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From $E=m_0c^2$ in normal space to $E=m_0c_Ic_K$ inthe complex configuration Spaces

André Michaud
Service de Recherche Pédagogique

Abstract: *Establishment of the electromagnetic oscillation characteristics of the stabilized energy quantum of the electron invariant rest mass and of that of its varying carrying energy within their complex configuration spaces, that admit the presence of no physical singularities, given that all energy quanta of which matter and free moving energy are made reach stable stationary action oscillation states at energy levels way below the range at which singularities could develop.*

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I. INTRODUCTION

Equation $E=mc^2$ has both captivated and fascinated the world ever since it was justified by Albert Einstein in his third 1905 article [1].The concept of *mass*, symbolized by *m*, was initially defined in the 1600's as the *amount of matter* of which a ponderable body is made, that determines its volume and that will inertially resist its state of motion being modified whichever direction a force is applied to it, which is a characteristic of ponderable bodies that was defined by Newton as the first law of motion. *Matter* on its part was defined as any physically existing *substance*.

It was then progressively established over the course of the 1800's that every *substance* categorized as *matter* was made of a variety of atoms that were eventually classified in the Periodic Table of Elements, and that were found to assemble in the variety of molecular combinations of which the various *substances* in our environment are made.

Each atom being found to be the seat of a specific amount of mass, their sum could then be satisfactorily calculated to account for the weight and mass of all known substances. It was then established that all atoms are so small as to be impossible to be directly observed and measured individually even with the most powerful optical microscopes available in the first decade of the 1900's.

With the progressive understanding that atoms are not themselves elementary particles, but are actually *systems of still smaller massive particles*, the electron came to be understood as being such an elementary massive particle, that exists at a level of magnitude still farther down, now understood as the *ultimate subatomic level of magnitude*, the characteristic of being *elementary* now specifically meaning that the electron is not itself made of smaller components.

This is how it came to be understood that there exists two types of ponderable mass in the universe, masses made of *amounts of matter* whose various substances consist in molecules that are assemblies of atoms down to the atomic level of magnitude, and the ponderable masses of the elementary particles themselves of which all atoms and nucleons are made, that could only consist in some amount of *continuous fundamental energy substance* yet to be understood, but that must possess characteristics/properties still to be identified that allow any isolated amount of this substance to stabilize into the massive configurations of these elementary particles of which all nucleons and atoms are made, and also into the seemingly non-massive stable configuration of free moving electromagnetic photons, according to Maxwell's electromagnetic equations.

Louis de Broglie defined such a set of conditions in the 1930's [2] according to which free moving photons would obey Maxwell's equations. A set of 4 such characteristics/properties was identified in References [3][4] to characterize this *continuous energy substance* according to which freshly emitted amounts of this continuous substance, as for example a magnetic pulse emitted by a fixed-length dipole antenna, would automatically *self-structure* as an electromagnetic photon according to the required triply perpendicular structure that can move only at the speed of light in vacuum, in complete agreement with Maxwell's equations and *self-guide* in straight line unless interfered with by some external agent such as the density of the local gravitational gradient that could cause a deflection of its trajectory.

These properties also allowed establishing a clear conversion mechanics of an electromagnetic photon of minimum energy 1.022 MeV to a pair of massive electron-positron [5], as we will shortly see, but the question as to why the square of the speed of light is involved in the process remained to be clarified. This involvement of the speed of light with the rest mass of the electron will be analyzed in this article.

The electron became highly intriguing to most researchers when it was clearly established to be *elementary* with an invariant mass of m_0=9.10938188E-31 kg and an invariant negative unit charge e^- =1.6217462E-19 Coulomb.

II. PONDERABLE MASSES DIRECTLY MADE OF A CONTINUOUS ENERGY SUBSTANCE QUANTUM

In the first decade of the 1900's, many unexpected characteristics were observed for free moving electrons that had never been predicted by any theory, except regarding a possible difference between the rest mass of the electron and an expected larger apparent mass of the electron in motion as calculated from the electromagnetic perspective by G.F.C. Searle in 1897 [6], as mentioned by Walter Kaufmann in Section *9) Wahre und scheinbare Masse* of his 1901 paper [7] and in Wilhelm Wien's 1901 paper [6], as analyzed in References [8][9], but that covered only partially even this possible mass increase characteristic that seemed to contradict the then current classical mechanics perception of the electron as a very small solid invariant mass potentially deformable as it moves.

In fact, none of the characteristics observed during these first ever experiments carried out with free moving electron beams made to move on measurable curved trajectories inside a bubble chamber by means of finely calibrated ***E***- and ***B***-fields had ever been observed for larger masses made of atoms and molecules[7][10][11][12]. To understand how perplexing these characteristics were with respect to all of the established theories of the time for the leading edge experimentalists and theoreticians of the first decade of the 1900's, one only has to read Section *7e Electromagnetic Mass; The First Century* of Abraham Pais' biography of Einstein [13].

