International Journal Of Engineering Research And Development
e- ISSN: 2278-067X, p-ISSN: 2278-800X, www.ijerd.com
Volume 20, Issue 2 (February. 2024), PP. 57-64

Condition of Time when Acceleration is Twice the Velocity in Linear Motion: SD Hypothesis

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ABSTRACT
A standard case of rectilinear motion states the condition when a particle is moving in a straight line and one is supposed to calculate, observe and manipulate among several scalar and vectorial factors which includes initial velocity, final velocity, time period, distance covered, displacement, acceleration, retardation and speed. All these factors are scaled from average to instantaneous level. This is a theoretical and Mathematical approach in order to show that in an event of rectilinear motion when a particle is moving in a straight line, there is a possible instant when the acceleration becomes twice the initial velocity. Not only a linear equation between acceleration and velocity is being established, but also, the condition of displacement and time and their proportion with acceleration is shown along with a formulated equation further. Experimental instances have been initiated and possible graphs and data have been plotted and discussed further. The paper caters a hypothetical scenario whose mathematical and theoretical feasibility is checked in order to validate a further study on this particular condition and developments can be made in real life scenario by its applications.

Keywords: *Linear Motion, Time, Kinematics, Classical Mechanics, SD hypothesis*

Date of Submission: 12-02-2024 Date of Acceptance: 26-02-2024

I. INTRODUCTION

Rectilinear motion has always been a gateway to Mechanics and a condition of a particle in motion where vector and scalar can be manipulated easily and almost all the vector and scalar equations have the same representation. It justifies the equations of motion which clarifies the relation among different vectorial and scalar parameters of motion, namely, velocity, speed, acceleration, displacement, distance and time. In this condition of rectilinear motion, a special condition exists where the particle has a uniform and non-zero acceleration; for example, a freely falling body is an instance of such motion. Focusing on such an instance of uniformly accelerated motion, there is a particular instance which shows that the acceleration of a body has the value which is twice its velocity. That particular moment of time can be calculated and this is a theoretical approach on what effect does this particular time bring on the further state of motion of the particle. The establishment of the relation gets possible by nurturing the second equation of motion which states

$$x = ut + \frac{1}{2}wt^2$$

This helps us to achieve the instant of time:

$$t = \left(\frac{\sqrt{|\vec{u}| + 4|\vec{x}|}}{2\sqrt{|\vec{u}|}} - \frac{1}{2}\right)$$

This is satisfied if and only if

$$|\vec{w}| = 2|\vec{u}|$$

The proof for the functions along with probable constraints and the pattern of motion of the particle after this particular time instant is being provided and explained further.

II. SYMBOLS AND THEIR REPRESENTATIONS

The following table contains the symbols and the physical/derived quantity that it represents:

Abbreviation	Represented quantity
t	Time
x	Distance ($\vec{x}$ is used for displacement and $\|\vec{x}\|$ is used for magnitude of displacement)
u,v	u represents initial speed and v represents final speed ($\vec{u}$ and $\vec{v}$ are used for representing initial and final velocities respectively whereas $\|\vec{u}\|$ and $\|\vec{v}\|$ represent magnitude of initial and velocities respectively)
w or $\|\vec{w}\|$	Magnitude of acceleration ($\vec{w}$ is used for representing acceleration)

III. ASSUMPTIONS

- Uniform acceleration is assumed throughout the hypothesis. Hence $\frac{dw}{dt} = 0$.
- The parameters are all instantaneous. This means, for a parameter r, we have considered
$$\lim_{r \to 0} \frac{\Delta r}{\Delta t}$$
- The particle is already in motion. Hence, t > 0 and x≠0. Also, instantaneous velocity $v_{inst} \neq 0$.
- Acceleration is twice the initial velocity and hence
$$\vec{w} = 2\vec{u}$$

IV. KINEMATICS AND TIME

The equations of motion that define the parameters of a particle at an interval or an instant involves the following five equations of motion [1]:

$$v = u + wt$$
$$x = ut + \frac{1}{2}wt^2$$
$$v^2 - u^2 = 2wx$$
$$x = vt - \frac{1}{2}wt^2$$
$$x = \frac{1}{2}(u + v)t$$

These equations can be validated vectorially for classical mechanics and have the following forms:

$$\vec{v} = \vec{u} + \vec{w}t$$
$$\vec{x} = \vec{u}t + \frac{1}{2}\vec{w}t^2$$
$$(\vec{v} \cdot \vec{v}) - (\vec{u} \cdot \vec{u}) = 2(\vec{w} \cdot \vec{x})$$
$$\vec{x} = \vec{v}t - \frac{1}{2}\vec{w}t^2$$
$$\vec{x} = \frac{1}{2}(\vec{u} + \vec{v})t$$

