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Characterization of Bayes Risk under Different Loss Functions using Informative Priors

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Abstract *In this article, the characterization of the Bayes risk of the shape parameter of Pareto type – I distribution using informative priors namely Exponential, Inverted Gamma distribution under different loss functions such as Squared Error Loss Function (SELF), Quadratic Loss Function (QLF) and Precautionary Loss Function (PLF) are studied by using data, as per the result, it is found that the Bayes risk is minimum under SELF the QLF and PLF using Exponential prior the Inverted gamma prior*
Keywords *Bayes Risk, Exponential distribution, Informative priors, Inverted Gamma distribution, Loss function, , Pareto distribution.*

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I. INTRODUCTION

The Pareto probability distribution is a simple model for non- negative data with positively Skewed distribution. This distribution was introduced by Wilfredo Pareto (1848-1923) especially for wealth distribution of the population of a city within a given area. The use of the Pareto distribution as model to analyses stock prize and instability in business and economic, field of bio medical science, risk factor in insurance company, migration of population, survival time in quadratic system, Geophysical phenomena in society, reliability and life testing. Sankudey and Sudhansu A.Maiti (2012), have studied the Bayes estimators of Rayleigh parameter and its associated risk based on extended Jeffrey's prior under the assumptions of both symmetric and asymmetric loss function. R.K.Radha,(2015), studied the Bayesian analysis of exponential distribution using informative prior. Gaurav Shukla, Umesh Chandra and Vinod kumar (2020), derived and examined the expression for risk function under three different loss function. It is remarkable that the development of appropriate Bayesian inference procure has been very limited. Bayesian inference in the Pareto type I distribution for the special case in which the scale parameter is known. In this study the characterization of Bayes risk using exponential prior and inverted gamma prior different loss function such as SELF,QLF and PLF is to described.

The probability density function (pdf) of Pareto type – I distribution is defined as

$$f(t;\alpha,\theta) = \left\{\frac{\theta\alpha^{\theta}}{t^{\theta+1}}\right. ; \quad t > \alpha ;\ \theta>0;\ \alpha>0 \qquad (1)$$

Where t is a random variable, θ is the shape parameter and α is the scale parameter, which is known.
The moments of Pareto type –I distribution, were given by

Mean, $E(t) = \frac{\alpha\theta}{\theta-1} ; \theta > 1$

Variance, $V(t) = \frac{\theta\alpha^2}{(\theta-1)^2(\theta-2)} ; \theta > 2$

II. METHODOLOGY

2.1 Maximum Likelihood Estimation of Pareto Type I Distribution

Let $t_1, t_2, \ldots.. t_n$ be a set of 'n' random variables from Pareto Type I distribution with Parameters θ and α having the probability density function defined in (1), then the likelihood function,

$$L = \prod_{i=1}^{n} f(t_i; \alpha, \theta) \qquad (2.1.1)$$

$$= \prod_{i=1}^{n} \frac{\theta\alpha^{\theta}}{t_i^{\theta+1}}$$

$$L = \theta^n e^{n\theta \log\alpha} \; e^{-(\theta+1)\sum_{i=1}^{n} \log t_i}$$

$$\frac{\partial \log L}{\partial\theta} = \frac{n}{\theta} + n\log\alpha - \sum_{i=1}^{n} \log t_i$$

Using the Maximization Likelihood Principle, we get the estimated value of θ as

$$\hat{\theta} = \left[\frac{\sum_{i=1}^{n} \log t_i}{n} - \log\alpha\right]^{-1}$$

and $\hat{\alpha} = \underset{1\leq i\leq n}{mini}\,(t_i)$.

