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Application of a Mathematical Model that describes the importance of Randomness and Frequency to the Volatility Stock Prices and its positive Correlation to the Volume of Buy and Sell Orders

Luka Mënalla[1], Helios Peti[2], Monika Mënalla[3]
[1]*Turgut Ozal College, Durrës Albania*
[2]*University of Tirana, Faculty of Natural Sciences, Informatics Department, Tirana Albania*
[3]*Kastrati Group Company, Durrës Albania*
Corresponding Author:Luka Mënallalukamenalla@gmail.com

ABSTRACT
Many researchers in their articles suppose that there is an indirect suggestion supposing Brownian Movement is the main force behind the stock price Volatility, but is it true? The following conclusions are drawn from an analysis made to the daily stock prices and their volumes:
1) There is a strong positive correlation between the logarithms of the moving averages of stock prices and their volumes, but on the other hand, the price and volume do not have the same volatility. 2) Price and volume Volatility is chaotic in nature, but the time series are not kind of Brownian movements: this is because they replace the daily value by 1 when stock price rises or -1 when it falls, comparing the results to the previous day's value. 3) The difference between the volume on the previous day and that on the current day is cyclic. As a result of this, we applied differential equations for stock prices, The equations applied incorporate terms for randomness and periodicity. Randomness and periodicity are very important for stock price volatility to be sustainable; also, the stock prices repeat themselves over a certain period of time. We found a positive correlation between Stock prices and the volumes of buy and sell orders. 4) There are other macroeconomic and microeconomic factors that affect the Stock Prices. The Stock Prices are also affected by the Investor's psychologic behavior and their willingness to take a certain level of risk.

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I. INTRODUCTION

The Although the world now is armed with a sophisticated technology, it is very difficult to forecast the stock prices. Analysis used to forecast the stock prices are fundamental analysis and technical analysis. Fundamental analysis means analyzing data that are supposed to impact the price of a stock. While technical analysis is based on the historical price data of a stock. The technical analysis states that the market price is affected by macroeconomic factors and actors, which influence the stock market. As a result, there is no need to take new economic developments into account because they are already priced into a given security. Technical analysts suppose that prices move in trends, and that they repeat themselves. The widely used technical indicators are moving averages, that are supposed to smooth price data. The main difficulty to predict stock prices is their randomness. The concept of using a Brownian movement process is that of independent increases or decreases in Stock Prices. This supposes that the present price does not influence future prices. From this the conclusion drown is that The Brownian movement is not accurate for explaining stock price volatility. Also, it is concluded that there is a positive relationship between stock price and volume. Any movement of price up or down with a big volume is seen as a high influence, it is the opposite with the same move with a smaller volume. As we highlighted above, the trend of the stock prices is random. But as volume is a measure of the interest sellers and buyers have in the stock, it can give a better picture of stock price volatility. The volatility of the Stock Price is also influenced by other macroeconomic and microeconomic factors and actors which imply risk. The main factors taken into consideration are exchange rates taken as a reference in a previous study, and other macroeconomic factors like interest rate, inflation, term structure, spread or other factors (represented by the S&P 500). The Governments try to use different policy tools and policy instruments on the other hand to affect the market prices. These include, Monetary Policy, Fiscal Policy, Exchange Rate Policy, External Trade Policy which also affect the Stock Price and Stock Returns. Investors who take a higher risk are rewarded for this risk with a higher return. So, measuring the volatility of the Stock Prices means measuring the firm's value. A firm's value is the market value of the whole business. It is a sum of the claims of all claimants, creditors and equity holders, which include equity holders of preferred and common stock. Considering the fact, we are am from Albania, we wanted to see application of the study even in the Albanian market. Considering the fact that Albania has not companies listed in the Stock Exchange, we took reference to a previous study made for GDP growth and Okun Misery Index using DATA from INSTAT. GDP growth is a sign of a good performance of the companies in Albania, which in other words make the Stock prices of the company go up. This explains the fact that other factors may be taken as reference.

