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Comparison of a tower geodetic micro-network optimization results obtained using the MABAC, MAIRCA, COCOSO and ROV methods with those obtained applying the VIKOR method

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ABSTRACT
In this study, the author deals with the optimization of a tower geodetic micro-network. He uses the MABAC, MAIRCA, COCOSO and ROV methods, that have not yet been applied in geodetic tasks so far. These methods all use the same data normalization method, which makes them perfectly comparable. The results obtained applying the four methods were compared with the reference results provided by the VIKOR method, which was used as a reference in the study, as already proven and effective mathematical tool in many areas, including geodesy as well. First, criteria functions related to precision and reliability were established and some parameter constraints were introduced. Then, several variants of the geodetic network were tested whether they fulfilled all the requirements preset. Among those variants, four acceptable solutions were chosen for the study as alternative solutions, that were then ranked using the aforementioned methods. It turned out the results obtained using the MABAC, MAIRCA, COCOSO and ROV methods matched well with those obtained using the VIKOR method. Besides, the MABAC, MAIRCA and ROV methods provided exactly the same optimization results. Comparing to those results, the results produced by the COCOSO method were a little different. It was shown changing the criteria weights had a different impact on the COCOSO method performance, which was reflected in a different ranking list comparing to the corresponding ranking lists provided using the MABAC, MAIRCA and ROV methods. Anyway, since the four analyzed methods from this study produced logical optimization results, it can freely be said they also represent useful tools when it comes to finding the optimal design for a geodetic micro-network.
Keywords: *geodetic micro-network, alternative solutions, MCDM methods, MABAC, MAIRCA, COCOSO, ROV, VIKOR, comparison*

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I. INTRODUCTION

There is great number of works dealing with different applications of the Multi-Criteria Decision-Making (MCDM) methods, among which are also the following four methods: Multi-Attributive Border Approximation Area Comparison (**MABAC**), Multi-Attributive Ideal-Real Comparative Analysis (**MAIRCA**), COmbined COmpromise SOlution (**COCOSO**) and Range Of Value (**ROV**). These methods are, in fact, some of the MCDM methods that have not often been used in the literature so far. However, there are some of the recent presentations dealing with them. For instance, the MABAC method is presented in: Chakraborty et al. (2023), Widodo, Prathivi and Hadi (2023), Kalem and Akpinar (2022), Lukić (2021), Wang et al. (2020). When it is about the MAIRCA method, the following recent works can be mentioned: Hadian, Tabarestani and Pham (2022), Huy et al. (2022), Trung and Thinh (2021), Aksoy (2021), Altintaş (2021). As for the the COCOSO and ROV methods, there are also many papers. For the former one, the papers that can be listed are, e.g. Kesarwani, Verma and Xu (2024), Ersoy (2023), Panchagnula et al. (2022), Popović (2021), Lai et al. (2020). And, when it is about the latter one, the works such as Ersoy and Taslak (2023), Mitra (2021), Ersoy (2021), Rajput, Khan and Fazal (2020), Madić, Radovanović and Manić (2016) are worth mentioning.

As far as the author of this paper is aware, none of the works regarding application of the MABAC, MAIRCA, COCOSO and ROV methods have considered a topic regarding geodetic networks so far. Anyway, one can assert that there is not a great number of the studies dealing with the use of MCDM methods in the consideration of geodetic networks. However, recent studies that can be mentioned are e.g. Anđić (2010), Anđić and Đurović (2023) and Kobryń (2019). The first two deals with the use of the Multi-Criteria Optimization and

Compromise Ranking/Solution (VIKOR; in Serbian: VIšekriterijumsko KOmpromisno Rangiranje/Rešenje) (Opricović, 1986) method in finding the optimal design solution for a special-purpose geodetic network, whilst the third one provides a comparative analysis of application of the Analitics Hierarchy Process (AHP), Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETHEE), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and PROcessing TEchnique of Ratings for Ranking of Alternatives (PROTERRA) methods, also in finding the optimal design for a chosen geodetic network.

In view of the foregoing, the author presents multi-criteria decision-making based on the application of the methods MABAC, MAIRCA, COCOSO and ROV in this study. The results obtained using these four methods are compared to those obtained after applying the VIKOR method, that was chosen as a reference. All the five methods are based on the same data normalization method, i.e. Max-Min Linear Normalization, which makes them perfectly comparable. Such a choice for reference method is made due to the fact the VIKOR method is one of the most used MCDM methods in science and practice and, in addition, it represents a proven mathematical tool in special-purpose geodetic network optimization tasks, what was shown in Anđić (2010) and Anđić and Đurović (2023).

This study is based on nine criteria functions related to precision and reliability, and four alternative geodetic network solutions that are ranked using different preferential approaches.

II. MATERIAL AND METHODS

In this section, the theoretical basis of the MABAC, MAIRCA, COCOSO, ROV and VIKOR methods, the method chosen for calculating criteria weights, as well as basic characteristics of the system being optimized in the study are presented.

2.1 The MABAC method

This method considers a six-step procedure. The steps are the following (after Pamučar and Ćirović (2015)):

Step 1. Evaluation each of m alternatives by each of n criteria, i.e. obtaining values x_{ij}, with $i \in \{1,2,\dots,m\}$, $j \in \{1,2,\dots,n\}$, and then establishing the initial decision matrix (**X**) as follows:

$$\mathbf{X}_{m\times n} = (x_{ij})_{1\le i\le m, 1\le j\le n} = \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{pmatrix}; \tag{1}$$

Step 2. Determination of the normalized matrix (**N**) in the following way:

$$\mathbf{N}_{m\times n} = (n_{ij})_{1\le i\le m, 1\le j\le n} = \begin{pmatrix} n_{11} & \cdots & n_{1n} \\ \vdots & \ddots & \vdots \\ n_{m1} & \cdots & n_{mn} \end{pmatrix}, \tag{2}$$

where elements n_{ij} are calculated depending on whether the criterion is minimized (Eq. (2a)) or maximized (Eq. (2b)), which is to say:

$$n_{ij} = (x_{ij} - \max_{i\in\{1,2,\dots,m\}} x_{ij})/(\min_{i\in\{1,2,\dots,m\}} x_{ij} - \max_{i\in\{1,2,\dots,m\}} x_{ij}), \tag{2a}$$

when a less value is preferable, and

$$n_{ij} = (x_{ij} - \min_{i\in\{1,2,\dots,m\}} x_{ij})/(\max_{i\in\{1,2,\dots,m\}} x_{ij} - \min_{i\in\{1,2,\dots,m\}} x_{ij}), \tag{2b}$$

in the case when a greater value is preferable;

Step 3. Determination of the weighted matrix (**V**) as follows:

$$\mathbf{V}_{m\times n} = (v_{ij})_{1\le i\le m, 1\le j\le n} = \begin{pmatrix} w_1(n_{11}+1) & \cdots & w_n(n_{1n}+1) \\ \vdots & \ddots & \vdots \\ w_1(n_{m1}+1) & \cdots & w_n(n_{mn}+1) \end{pmatrix}, \tag{3}$$

where elements w_j, with $j \in \{1,2,\dots,n\}$, represent the criteria weight coefficients (see subsection 2.6);

Step 4. Determination of the border approximation area (BAA) matrix (**B**) according to the following equation:

$$\mathbf{B}_{m\times n} = (b_{ij})_{1\le i\le m, 1\le j\le n} = \begin{pmatrix} \prod_{i=1}^{m} \sqrt[m]{v_{i1}} & \cdots & \prod_{i=1}^{m} \sqrt[m]{v_{in}} \\ \vdots & \ddots & \vdots \\ \prod_{i=1}^{m} \sqrt[m]{v_{i1}} & \cdots & \prod_{i=1}^{m} \sqrt[m]{v_{in}} \end{pmatrix}, \tag{4}$$

where, as can be noticed, $(\forall (i,j) \in \{1,2,\dots,m\} \times \{1,2,\dots,n\})(b_{ij} \equiv b_j)$;

Step 5. Calculating the distance of the alternatives from the border approximation area, i.e. establishing the matrix **Q**:

$$\mathbf{Q}_{m\times n} = \mathbf{V}_{m\times n} - \mathbf{B}_{m\times n} = (q_{ij})_{1\le i\le m, 1\le j\le n} = \begin{pmatrix} q_{11} & \cdots & q_{1n} \\ \vdots & \ddots & \vdots \\ q_{m1} & \cdots & q_{mn} \end{pmatrix}; \tag{5}$$

Step 6. Ranking the alternatives using the following sums by rows of the matrix **Q**:

$$RQ_i = \sum_{j=1}^{n} q_{ij}, \tag{6}$$

where $i \in \{1,2,\dots,m\}$, $j \in \{1,2,\dots,n\}$.