In case of frequency distribution

$$\hat{\theta} = \left[\frac{\sum_{i=1}^{n} f_i \log t_i}{N} - \log\alpha\right]^{-1} \quad (2.1.2)$$

Where $N = \sum f$

2.2 Bayesian Estimation and Bayes Risk using Informative Prior

Bayesian estimation is an estimation of an unknown parameter $\boldsymbol{\theta}$ that minimizes the expected loss for all observations $\boldsymbol{x\ of\ X}$. The Bayes approach is an average case analysis by taking the average risk of an estimator for all the parameters involved in the distribution under study. Suppose we take the prior probability distribution $\boldsymbol{\pi}$, on the parameter space $\boldsymbol{\omega}$ then the average risk is defined as

$$\boldsymbol{R_{\pi}(\hat{\theta}) = E_{\theta,x}\left[L(\theta,\hat{\theta})\right]}$$

and the Bayes risk for a prior $\boldsymbol{\pi}$ is the minimum that the average risk can achieve

$$\boldsymbol{\hat{R}_{\pi} = \underset{\theta}{inf}\left[R_{\pi}(\hat{\theta})\right]}$$

2.3 Bayes estimation and Bayes risk using Informative Prior (Exponential prior)
The posterior pdf of exponential prior:

Assuming that θ has informative prior as exponential prior which takes the following form

$$g(\theta) = \frac{1}{\lambda} e^{-\theta/\lambda}; \theta, \lambda > 0$$

The posterior pdf of exponential prior is defined as

$$h(\theta/t) = \frac{L(t_1,t_2,\ldots\ldots,t_n)g(\theta)}{\int_0^{\infty} L(t_1,t_2,\ldots\ldots,t_n)g(\theta)d\theta}$$

$$= \frac{\theta^n e^{-(\theta+1)p} \frac{1}{\lambda} e^{-\theta/\lambda}}{\int_0^{\infty} \theta^n e^{-(\theta+1)p} \frac{1}{\lambda} e^{-\theta/\lambda} d\theta}$$

Consider,

$$\int_0^{\infty} \theta^n e^{-(\theta+1)p} \frac{1}{\lambda} e^{-\theta/\lambda} d\theta = \int_0^{\infty} \theta^n e^{-\theta p} e^{-p} \frac{1}{\lambda} e^{-p/\lambda} d\theta$$

$$= \int_0^{\infty} \theta^n e^{-\theta p - \theta/\lambda} \frac{1}{\lambda} e^{-p/\lambda} d\theta$$

$$= \frac{e^{-p}}{\lambda} \frac{\sqrt{n+1}}{\left(p + 1/\lambda\right)^{n+1}}$$

∴The posterior pdf of exponential prior is

$$h(\theta/t_1, t_2, \ldots\ldots, t_n) = \frac{\left(p + 1/\lambda\right)^{n+1}}{\sqrt{n+1}} \theta^n e^{-\theta}(p + 1/\lambda)$$

2.3. Bayes Estimation and Bayes Risk using Informative Prior under different Loss Function
2.3.1. Bayes Estimation under Squared Error Loss Function

The SELF is defined as $L(\theta,\hat{\theta}) = (\hat{\theta} - \theta)^2$

The Bayes estimator under squared error loss function is

$$\hat{\theta}_{SELF} = E(\theta) = \int_0^{\infty} \theta\, h(\theta/t_1, t_2, \ldots\ldots, t_n) d\theta$$

$$= \int_0^{\infty} \theta\, \frac{\left(p+{}^{1}/_{\lambda}\right)^{n+1} \theta^n e^{-\theta(p+{}^{1}/_{\lambda})}}{\sqrt{n+1}} d\theta$$

∴ The Bayes Estimator Squared Error Loss Function is

is

$$\hat{\theta}_{SELF} = E(\theta) = \frac{n+1}{p+\frac{1}{\lambda}} \qquad (2.3.1)$$

2.3.2 Bayes Risk under Squared Error Loss Function

The Bayes risk $R(\theta, \hat{\theta})$ under SELF is defined as the expected loss under SELF

(ie) $R(\theta, \hat{\theta}) = E[L(\hat{\theta}, \theta)] = (\hat{\theta} - \theta)^2 = [E(\theta) - \theta]^2$

$$= \left[\frac{n+1}{\left(p+{}^{1}/_{\lambda}\right)} - \theta\right]^2$$

$$R(\theta, \hat{\theta}) = \left\{\frac{1}{\left(p+{}^{1}/_{\lambda}\right)^2} (n^2 + 1 + 2n) + \theta^2 - \frac{2\theta(n+1)}{(p+{}^{1}/_{\lambda})}\right\}$$