The equations can be very easily jotted down as functions of time and distinct differential equations can be created for these metrics

$$x = f(t)$$
$$v = f'(t)$$

(Here v denotes velocity in general and does not imply initial or final velocity)

$$w = f''(t)$$

Our assumption for this whole experiment is the condition that

$$w = 2u$$

This can also be interpreted as:

$$|\vec{w}| = 2|\vec{u}|$$

Or

$$\frac{\vec{w}}{\hat{w}} = 2\left(\frac{\vec{u}}{\hat{u}}\right)$$

Considering the scalar form, the first formation will be referred throughout the paper. Out of the five equations of motion [1], we consider the second equation of motion for the evaluation and will validate the condition by substituting the expression in the other equations of motion. For considering the particular instant, the instant is referred to SD time henceforth.

Hence, SD time can be referred to the particular instant when

$$|\vec{w}| = 2|\vec{u}|$$

is satisfied. Also, the hypothesis that is being assumed throughout is referred to as SD hypothesis throughout.

V. SD HYPOTHESIS

There are instances in nature where we can find movement of particles in such a way where acceleration is twice the velocity. The acceleration in such cases is not always uniform as the continuous force required for sustaining that magnitude will be required in a continuous frame and hence, currently, this hypothesis limits to uniformly accelerated motion only. As per the hypothesis, in a condition when acceleration is twice the velocity and the particle is already in motion, then the displacement is highly influenced by acceleration and not by time. Which means, time will not influence displacement or will have very low influence on displacement. The main point of studying this particular condition is that, we can provide such an acceleration to a body but what is the time required for a body to attain the velocity, provided acceleration is constant. Further, this can be used to correlate to real-life scenarios and calorimetric and thermodynamic studies can be done to develop mechanisms that can help in achieving SD velocity in SD time and can help in reframing the structure of fuel consumption mechanism especially in spaceflights.

Although the initial proof is done on such a simple condition, soon it will be imposed on more complex condition with non-uniform motion in order to study the patterns and move further on concluding the mechanical and thermodynamic constraints that can be faced while using SD hypothesis and also, we will come across the factors belonging to the same fields which we can overcome by using SD hypothesis.

VI. CONDITION OF VELOCITY AT SD TIME

Considering our hypothesis and applying it in the second equation of motion, where acceleration is substituted with the velocity expression, we get,

$$|\vec{x}| = |\vec{u}|t + \frac{1}{2}(2|\vec{u}|)t^2$$

Or,

$$|\vec{x}| = |\vec{u}|(t + t^2)$$

Or,

$$\frac{|\vec{x}|}{t(1+t)} = |\vec{u}|_{SD}$$

Considering SD hypothesis,

$$|\vec{w}|_{SD} = \frac{2|\vec{x}|}{t(1+t)}$$

These particular parameters can be called SD velocity and SD acceleration respectively and can be denoted as u_{SD} and w_{SD} respectively.

This hypothesis is valid if and only if the particle is in motion and cannot be applied in rest phase; i.e., t > 0. Also, for this phenomenon to be applicable, a particle cannot pass through origin at that instant, i.e., at t=t_{SD}, $|\vec{x}| \neq 0$.

Since the acceleration is constant, the magnitude of displacement can be written as

$$|\vec{x}| = f(t)$$

where

$$f(t) = \frac{|\vec{w}|(t + t^2)}{2}$$

Hence,

$$|\vec{x}|_{SD} = \frac{|\vec{w}|(t + t^2)}{2}$$

Also, the relation between acceleration and displacement

$$\vec{w} = \frac{d}{dt}\left(\frac{d(\vec{x})}{dt}\right)$$

Hence,

$$|\vec{w}| = \frac{d^2}{dt^2}(f(t))$$

Using this equation, the graph of SD displacement vs time is plotted further.

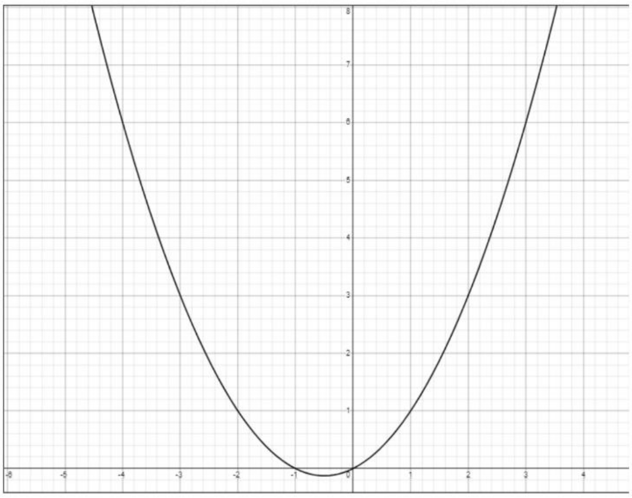

Figure 1: Plot for Displacement vs Time for SD Hypothesis

This graph has time t in x axis and displacement $|\vec{x}|$ in y axis and plots the variation with changes in displacement and time. The acceleration is kept constant throughout to obtain the result.