The best alternative, i.e. the optimal solution, is the one having the greatest value of RQ.

2.2 The MAIRCA method

It is also about a six-step procedure. Considering the use of the same index denotations, the steps are as presented in the continuation (following Pamučar, Pejčić Tarle and Parezanović (2018)):

Step 1. The same as Step 1 of the MABAC method (see subsection 2.1);

Step 2. Defining preferences for the choice of alternatives.

Assuming the probability of choosing any particular alternative is not taken into account, the preferences for the selection of individual alternatives are equal and calculated as follows:

$$(\forall(i,j) \in \{1,2,\dots,m\} \times \{1,2,\dots,n\})(P_{ij} = 1/m), \tag{7}$$

whereby $\sum_{i=1}^{m} P_{ij} = 1$;

Step 3. Determination of the theoretical ratings matrix ($\mathbf{T_p}$) as shown below:

$$\mathbf{T}_{\mathbf{p},m\times n} = \left(t_{p,ij}\right)_{1\le i\le m, 1\le j\le n} = \left(P_{ij}w_j\right)_{1\le i\le m, 1\le j\le n} = \begin{pmatrix} w_1/m & \cdots & w_n/m \\ \vdots & \ddots & \vdots \\ w_1/m & \cdots & w_n/m \end{pmatrix}, \tag{8}$$

where it can be seen that $(\forall(i,j) \in \{1,2,\dots,m\} \times \{1,2,\dots,n\})(t_{p,ij} \equiv t_{p,j})$;

Step 4. Determination of the real ratings matrix ($\mathbf{T_p}$) according to the following:

$$\mathbf{T}_{\mathbf{r},m\times n} = \left(t_{r,ij}\right)_{1\le i\le m, 1\le j\le n} = \left(t_{p,ij}n_{ij}\right)_{1\le i\le m, 1\le j\le n} = \begin{pmatrix} t_{p,1}n_{11} & \cdots & t_{p,n}n_{1n} \\ \vdots & \ddots & \vdots \\ t_{p,1}n_{m1} & \cdots & t_{p,n}n_{mn} \end{pmatrix}, \tag{9}$$

whereby factors n_{ij}, with $i \in \{1, 2, \dots, m\}$, $j \in \{1, 2, \dots, n\}$, are calculated as in Step 2 of the MABAC method (see subsection 2.1, Eq. (2a) and Eq. (2b));

Step 5. Calculation of the total gap matrix (**G**) as follows:

$$\mathbf{G}_{m\times n} = \mathbf{T}_{\mathbf{p},m\times n} - \mathbf{T}_{\mathbf{r},m\times n} = \left(g_{ij}\right)_{1\le i\le m, 1\le j\le n} = \begin{pmatrix} g_{11} & \cdots & g_{1n} \\ \vdots & \ddots & \vdots \\ g_{m1} & \cdots & g_{mn} \end{pmatrix}; \tag{10}$$

Step 6. Ranking the alternatives using the following sums by rows of the matrix **G**:

$$RG_i = \sum_{j=1}^{n} g_{ij}, \tag{11}$$

where $i \in \{1, 2, \dots, m\}$, $j \in \{1, 2, \dots, n\}$.

Here, the best alternative is the one that the least value of RG is related to.

2.3 The COCOSO method

The COCOSO method is carried out in the following five steps (according to Yazdani et al. (2019)):

Step 1. The same as Step 1 of the MABAC method (see subsection 2.1);

Step 2. The same as Step 2 of the MABAC method (see subsection 2.1);

Step 3. Calculation of the weighted comparability sequences' sum (S_i) and the power-weighted comparability sequence (P_i) for each alternative using the following equations:

$$S_i = \sum_{j=1}^{n} w_j n_{ij}, \tag{12}$$

$$P_i = \sum_{j=1}^{n} n_{ij}^{w_j}, \tag{13}$$

where $i \in \{1, 2, \dots, m\}$, $j \in \{1, 2, \dots, n\}$;

Step 4. Calculation of three aggregate evaluation scores with the aim to produce relative performance scores of the alternatives. It is done as follows:

$$k_{a,i} = (S_i + P_i)/\sum_{i=1}^{m}(S_i + P_i), \tag{14}$$

$$k_{b,i} = S_i \Big/ \min_{i\in\{1,2,\dots,m\}} S_i + P_i \Big/ \min_{i\in\{1,2,\dots,m\}} P_i, \tag{15}$$

$$k_{c,i} = [\lambda S_i + (1-\lambda)P_i] \Big/ [\lambda \max_{i\in\{1,2,\dots,m\}} S_i + (1-\lambda) \max_{i\in\{1,2,\dots,m\}} P_i], \quad 0 \le \lambda \le 1, \tag{16}$$

with $\lambda = 0.5$ (herein) and $i \in \{1, 2, \dots, m\}$, $j \in \{1, 2, \dots, n\}$;

Step 5. Ranking the alternatives using the final prioritisation of them that is calculated as shown below:

$$k_i = \sqrt[3]{k_{a,i}k_{b,i}k_{c,i}} + (k_{a,i} + k_{b,i} + k_{c,i})/3. \tag{17}$$

The optimal solution is represented by the alternative with the greatest value of k.

2.4 The ROV method

This method considers a three-step procedure. These steps are the following (after Yakowitz, Lane and Szidarovszky (1993)):

Step 1. The same as Step 1 of the MABAC method (see subsection 2.1);

Step 2. The same as Step 2 of the MABAC method (see subsection 2.1);

Step 3. Ranking the alternatives using the weighted comparability sequences' sum, S_i, with $i \in \{1, 2, ..., m\}$, for each alternative (as done in Step 3 of the COCOSO method, see Eq. (14)), and choosing the best alternative solution. Namely, the one with the highest value of S represents the optimal solution, when it comes to application of this method.

2.5 The VIKOR method

This method is applied in the following seven steps (after Opricović (1998)):

Step 1. The same as Step 1 of the MABAC method (see subsection 2.1);

Step 2. The same as Step 2 of the MABAC method, with the only difference implying switched using of Eqs. (2a) and (2b). Namely, Eq. (2a) is now used for a criterion function to be maximized and Eq. (3) for a criterion function to be minimized (see subsection 2.1);

Step 3. Calculation of the S_i values according to Eq. (12), given in Step 3 of the COCOSO method (see subsection 2.3);

Step 4. Calculation of the R_i values as follows.

$$R_i = \begin{cases} \max\limits_j (w_j n_{ij}), & \text{if } R_i = \max\limits_i R_i \text{ for less than two } i \text{ indices} \\ \max\limits_j (w_j n_{ij}) + \left(S_i - \max\limits_i R_i\right)/100, & \text{if } R_i = \max\limits_i R_i \text{ for two or more } i \text{ indices} \end{cases}; \tag{18}$$

Step 5. Calculation of the measure Q_i which is the VIKOR method ranking based on in the following way:

$$Q_i = \upsilon QS_i + (1 - \upsilon) QR_i, \tag{19}$$

where υ represents the weight of the strategy of fulfilling most of the criteria (the value of 0.5 is assumed) and

$$QS_i = \left(S_i - \min_i S_i\right) / \left(\max_i S_i - \min_i S_i\right), \tag{19a}$$

$$QR_i = \left(R_i - \min_i R_i\right) / \left(\max_i R_i - \min_i R_i\right); \tag{19b}$$

Step 6. Forming three ranking list, namely:

- First ranking list, according to QS-values (first-ranked alternative has the least QS-value),
- Second ranking list, according to QR-values (first-ranked alternative has the least QR-value),
- Third, compromise ranking list, according to Q-values (first-ranked alternative is the one having the least Q-value);

Step 7. Checking whether the required conditions are met and decision making.