2.3.3 Bayes Estimation under Quadratic Loss Function

The Quadratic Loss Function is defined as

$$L(\theta, \hat{\theta}) = \left(\frac{\theta - \hat{\theta}}{\theta}\right)^2 = \left(1 - \frac{\hat{\theta}}{\theta}\right)^2$$

Consider the risk function $R(\theta, \hat{\theta})$ to estimate the parameter θ under quadratic loss function

where, $R(\theta, \hat{\theta}) = E\left(1 - \frac{\hat{\theta}}{\theta}\right)^2$

$$= \int_0^{\infty} \left(1 - \frac{\hat{\theta}}{\theta}\right)^2 h(\theta/t_1, t_2, \ldots\ldots, t_n) d\theta$$

$$\hat{\theta}_{QLF} = \frac{E({}^{1}/_{\theta})}{E({}^{1}/_{\theta^2})} \qquad (2.3.2)$$

Where,

$$E\left(\frac{1}{\theta}\right) = \int_0^{\infty} \left(\frac{1}{\theta}\right) h(\theta/t_1, t_2, \ldots\ldots, t_n) d\theta$$

$$= \int_0^{\infty} \left(\frac{1}{\theta}\right) \frac{\left(p+{}^{1}/_{\lambda}\right)^{n+1}}{\sqrt{n+1}} \theta^n e^{-\theta\left(p+{}^{1}/_{\lambda}\right)}$$

$$= \frac{\left(p+{}^{1}/_{\lambda}\right)^{n+1}}{\sqrt{n+1}} \int_0^{\infty} \theta^{n-1} e^{-\theta\left(p+{}^{1}/_{\lambda}\right)} d\theta$$

$$E\left(\frac{1}{\theta}\right) = \frac{\left(p+{}^{1}/_{\lambda}\right)}{n}$$

Also,

$$E\left(\frac{1}{\theta^2}\right) = \int_0^{\infty} \left(\frac{1}{\theta^2}\right) h\left({}^{\theta}/_{x}\right) d\theta$$

$$= \int_0^{\infty} \left(\frac{1}{\theta^2}\right) \frac{\left(p+{}^{1}/_{\lambda}\right)^{n+1}}{\sqrt{n+1}} \theta^n e^{-\theta\left(p+{}^{1}/_{\lambda}\right)} d\theta$$

$$= \frac{\left(p+{}^{1}/_{\lambda}\right)^n \left(p+{}^{1}/_{\lambda}\right)}{\sqrt{n+1}} \frac{\sqrt{n+1}}{\left(p+{}^{1}/_{\lambda}\right)^{n+1}}$$

$$= \frac{\left(p+{}^{1}/_{\lambda}\right)^2}{n(n-1)}$$

∴ The Bayes Estimator under Quadratic Loss Function is

$$\hat{\theta}_{QLF} = \frac{(n-1)}{(p+\frac{1}{\lambda})} \qquad (2.3.3)$$

2.3.4 Bayes risk under the Quadratic Loss Function

The Bayes risk $R(\theta,\hat{\theta})$ under Quadratic Loss Function defined as the expected loss under QLF,

$$R(\theta,\hat{\theta}) = E\left[L(\hat{\theta},\theta)\right] = \left(\frac{\theta-\hat{\theta}}{\theta}\right)^2 = \left(1-\frac{\hat{\theta}}{\theta}\right)^2$$

Where $\hat{\theta} = \frac{(n-1)}{(p+\frac{1}{\lambda})}$

$$R(\theta,\hat{\theta}) = \left[1-\frac{\left(\frac{n-1}{p+\frac{1}{\lambda}}\right)}{\theta}\right]^2$$

$$R(\theta,\hat{\theta}) = 1+\left[\frac{n-1}{\theta\left(p+\frac{1}{\lambda}\right)}\right]^2 - 2\frac{n-1}{\theta\left(p+\frac{1}{\lambda}\right)}$$

$$= 1+\left\{\frac{1}{\theta\left(p+\frac{1}{\lambda}\right)}\left[\frac{(n-1)^2}{\theta\left(p+\frac{1}{\lambda}\right)}-2(n-1)\right]\right\}$$