II. THE OBJECTIVES OF THE STUDY

The objective of the of this study is to find the driving force of the volatility in the price of a stock using a mathematical model. The data used are from Stock Prices of Companies listed in NYSE, NASDAQ and Japan Stock Exchange based in the previous studies showed in the section of references show us the following conclusions.

1. It is noticed that in certain days the prices are not always connected to a change in the volume.
2. And it is also noticed that the present price is not correlated with the previous stock price, nor correlated with the volume, but it is correlated to the volume and prices over several days.
3. There exists a high positive correlation between the prices and its volumes from the logarithm of moving average.
4. The Stock prices volatility depends on other macroeconomic and microeconomic factors. There are small countries like Albania with no Stock Exchange market, where other variables can be taken into consideration.

In order to construct a mathematical model, we used the following statement: Volatility in price and volume is random and it changes time after time.

III. MATHEMATICAL MODEL

The Lanchester's combat [11] approach to predict the stock prices of different companies in different stock exchange markets is used by a range of researchers. The primary system linear non autonomous differential equations model was as follows:

$$\begin{cases} \frac{dx}{dt} = -ay + f(t) \\ \frac{dy}{dt} = -bx + g(t) \end{cases}$$

where $x(t)$ and $y(t)$ refer to the amount of ready-for-use product items for sale of firm A and B correspondingly, $f(t)$ and $g(t)$ refer to their respective increase and decrease rates, while $ay(t)$, $bx(t)$ correspond to the handy product items' rates. In this paper, we propose a next system differential equations model to predict stock prices. The data analysis was based on a 3×3 differential equations model. The experimental results by showed an equal distribution and the same median as the real data, demonstrating a good fit of the model. Thus, the method used can be applied in stock price prediction cases. However, this kind of model is used in stock price prediction cases. Additionally, other models for stock could take into account various factors such as a country's economy, political structure, or psychological factors, which comprise macroeconomic factors and monetary, fiscal or other policies the governments use.

Stock prices, the volume of buy and sell orders are defined by *P*, *B*, and *S*, respectively. Relationships among prices, volume of buy or sell orders are widely used in research literature. The formulas of these relationships are shown below:

$$\frac{dP}{dt} = a(B - S) - r_1 \cdot RND_1 P + t_1 \cdot sign(RND_4 - t_2) \quad (1)$$

$$\frac{dB}{dt} = b\left(1 - r_2 \cdot RND_2 sin\,(2\pi\sigma t)\right)P - B \quad (2)$$

$$\frac{dS}{dt} = c\left(1 - r_3 \cdot RND_3 sin\,(2\pi\sigma t)\right)P - S, \quad (3)$$

where $a, b, c, t_1, t_2, r_1, r_2, r_3$ and σ are positive constants,

$$sign(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

and $RND_1, RND_2, RND_3, RND_4$ are uniformly distributed random numbers in the interval $(0,1)$, each of which changes every time step on solving these differential equations (ODE system) in MATLAB using function *ode45*. In equation (1), $B - S$ represents the volume V.

If $t_1 \cdot sign\ (RND_4 - t_2) > 0$, prices are increasing, if $t_1 \cdot sign\ (RND_4 - t_2) = 0$, prices display no trend, and if $t_1 \cdot sign\ (RND_4 - t_2) < 0$, prices are decreasing. In Equations (1), (2), and (3), $-r_1 \cdot RND_1\, P$, $-B$, and $-S$ are necessary to prevent these variables from diverging to infinity. A preliminary study (see Osaka [7]) indicated that RND_1, RND_2, RND_3 and RND_4 are required for P, B, and S to fluctuate randomly in various changes, similar to real data. This finding suggests that continuous up-and-down changes are the result of the randomness of prices.