To consider alternative a_i the best (optimal) solution, it must be first-ranked on the compromise ranking list (according to Q-values), having the least Q-value, and, at the same time, fulfill the following conditions (Opricović, 1998):

Condition C1. The first-ranked alternative on the compromise ranking list for $\upsilon = 0.50$, denoted as $a^{(1)}$, must have a "sufficient advantage" over the alternative from the next position (the second-ranked one), denoted as $a^{(2)}$, which implies that

$$Q_{a^{(2)}} - Q_{a^{(1)}} \geq \min(0.25;\ 1/(m - 1)), \tag{20}$$

Condition C2. The first-ranked alternative on the compromise ranking list for $\upsilon = 0.50$, i.e. $a^{(1)}$, must have a "sufficiently stable" first position. It is about the case when at least one of the following requirements is fulfilled:

(1) $a^{(1)}$ is first-ranked on the first ranking list;
(2) $a^{(1)}$ is first-ranked on the second ranking list;
(3) $a^{(1)}$ is first-ranked on the third ranking list (for $\upsilon = 0.25$ and $\upsilon = 0.75$).

Conclusions are made as follows (Anđić and Đurović, 2023):

- *If $a^{(1)}$ fulfills both conditions (C1 and C2), it is considered the only and best solution*;
- *If $a^{(1)}$ does not fulfill only the condition C2, it is considered not "sufficiently" better than $a^{(2)}$, and then a set of compromise solutions is formed consisting of these two alternatives*;
- *If $a^{(1)}$ does not fulfill only the condition C1 or both conditions (C1 and C2), it is considered not "sufficiently" better than $a^{(2)}$ and any other alternative, denoted as $a^{(k)}$, on the list that fulfills*:

$$Q_{a^{(k)}} - Q_{a^{(1)}} < \min(0.25;\ 1/(m - 1)), \tag{21}$$

and then a set of compromise solutions is formed so that it includes $a^{(1)}$, $a^{(2)}$ and any other alternative $a^{(k)}$ for which the above inequality is valid.

2.6 The method of determining the criteria weights used in the study

The SWING Weighting Technique (SWING) method (von Winterfeldt and Edwards, 1986) was chosen to be used herein. The method is applied through the following four steps:

Step 1. The criteria are ranked according to preference, i.e. with regard to importance, by assigning a number of points to each of them (n-tuple $(b_1, b_2, ..., b_n)$ is established);

Step 2. The most important criterion is assigned the greatest number of points ($\max\{b_1, b_2, ..., b_n\}$);

Step 3. Numbers of points are assigned to each of the remaining criteria in such way to provide an appropriate representation of the degree of their importance relative to the most important criterion (from Step 2);
Step 4. Calculation of criterion weight value for each criterion as the ratio of the number of points assigned to that criterion to the total number of points, namely:

$$w_j = b_j / \sum_{j=1}^{n} b_j. \tag{22}$$

In this study, the author uses four preference approaches (see section IV), each of which is represented by a particular nonuple of criteria weight coefficients, $(w_1, w_2, \ldots, w_9)$.

III. DESCRIPTION OF THE SYSTEM TO BE OPTIMIZED HEREIN

It is about a tower geodetic micro-network (in the following text: geodetic network) that is to be optimized in this study. The tower is located in Montenegro and discretized by nine control points, marked by special bolts. These points are denoted herein as 1a, 2a, 3a, 1b, 2b, 3b, 1c, 2c and 3c (see Figs. 1 to 4).

The geodetic network is represented by four acceptable variants (with varying number of reference points, n_r), that fulfill all requirements and constraints preset, which are presented in subsection 3.2. These variants take on the role of four competitive alternative solutions (in the following text: alternative) to be ranked. The alternatives are presented separately through subsections 3.3, 3.4, 3.5 and 3.6.

It is assumed that all reference points are materialized by pillars, each of which is with a built-in forced-centering device on the top. That way, instrument and signal (i.e. prism) centering error at a pillar is reduced to a negligible value and, therefore, not considered in calculations. On the other hand, there is not any impact of centering a signal on the horizontal direction measurements, taken from the pillars to the control points, since the notched crosses on the bolt heads are targeted.

3.1 Mathematics of geodetic network adjustment in brief

The adjustment of the geodetic network is performed using Least Squares method (for a detailed insight into the method, see e.g. Perović (2005)) and starts by forming two main matrices, namely $\mathbf{A}$ (design matrix) and $\mathbf{P}$ (wieght matrix) in a well-known way. Then, normal equation coefficient matrix ($\mathbf{N}$), cofactor matrix for the estimates of unknowns ($\mathbf{Q}_{\hat{\mathbf{x}}}$), cofactor matrix for the estimates of values of measured lengths and horizontal directions ($\mathbf{Q}_{\hat{\mathbf{l}}}$), cofactor matrix for the estimates of corrections of measured lengths and horizontal directions ($\mathbf{Q}_{\hat{\mathbf{v}}}$) and redundancy matrix ($\mathbf{R}$) are calculated.
After obtaining the singular matrix $\mathbf{N}$ as

$$\mathbf{N} = \mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}, \tag{23}$$

the cofactor matrix $\mathbf{Q}_{\hat{\mathbf{x}}} = \mathbf{N}^{+}$ (i.e. a pseudoinverse of $\mathbf{N}$) is extracted from the inverse of the matrix formed by expanding the matrix $\mathbf{N}$ by a datum constraint matrix $\mathbf{R}'$, i.e

$$\begin{pmatrix} \mathbf{N} & \mathbf{R}' \\ \mathbf{R}'^{\mathrm{T}} & \mathbf{0} \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{N}^{+} & (\mathbf{R}'^{+})^{\mathrm{T}} \\ \mathbf{R}'^{+} & \mathbf{0} \end{pmatrix}. \tag{24}$$

Herein, the datum constraint matrix is chosen to provide a minimal trace of the cofactor matrix for estimates of the unknown coordinates for all of $n_p = n_r + n_c = n_r + 9$ points in the geodetic network (due to a negligible influence of unknown parameters related to deviations of the horizontal circle zero from north, n_{zd} of them in total, they are not considered), and is obtained as follows:

$$\mathbf{R}'^{\mathrm{T}} = \left(\underbrace{\begin{matrix} 1/\sqrt{n_p} & 0 & \cdots & 1/\sqrt{n_p} & 0 \\ 0 & 1/\sqrt{n_p} & \cdots & 0 & 1/\sqrt{n_p} \\ -\xi_1 & \eta_1 & \cdots & -\xi_{n_p} & \eta_{n_p} \end{matrix}}_{2n_p} \quad \underbrace{\begin{matrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{matrix}}_{n_{zd}} \right), \tag{25}$$

where

$$\eta_j = (Y_{j,0} - \sum_{j=1}^{n_p} Y_{j,0}/n_p) / \sqrt{\sum_{j=1}^{n_p} (Y_{j,0} - \bar{Y}_0)^2 + \sum_{j=1}^{n_p} (X_{j,0} - \bar{X}_0)^2}, \tag{25a}$$

$$\xi_j = (X_{j,0} - \sum_{j=1}^{n_p} X_{j,0}/n_p) / \sqrt{\sum_{j=1}^{n_p} (Y_{j,0} - \bar{Y}_0)^2 + \sum_{j=1}^{n_p} (X_{j,0} - \bar{X}_0)^2}. \tag{25b}$$

Then, we calculate the remaining cofactor matrices ($\mathbf{Q}_{\hat{\mathbf{l}}}$ and $\mathbf{Q}_{\hat{\mathbf{v}}}$), and redundancy matrix ($\mathbf{R}$), as follows:

$$\mathbf{Q}_{\hat{\mathbf{l}}} = \mathbf{A}\mathbf{N}^{+}\mathbf{A}^{\mathrm{T}} = \mathbf{A}\mathbf{Q}_{\hat{\mathbf{x}}}\mathbf{A}^{\mathrm{T}}, \tag{26}$$

$$\mathbf{Q}_{\hat{\mathbf{v}}} = \mathbf{P}^{-1} - \mathbf{Q}_{\hat{\mathbf{l}}} = \mathbf{P}^{-1} - \mathbf{A}\mathbf{Q}_{\hat{\mathbf{x}}}\mathbf{A}^{\mathrm{T}}, \tag{27}$$

$$\mathbf{R} = \mathbf{Q}_{\hat{\mathbf{v}}}\mathbf{P} = (\mathbf{P}^{-1} - \mathbf{A}\mathbf{Q}_{\hat{\mathbf{x}}}\mathbf{A}^{\mathrm{T}})\mathbf{P} = \mathbf{E} - \mathbf{A}\mathbf{Q}_{\hat{\mathbf{x}}}\mathbf{A}^{\mathrm{T}}\mathbf{P}. \tag{28}$$