Therefore,

$$R_{QLF}(\theta,\hat{\theta}) = 1+\left\{\frac{1}{\theta\left(p+\frac{1}{\lambda}\right)}\left[\frac{n^2+1-2n-2n\theta\left(p+\frac{1}{\lambda}\right)+\theta\left(p+\frac{1}{\lambda}\right)}{\theta\left(p+\frac{1}{\lambda}\right)}\right]\right\}$$

2.3.5 Bayes Estimation under Precautionary Loss Function

The Precautionary Loss Function is defined as

$$L_{PLF}(\theta,\hat{\theta}) = \left(\frac{\hat{\theta}-\theta}{\theta}\right)^2 = \left(1-\frac{\theta}{\hat{\theta}}\right)^2$$

Consider the risk function $R(\theta,\hat{\theta})$ to estimate the parameter θ under quadratic loss function

Where, $R(\theta,\hat{\theta}) = E\left[L(\hat{\theta},\theta)\right]$

$$R(\theta,\hat{\theta}) = \int_0^\infty L(\hat{\theta},\theta)h(\theta/x)d\theta$$

$$= \int_0^\infty \left(1-\frac{\theta}{\hat{\theta}}\right)\left(-\frac{1}{\hat{\theta}^2}\right)h(\theta/x)d\theta = 0$$

$$\frac{1}{\hat{\theta}^3}\int_0^\infty \theta\, h(\theta/x)d\theta = \frac{1}{\hat{\theta}^2}\int_0^\infty h(\theta/x)d\theta$$

∴ The Bayes Estimator under Precautionary Loss Function is

$$\hat{\theta}_{PLF} = \frac{(n-1)}{(p+\frac{1}{\lambda})} \qquad (2.3.5)$$

2.3.6 Bayes Risk under Precautionary Loss Function

The Bayes risk $R(\theta,\hat{\theta})$ under precautionary loss function is defined as the expected loss under PLF

(ie) $R(\theta,\hat{\theta}) = E(L(\theta,\hat{\theta})) = \frac{(\hat{\theta}-\theta)^2}{\hat{\theta}}$,

Where $\hat{\theta} = \frac{n+1}{p+\frac{1}{\lambda}}$

$$\therefore R(\theta,\hat{\theta}) = \frac{1}{\left(\frac{n+1}{p+\frac{1}{\lambda}}\right)}\left[\left(\frac{n+1}{p+\frac{1}{\lambda}}\right)^2+\theta^2-2\theta\left(\frac{n+1}{p+\frac{1}{\lambda}}\right)\right]$$

2.4. Bayes Estimation and Bayes risk using Informative Prior (Inverted Gamma Prior)

The pdf of Pareto type – I distribution is defined in (1) as follows

$$f(x,\alpha,\theta) = \frac{\theta\alpha^\theta}{x^{\theta+1}}; x>\alpha; \theta>0$$

The likelihood function is defined as

$$L = \prod_{i=1}^{n} f(x, \alpha, \theta)$$

$$L = \theta e^{n\theta log\alpha}\ e^{-(\theta+1)\sum_{i=1}^{n} logx}$$

The Inverted gamma prior is defined as

$$g(\theta) = \begin{cases}\frac{\beta^{\alpha}}{\sqrt{\alpha}}\ \theta^{-(\alpha+1)} e^{-\beta/\theta}; if\theta > 0, (\alpha, \beta) > 0\end{cases}$$

Therefore, the posterior pdf of inverted gamma prior is to be obtained as follows

$$h(\theta/x) = \frac{L(x_1, x_2, \ldots\ldots, x_n) g(\theta)}{\int_0^{\infty} L(x_1, x_2, \ldots\ldots, x_n) g(\theta) d\theta}$$

$$= \frac{\theta^n\ e^{n\theta log\alpha\ -(\theta+1)\sum logx}\ \frac{\beta^{\alpha}}{\sqrt{\alpha}} \theta^{-(\alpha+1)} e^{-\beta/\theta}}{\int_0^{\infty} \theta^n\ e^{n\theta log\alpha\ -(\theta+1)\sum logx}\ \frac{\beta^{\alpha}}{\sqrt{\alpha}} \theta^{-(\alpha+1)} e^{-\beta/\theta} d\theta}$$