The following table can be concluded for studying the trend of values of the curve:

S No.	$\lvert\vec{w}\rvert$	t_i	$\lvert\vec{x}\rvert_i$
1	1.42	0.100	0.078
		0.200	0.170
		0.500	0.532
		0.950	1.315
		1.102	1.645
2	1.45	0.100	0.0798
		0.200	0.1740
		0.500	0.5438
		0.950	1.3431
		1.102	1.6794
3	5.1	0.100	0.281
		0.200	0.612
		0.500	1.912
		0.950	4.724
		1.102	5.907
4	6.1	0.100	0.3355
		0.200	0.7320
		0.500	2.2875
		0.950	5.6500
		1.102	7.0650
5	-0.5	0.100	-0.0280
		0.200	-0.0600
		0.500	-0.1875
		0.950	-0.4631
		1.102	-0.5791

Table 1

Table 1 contains the computations and results of displacement based on five chosen instants of time and five chosen values of acceleration. The cases of variation of acceleration can be compared by grouping them in a group of two or rather coupling them.

a. CASE 1: S NO. 1 AND 2:
Under CASE 1, the value of $|\vec{w}|$ has been changed in a very minor unit where $\Delta|\vec{w}|$=0.03. To analyse this case, we have to compare $|\vec{x}|_i$ with their respective t_i across the different observation numbers.

b. CASE 2: S NO. 3 AND 4:
CASE 2 shows the variation when the value of $|\vec{w}|$ has been changed to a significant extent. It can be seen how both the cases have been showcasing the variation and showing a proportion with a gentle slope.

c. CASE 3: S NO. 5 (RETARDATION):
This case is just for showing the condition of retardation and completes the analysis from all the aspects.

Based on Table 1, we can find the correlation between acceleration and displacement. Firstly, we need to find the standard deviation of acceleration and displacement separately. Using the population standard deviation formula

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}}$$

Where X_i is the reading, $\bar{X}$ is the arithmetic mean of the class and n is the number of readings or datapoints. Similarly, the following is used for covariance of both the classes; acceleration and displacement respectively:

$$Cov(X,Y) = \frac{\sum_{i=1}^{n}\left((X_i - \bar{X}) \times (Y_i - \bar{Y})\right)}{n}$$

Finally, the correlation coefficient [2] is obtained by

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sigma_X \times \sigma_Y}$$

Calculating the mean, standard deviation and covariance, we arrive at Table 2 which contains the corresponding metrics of the individual classes in a time-series distribution.

	$\lvert\vec{w}\rvert$	$\lvert\vec{x}\rvert$ **Time**	Output
Mean	2.714	0.100	0.14926
		0.200	0.3256
		0.500	5.0878
		0.950	2.5138
		1.102	3.14346
Standard Deviation	2.480319334	**Time**	Output
		0.100	2.2121079
		0.200	2.199534197
		0.500	2.131066559
		0.950	2.578982466
		1.102	3.1852248
Covariance		**Time**	Output
		0.100	0.33893236
		0.200	0.7383416
		0.500	2.30687216
		0.950	5.6982958
		1.102	7.12520556

Table 2

Using the outputs of Table 2, we can plot Table 3 which contains the correlation coefficient across different time intervals.

Table 3

Time(t_i)	**Correlation coefficient between $\|\vec{w}\|$ and $\|\vec{x}\|$**
0.100	0.06177306216
0.200	0.1353377715
0.500	0.4364343402
0.950	0.8908180366
1.102	0.901881983

Analyzing Table 1, Table 2 and Table 3, it leads us to the question: If we are focusing on kinematic frame of the event, what happens to time? Throughout, the factors like displacement, velocity and acceleration have been discussed but what about time? Well, so this leads us to the establishment of the equation of SD time.