3.2 Requirements and constraints preset

The main requirement the geodetic network should have fulfilled is related to minimal movements of each of the nine control points that must be detected in the geodetic network figure congruence testing between two epochs of measurements. These movement of the control point c is calculated as follows:

$$dp_c = \sigma_0\sqrt{\lambda}/\sqrt{\mathbf{c}_c^{\mathrm{T}}\mathbf{Q}_{\mathbf{d}}^{+}\mathbf{c}_c}, \quad (29)$$

whereby σ_0 the is square root of an adopted dispersion coefficient *a priori*, λ is the non-centrality parametar, $\mathbf{c}_c^{\mathrm{T}} = [0 \quad 0 \quad \cdots \quad \sin\theta_c \quad \cos\theta_c \quad \cdots \quad 0 \quad 0]$, with trigonometric terms at the places corresponding to the places of the coordinates y_c and x_c, respectively, and $\mathbf{Q}_{\mathbf{d}}^{+}$ is the pseudoinverse of the cofactor matrix that is, considering the same observation plan in two consecutive epochs (which means $\mathbf{Q}'_{\hat{\mathbf{x}}} = \mathbf{Q}''_{\hat{\mathbf{x}}} \equiv \mathbf{Q}_{\hat{\mathbf{x}}}$), calculated as $\mathbf{Q}_{\mathbf{d}} = \mathbf{2Q}_{\hat{\mathbf{x}}}$.

In this study, minimal movements that must have been detected in control points were the following:

- For control points 1a, 2a, 3a: 24 mm, 32 mm and 24 mm, respectively;
- For control points 1b, 2b, 3b: 24 mm, 32 mm and 24 mm, respectively;
- For control points 1c, 2c, 3c: 24 mm, 32 mm and 24 mm, respectively.

The test power in detecting predetermined minimal movement (dp_c^{pd}) is obtained using

$$(1-\beta)_c = \mathrm{normsdist}\left(dp_c^{pd}\sqrt{\mathbf{c}_c^{\mathrm{T}}\mathbf{Q}_{\mathbf{d}}^{+}\mathbf{c}_c}/\sigma_0 - t_{1-\alpha_{tab}/2}\right), \quad (30)$$

with $\alpha_{tab} = 0.02367$ representing the significance level derived from the tabular value (Perović, 2005) for the non-centrality parametar used in the test of congruence for 2D control point position, for adopted global test power and significance level of 0.80 and 0.05, respectively, and figure rank (degrees of freedom) equal to 2.

As for the constraints, six of them are introduced (with each variant eastablished for the geodetic network) in this study, namely:

$$(\forall r \in \{1,2,\ldots,n_r\})(A_r/B_r < 2); \quad (31)$$

$$(\forall c \in \{1\mathrm{a}, 2\mathrm{a}, 3\mathrm{a}, 1\mathrm{b}, 2\mathrm{b}, 3\mathrm{b}, 1\mathrm{c}, 2\mathrm{c}, 3\mathrm{c}\})(A_c/B_c < 3.5); \quad (32)$$

$$(\forall i \in \{1, 2, \ldots, n_m\})(r_{ii} \geq 0.20), \quad n_m = n_l + n_{hd}; \quad (33)$$

$$(\forall i \in \{1, 2, \ldots, n_m\})\left(|G_i^*| = \left(t_{1-\beta_0} + t_{1-\alpha_0/2}\right)\sigma_i/\sqrt{r_{ii}} < 7.64\sigma_i\right); \quad (34)$$

$$(\forall i \in \{1, 2, \ldots, n_m\})((1-\beta_0)_i \geq 0.80); \quad (35)$$

$$(\forall c \in \{1\mathrm{a}, 2\mathrm{a}, 3\mathrm{a}, 1\mathrm{b}, 2\mathrm{b}, 3\mathrm{b}, 1\mathrm{c}, 2\mathrm{c}, 3\mathrm{c}\})((1-\beta)_c \geq 0.80), \quad (36)$$

where, in addition to the previously introduced, the following denotations are present:

- r, c, i – a reference point in the geodetic network, a control point on the tower body and the ordinal number of a measurement, respectively;
- $A_{r(c)}$, $B_{r(c)}$ – the semi-major axis and semi-minor axis of the standard error ellipse in the point $r(c)$, respectively;
- n_r, n_l, n_{hd}, n_m – the number of reference points, the number of measured lengths, the number of measured horizontal directions, the total number of measurements, respectively;
- r_{ii}, $|G_i^*|$, σ_i – the redundancy coefficient for the measurement i, the detectable marginal gross-error value in the measurement i (for $1-\beta_0 = 0.80$, $\alpha_0 = 0.01$ and $r_{ii} = 0.20$, the limit value of $7.64\sigma_i$ is obtained), the standard deviation of the measurement i, respectively;
- $t_{1-\beta_0}$, $t_{1-\alpha_0/2}$ – the normal distribution quantiles for the test power $(1-\beta_0)$ and significance level (α_0) for one-dimensional hypotheses in the *Data Snooping Test*;
- $(1-\beta_0)_i$ – the test power in detecting gross-error limit value in the measurement i.

3.3 Criteria functions and alternatives to be ranked

Nine criteria functions are used with each alternative. These functions are:

- **CF1:** *Sum of the cofactors for the estimates of the coordinates of the control points*:

$$\sum_{c=1}^{n_c} \mathrm{tr}(\mathbf{Q}_{\hat{\mathbf{x}},c}), \quad (37)$$

  where $\mathbf{Q}_{\hat{\mathbf{x}},c}$ is a submatrix of the matrix $\mathbf{Q}_{\hat{\mathbf{x}}}$ related to the control point c;

- **CF2:** *Maximal ratio of semi-major axis to semi-minor axis of the standard error ellipse within the control points*:

$$\max(A_c/B_c), \quad (38)$$

  whereby:

$$A_c = Q_{\hat{x}_c\hat{x}_c} + Q_{\hat{y}_c\hat{y}_c} + \sqrt{\left(Q_{\hat{x}_c\hat{x}_c} - Q_{\hat{y}_c\hat{y}_c}\right)^2 + 4Q_{\hat{y}_c\hat{x}_c}^2}, \quad (38a)$$

$$B_c = Q_{\hat{x}_c\hat{x}_c} + Q_{\hat{y}_c\hat{y}_c} - \sqrt{\left(Q_{\hat{x}_c\hat{x}_c} - Q_{\hat{y}_c\hat{y}_c}\right)^2 + 4Q_{\hat{y}_c\hat{x}_c}^2}; \quad (38b)$$

- **CF3:** *Maximal control point standard positional error*:

$$\max(m_{p,c}) = \max\sqrt{\sigma_0^2(Q_{\hat{y}_c\hat{y}_c} + Q_{\hat{x}_c\hat{x}_c})}; \tag{39}$$

- **CF4:** *Minimal value obtained for the test power in detecting gross error limit value in the measurement*:
$$\min(1-\beta_0)_i, \tag{40}$$
whereby
$$(1-\beta_0)_i = \text{normsdist}\left(7.64\sqrt{r_{ii}} - t_{1-\alpha_0/2}\right); \tag{40a}$$
- **CF5:** *Sum of the minimal detectable marginal gross-error value in the measured lengths and minimal detectable marginal gross-error value in the measured horizontal directions*:
$$\min|G_{i_l}^*| + \min|G_{i_{hd}}^*|, \tag{41}$$
with
$$|G_{i_l}^*| = \left(t_{1-\beta_0} + t_{1-\alpha_0/2}\right)\sigma_{i_l}/\sqrt{r_{i_l i_l}}, \quad i_l \in \{1,2,\ldots,n_l\}, \tag{41a}$$
$$|G_{i_{hd}}^*| = \left(t_{1-\beta_0} + t_{1-\alpha_0/2}\right)\sigma_{i_{hd}}/\sqrt{r_{i_{hd}i_{hd}}}, \quad i_{hd} \in \{1,2,\ldots,n_{hd}\}; \tag{41b}$$
- **CF6:** *Deviation of the average redundancy coefficient from the related adopted optimal value*:
$$\bar{r} - r_{opt} = \bar{r} - 0.40, \tag{42}$$
with $\bar{r}$ obtained as follows:
$$\bar{r} = \sum_{i=1}^{n_m}(\mathbf{R})_{ii}/n_m = \sum_{i=1}^{n_m}(\mathbf{E} - \mathbf{A}\mathbf{Q}_{\hat{\mathbf{x}}}\mathbf{A}^{\mathrm{T}}\mathbf{P})_{ii}/n_m; \tag{42a}$$
- **CF7:** *Sum of influences on adjusted observations*:
$$\mathrm{tr}(\mathbf{P}\mathbf{Q}_{\hat{\mathbf{l}}}\mathbf{P})/n_m = \mathrm{tr}(\mathbf{P}\mathbf{A}\mathbf{Q}_{\hat{\mathbf{x}}}\mathbf{A}^{\mathrm{T}}\mathbf{P})/n_m = \sum_{i=1}^{n_m} P_i^2 Q_{\hat{l}_i\hat{l}_i}/n_m; \tag{43}$$
- **CF8:** *Cook-Perović's distance average value*:
$$\overline{CP} = \sum_{i=1}^{n_m} CP_i/n_m = \sum_{i=1}^{n_m} t_{1-\alpha_0/2}^2(1-r_{ii})\mathrm{r}(\mathbf{Q}_{\hat{\mathbf{x}}})^{-1} r_{ii}^{-1}/n_m; \tag{44}$$
- **CF9:** *Maximal value among the minimal detectable movements in the control points between two epochs*:
$$\max(dp_c) = \max\left(\sigma_0\sqrt{\lambda}/\sqrt{\mathbf{c}_c^{\mathrm{T}}\mathbf{Q}_{\mathbf{d}}^{+}\mathbf{c}_c}\right). \tag{45}$$