$$= \frac{\theta^n\ e^{n\theta log\alpha\ -(\theta+1)} e^{-\sum logx}\ \theta^{-(\alpha+1)} e^{-\beta/\theta}}{\int_0^{\infty} \theta^n\ e^{n\theta log\alpha\ -(\theta+1)} e^{-\sum logx}\ \theta^{-(\alpha+1)} e^{-\beta/\theta} d\theta}$$

$$= \frac{\theta^n\ e^{n\theta log\alpha\ -(\theta+1)}\ \theta^{-(\alpha+1)} e^{-(1/\theta)\beta}}{\int_0^{\infty} \theta^n\ e^{n\theta log\alpha\ -(\theta+1)}\ \theta^{-(\alpha+1)} e^{-(1/\theta)\beta} d\theta}$$

$$= \frac{\theta^n\ e^{n\theta log\alpha\ -(\theta+1)}\ \theta^{-(\alpha+1)} e^{-(1/\theta)} e^{\beta}}{\int_0^{\infty} \theta^n\ e^{n\theta log\alpha\ -(\theta+1)}\ \theta^{-(\alpha+1)} e^{-(1/\theta)} e^{\beta} d\theta}$$

$$= \frac{\theta^{n-\alpha-1}\ e^{-\theta p}}{\int_0^{\infty} \theta^{n-\alpha-1}\ e^{-\theta p}\ d\theta}$$

Thus,

$$h(\theta/x_1, x_2, \ldots\ldots, x_n) = \frac{p^{n-\alpha}}{\sqrt{n-\alpha}} \theta^{n-\alpha-1}\ e^{-\theta p}$$

Hence, the posterior pdf under inverted gamma prior is

$$h(\theta/x_1, x_2, \ldots\ldots, x_n) = \frac{p^{n-\alpha}}{\sqrt{n-\alpha}} \theta^{n-\alpha-1}\ e^{-\theta p} \qquad (2.4)$$

2.4.1 Bayes Estimation under Squared Error Loss Function

The Bayes estimator using squared error loss function is

$$E(\theta) = \hat{\theta}$$

$$E(\theta) = \int_0^{\infty} \theta\ h(\theta/x_1, x_2, \ldots\ldots, x_n) d\theta$$

$$= \int_0^{\infty} \theta\ \frac{p^{n-\alpha}}{\sqrt{n-\alpha}} \theta^{n-\alpha-1}\ e^{\theta p}\ d\theta$$

$$= \frac{p^{n-\alpha}}{\sqrt{n-\alpha}}\ \frac{\sqrt{n-\alpha+1}}{p^{n-\alpha+1}}$$

$$E(\theta) = \frac{n-\alpha}{p} \qquad (2.4.1)$$

2.4.2 Byes Risk under Squared Error Loss Function

The Bayes risk under Squared Error Loss Function

$$\text{L}\ (\theta, \hat{\theta}) = E(L(\hat{\theta} - \theta)^2)$$

$$= \left[\frac{n-\alpha}{p} - \theta\right]^2$$

$$= \left\{\frac{1}{p^2}\ (n^2 + \alpha^2 - 2n\alpha) + \theta^2 - 2\theta\left(\frac{n-\alpha}{p}\right)\right\}$$

$$= \left\{\frac{n^2+\alpha^2-2n\alpha}{p^2}\ + \theta^2 - 2\theta\left(\frac{n-\alpha}{p}\right)\right\}$$

∴ Bayes Risk under Squared Error Loss Function is

$$L\,(\theta, \hat{\theta}) = \left\{\frac{n^2+\alpha^2-2n\alpha}{p^2} + \theta^2 - 2\theta\left(\frac{n-\alpha}{p}\right)\right\}$$

2.4.3 Bayes Estimation under Quadratic Loss Function

The Bayes estimator using quadratic loss function is

$$L(\theta, \hat{\theta}) = \left(\frac{\theta-\hat{\theta}}{\theta}\right)^2 = \left(1-\frac{\hat{\theta}}{\theta}\right)^2$$