VII. SD TIME: THE SPOTLIGHT OF SD HYPOTHESIS

Based on our findings of SD velocity, SD acceleration and SD displacement as well, we come up to the equation that will provide the exact time instant when the SD hypothesis is valid. For proceeding further, we will have to recall the second equation of motion and validate it using our SD hypothesis. Time will be firstly established as a function of velocity and displacement i.e.,

$$t = f(|\vec{x}|, |\vec{u}|)$$

Starting with second equation of motion and using $|\vec{w}|$=2$|\vec{u}|$ in it,

$$|\vec{x}| = |\vec{u}|t + \frac{1}{2}2|\vec{u}|t^2$$

Or, it can be written as

$$\frac{|\vec{x}|}{|\vec{u}|} = t + t^2$$

If we consider instantaneous displacement and velocity for the estimation, then we can consider this as a quadratic equation where $\frac{|\vec{x}|}{|\vec{u}|}$ is a constant. Hence, the time can be found by finding the roots of the quadratic equation. Using the expression for roots of a quadratic equation, we get,

$$t = \frac{-1 \pm \sqrt{1 + 4\frac{|\vec{x}|}{|\vec{u}|}}}{2}$$

This can be rearranged and written as,

$$t = \frac{-\sqrt{|\vec{u}|} \pm \sqrt{|\vec{u}| + 4|\vec{x}|}}{2\sqrt{|\vec{u}|}}$$

Simplifying further and referring time as SD time,

$$t_{SD} = \frac{\pm\sqrt{|\vec{u}| + 4|\vec{x}|}}{2\sqrt{|\vec{u}|}} - \frac{1}{2}$$

The rearrangement was made because clearly,

$$\left|\sqrt{|\vec{u}| + 4|\vec{x}|}\right| > \left|\sqrt{|\vec{u}|}\right|$$

Also, since we are considering time to be non-negative; $t_{SD}>0$ and hence, the equation of t_{SD} can be written as,

$$t_{SD} = \frac{\sqrt{|\vec{u}| + 4|\vec{x}|}}{2\sqrt{|\vec{u}|}} - \frac{1}{2}$$

This equation of t_{SD} marks the mathematical validity and the theoretical existence of such a time instant and we can further study its validity for intervals and non-uniform mechanical events, involving non-linear scenarios, which will be studied further for catering real-life incidents.

VIII. OBSERVATIONS

Calculations and estimations of the mechanical condition of the physical factors of a body during the validation of SD hypothesis shows that it is mathematically possible to obtain such a condition of velocity and acceleration at a particular position and at a particular time instant. Since instantaneous aspects have been used throughout, its occurrence can e seen at the instants that satisfy the equations of $|\vec{x}|_{SD}$, $|\vec{v}|_{SD}$, $|\vec{w}|_{SD}$ and t_{SD}. Another notable observation can be pointed by observing Table 3 which shows that the correlation between acceleration and displacement increases with time and hence showing how the displacement is getting influenced strongly with an increase of time. Also, one can say on this that the higher the time instant is considered for observation, more accurate will be the observation as the correlation is expected to increase steadily in that manner. In contrast, displacement is supposedly less influenced by time.

IX. GRAPHS

The graphs of velocity vs time, acceleration vs time and displacement vs acceleration are provided to observe the curve and proving the validation of the equations in the subsequent figures. For graphing, all constant factors are set to 1.

