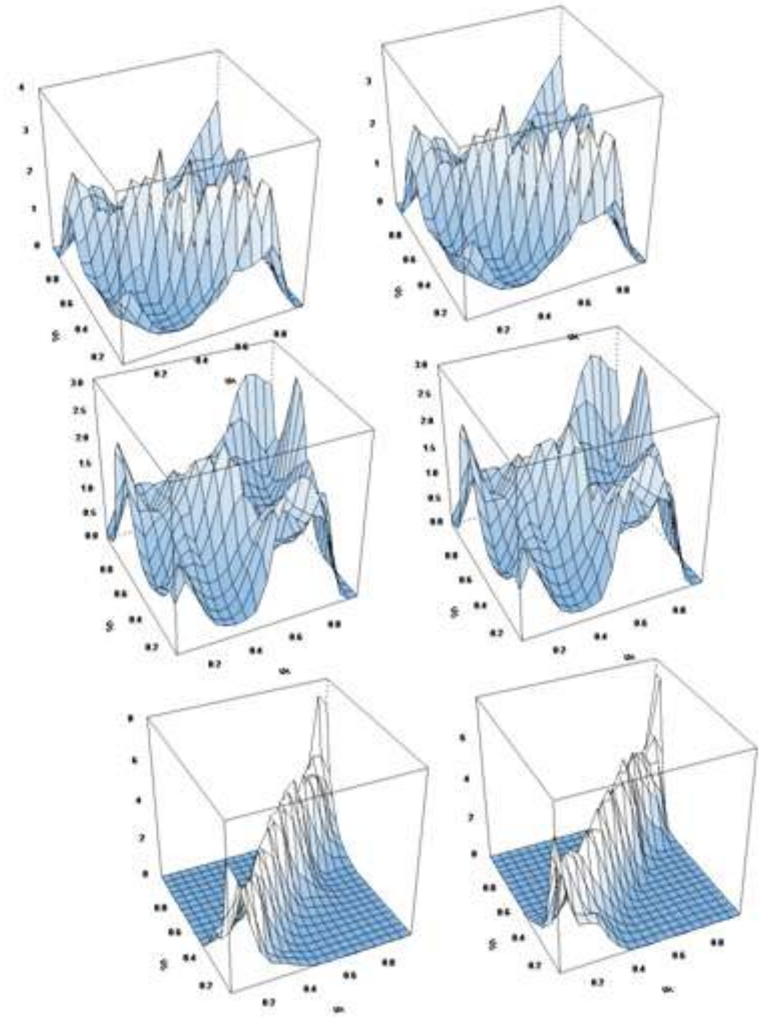


Figure 4. Kernel distrubution function of Keban, Sürgü and SultanSuyudamVolume and Kot respectively

Table 4. Copula Modelling

	Keban-Sürgü vol	Keban-Sultan Suyu vol	Sürgü-Sultan Suyu vol	Keban-Sürgü kot	Keban- Sultan Suyu kot	Sürgü-Sultan Suyu kot
τ	0,060	0,024	0,797	0,055	0,021	0,794
ρ	0,123	0,061	0,947	0,115	0,057	0,944
Clayton θ	0,13	0,04	7,8	0,12	0,04	7,65
Logl	-3,66	-7,06	-395,96	-3,89	-6,32	-363,61
AIC	9,32	16,11	793,91	9,78	14,63	729,21
BIC	14,83	21,62	799,42	15,29	20,14	734,72
λ_u	0	0	0	0	0	0
λ_l	0	0	0,92	0	0	0,91
Gumbel θ	1,06	1,02	4,9	1,06	1,02	4,83
Logl	29,79	7,97	1247,69	26,97	7,4	1276,37
AIC	-57,59	-13,94	-2493,38	-51,95	-12,8	-2550,74
BIC	-52,08	-8,43	-2487,87	-46,44	-7,29	-2545,23
λ_u	0,08	0,03	0,85	0,07	0,03	0,85
λ_l	0	0	0	0	0	0
Frank θ	0,54	0,2	17,79	0,49	0,18	17,49
Logl	12,69	2,59	1889,05	10,91	2,18	1866,63
AIC	-23,39	-3,19	-3776,09	-19,81	-2,35	-3731,26
BIC	-17,88	2,32	-3770,05	-14,3	3,16	-3725,75
λ_u	0	0	0	0	0	0
λ_l	0	0	0	0	0	0
Joe θ	1,11	1,04	8,57	1,1	1,03	8,42
Logl	38,06	12,08	534,23	34,89	11,3	607,87
AIC	-74,13	-22,17	-1066,46	-67,78	-20,59	-1213,74
SIC	-68,62	-16,66	-1060,95	-62,27	-15,08	-1208,23
λ_u	0,13	0,05	0,92	0,12	0,05	0,91
λ_l	0	0	0	0	0	0
Gaussian θ	0,09	0,03	0,95	0,09	0,03	0,95
Logl	24,8	4,85	995,17	21,88	4,09	985,23
AIC	-47,59	-7,69	-1988,34	-41,75	-6,17	-1968,46
BIC	-42,08	-2,18	-1982,83	-36,24	-0,66	-1962,95
λ_u	0	0	0	0	0	0
λ_l	0	0	0	0	0	0