Only the criteria function **CF4** is maximized and the remaining ones are minimized. Their values by each alternative solution are given in Table 2.

To show how design differs from one to another alternative, Table 1 is given. Namely, it shows main design characteristics for all alternatives (denoted as A, B, C and D), established as acceptable solutions for the geodetic network of tower considered in this study.

Table 1: Comparison of main design characteristics of the alternatives

Parameter	Values			
	Alternative A	**Alternative B**	**Alternative C**	**Alternative D**
Number of reference points	6	6	5	4
Number of control points	9	9	9	9
Total number of points	15	15	14	13
Number of length measurements	15	11	9	12
Number of horizontal direction measurements	69	57	56	48
Total number of measurements	**84**	**68**	**65**	**60**
Number of unknowns	36	36	33	30
Network datum defect	3	3	3	3
Degrees of freedom	51	35	35	33

All the alternatives imply using of a total station providing accuracy of $10''$ for horizontal directions and $2\ \mathrm{mm} + 2\ \mathrm{mm/km}$ for lengths, e.g. Leica TC410ℂ (Leica Geosystems, 2016). Horizontal directions are all measured in one gyrus.

An adequate graphical presentation of the alternatives A, B, C and D, including the corresponding stndard error ellipses, is provided by Figs. 1 to 4.