As $t_1 = 0$, Equation (1) is expressed as follows,

$$\frac{dP}{dt} = aV - r_1 \cdot RND_1 \cdot P. \tag{4}$$

Equation (2) minus Equation (3) is expressed as follows,

$$\frac{dB}{dt} - \frac{dS}{dt} = b\left(1 - r_2 \cdot RND_2 sin\ (2\pi\sigma t)\right)P - B - c\left(1 - r_3 \cdot RND_3 sin\ (2\pi\sigma t)\right)P + S.$$

So, we have:

$$\frac{dV}{dt} = [RNsin(2\pi\sigma t)(cr_3D_3 - br_2D_2) + (b - c)]P - V \tag{5}$$

here $r_1 \cdot RND_1, RNsin\ (2\pi\sigma t)(cr_3D_3 - br_2D_2) + (b - c)$ are denoted as β, γ respectively.

Let's assume that β is distributed uniformly in (0, 0.4).Then Equations (4) and (5) are considered to be ordinary differential equations,

$$\begin{pmatrix} \frac{dP}{dt} \\ \frac{dV}{dt} \end{pmatrix} = \begin{pmatrix} -\beta & a \\ \gamma & -1 \end{pmatrix}\begin{pmatrix} P \\ V \end{pmatrix} \tag{6}$$

or same system as follows,

$$\begin{cases} \frac{dP}{dt} = -\beta P + aV \\ \frac{dV}{dt} = \gamma P - V. \end{cases}$$

As is known by this system,

$$\begin{cases} -\beta P + aV = 0 \\ \gamma P - V = 0, \end{cases}$$

we have fixed point is $(P^*, V^*) = (0,0)$ for $\gamma \neq \frac{\beta}{a}$.

The characteristic polynomial of Equation (6) is given by

$$(-\beta - \lambda)(-1 - \lambda) - a\gamma = 0$$

$$\lambda^2 + (\beta + 1)\lambda + \beta - a\gamma = 0 \tag{7}$$

$$\lambda = \frac{-(\beta + 1) \pm \sqrt{(\beta + 1)^2 - 4(\beta - a\gamma)}}{2} = \frac{-(\beta + 1) \pm \sqrt{(\beta - 1)^2 + 4a\gamma}}{2}. \tag{8}$$

Case 1.$(\beta - 1)^2 + 4a\gamma > 0$or $\gamma > \frac{-(\beta-1)^2}{4a}$then the equation has two different real eigenvalues, but if $\beta - a\gamma < 0$ or otherwise $\gamma > \frac{\beta}{a}$ one of two real eigenvalues is positive and the other is negative. Since β is distributed uniformly in (0, 0.4), then $0 < \frac{\beta}{a} < \frac{2}{5a}$. If $\gamma > \frac{2}{5a}$ one of the two eigenvalues is always positive.

As $\gamma = RNsin\ (2\pi\sigma t)(cr_3D_3 - br_2D_2) + (b - c)$ then $\frac{\gamma + c - b}{RN(cr_3D_3 - br_2D_2)} = sin\ (2\pi\sigma t)$.

Let's substitute $\delta = RN(cr_3D_3 - br_2D_2)$, then

$$\gamma = \delta sin\ (2\pi\sigma t) + b - c > \frac{2}{5a}.$$

So, for now $-1 < \delta < 1$, we can distinguish two cases.

If, $-1 \leq sin\ (2\pi\sigma t) < 0$ we have,

$$\delta < \frac{2 + 5a(c-b)}{5asin\ (2\pi\sigma t)} \leq \frac{-2}{5a} - (c-b) = a_1.$$

If, $1 \geq sin\ (2\pi\sigma t) > 0$ we have,

$$\delta > \frac{2 + 5a(c-b)}{5asin\ (2\pi\sigma t)} \geq \frac{2}{5a} + (c-b) = a_2.$$

Case 2.$(\beta - 1)^2 + 4a\gamma < 0$or $\gamma < \frac{-(\beta-1)^2}{4a}$then, the equation has two different complex eigenvalues.

Case 3.$(\beta - 1)^2 + 4a\gamma = 0$or $\gamma = \frac{-(\beta-1)^2}{4a}$then, real eigenvalues, both are equal to $\frac{-(\beta+1)}{2}$.