Even Wilhelm Wien, the staunchest advocate of establishing electromagnetic mechanics as a common foundation to define a set of equations from which both electromagnetic mechanics and kinematic mechanics could be described [6], had this comment in 1912:

> *"Concerning the new experiments on cathode and β-rays, I would not consider them to have decisive power of proof. The experiments are very subtle, and one cannot be sure whether all sources of error have been excluded."*Wilhelm Wien (1912) ([13], p. 159)

It must be understood that at this point in time, the main cause of this uncertainty about the validity of the conclusive Kaufmann experiments and of the confirming experiments carried out by other experimentalists between 1905 and 1907 was that the *invariance of the electron unit charge*, and even its exact value, as well as the *invariance of its rest mass* had not yet been established, which maintained doubts about the precision of all calculations. These invariant characteristics of the electron were experimentally established and proven by Robert Millikan only in 1913 [14].

So, Wien's comment made in 1912 is possibly the best explanation and justification for the decision taken in 1907 by the whole community to stick with the kinematic trend that led to the adoption of the Special Relativity theory. But this decision nevertheless had unexpectedly negative consequences on the coming development of Quantum Mechanics as we will shortly see.

But let us first overview the observed characteristics of the free moving electron observed during Kaufmann's experiments that no prior theory had predicted. The first characteristic pertains in fact to the means that Kaufmann used to propel electrons at his chosen velocities and trajectory curvatures, that is, ***E***- and ***B***-fields.

2.1 Kinetic energy continuously induced in electrons by the *E*-field

According to classical mechanics, ponderable bodies can be set in motion only via an initial impulse. In fact, the concept of momentum is so narrowly related to the idea that bodies can be set in motion only via an initial impulse that the very word *momentum* is usually translated in the German language for example by the word "*Impuls*". This conclusion led to the perception that *momentum* is a measure of the inertia of a mass in motion, and that its intensity depends on both the velocity and of the amount of mass of massive bodies in motion:

$$p = mv \tag{1}$$

This means that from the classical mechanics perspective, the momentum of a moving mass is considered as being caused by its velocity, which is diametrically opposite to what the Kaufmann experiments allows observing from the electromagnetic mechanics perspective, since the momentum energy of the electron is constantly induced in the invariant charge of the electron by means of the calibrated ***E***-field – the external calibrated ***B***-field used contributing no energy to the electron but only defines the curvature of its trajectory, as analyzed in References [8][9] – which means that it was the velocity of the electron during the Kaufmann experiments that depended on the amount of unidirectional momentum kinetic energy provided by the

adjustable ***E***-field, and not the reverse as conceptualized in classical mechanics.

Introductory textbooks to classical mechanics generally do not immediately relate momentum to the unidirectional kinetic energy involved. We will establish this direct relation here because what the Kaufmann data brings to attention is precisely that ***E***-fields can be used to cause the velocity of electrons to gradually vary by progressively varying the amount of kinetic energy induced in the charge of the electron, either by adiabatically decreasing or increasing this amount of induced kinetic energy in an infinitesimally progressive manner [15][16].

It turns out that kinetic energy is constantly being induced in elementary charged particles as a function of the inverse of the distances separating them at the subatomic level of magnitude, and that this amount varies in an infinitesimally progressive manner as these distances vary, which is a process that has no equivalent in classical mechanics, given that the interaction law governing the force relation between point-like behaving electric charges were discovered by Coulomb about a hundred years after classical mechanics was fully developed mainly by Newton to describe the relations between ponderable masses as observable at our macroscopic level of magnitude.

The amount of kinetic energy related to the classical momentum of a massive body is easy to experimentally calculate and directly experimentally measure because it is totally communicated to the environment when a body is suddenly stopped in its motion. Its equation was defined by Newton as:

$$K = \frac{mv^2}{2} \tag{2}$$

Isolating the definition of velocity v in classical momentum Equation (1) $v=p/m$, and substituting v by this definition in Equation (2) then leads to:

$$K = \frac{\mathrm{m}}{2}\left(\frac{\mathrm{p}}{\mathrm{m}}\right)^2 \quad \rightarrow \quad K = \frac{\mathrm{p}^2}{2\mathrm{m}} \tag{3}$$

While energy is perceived as constant in the universe from the classical mechanics perspective, diminishing kinetic energy being perceived as converting to potential energy and the reverse, kinetic energy is observed as adiabatically varying with the local intensity of ***E***-field in free moving electrons, without any trace of any implication of the concept of potential energy of classical mechanics in the acceleration method used by Kaufmann.

This difference between Kaufmann's method and classical mechanics – adiabatic induction of kinetic energy in charged particles by the Coulomb force, instead of the assumed conversion of potential energy into kinetic energy – seems not to have drawn attention at the time, and the adiabatic nature of energy induction in all elementary charged particles via ***E***-fields analyzed in depth in References[15][16]was understood and explained only much later by Aram d'Abro, in the 1930's [17] as put in perspective in Reference [18].