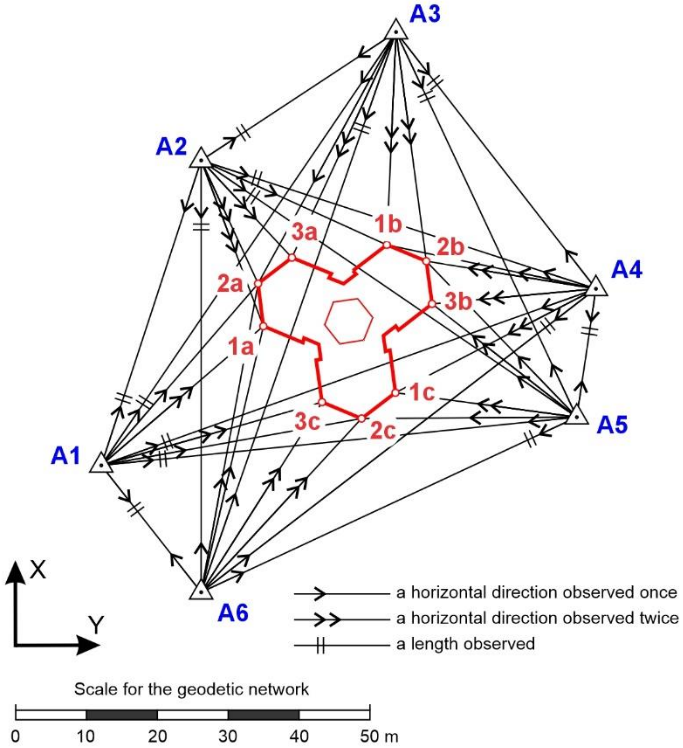

Figure 1: The Dajbabska Gora Tower geodetic network design for alternative A

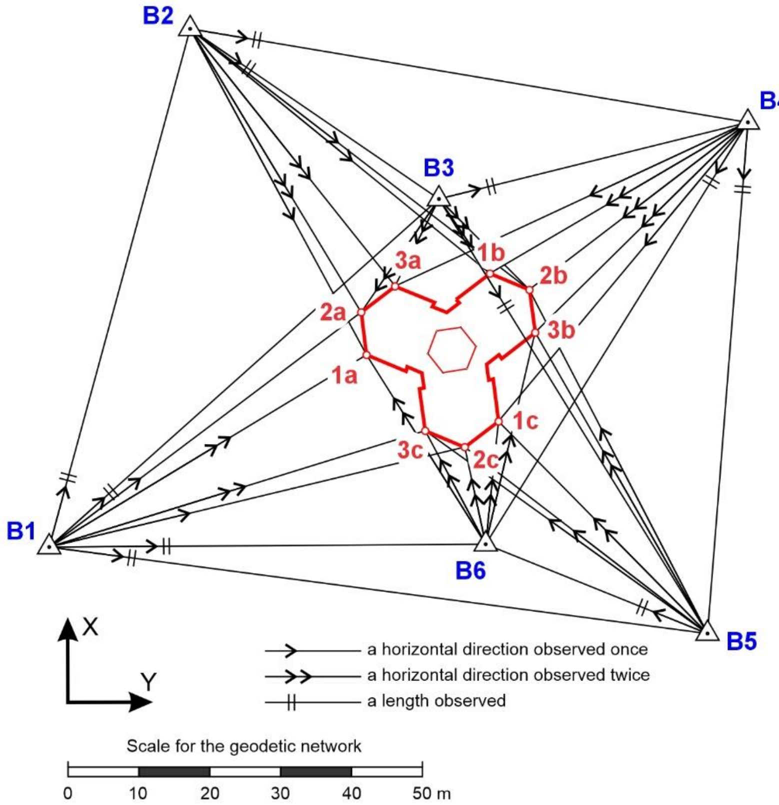

Figure 2: The Dajbabska Gora Tower geodetic network design for alternative B

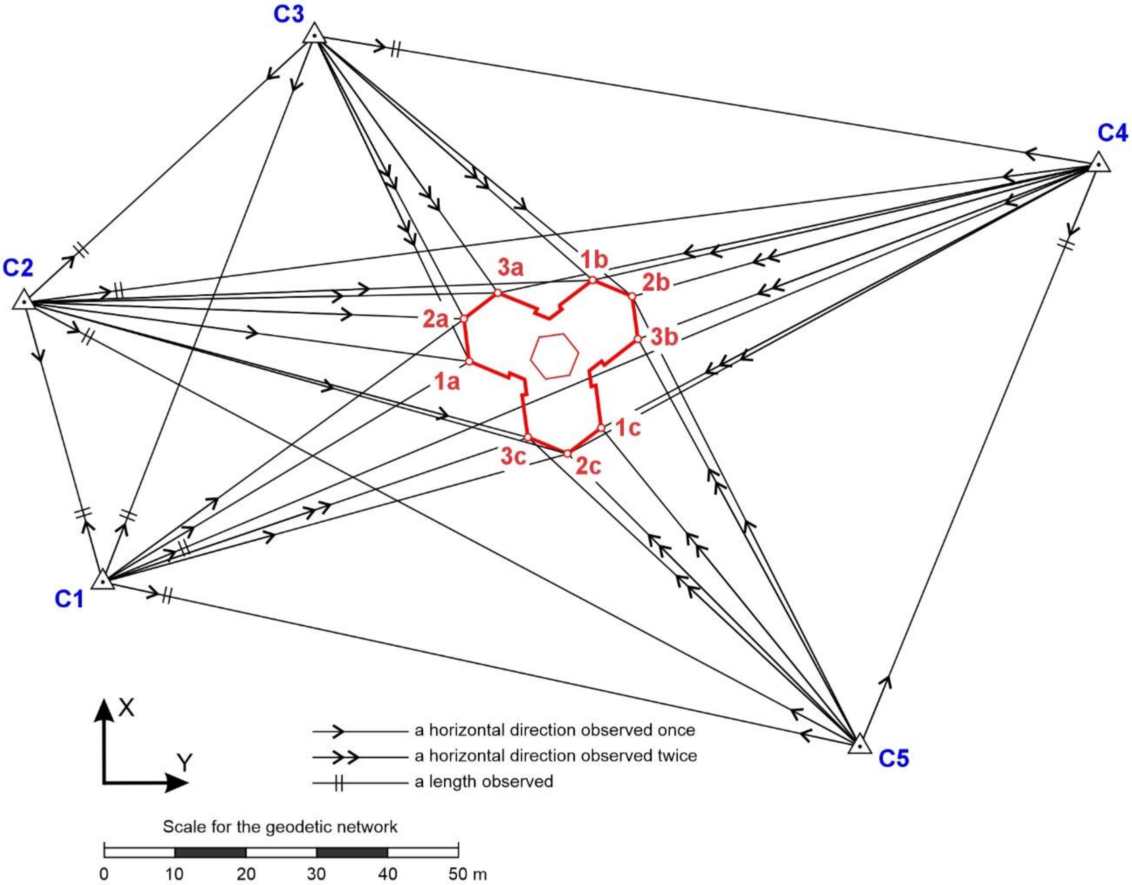

Figure 3: The Dajbabska Gora Tower geodetic network design for alternative C

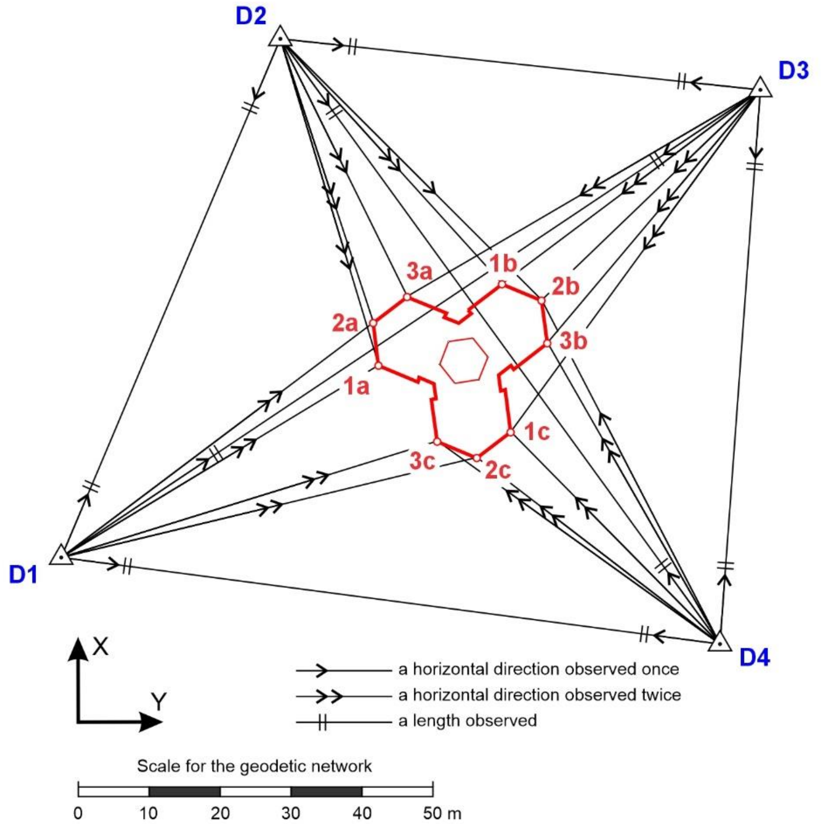

Figure 4: The Dajbabska Gora Tower geodetic network design for alternative D

IV. RESULTS AND DISCUSSION

The values of the nine criteria functions, given by Eqs. (37) to (45), for each alternative are presented in Table 2.

Table 2: Values of criteria functions by alternatives

Criteria function	Values			
	Alternative A	**Alternative B**	**Alternative C**	**Alternative D**
CF1	0.3883 mm^2 *	0.4531 mm^2	1.1438 mm^2 **	0.7806 mm^2
CF2	3.1 **	2.7	2.1 *	2.3
CF3	2.8 mm	2.6 mm *	4.3 mm **	3.7 mm
CF4	0.897	0.855 **	0.857	0.966 *
CF5	46.5 *	49.0	48.5	50.3 **
CF6	0.21 **	0.11 *	0.14	0.15
CF7	1.5063 *	2.5322 **	1.9159	1.8746
CF8	0.1475 *	0.2065	0.2145 **	0.2104
CF9	14.85 mm *	14.88 mm	23.17 mm **	17.25 mm

* *Best criterion value*
** *Worst criterion value*

Considering the values given in Table 2, initial ranking lists of the four alternatives can be established for each of the nine criteria functions (Table 3).

Table 3: Initial ranking lists of the alternatives

Position on the initial ranking list	Initial ranking lists of the alternatives								
	CF1	**CF2**	**CF3**	**CF4**	**CF5**	**CF6**	**CF7**	**CF8**	**CF9**
First	A	C	B	D	A	B	A	A	A
Second	B	D	A	A	C	C	D	B	B
Third	D	B	D	C	B	D	C	D	D
Fourth	C	A	C	B	D	A	B	C	C

The alternatives in this study are, as previously said in subsection 2.6, ranked using four preferential approaches. These approaches, denoted as **Preferential approach I**, **II**, **III** and **IV** (hereinafter: **PA-I**, **PA-II**, **PA-III** and **PA-IV**, respectively), and used for all the four methods herein, imply the following nonuples of points and weight coefficients, associated to the nonuple of the crtiteria functions:

- **PA-I**: Points: $(1, 1, 1, 1, 1, 1, 1, 1, 1)$,
  Weight coefficients: $(0.1111, 0.1111, 0.1111, 0.1111, 0.1111, 0.1111, 0.1111, 0.1111, 0.1111)$;
- **PA-II**: Points: $(4, 4, 4, 6, 4, 6, 4, 4, 8)$,
  Weight coefficients: $(0.0909, 0.0909, 0.0909, 0.1364, 0.0909, 0.1364, 0.0909, 0.0909, 0.1818)$;
- **PA-III**: Points: $(3, 6, 5, 7, 6, 6, 3, 3, 5)$,
  Weight coefficients: $(0.0682, 0.1364, 0.1136, 0.1591, 0.1364, 0.1364, 0.0682, 0.0682, 0.1136)$;
- **PA-IV**: Points: $(5, 4, 5, 10, 3, 12, 5, 5, 7)$,
  Weight coefficients: $(0.0893, 0.0714, 0.0893, 0.1786, 0.0536, 0.2143, 0.0893, 0.0893, 0.1250)$.

4.1 Results after applying the MABAC, MAIRCA, COCOSO and ROV methods

As stated in subsections 2.1, 2.2, 2.3 and 2.4, the MABAC, MAIRCA, COCOSO and ROV methods use the same data normalization method, with elements n_{ij}, $i \in \{1, 2, \dots, m\}$, $j \in \{1, 2, \dots, n\}$, calculated the same way, as shown by Eqs. (2a) and (2b). Therefore, all of these four methods have the same normalized matrix. It is presented as Table 4.

Table 4: Normalized matrix values (MABAC, MAIRCA, COCOSO, ROV)

Alternative	Normalized values of the criteria functions								
	CF1	**CF2**	**CF3**	**CF4**	**CF5**	**CF6**	**CF7**	**CF8**	**CF9**
A	1	0	0.8524	0.3752	1	0	1	1	1
B	0.9143	0.4276	1	0	0.3551	1	0	0.1188	0.9973
C	0	1	0	0.0171	0.4894	0.7430	0.6008	0	0
D	0.4807	0.8490	0.3429	1	0	0.6182	0.6410	0.0604	0.7123

Due to the rather limited scope of the presentation in the paper, only the final results for the four methods are shown. Namely, the main results obtained using the PA-I, PA-II, PA-III and PA-IV are, respectively, presented in Tabs. 5, 6, 7 and 8.