Table 5. Goodness-of-fit tests for multivariate copula models

	Clayton		Gumbel		Frank		Joe		Gaussian	
	KS_C	CvM_C	KS_C	CvM_C	KS_C	CvM_C	KS_C	CvM_C	KS_C	CvM_C
Keban-Sürgü vol	5,55224 0,0000	6,1599 0,0000	6,28048 0,0000	7,52521 0,0000	6,19496 0,0000	7,56279 0,0000	10,4947 0,00000	31,369 0,0000	7,15571 0,00000	10,2968 0,00000
Keban Sultan Suyu vol	3,95253 0,0000	5,1821 0,0000	4,83001 0,00000	6,94488 0,0000	4,59754 0,0000	6,505281 0,0000	8,66451 0,0000	26,371 0,0000	4,61926 0,0000	6,68094 0,0000
Sürgü-Sultan Suyu vol	6,18410 0,0000	14,181 0,000	3,07840 0,0000	2,92856 0,0000	1,31847 0,00000	0,420063 0,0000	3,43769 0,0000	3,9769 0,0000	2,89529 0,00000	3,08591 0,0000
Keban-Sürgü kot	5,38117 0,000	5,8513 0,0000	6,16604 0,0000	7,35611 0,0000	6,05467 0,0000	7,29844 0,0000	10,2940 0,0000	30,475 0,0000	7,08127 0,0000	10,2699 0,0000
Keban-Sultan	3,88652 0,00000	5,0270 0,0000	4,84714 0,00000	6,80603 0,0000	4,53084 0,0000	6,256857 0,00000	8,74061 0,0000	26,208 0,000	4,87967 0,0000	6,99591 0,0000

Suyu kot										
Sürgü-Sultan Suyu kot	6,18692 0,0000	13,9396 0,0000	2,846775 0,0000	2,730498 0,00000	1,296828 0,0000	0,3918369 0,0000	3,437761 0,0000	4,12514 0,0000	3,173693 0,0000	3,489497 0,00000

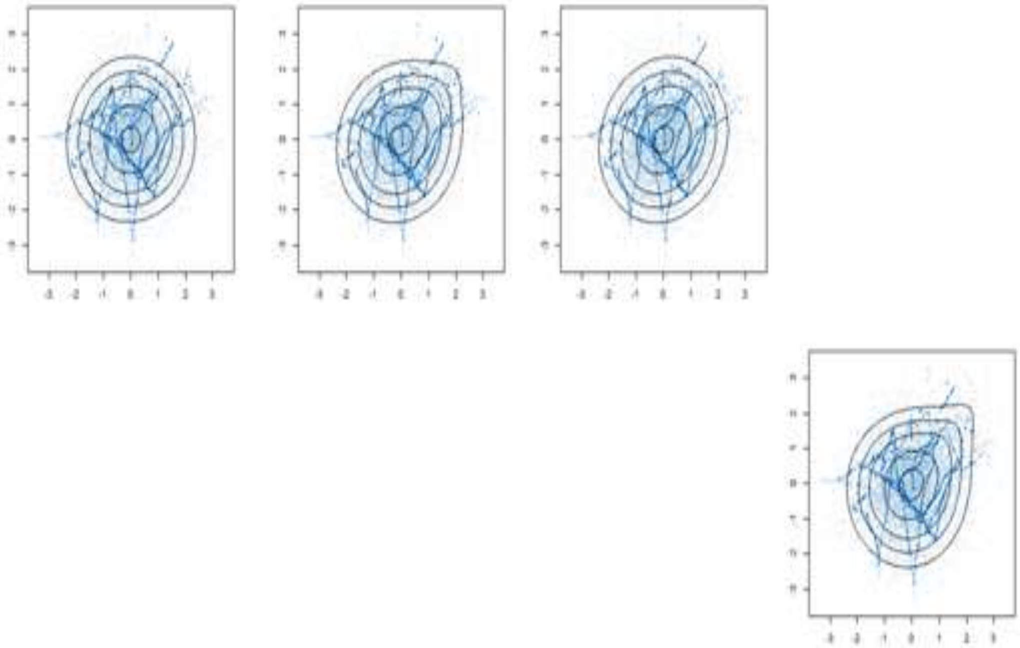


Figure 5. According to Clayton, Gumble, Frank, Joe and Gaussian family, Scatter graph of Keban and Sürgü dam volume respectively