2.2 The electromagnetic nature of the mass of the electron

In his 1904 paper [19], famous for having defined the mathematical transformations that enabled the hypothesis of a potential speed-related contraction of the length of the mass of the electron perceived as a small rigid body from the point of view of classical mechanics, subsequently used in the establishment of the theory of Special Relativity, Lorentz also came to the conclusion in the same paper, following his analysis of Kaufmann's data, that the mass of the electron must be electromagnetic in nature:

> *"Hence, in phenomena in which there is an acceleration in the direction of motion, the electron behaves as if it had a mass m_1, those in which the acceleration is normal to the path, as if the mass were m_2. These quantities m_1 and m_2 may therefore properly be called the "longitudinal" and "transverse" electromagnetic masses of the electron. I shall suppose that there is no other, no 'true' or 'material' mass."*H.A. Lorentz (1904) [19]

This concept was related to the fact that the data collected by Kaufmann was due to electrons being made to accelerate, not by means of some initial impulse, but by means of calibrated electromagnetic ***E***- and ***B***-fields known to be acting on the charge of the electron instead of on its mass, which seemed to imply that the very substance of which the electron mass was made and to which the electron charge was related had to also be electromagnetic in nature.

But this interpretation was not retained at the time. In 1907, the community chose to maintain the classical conception that the electron as being a rigid body lengthwise deformable as also alternately proposed by Lorentz in the same article [19].

Let us note here that opinions were split at the time as to how the *longitudinal* and *transverse* mass of the electron should be calculated. Einstein for example, in his third 1905 paper [1] established this calculation in such a way that he ended up numerically calculating the total kinetic energy induced in the accelerating electron

as $K=mc^2(\gamma-1)$ – which matches the momentum energy emitted as a photon when an electron is captured in the ground state of a hydrogen atom and is recorded in the spectrum of the hydrogen atom, but which is only half the kinetic energy induced in the electron at any velocity, and that de Broglie used in this 1924 thesis to calculate the energy of the electron on the Bohr orbit, which led him to use a wrong frequency in his calculation of the phase wave velocity as analyzed in Reference [20] – instead of $K=2mc^2(\gamma-1)$ like Lorentz with his Equation (30) in his 1904 paper [19] – see also Equation (43) in Reference [21].

Einstein was aware that the Lorentz calculation from Kaufmann's data was different from his own method, because in his third 1905 paper [1], he clearly alludes to the fact that different definitions of force and acceleration lead to different values for *longitudinal* and *transverse* mass of the electron:

> *"Natürlich würde man bei anderer Definition der Kraft und der Beschleunigung andere Zahlen für die Massen erhalten; man ersieht daraus, daß man bei der Vergleichung verschiedener Theorien der Bewegung des Elektrons sehr vorsichtig verfahren muß."*Albert Einstein (1905), [1]
>
> *"Of course, with a different definition of force and acceleration, one would obtain different numerical values for the masses; one can see from this that one must proceed very carefully when comparing different theories on the motion of the electron."*

At the time, Einstein seemed not to have realized the importance of the fact that Lorentz's calculation was not based on a personal theory by Lorentz, but rather on his analysis of the experimental data collected by Kaufmann.

It was only in 1933 that it was conclusively observed by C.D. Anderson that the electron mass was actually made of the same *energy substance* as electromagnetic photons, as he established experimentally that localized photons of energy exceeding the 1.022 MeV energy level, easily converted to *charged* and *massive* electron-positron pairs moving separately in space, the two particles being eventually measured as identical in all respects, except for the signs of their *equal and invariant charges*[22] as previously hypothesized by Paul Dirac in 1928[23].

This conversion process was confirmed the same year by Blackett and Occhialini as they proved experimentally that cosmic radiation by-product photons of energy 1.022 MeV or more spontaneously convert to electron/positron pairs when grazing atomic nuclei [24], a process that was then named *materialization*.

Moreover, a team led by Kirk McDonald at the *Stanford Linear Accelerator* (SLAC), confirmed in 1997 that by converging two sufficiently concentrated photon beams toward a single point in space, one beam comprising photons exceeding the 1.022 MeV threshold, electron/positron pairs were created without any atomic nuclei being close by [25].

As a double confirmation that the electron mass is made of *the very same electromagnetic energy substance* as free moving photons, it was observed, also by Blackett and Occhialini that the reverse process of an electron and a positron interacting with insufficient energy to escape from each other, reconvert to 2 or more free moving electromagnetic photons ([26], p. 215) after having metastabilized for a moment in bound positronium configuration.

Finally, a barely believable streak of luck allowed both processes of dematerialization and rematerialization of an electron-positron pair to be captured on a single photograph of experiment E632 in the 15 foot FERMILAB bubble chamber, as the bubble trail left by a high energy positron reveals its dematerializing as it enters in head on collision with an electron – at point **A**in **Figure 1 –** and a little further on, in direct line with the trajectory that the positron had been following before dematerializing, a new electron-positron pair is recorded as appearing– at point **B**– which means that the positron-electron annihilation at point **A** mandatorily resulted in the production of a single electromagnetic photon carrying the total amount of energy of the rest masses of both particles plus the momentum energy of the incoming positron, a total amount that obviously exceeds the minimum decoupling threshold level of 1.022 MeV.