As $br_2 \cdot RND_2$ and $cr_3 \cdot RND_3$ are distributed uniformly in (0,1), the probability density function, $f(\delta)$, of $\delta = RN(cr_3D_3 - br_2D_2)$ is expressed as shown in Fig.1.

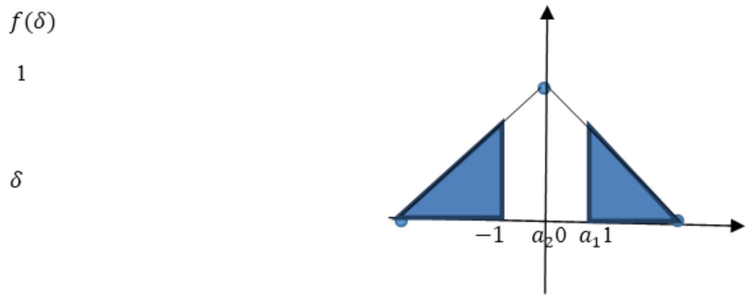

Figure1:Probability density function is the area of each blue triangle

The probability of $\delta \leq \frac{-2}{5a} - (c-b)$ is$\frac{\left(1+\frac{2}{5a}+c-b\right)^2}{2}$, and the probability of $\delta \geq \frac{2}{5a} + (c-b)$ is $\frac{\left(1+\frac{2}{5a}+c-b\right)^2}{2}$. The total time of $sin\ (2\pi\sigma t) > 0$ is 50% in any observation time. Similarly, the total time of $sin\ (2\pi\sigma t) < 0$ is 50% in any observation time. Hence the total time of $\gamma > \frac{2}{5a}$ is at most $\frac{\left(1+\frac{2}{5a}+c-b\right)^2}{2}$in any observation time.

Conversely, if $\gamma > \frac{-(\beta-1)^2}{4a}$ and $\gamma < \frac{2}{5a}$, the two eigenvalues are always negative real numbers.

If $\gamma < \frac{-(\beta-1)^2}{4a}$, the two eigenvalues are always complex numbers with negative real parts, as $-\frac{(\beta+1)}{2}$. In the same way as $\gamma > \frac{2}{5a}$, the total time of $\gamma < 0$ is at most $\frac{\left(1+\frac{2}{5a}+c-b\right)^2}{2}$ in any observation time.

In the remaining time, when $0 < \gamma < \frac{2}{5a}$, there are various combinations of the two eigenvalues: (A) one positive and the other negative, real number, (B) two negative real numbers, and (C) two complex numbers with negative real parts. In no cases do two positive real eigenvalues or two complex eigenvalues with positive real parts occur.

IV. DISCUSSION

We will classify our fixed pointdepending on the eigenvalues. Before we do this, we will briefly review fixed points. From the well-known theory of dynamical systems we know that, the classification of fixed points depends on the eigenvalues of the Jacobian matrix evaluated at the given fixed point. If both eigenvalues are negative, we have a *sink*, see case (B). If the eigenvalues are complex with negative real parts, then we have a *spiral sink*. If both eigenvalues are positive, we have a *source*. If the eigenvalues are complex with positive real parts, then we have a *spiral source*. If the eigenvalues are purely imaginary the fixed point of the linear system is a *center*. Trajectories about a center are closed orbits that neither spiral into a sink nor spiral out like a source. In this situation, the classification of fixed points based on the linear system is inconclusive. Another possibility is a system for which one eigenvalue is positive and one eigenvalue is negative. When this is the case, the fixed point is called a *saddle*, see case (A). A saddle is considered a semi stable fixed point since we have one set of straight line solutions approaching it and one set of straight line solutions moving away from it. A sink is a fixed point where all solutions near (P^*, V^*) approach (P^*, V^*) as $t \to \infty$. If no straight line solutions exist, then the system has complex eigenvalues. Thus we have trajectories of the form $x(t) = c_1e^{\varphi t}cos\omega t + c_2e^{\varphi t}sin\omega t$