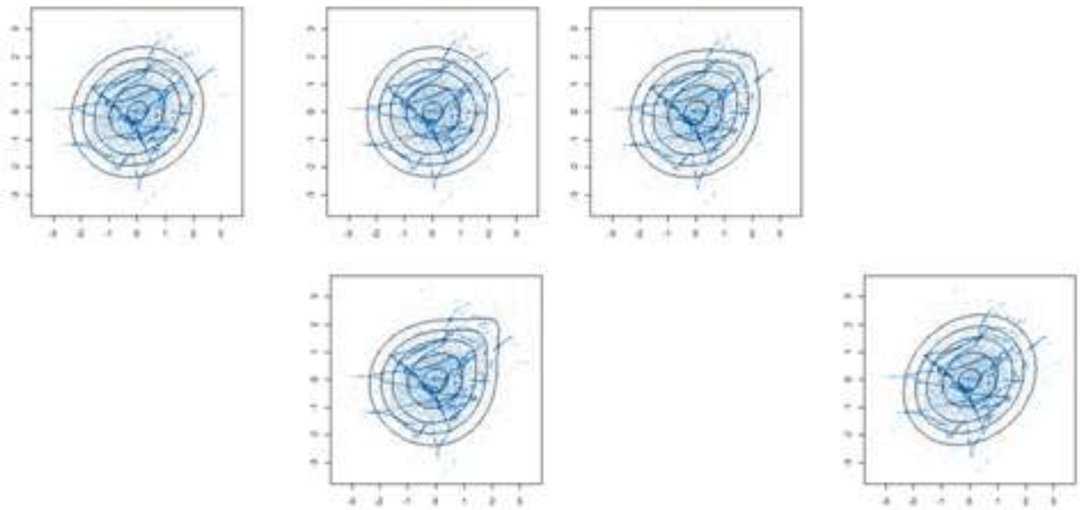


Figure 6. According to Clayton, Gumble, Frank, Joe and Gaussian family, Scatter graph of Keban and Sürgüdam Kot respectively

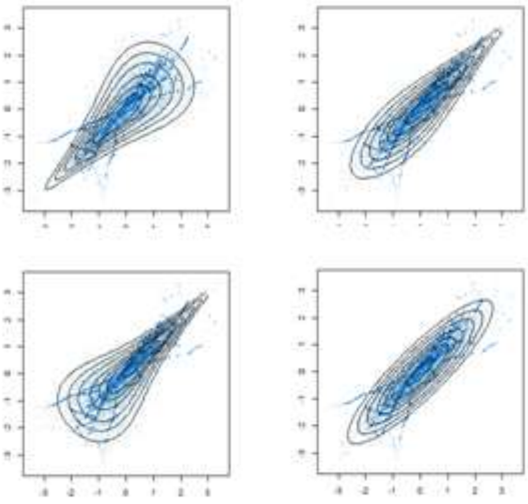


Figure 7. According to Clayton, Gumble, Frank, Joe and Gaussian family, Scatter graph ofSurgu and Sultan Suyudam volume respectively

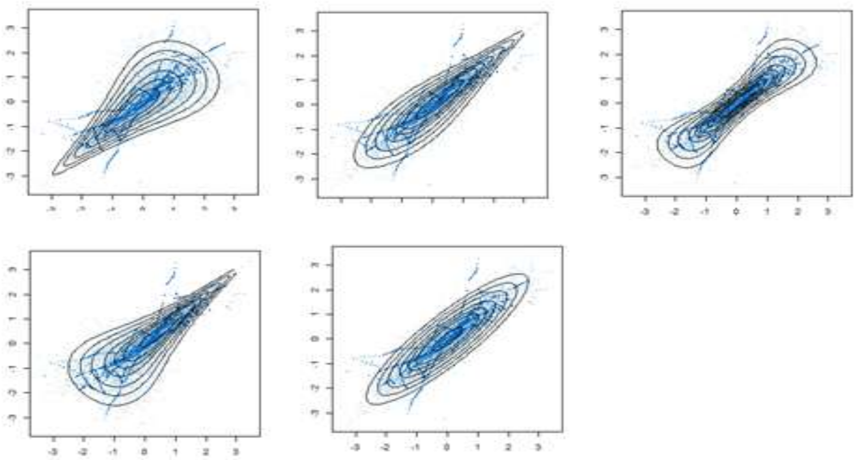


Figure 8. According to Clayton, Gumble, Frank, Joe and Gaussian family, Scatter graph of Sürgü and Sultan suyu Kot respectively