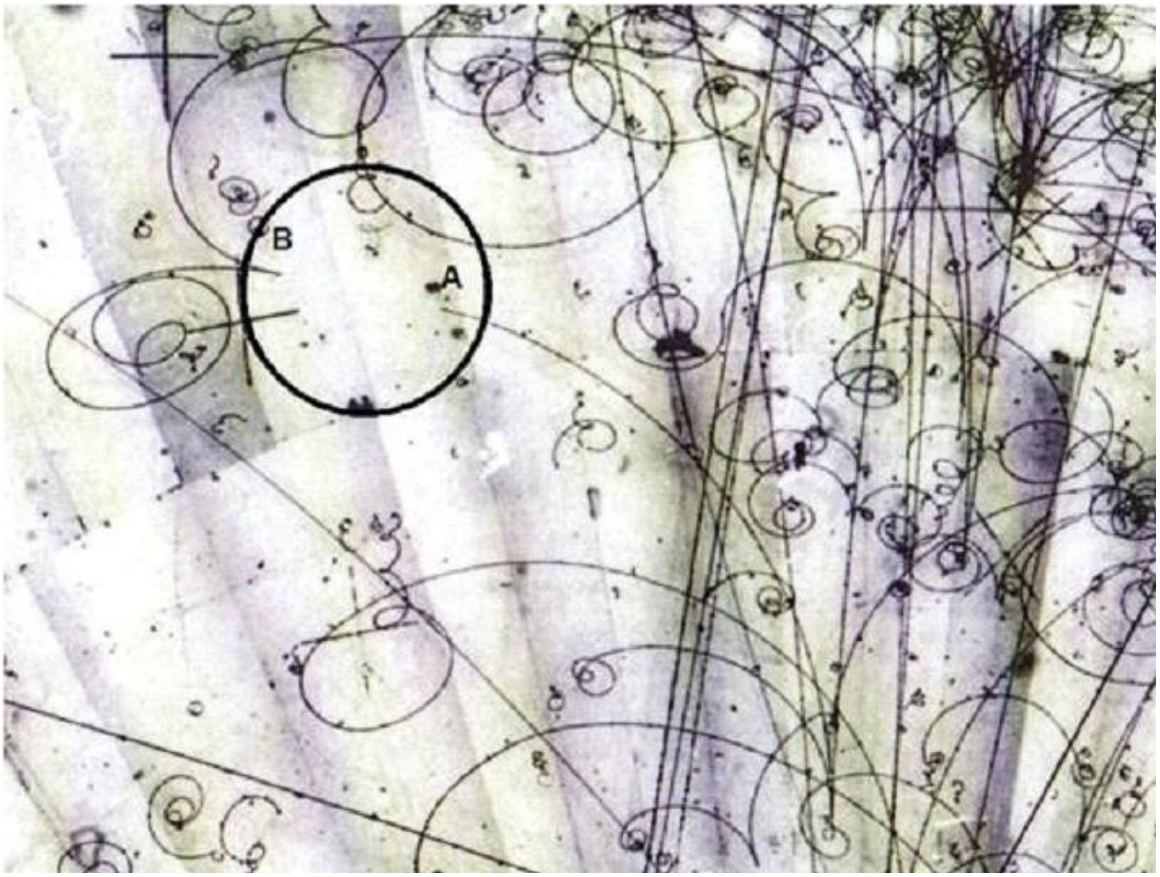


Figure 1: Bubble chamber photograph of FERMILAB experiment E632

So it could henceforth be strongly asserted that the electron mass is effectively electromagnetic in nature.

2.3 The varying mass of the electron in process of acceleration

Although it had been established since Newton that the mass of all bodies seemed invariant at all rather low velocities – with respect to the speed of light – at which it was possible to propel these bodies during all past experiments with masses consisting of atoms and molecules, Kaufmann's data revealed that at the extremely high velocities at which electrons were accelerated in his bubble chamber, whose masses are made of a stabilized electromagnetic energy quanta – i.e. from 1/5 to 1/3 of the speed of light – the electron's effective mass could be measured as having been increasing as a function of its velocity as the energy constantly supplied to it by the calibrated ***E***-field of the Lorentz method was increasing, as analyzed in depth in this 1904 article [19].

In fact, The Kaufmann experiments were the first experiments ever carried out in which masses directly made of the fundamental *energy substance* were set in motion and kept in motion by a means different from the traditional initial impulse used to propel masses made of atoms and molecules. It is well established that ***E***-fields can be continuously calibrated to progressively vary the amount of energy provided to charged particles to progressively diminish or increase their velocity and to continuously vary the intensity of **B**-fields to progressively increase or decrease the curvature of individual charged particles or charged particle beams trajectories, such as the trajectories of electrons in CRT tubes.

So, contrary to the historical perception in classical mechanics that the momentum of a massive body would be due to its velocity, the Kaufmann data reveals that at least for charged and massive electrons, whose rest mass is directly made of the fundamental *energy substance*, this perception needs to be reversed, and that it is the velocity of the electron during the Kaufmann experiments that depended at any given moment on the instantaneous amount of physically existing momentum kinetic energy constantly being induced by the calibrated ***E***-field.

Kaufmann's data also revealed that the increasing transverse inertia with velocity of the accelerating electron was different from its increasing longitudinal inertia by the amount of its momentum energy, which was measurable only longitudinally, both inertia conditions being represented by electromagnetic masses m_1 and m_2 in Lorentz's article. It is to be noted that Kaufmann was expecting an effective mass increase of the electron with velocity due to G.F.C. Searle's calculation to this effect published only a few years earlier in 1897 [27], but not

that this increase would be different longitudinally and transversely, which is what drew attention to the fact that the propelling energy of the momentum of the electron was the outstanding factor that differentiated the measurements in both directions, and that moreover, that this amount of momentum energy remained exactly equal in quantity all through the acceleration process to the amount of energy that was measurable as the effective electron transverse mass increment.