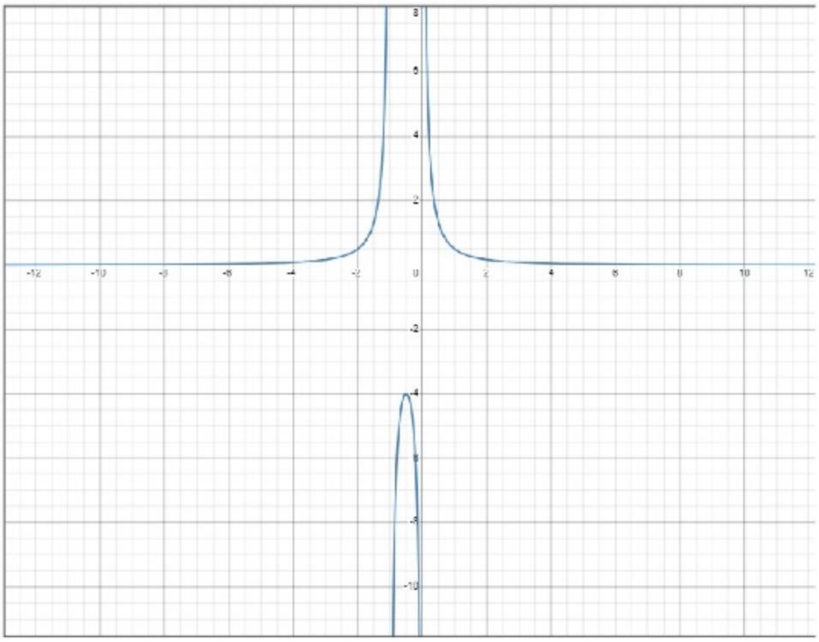

Figure 2: Velocity vs Time under SD Hypothesis

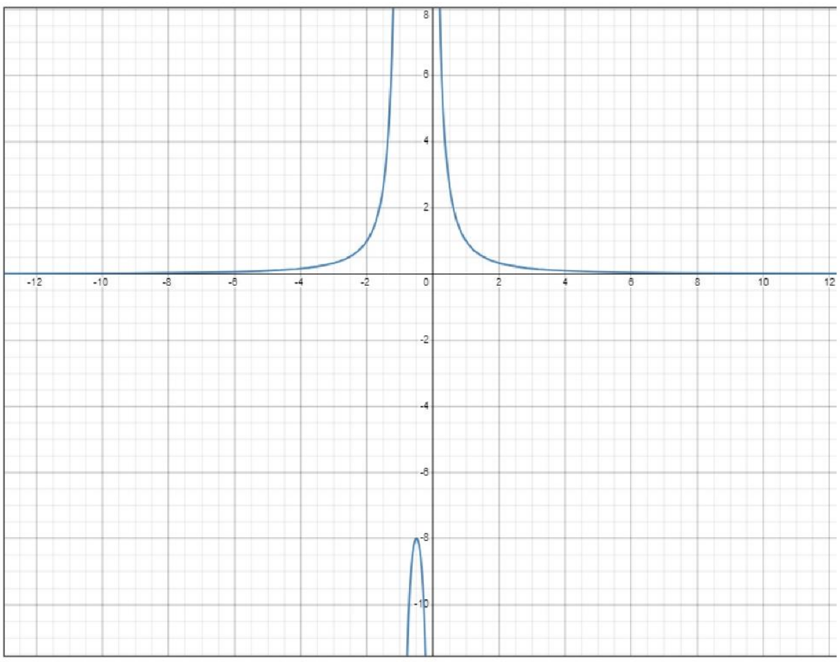

Figure 3:Acceleration vs Time under SD Hypothesis

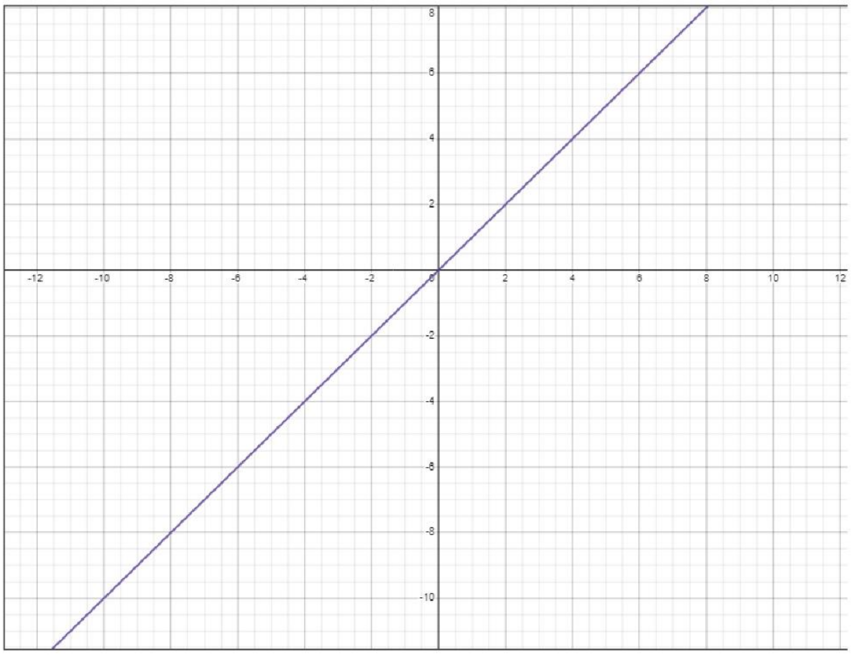

Figure 4:Displacement vs Acceleration under SD Hypothesis

X. CONCLUSION

Based on the observations conducted graphically, mathematically and statistically, this conclusion can be reached that it is theoretically possible to have an instant where a body is in uniformly accelerated motion and possessed some initial velocity such that the acceleration is twice the velocity; provided that already some time has passed after leaving starting point of motion and at that particular instant, the particle is not present in origin. Further work is going on in this field in order to observe its stability and validity in a bigger time interval. The current formulae of SD hypothesis can be used for estimations of the features of the particle at a particular time instant or position instant.

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