Table 5: The final results after applying the MABAC, MAIRCA, COCOSO and ROV methods (PA-I)

Method		**Final results (PA-I)**			
		Alternative A	**Alternative B**	**Alternative C**	**Alternative D**
MABAC	RQ_i	0.2279	0.0707	-0.1473	0.0587
	Rank:	***First***	Second	Fourth	Third
MAIRCA	RG_i	0.0770	0.1163	0.1708	0.1193
	Rank:	***First***	Second	Fourth	Third
COCOSO	k_i	2.6336	2.3311	1.5145	2.5007
	Rank:	***First***	Third	Fourth	Second
ROV	S_i	0.6920	0.5348	0.3167	0.5227
	Rank:	***First***	Second	Fourth	Third

Table 6: The final results after applying the MABAC, MAIRCA, COCOSO and ROV methods (PA-II)

Method		**Final results (PA-II)**			
		Alternative A	**Alternative B**	**Alternative C**	**Alternative D**
MABAC	RQ_i	0.2004	0.1000	-0.1800	0.0923
	Rank:	***First***	Second	Fourth	Third
MAIRCA	RG_i	0.0815	0.1066	0.1766	0.1085
	Rank:	***First***	Second	Fourth	Third
COCOSO	k_i	2.6756	2.4682	1.5024	2.6408
	Rank:	***First***	Third	Fourth	Second
ROV	S_i	0.6741	0.5737	0.2937	0.5660
	Rank:	***First***	Second	Fourth	Third

Table 7: The final results after applying the MABAC, MAIRCA, COCOSO and ROV methods (PA-III)

Method		**Final results (PA-III)**			
		Alternative A	**Alternative B**	**Alternative C**	**Alternative D**
MABAC	RQ_i	0.1487	0.0781	-0.1143	0.0972
	Rank:	***First***	Third	Fourth	Second
MAIRCA	RG_i	0.0972	0.1149	0.1630	0.1101
	Rank:	***First***	Third	Fourth	Second
COCOSO	k_i	2.4444	2.2960	1.5010	2.5317
	Rank:	Second	Third	Fourth	***First***
ROV	S_i	0.6111	0.5405	0.3481	0.5597
	Rank:	***First***	Third	Fourth	Second

Table 8: The final results after applying the MABAC, MAIRCA, COCOSO and ROV methods (PA-IV)

Method		**Final results (PA-IV)**			
		Alternative A	**Alternative B**	**Alternative C**	**Alternative D**
MABAC	RQ_i	0.1249	0.1054	-0.1511	0.1322
	Rank:	Second	Third	Fourth	***First***
MAIRCA	RG_i	0.1026	0.1075	0.1716	0.1008
	Rank:	Second	Third	Fourth	***First***
COCOSO	k_i	2.5012	2.4437	1.4984	2.6565
	Rank:	Second	Third	Fourth	***First***
ROV	S_i	0.5895	0.5700	0.3136	0.5969
	Rank:	Second	Third	Fourth	***First***

As for the results shown in Tabs. 5, 6, 7 and 8, first of all, it can be concluded the MABAC, MAIRCA and ROV methods provided the same ranking results, regardless of the preferential approach used, but with a difference between the approaches. Namely, the order of alternatives is the same in PA-I and PA-II, but different when comparing to PA-III and PA-IV, which are also defferent from one another. Also, when it comes to these three methods, alternative A is the first on the ranking lists related to PA-I, PA-II and PA-III, but the second when it comes to PA-IV, where alternative D took the leading position.

The COCOSO method provided the same ranking results as the remaining three methods used herein but only in PA-IV. However, when it is about PA-I and PA-II, this method produced rankings that differed from those obtained by the MABAC, MAIRCA and ROV methods, but only in terms of switching the second and fourth positions (shaded cells in Tabs. 5 and 6). In addition, after applying PA-III, the COCOSO method declared alternative D the optimal and alternative A the second-ranked (shaded cells in Table 7), which is the only difference compared to what was obtained using the remaining three methods, when A was the optimal and D the second-ranked.

On the other hand, it turned out that alternative C is the worst, no matter what method and preferential approach was used in ranking.

With the aim to analyse advantages between alternatives A, B, C and D, the following measures (relative distances) for the four methods used are introduced:

$$\Delta RQ_{(r-s)} = 100\% \cdot \left(RQ_{a^{(r)}} - RQ_{a^{(s)}}\right)/\left(RQ_{a^{(1)}} - RQ_{a^{(4)}}\right), \quad (46)$$

$$\Delta RG_{(r-s)} = 100\% \cdot \left(RG_{a^{(r)}} - RG_{a^{(s)}}\right)/\left(RG_{a^{(1)}} - RG_{a^{(4)}}\right), \quad (47)$$

$$\Delta k_{(r-s)} = 100\% \cdot \left(k_{a^{(r)}} - k_{a^{(s)}}\right)/\left(k_{a^{(1)}} - k_{a^{(4)}}\right), \quad (48)$$

$$\Delta S_{(r-s)} = 100\% \cdot \left(S_{a^{(r)}} - S_{a^{(s)}}\right)/\left(S_{a^{(1)}} - S_{a^{(4)}}\right), \quad (49)$$

where $r \in \{1,2,3,4\}$, $s \in \{2,3,4\}$, $r < s$ and $a^{(1)}$, $a^{(r)}$, $a^{(s)}$, $a^{(4)}$ representing alternatives that took 1^{st}, r^{th}, s^{th}, 4^{th} positions on the rank list, respectively.

Using the values for S_i, Q_i, k_i and CS_i, given in Tabs. 5 to 8, the relative distances (46), (47), (48) and (49) are calculated and presented in Tabs. 9 to 12.

Table 9: Advantages higher-ranked than other alternatives for all preferential approaches (MABAC)

Preferential approach	Relative distance					
	$\Delta RQ_{(1-2)}$	$\Delta RQ_{(1-3)}$	$\Delta RQ_{(1-4)}$	$\Delta RQ_{(2-3)}$	$\Delta RQ_{(2-4)}$	$\Delta RQ_{(3-4)}$
PA-I	41.9%	45.1%	100.0%	3.2%	58.1%	54.9%
PA-II	26.4%	28.4%	100.0%	2.0%	73.6%	71.6%
PA-III	19.6%	26.8%	100.0%	7.3%	80.4%	73.2%
PA-IV	2.6%	9.5%	100.0%	6.9%	97.4%	90.5%

Table 10: Advantages higher-ranked than other alternatives for all preferential approaches (MAIRCA)

Preferential approach	Relative distance					
	$\Delta RG_{(1-2)}$	$\Delta RG_{(1-3)}$	$\Delta RG_{(1-4)}$	$\Delta RG_{(2-3)}$	$\Delta RG_{(2-4)}$	$\Delta RG_{(3-4)}$
PA-I	41.9%	45.1%	100.0%	3.2%	58.1%	54.9%
PA-II	26.4%	28.4%	100.0%	2.0%	73.6%	71.6%
PA-III	19.6%	26.8%	100.0%	7.3%	80.4%	73.2%
PA-IV	2.6%	9.5%	100.0%	6.9%	97.4%	90.5%

Table 11: Advantages higher-ranked than other alternatives for all preferential approaches (COCOSO)

Preferential approach	Relative distance					
	$\Delta k_{(1-2)}$	$\Delta k_{(1-3)}$	$\Delta k_{(1-4)}$	$\Delta k_{(2-3)}$	$\Delta k_{(2-4)}$	$\Delta k_{(3-4)}$
PA-I	11.9%	27.0%	100.0%	15.2%	88.1%	73.0%
PA-II	3.0%	17.7%	100.0%	14.7%	97.0%	82.3%
PA-III	8.5%	22.9%	100.0%	14.4%	91.5%	77.1%
PA-IV	13.4%	18.4%	100.0%	5.0%	86.6%	81.6%

Table 12: Advantages higher-ranked than other alternatives for all preferential approaches (ROV)

Preferential approach	Relative distance					
	$\Delta S_{(1-2)}$	$\Delta S_{(1-3)}$	$\Delta S_{(1-4)}$	$\Delta S_{(2-3)}$	$\Delta S_{(2-4)}$	$\Delta S_{(3-4)}$
PA-I	41.9%	45.1%	100.0%	3.2%	58.1%	54.9%
PA-II	26.4%	28.4%	100.0%	2.0%	73.6%	71.6%
PA-III	19.6%	26.8%	100.0%	7.3%	80.4%	73.2%
PA-IV	2.6%	9.5%	100.0%	6.9%	97.4%	90.5%

Based on what is shown in Tabs. 9 to 12, it can be noticed that the MABAC, MAIRCA and ROV methods produce ranking lists with identical advantages of alternatives, which is an important fact, but the COCOSO method provide totally different relative distances.