Moreover, Lorentz observed that the famous gamma factor ($\gamma=1/\sqrt{(1-(v/c)^2)}$) – afterwards named after him – that was previously developed from theory by Woldemar Voigt [28] and had been tentatively related to a hypothetical time dilation and length contraction of moving masses, emerges directly out of the Kaufmann data for both the transverse and the longitudinal inertia of the accelerating electrons, as applying not to a hypothetical time dilation and mass length contraction, but to the rate of progressive energy increase in the electrons in process of accelerating, an energy provided by the calibrated ***E***-field used during the experiments.

In his 1904 article [19] Lorentz identified the γ-factor with symbol k as defined in his Equation (3), and used it in both equations for m_1 and m_2 (his Equations (30)), that is, to calculate separately the longitudinal inertia and the transverse inertia of the electron. His equation for longitudinal inertia m_1 reveals a detail that apparently went unnoticed at the time, which is that the ***E***-field which is part of the first term of the Lorentz force equation:

$$\mathrm{F} = \mathrm{q}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{4}$$

is actually the Coulomb equation minus one of its charges:

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 d^2} \quad \rightarrow \quad \boldsymbol{E} = \frac{q_2}{4\pi\varepsilon_0 d^2} \tag{5}$$

And that when applied to Lorentz's Equation (30) for longitudinal inertia m_1, the total amount of energy induced at any given velocity of the electron is provided:

$$E = d\, e\, \boldsymbol{E} = \frac{e\, q_2}{4\pi\varepsilon_0 d} \tag{6}$$

which reveals, according to calculations based on data collected by Kaufmann, that the Coulomb force induces in the electron a quantity of energy twice that of its momentum energy, and that this second quantity of induced energy apparently self-transposes perpendicularly to the direction of motion of the particle, so that it becomes measurable as an increment of mass in the longitudinal direction as well as in the transverse direction.

This critically important bit of information not having been noticed, and consequently not having been integrated in the textbooks of the era, the whole community remained under the impression that the frequencies recorded in the spectra of atoms, that were known to correspond only to the momentum energy released when electrons were captured at their least action distance from atomic nuclei, was the total amount of energy induced in the electron for any given velocity.

The fact that in the traditional reference case of the hydrogen atom on this issue, for example, this amount of momentum kinetic energy exactly defines the theoretical classical velocity of the electron on the theoretical ground state orbit of the Bohr model of the hydrogen atom only seemed to confirm this impression:

$$E = \mathrm{a}_0\, e\, \boldsymbol{E} = \frac{e^2}{4\pi\varepsilon_0\, \mathrm{a}_0} = \mathrm{m}_0 \mathrm{v}^2 \tag{7}$$

in which a_0=5.291772083E-11 m is the Bohr radius, leading to calculate the electron classical velocity on the theoretical Bohr orbit:

$$v = \sqrt{\frac{\mathrm{e}^2}{4\pi\varepsilon_0 \mathrm{m}_0\, \mathrm{a}_0}} = 2187691.252\,\mathrm{m/s} \tag{8}$$

This unfortunate circumstance is what caused de Broglie to calculate a wrong velocity for the phase wave of the electron in his 1924 thesis [29][30] from only the energy related to the momentum of the electron on the Bohr orbit, as analyzed in Reference [20], without anybody noticing the problem, due to Kaufmann's experimental results never having been referenced in any of the textbooks and reference works of the remainder of the 20th century, for even potential chance re-assessment, which entertained the uncertainty as to the precise location of the electron on its trajectory in the hydrogen atom that unduly caused the eventual formalization by Heisenberg of the Uncertainty principle, as put in perspective in Reference [20].

2.4 The Marmet discovery

It was only in 2003 that attention was again brought back to the Kaufmann data when Paul Marmet re-derived a harmonized electromagnetic/kinematic equation from the Biot-Savart equation that led to the same

conclusion historically drawn by Searle in 1897 [27]. Actually, Marmet simply rediscovered and published in 2003 what Searle already established by other means more than one century before:

> *"We notice in equation 23 that both phenomena (magnetic energy and relativistic energy) produce an increase of mass"* Paul Marmet (2003) [31]

His clear Equations 23 – reproduced here as Equations (9) – reveal that the known increasing magnetic ***B***-field of the electron with velocity was the sum of the invariant $\mathbf{B}_e$-field related to half of the invariant rest mass of the electron plus a $\Delta\boldsymbol{B}$-field increment of a magnetic field energy of unclear origin, that could be related only to the transverse mass increase of Lorentz's Equation (30) for the transverse inertia m_2 of the electron in motion [19] that emerged from Kaufmann's data [7][10][11][12]:

$$\frac{\mu_0\left(e^-\right)^2}{8\pi}\frac{1}{r_e}\frac{v^2}{c^2}\Rightarrow\frac{M_e}{2}\frac{v^2}{c^2}\quad\rightarrow\quad\frac{\mu_0\left(e^-\right)^2}{8\pi}\frac{1}{r_e}\Leftrightarrow\frac{M_e}{2} \tag{9}$$

Marmet's final conclusion was:

> *"Therefore the increase of the so-called relativistic mass is in fact nothing more that the mass of the magnetic field generated due to the electron velocity. In fact, the real fundamental nature of the kinetic mass, which increases with velocity, is nothing else than the magnetic energy, as given by the Biot-Savart equation."*Paul Marmet (2003) [31]

His discovery made it possible to relate the longitudinal electromagnetic wavelength of the free-moving photon energy to the electron's rest mass and to the oscillation amplitude on a plane transverse to the particle's direction of motion of the magnetic energy that causes the measurable transverse inertia of all elementary particles via the ratio provided by the fine structure constant α, and to the derivation of the set of local ***E*** and ***B*** field equations in accordance with Maxwell's equations, which account for the rest mass energy of each stabilized elementary particle, in the first series of derivations to emerge from Marmet's conclusion, published in 2007 in Reference [32].

The analysis of the stability of the electron in the rest orbital of the hydrogen atom realized in References [33][34] moreover revealed that when an electron is prevented from moving in the vectorial direction in which its $\Delta\boldsymbol{K}$ momentum energy tends to propel it, this $\Delta\boldsymbol{K}$ momentum energy remains induced and continues applying an equivalent pressure in the same vectorial direction, which can also be directly related to gravitation, as analyzed in Reference [18].

Masses made of atoms have been extensively studied over the course of the past centuries, but due to the manner in which Quantum Mechanics was established in the 1920's [20], the attention of the community was drawn away from studying more closely the nature of the electromagnetic energy of which the localized mass of electrons is made.

The outcome is that the consequences of the fact that the effective masses of all atoms made up of these elementary masses in which these quantities of momentum energy $\Delta\boldsymbol{K}$ and field energy increments $\Delta\boldsymbol{B}$ are permanently induced, that vary as a function of the inverse of the distances separating them, have also not been studied. We'll come back to this later, after completing the historical overview of the evolution of our understanding of the nature of the mass of charged elementary particles.

III. MASSES MADE OF SEPARATE ATOMS AND MOLECULES

The true order of magnitude of atoms and molecules was established in the first decade of the 1900's from calculations made by Albert Einstein and Marian von Smoluchowski as they were separately studying Brownian motion, an apparently stochastic and very irregular motion observable of microscopic particles immersed in liquids which was due, according to their conclusion, to a constant motion of the atoms and molecules of the liquids in which the microscopic particles were immersed [35][36]. The first English translation of Smoluchowski's work was recently made available by the *Minkowski Institute Press* [37].

Reference [18] also analyzes Brownian motion, highlighting the fact that this permanent motion of atoms and molecules in all liquids and gases is the proof that the kinetic energy of their momentum has a *permanent kinetic presence*, as Kaufmann's experiments seemed to reveal, thus a *permanent physical existence*, which established that it could be a *physically existing substance* whose properties remained to be established.

It was then established that atoms are practically empty structures involving particles much smaller yet and really elementary, that were individually electrically charged and massive, that are many orders of magnitude smaller than the atoms themselves and whose stabilized stationary local configurations define the atomic volumes.

To give an idea of how relatively "empty" atoms prove to be and how much smaller their constituting elementary subcomponents really are, and finally, how relatively distant from each other they are within each atom, if for example a hydrogen atom, made of only one proton and one electron, was metaphorically enlarged

until the central proton – of established diameter of about 1.5E-15 m – became as large as the Sun – of diameter of about 1.4 million km – (an increase in size of 9-followed-by-23-zeros times), then the electron would be located as far from the proton as Neptune is from the Sun in the solar system, which means that the hydrogen atom would become as large as the whole solar system!

This meant that the total mass that can be measured for each atom is in reality the sum of the physically existing local masses of each of the continuous energy substance quanta of which each of the subatomic sized elementary particles themselves of which atoms are made. Effectively, the total mass of the hydrogen atom, despite the *relatively astronomical distance* that separates the electron from the proton, turns out to be 1.673532518E-27 kg, which is the sum of the now well established mass of the localized electron 9.10938188E-31 kg and of that of the well established mass of the localized proton 1.67262158E-27 kg, which itself is not elementary but is also an almost empty system of smaller elementary massive components, that themselves are of the same order of magnitude as the electron, as demonstrated in highly inelastic collision experiments with highly energetic electron and positron beams in the early years of operation of the *Stanford Linear Accelerator* (SLAC) [38].

Now, with all macroscopic masses being understood to be made of matter, every substance of which is known to be made of practically empty atoms and molecules, themselves made of subatomic sized massive elementary electrons located relatively very far from the protons and neutrons that make up their subatomic sized nuclei, that are themselves systems made of elementary particles of the same order of magnitude of electrons, the question that now comes to mind is:

What is exactly the nature of this continuous substance of which the masses of these subatomic elementary charged particles are made of?

IV. MASSES MADE OF STABILIZED CONTINUOUS ELECTROMAGNETIC ENERGY

As just mentioned, besides electrons that stabilize at various distances from atomic nuclei and that thus define their volume, the remaining components of atoms, that is, protons and neutrons, were also confirmed as being themselves systems of elementary particles when the charged and massive subcomponents of nucleons were rebounded against in a *non-destructive* highly inelastic manner by highly energized electron and positron beams, in the first 2 years of operation of the *Stanford Linear Accelerator* (SLAC) [38] that entered service in 1966, i.e., highly inelastic rebound patterns that are an unmistakable telltale that these internal subcomponents of nucleons have masses in the very same range as that of the electron. Incidentally, this type of *non-destructive* highly inelastic scattering against nucleons is due to resume in a not too distant future in the projected *Electron-Ion Collider* (EIC) [39][40].