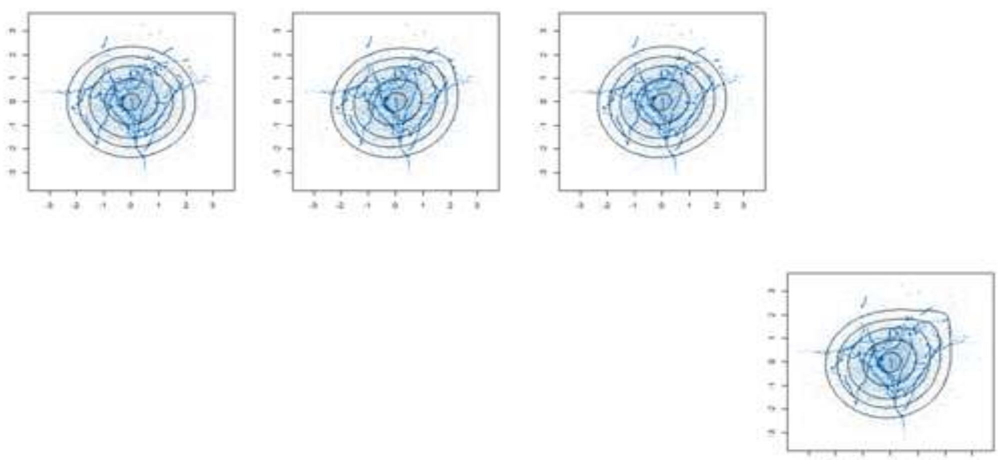


Figure 9. According to Clayton, Gumble, Frank, Joe and Gaussian family ,Scatter graph of Keban and Sultan dam volume respectively

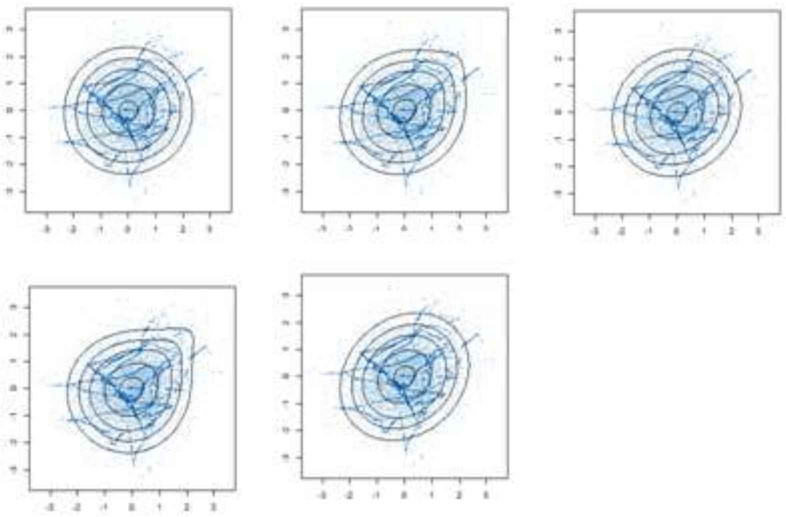


Figure 10. According to Clayton, Gumble, Frank, Joe and Gaussian family, Scatter graph of Keban and Sultan damKot respectively

V. CONCLUSION

Our analysis supports where no strong dependence was found between Keban and other dams. But, relationship between Sultan Suyudam and Sürgü dam was observed to be much strong (table 4). These results indicate that the water amount of the Keban dam is less than the other dams.This indicates that the Keban dam is likely to suffer in terms of the amount of water in the future (figure 2).

Compliance with ethical standards

Conflict of Interest: There is no funding for this study, and the authors declare that they have no conflict of interest**.**

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Ayşe Metin Karakaş" Modelling of Volumes and Kots of Keban, Sürgüand Sultan Suyu Dams with Copula Method" International Journal Of Engineering Research And Development , vol. 14, no. 10, 2018, pp 28-38