4.2 Results after applying the VIKOR method

Going back to what is stated in subsection 2.5, it can be easily concluded the VIKOR method use the same data normalization method as the MABAC, MAIRCA, COCOSO and ROV methods, but switched calculation of elements n_{ij}, $i \in \{1,2,\dots,m\}$, $j \in \{1,2,\dots,n\}$. So, it is not about the same normalized matrix as that obtained in the remaining four methods. The normalized matrix obtained in using the VIKOR method is given as Table 13.

Table 13: Normalized matrix values (VIKOR)

Alternative	Normalized values of the criteria functions								
	CF1	**CF2**	**CF3**	**CF4**	**CF5**	**CF6**	**CF7**	**CF8**	**CF9**
A	0	1	0.1476	0.6248	0	1	0	0	0
B	0.0857	0.5724	0	1	0.6449	0	1	0.8812	0.0027
C	1	0	1	0.9829	0.5106	0.2570	0.3992	1	1
D	0.5193	0.1510	0.6571	0	1	0.3818	0.3590	0.9396	0.2877

The final results obtained applying the VIKOR method, for all the four preferential approaches, are presented in Tabs. 14, 15, 16 and 17.

Table 14: The final results after applying the VIKOR method (PA-I)

Measure	Final results (PA-I)			
	Alternative A	**Alternative B**	**Alternative C**	**Alternative D**
QS_i	0	0.4188	1	0.4510
Rank:	First	Second	Fourth	Third
QR_i	0	0.4188	1	0.4510
Rank:	First	Second	Fourth	Third
$Q_i(\upsilon = 0.50)$	0	0.4188	1	0.4510
Rank:	First	Second	Fourth	Third
$Q_i(\upsilon = 0.25)$	0	0.4188	1	0.4510
Rank:	First	Second	Fourth	Third
$Q_i(\upsilon = 0.75)$	0	0.4188	1	0.4510
Rank:	First	Second	Fourth	Third

Table 15: The final results after applying the VIKOR method (PA-II)

Measure	Final results (PA-II)			
	Alternative A	**Alternative B**	**Alternative C**	**Alternative D**
QS_i	0	0.2640	1	0.2842
Rank:	First	Second	Fourth	Third
QR_i	0.5000	0.5000	1	0
Rank:	Second	Third	Fourth	First
$Q_i(\upsilon = 0.50)$	0.2500	0.3820	1	0.1421
Rank:	Second	Third	Fourth	First
$Q_i(\upsilon = 0.25)$	0.3750	0.4410	1	0.0711
Rank:	Second	Third	Fourth	First
$Q_i(\upsilon = 0.75)$	0.1250	0.3230	1	0.2132
Rank:	First	Third	Fourth	Second

Table 16: The final results after applying the VIKOR method (PA-III)

Measure	Final results (PA-III)			
	Alternative A	**Alternative B**	**Alternative C**	**Alternative D**
QS_i	0	0.2684	1	0.1956
Rank:	First	Third	Fourth	Second
QR_i	0	1	0.8802	0
Rank:	First	Fourth	Third	Second
$Q_i(\upsilon = 0.50)$	0	0.6342	0.9401	0.0978
Rank:	First	Third	Fourth	Second
$Q_i(\upsilon = 0.25)$	0	0.8171	0.9102	0.0489
Rank:	First	Third	Fourth	Second
$Q_i(\upsilon = 0.75)$	0	0.4513	0.9701	0.1467
Rank:	First	Third	Fourth	Second

Table 17: The final results after applying the VIKOR method (PA-IV)

Measure	Final results (PA-IV)			
	Alternative A	**Alternative B**	**Alternative C**	**Alternative D**
QS_i	0.0259	0.0947	1	0
Rank:	Second	Third	Fourth	First
QR_i	1	0.7261	0.7027	0
Rank:	Fourth	Third	Second	First
$Q_i(\upsilon = 0.50)$	0.5130	0.4104	0.8513	0
Rank:	Third	Second	Fourth	First
$Q_i(\upsilon = 0.25)$	0.7565	0.5682	0.7770	0
Rank:	Third	Second	Fourth	First
$Q_i(\upsilon = 0.75)$	0.2694	0.2525	0.9257	0
Rank:	Third	Second	Fourth	First

By means of the results shown in Tabs. 14, 15, 16 and 17, the following decisions for the four preferential approaches are made:

- **PA-I**: The only and best solution, i.e. the optimal solution, is ***alternative A***;
- **PA-II**: The solution is the set of compromise solutions consisting of ***alternative D, A and B*** (respecting the order on the compromise ranking list, established according to Q-values);
- **PA-III**: The solution is the set of compromise solutions consisting of ***alternative A and D*** (respecting the order on the compromise ranking list, established according to Q-values);
- **PA-IV**: The only and best solution, i.e. the optimal solution, is ***alternative D***.

Alternative C is the worst here as well, regardless of the preferential approach used.

4.3 Comparison of the optimization results obtained using the MABAC, MAIRCA, COCOSO and ROV method with the reference ones provided by applying the VIKOR method

A comparative presentation of the results shown in subsections 4.1 and 4.2 is given at the same place, in Table 18. Although this does not avoid some repetitions from the previous subsections, the author decided on such a presentation way, because it is very representative.

Table 18: Comparison of the final results obtained using the MABAC, MAIRCA, COCOSO and ROV methods with those obtained using the VIKOR method

Preferential approach	Optimal solutions declared in the four methods analized				Optimal solution declared in the reference method
	MABAC	**MAIRCA**	**COCOSO**	**ROV**	**VIKOR**
PA-I	A	A	A	A	A
PA-II	A	A	A	A	Compromise solutions set: {D, A, B}
PA-III	A	A	D	A	Compromise solutions set: {A, D}
PA-IV	D	D	D	D	D

After looking at Table 18, it is noticed that in PA-I and PA-IV all the methods, including the reference (VIKOR) method as well, provided the same optimal solution (alternative A).
In PA-II, the MABAC, MAIRCA, COCOSO and ROV methods declared alternative A the optimal, and that alternative is included in the set of compromise solutions in VIKOR decision making.
On the other hand, when it is about PA-III, the MABAC, MAIRCA and ROV methods declared alternative A the optimal, while it was alternative D in the case of the COCOSO method. However, both alteranitive A and D are included in the set of compromise solutions in VIKOR decision making.
Finaly, all the five methods declared the same optimal solution (alternative D).

V. CONCLUSION AND DIRECTIONS FOR FURTHER RESEARCH

On the basis of the outcome of the research that was carried out in this study, one can be said that the MABAC, MAIRCA and ROV, provided exactly the same results. The results provided by the COCOSO method were, however, slightly different, but not significantly. Namely, the results obtained in PA-I, PA-II and PA-III showed that changing the criteria weights affected the COCOSO results differently, so a difference in the ranking list comparing to the corresponding ranking lists provided applying the three remaining methods arose. Anyway, each of these four methods can be used as a very efficient tool in finding the best solution for the design of a geodetic micro-network.

When comparing the final results obtained using the four mentioned methods, on the one hand, with the results obtained using the VIKOR method, on the other hand, it is easy to conclude that the results match well. The only difference between the application of the four methods and application of the reference (VIKOR) method is reflected in the fact that the VIKOR method is more sensitive to the choice of the optimal solution, because it implies checking if the first-ranked alternative solution has a sufficient advantage over the second-ranked one, as well as whether it is sufficiently stable on the first position on the ranking list, which can lead to a final solution choice that is represented by a set of compromise solutions.

As a final conclusion, the author would like to state that, apart from the already tested VIKOR method in earlier studies, each of the four remaining methods from this research can also be used successfully in the optimization of geodetic micro-networks due to their effectiveness in solving a number of conflicting requirements, especially those related to precision and reliability, at once.

Further research directions could imply analysing results provided by some other comparable MCDM methods, not yet applied in geodetic tasks. For instance, a future study could deal with the Simple Additive Weighting (SAW), Preference Selection Index (PSI), Measurement Alternatives and Ranking According to COmpromise Solution (MARCOS) and COmbinative Distance-Based ASsessment (CODAS**)** methods, each of which use linear normalization method for normalizing data.

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