The possible mass range of the two types of charged elementary particles detected inside protons and neutrons, that is, up and down quarks, was established to be between 1.5 and 5 MeV/$c^2$with an exact electric charge of +2/3 of the unit charge for the up quark, and between 3 and 10 MeV/$c^2$with an exact electric charge of -1/3 of the unit charge for the down quark ([41], p. 11-6).

So it can be expected that more precise rest masses will be experimentally measured for these elementary charged and massive subcomponents of nucleons in the near future at the coming EIC facility, possibly confirming the rest masses predicted from theory in 2013 in context of the development of Electromagnetic Mechanics, that fall precisely within the possible energy ranges experimentally established during the SLAC experiments, that is, 2.049610923E-30 kg for the up quark and 8.198443779E-30 kg for the down quark.

These masses so close to the electron rest mass accredited the possibility that up and down quarks could simply be very normal electrons and positrons whose charge and mass characteristics would have been warped in these altered states by the intensity of the ambient energy level within the confined structures of protons and neutrons, which is a possibility accredited by the deep analyses carried out in Reference [42], which resulted in the following general elementary mass equation for the whole set of the 3 truly elementary charged and massive particles directly made of quanta of the fundamental *continuous energy substance* of which all atoms are made:

$$m_{[d,u,e]} = \frac{k}{a_0}\left(\frac{3e}{n\alpha c}\right)^2 = \left(\frac{3}{n}\right)^2 \frac{e^2}{4\pi\varepsilon_0 r_e}\frac{1}{c^2} = \left(\frac{3}{n}\right)^2 \frac{e^2}{2\varepsilon_0\alpha\lambda_C c^2} \quad (n=1, 2, 3) \qquad (10)$$

in which $r_e = \alpha\lambda_c/2\pi = 2.817940285E\text{-}15m$ is the so-called *classical electron radius*.See Reference [32] for an explanation of the real nature of this so-called *electron radius*, which is in reality the transverse oscillation amplitude of the electron's oscillating rest mass energy within *complex* Y-space on a plane transverse to its direction of motion in *normal* X-space [5], that we will define more clearly further on.

As analyzed in Reference [43], up and down quarks proved to be the only two types of charged and massive elementary subcomponents that could be proven to physically exist inside protons and neutrons via non-destructive scattering, that is (uud) inside the proton with resultant positive unit charge e^+ for the proton,

and (udd) inside the neutron with resultant zero charge$e^0$for the neutron, which means that the only 3 *stable*, *charged* and *massive* elementary particles of which all atoms in the universe are made had henceforth been identified, which are the electron, the up quark and the down quark.

All other particles of the Standard Model that were eventually identified and listed after the 1960's via *destructive scattering*, proved to be unstable and to almost instantly degrade as the ultimate outcome of well established decay sequences [26][41] into one or other of these three stable stationary action energy resonance configurations of which all atoms of all larger masses are made in the universe, electrons, protons and neutrons. No other stable massive elementary component of atoms than this very restricted set was ever detected via scattering, with the positron being identical to the electron except for the sign of its charge.

For simplicity's sake, only the rest mass of the electron and its mass increment with velocity provided by its carrying energy will be discussed at length in this article, the case of the mass of the positron being identical to that of the electron as we will see, and the case of free moving photons, due to their similarity with the electron – or positron – carrying energy.

In context of the trispatial geometry, the case of the positron is discussed at length as to its origin in Reference [5] and as to its function in the establishment of nucleons in Reference [42]. On its part, neutrino energy emission is analyzed in Reference [44].

Now back to the question that remained to be answered and that can now be more clearly formulated:

What can the masses of these charged and massive elementary particles be made of and what has the square of the velocity of light to do with it?

V. THE REST MASS OF THE ELECTRON

In the case of the electron, the first general answer was provided with equation $E=m_0c^2$ in the first decade of the 1900's, that left many issues pending but nevertheless established that its precisely measured rest mass of m_0=9.10938188E-31 kg was made of a specific amount of an intriguingly elusive *substance* that could only be named *energy*, an amount of mass that can be calculated by dividing the precise amount of *energy* of$E$=8.18710414E-14 joules by the square of the speed of light:

$$m_0 = \frac{E}{c^2} = \frac{8.18710414\text{E-}14}{(299792458)^2} = 9.10938188\text{E} - 31\text{kg} \tag{11}$$

A dimensional analysis of this equation reveals that the velocity of light must be squared to convert a mass in kg to its equivalent amount of energy in joules, given that the dimensions of the joule were set by convention to $M\cdot L^2\cdot T^{-2}$ ($kg\cdot m^2\cdot s^{-2}$),and that those of a velocity were set to $T\cdot L^{-1}$ ($m\cdot s^{-1}$).Then of course, to be mathematically consistent, the conversion of kilograms into joules requires that the velocity involved be squared $(m\cdot s^{-1})^2$.Of course, this dimensional analysis does not provide any explanation as to why the speed of light needs to be involved in the first place in calculating the mass of a massive particle, which is well known not to be able to reach the speed of light in space, let alone requiring that this velocity be squared.