The risk function under the quadratic loss function is denoted by $R_{QLF}(\theta, \hat{\theta})$ is

$$R_{QLF}(\theta, \hat{\theta}) = E\left(1-\frac{\hat{\theta}}{\theta}\right)^2$$

$$= \int_0^\infty \left(1-\frac{\hat{\theta}}{\theta}\right)^2 h(\theta/x)d\theta$$

Differentiate with respect to $'\theta'$ and equating to zero, we get

$$\frac{dR_Q(\hat{\theta}, \theta)}{d\hat{\theta}} = 0$$

$$= 2\int_0^\infty \left(1-\frac{\hat{\theta}}{\theta}\right)^2 \left(-\frac{1}{\theta}\right) h(\theta/x)d\theta$$

⇨ $\int_0^\infty \left(\frac{\hat{\theta}}{\theta^2}\right) h(\theta/x)d\theta - \int_0^\infty \left(\frac{1}{\theta}\right) h(\theta/x)d\theta = 0$

⇨ $\int_0^\infty \left(\frac{\hat{\theta}}{\theta^2}\right) h(\theta/x)d\theta = \int_0^\infty \left(\frac{1}{\theta}\right) h(\theta/x)d\theta$

⇨ $\hat{\theta}.E\left(\frac{1}{\theta^2}\right) = E\left(\frac{1}{\theta}\right)$

⇨ $\hat{\theta} = \frac{E\left(\frac{1}{\theta}\right)}{E\left(\frac{1}{\theta^2}\right)}$

Consider

$$E\left(\frac{1}{\theta}\right) = \int_0^\infty \left(\frac{1}{\theta}\right) h(\theta/x)d\theta$$

$$= \int_0^\infty \left(\frac{1}{\theta}\right) \frac{p^{n-\alpha}}{\sqrt{n-\alpha}} \theta^{n-\alpha-1}\, e^{-\theta p} d\theta$$

$$= \frac{p^{n-\alpha}}{\sqrt{n-\alpha}} \int_0^\infty \theta^{-1}\, \theta^{n-\alpha-1}\, e^{-\theta p} d\theta$$

$$E\left(\frac{1}{\theta}\right) = \frac{p}{(n-\alpha-1)}$$

Also

$$E\left(\frac{1}{\theta^2}\right) = \int_0^\infty \left(\frac{1}{\theta^2}\right) h(\theta/x)d\theta$$

$$= \int_0^\infty \left(\frac{1}{\theta^2}\right) \frac{p^{n-\alpha}}{\sqrt{n-\alpha}} \theta^{n-\alpha-1}\, e^{-\theta p} d\theta$$

$$= \frac{p^{n-\alpha}}{\sqrt{n}-\alpha} \frac{\sqrt{n-\alpha-2}}{p^{n-\alpha-2}}$$

$$E\left(\frac{1}{\theta^2}\right) = \frac{P^2}{(n-\alpha-1)(n-\alpha-2)}$$

$$\hat{\theta}_{QLF} = \frac{n-\alpha-2}{p} \qquad (2.4.3)$$

2.4.4 Bayes Risk under Quadratic Loss Function

The Byes risk under Quadratic Loss Function is defined as

$$R(L(\hat{\theta}, \theta) = E\left(L\left(\frac{\theta-\hat{\theta}}{\theta}\right)^2\right)$$

$$= \left(1-\frac{\hat{\theta}}{\theta}\right)^2$$

$$= \left[1-\frac{\frac{n-\alpha-2}{p}}{\theta}\right]^2$$

$$= \left[1-\frac{n-\alpha-2}{p\theta}\right]^2$$

$$L(\hat{\theta}, \theta) = \left\{1+\left(\frac{n-\alpha-2}{p\theta}\right)^2 - 2\left(\frac{n-\alpha-2}{p\theta}\right)\right\}$$