where $\lambda = \varphi \pm i\omega$. A source is a fixed point where all solutions with an initial condition near (P^*, V^*) move away from (P^*, V^*)as $t \to \infty$. If $\varphi > 0$ the trajectories will approximate exponentially growing oscillations, thus solutions will spiral out from the fixed point. We call such a fixed point a spiral source. *A source is an unstable fixed point.* But we are in case (C), so if $\varphi < 0$, the trajectories will approximate exponentially decaying oscillations, thus solutions will spiral toward the fixed point. We call such a fixed point a spiral sink. *A sink is a stable fixed point*.

So, Stock prices$P$ does not diverge to infinity or converge to zero because the two eigenvalues are not always positive real numbers (or complex numbers whose real parts are positive numbers) and not always negative real numbers. As a result, P fluctuates within a certain range.

V. CONCLUSION

This study examines the relationship between prices and volume of stock Prices, it highlights the fact that price and volume fluctuations are random and independent, however, there is a significant positive correlation between the indicators for price and for the volumes. This correlation is influenced by macroeconomic and microeconomic factors. It is noted that the trade volume is cyclic, and this can be done by using frequency analysis, which has to do with the trade activity of the last days, especially to that of the previous day. A preliminary study suggests that several random variables (RND_1, RND_2, RND_3, and RND_4) are needed to simulate the daily fluctuations of prices (P), volume (B), and spreads (S) in order to reflect real data. These findings suggest that stock price fluctuations are random and periodic. Referring on theories of behavioral finance, investors often are subject to psychological influence, selling stocks to limit losses or buying back stocks in a repeated cycle based on the belief that prices will fall or rise. Some of the investors are risk takers and this is usually related to the risk premium they are willing to take. The risk factors are very important in the Stock Price volatility. So, the exchange rate fluctuations, interest rate, inflation, term structure, spread or other factors are very important for investors behavior. The study suggests that at times of volatility of the Stock Price, the company can choose the adequate hedging technique or derivative to profit from the volatility. This highlights a periodic factor in stock prices. The differential equations used in this study expresses the randomness and periodicity of the Stock Prices. It suggests that stock prices do usually move up and down within a certain circle. In fact, some technical analysts think that prices follow trends, research shows that price fluctuations are often stochastic in nature and occur without a clear upward or downward trend. Finally, if all the information that affects stock prices is known, they can be predicted. However, the randomness makes their prediction generally difficult. The randomness and Periodicity of the Stock prices is also affected by the risk factors which are usually related to the risk premium investors are willing to take and all other macroeconomic and microeconomic factors and actors.

The examination of daily prices and volumes for Stock prices in different Stock Exchange Markets in U.S.A and Japan using the mathematical Model above has led us to these conclusions:

1) Fluctuations in price and volume are random, and periodic and this has to do with the Volatility of the Stock Prices.

2) The difference between the trading volume of the previous day and that of the current day expresses the periodic pattern. In light of these observations, differential equations were developed to model stock prices which concluded that Stock Prices influenced by the volume of buy and sell orders. Simulations based on these equations demonstrate that both randomness and periodicity are very important for the Volatility in stock prices.

3) Stock Prices depend on the Volume of Buy and Sell orders. Usually, a high volume of buy and sell orders affects more a movement in the Stock Prices than a small volume.

4) Stock Prices are also affected by other macroeconomic and microeconomic factors, which imply risk. In this study we have aimed to clarify the conditions of the parameters within the mathematical model that allow stock prices to consistently fluctuate within a defined range. This exploration will identify which parameters have a significant influence on continuous fluctuations in stock prices within that range. Further Studies are needed to find out which other variables may impact stock Prices or correlate positively to the Stock Prices. These variables may be macroeconomic or microeconomic variables, dependent or independent variables. A further study we aim is to take these variables one by one and study their correlation to Stock Prices. Then to take all the factors as a whole and see which of them has the highest correlation and by which combination can risk be diversified away.

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