2.4.5 Bayes estimation under Precautionary Loss Function

The Precautionary Loss Function is defined as

$$L(\hat{\theta},\theta) = \frac{(\hat{\theta}-\theta)^2}{\hat{\theta}}$$

The Bayes estimator under a precautionary loss function is defined as

$$\hat{\theta}_{PLF} = [E(\theta^2)]^{1/2}$$

Consider

$$E(\theta^2) = \int_0^\infty \theta^2 h(\theta/x_1, x_2, \ldots\ldots, x_n) d\theta$$

$$= \int_0^\infty \theta^2 \frac{p^{n-\alpha}\theta^{n-\alpha-1}e^{-\theta p}}{\sqrt{n-\alpha}} d\theta$$

$$E(\theta^2) = \frac{(n-\alpha)(n-\alpha+1)}{p^2}$$

$$\hat{\theta}_{PLF} = \left[\frac{(n-\alpha)(n-\alpha+1)}{p^2}\right]^{1/2} \quad (2.4.5)$$

The risk function under precautionary loss function is denoted by $R_{PLF}(\hat{\theta},\theta)$ is

$$R_{PLF}(\hat{\theta},\theta) = E[L(\hat{\theta},\theta)]$$

$$= \int_0^\infty L(\hat{\theta},\theta) h(\theta/x)\, d\theta$$

Let $\frac{\partial R_{PLF}(\hat{\theta},\theta)}{\partial\hat{\theta}} = 0$

⇨ $2\int_0^\infty \left(1-\frac{\theta}{\hat{\theta}}\right)\left(-\frac{1}{\hat{\theta}^2}\right) h(\theta/x) d\theta = 0$

⇨ $\int_0^\infty \left(1-\frac{\theta}{\hat{\theta}}\right)\left(-\frac{1}{\hat{\theta}^2}\right) h(\theta/x) d\theta = 0$

⇨ $\frac{1}{\hat{\theta}^3}\int_0^\infty \theta\, h(\theta/x) d\theta = \int_0^\infty \left(\frac{1}{\hat{\theta}^2}\right) h(\theta/x) d\theta$

⇨ $\frac{1}{\hat{\theta}^3} E(\theta) = \frac{1}{\hat{\theta}^2}$

⇨ $E(\theta) = \frac{\hat{\theta}^3}{\hat{\theta}^2}$

⇨ $E(\theta) = \hat{\theta}$

Where $\hat{\theta}_{PLF} = \left[\frac{(n-\alpha)(n-\alpha+1)}{p^2}\right]^{1/2}$

2.4.6 Bayes Risk under Precautionary Loss Function

The Bayes risk under precautionary loss function is given by

$$R(\hat{\theta},\theta) = E\left(L\left(\frac{(\hat{\theta}-\theta)^2}{\hat{\theta}}\right)\right)$$

$$= \frac{\left(\left[\frac{(n-\alpha)(n-\alpha+1)}{p^2}\right]^{1/2} - \theta\right)^2}{\left[\frac{(n-\alpha)(n-\alpha+1)}{p^2}\right]^{1/2}}$$

$$= \frac{1}{\sqrt{(n-\alpha)(n-\alpha+1}} \frac{\sqrt{(n-\alpha)(n-\alpha+1}\sqrt{(n-\alpha)(n-\alpha+1}}{p} + \frac{\theta^2 p}{\sqrt{(n-\alpha)(n-\alpha+1)}} - 2\theta$$

$$= \frac{\sqrt{(n-\alpha)(n-\alpha+1)}}{p} + \frac{\theta^2 p}{\sqrt{(n-\alpha)(n-\alpha+1)}} - 2\theta$$

The, Bayes risk under precautionary loss function is

$$R(\hat{\theta},\theta) = \frac{[(n-\alpha)(n-\alpha+1)]^{1/2}}{p} + \frac{\theta^2 p}{[(n-\alpha)(n-\alpha+1)]^{1/2}} - 2\theta$$

III. Result and Discussion

In this study, we choose a sample size of n=25,50 and 100 to represent the small median and large data set. The Bayes estimation of the shape parameter of the Pareto type I distribution using simulation technique informative priors under different loss functions (SELF,QLF & PLF) thorough and presented in table 3.1

Table 3.1 Bayes Estimation and Bayes Risk of the Shape parameter $\boldsymbol{\theta = 0.1}$ and Scale parameter $\boldsymbol{\alpha = 0.2}$ & 0.4 (Pareto model)

Shape parameter = 0.1													
		SELF		QELF		PELF		SELF		QELF		PELF	
prior	N	Scale parameter =0.2						Scale parameter=0.4					
		BE	BR	BE	BR	BE	BR	BE	BR	BE	BR	BE	BR
Exponential	25	0.102089	0.0964541	0.101884	0.096454	0.0101789	0.095059	0.121913	0.0773328	0.0989542	0.563448	0.122013	0.640352
	50	0.0890885	0.0125242	0.078854	0.369947	0.0798088	0.142177	0.102977	0.0906792	0.0974547	0.572048	0.1060489	0.8190929
	100	0.0846372	0.0133089	0.0813181	0.351935	0.0794663	0.157242	0.0892945	0.0965379	0.0860928	0.617189	0.0892846	0.9908112
inverted gamma	25	0.0924656	0.0965541	0.092456	0.096454	0.0970233	0.109295	0.110951	0.0774321	0.0890472	0.5996208	0.120521	0.690899
	50	0.0846485	0.0126042	0.292157	0.373019	0.0863315	0.149662	0.102032	0.0870792	0.0938674	0.5997402	0.104049	0.8587901
	100	0.0829494	0.0133086	0.0796248	0.362255	0.0828854	0.161238	0.0880217	0.0970379	0.084113	0.614047	0.090395	0.909425

3.2 Bayes Estimation and Bayes Risk of the Shape parameter (Pareto model)

The Bayes risk of the shape parameter for Pareto type – I distribution is estimated using informative priors namely Exponential prior and Inverted Gamma prior under different loss functions. The survival data given in table 3.2 is used for estimating the bayes estimation and bayes risk of the shape parameter of Pareto Type- I distribution and presented in table 3.3.

Table: 3.2 Survival Time Distribution

Year of follow-up	Number alive at beginning of interval	Number dying in interval
0-1	1100	240
1-2	860	180
2-3	680	184
3-4	496	138
4-5	358	118
5-6	240	60
6-7	180	52
7-8	128	44
8-9	84	32
≥ 9	52	28

3.3 Bayes Estimation and Bayes Risk of the Shape parameter $\boldsymbol{\theta = 0.1}$ and Scale parameter $\boldsymbol{\alpha = 0.2}$ (Pareto model)

Shape parameter = 0.1						
	SELF		QELF		PELF	
Prior	scale=0.2					
	BE	BR	BE	BR	BE	BR
Exponential	0.5319	0.2578	0.4352	0.3328	0.4352	0.6604
Inverted gamma	0.4679	0.4313	0.3694	0.4508	0.4920	0.8133

IV. Discussion

From the table 3.1, it is estimated that the Bayes risk is decreased when the sample sizes are increased and it is minimum under Squared Error Loss Function than Quadratic Loss Function and Precautionary Loss Function using exponential prior and inverted gamma prior through simulation technique. If the shape parameter α is increased, the Bayes risk under SELF is decreased, but it is increased under QLF and PLF using the informative prior's namely exponential and inverted gamma prior distributions. Thus, the SELF is the best one than QLF and PLF to estimate the Bayes risk using informative priors.

From table 3.3 the Bayes risk under SELF is minimum than QLF and PLF using experimental prior than inverted gamma prior. Thus SELF is the best one to estimate the Bayes risk using exponential prior than inverted gamma prior. Finally, it is found that the Bayes risk under Squared Error Loss Function is minimum than QLF and PLF using the exponential prior distribution than inverted gamma prior distribution in both simulation technique and real life problem.

V. Conclusions

In this study, the characterization of the Bayes risk of shape parameter of Pareto type –I distribution using informative priors such as Exponential prior and Inverted Gamma prior under various loss functions proposed through simulation technique and real life problem have been estimated. By comparing the Bayes risk of the shape parameter, it is found that the Bayes risk under SELF is minimum than QLF and PLF using exponential prior than inverted gamma prior in both simulation techniques and real life problem. Finally, it is found that the SELF is the best one than QLF and PLF to estimate the Bayes risk using Exponential prior.

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