But there is a reason why Einstein and other major physicists of the era found that it made sense for the speed of light to be involved in calculating the mass of the electron, given that recent discoveries regarding confirmed interaction between electromagnetic light energy and matter revealed such a relation, as he clarified in this fourth 1905 article [45]:

> *"Gibt ein Körper die Energie L in Form von Strahlung ab, so verkleinert sich seine Masse um L/V^2… Die Masse eines Körpers ist ein Maß für dessen Energieinhalt; ändert sich die Energie um L, so ändert sich die Masse in demselben Sinne um $L/9.10^{20}$, wenn die Energie in Erg und die Masse in Grammen gemessen wird."* Albert Einstein ([45], p. 641)
>
> *"When a body emits energy L in the form of radiation, its mass decreases by L/V^2…. The mass of a body is a measure of its energy content; if the energy changes by L, the mass changes in the same manner by $L/9.10^{20}$, if the energy is measured in erg and the mass in grams."*

So, since he understood that the energy that radiates out of a massive body as it collides with some obstacle which causes its mass to diminish, is electromagnetic in nature, this obviously made Einstein suspect, pending experimental confirmation, that the remaining mass itself had to also be made of electromagnetic energy. Given that the speed of light, that Maxwell had related to free moving *electromagnetic energy*40 years earlier and calculated from second partial derivatives of the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields to be c=299792458 m/s – analyzed in References [8][9] – it only seemed logical that this velocity should be related to mass as the conversion factor.

There is also a physical reason for the square of this velocity to be directly related to *electromagnetic energy* itself. Although the velocity of elementary particles or of electromagnetic photons is understood as occurring in their direction of motion, some aspects of velocity are also known to occur on planes perpendicular

to this direction of motion when oscillating processes are involved, planes that remain parallel to the direction of motion of electromagnetic energy for continuous wave treatment as understood in electrodynamics, such as the varying transverse velocity of oscillation of shear-wave energy in rigid materials or of electromagnetic energy in Maxwell's theory of continuous electromagnetic waves for example, as traditionally represented with **Figure 2**.

This transverse velocity varies from a maximum at midpoint of the transverse amplitude of the motion of the energy to zero velocity at maximum transverse extent of the amplitude of the oscillation, as analyzed in **Subsection K** of Reference [46].

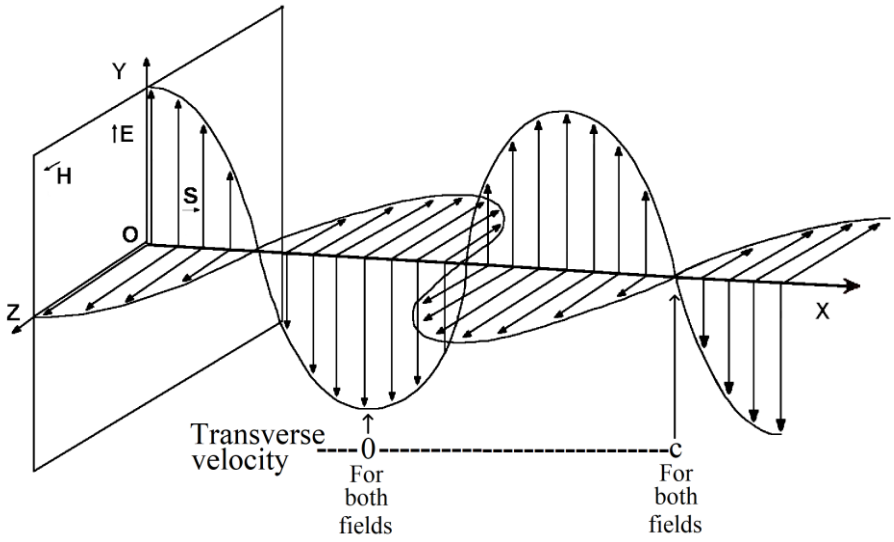


Figure 2: Traditional ***E*** and ***B*** fields transverse oscillation representation of an electromagnetic pulse propagating in an underlying elastic medium – defined by Maxwell as the aether – *spacewise* offset by 90°, and *timewise* simultaneously peaking in phase to a synchronous maximum amplitude, corresponding to the *Lorenz gauge* interpretation.

With regard to the representation of Maxwell's electromagnetic waves of **Figure 2**, let us remark that Maxwell was not entirely in agreement with the *Lorenz gauge* representation according to which both the ***E***-field and the ***B***-field would simultaneously peak to maximum in this manner, that leads to consider both states as being *a single electromagnetic field*, because this representation did away with the concept of *displacement current* that initially guided the development of his electromagnetic theory, and that he considered both fields as being separate due to different vectorial characteristics.

In summary, **Figure 2** is one of two possible representations of *continuous electromagnetic waves* established by Maxwell. The second representation involves an electromagnetic pulse of energy oscillating between an ***E***-field state and a ***B***-field state, the two fields being *spacewise* offset by 90° as in **Figure 2**, also oscillating transversely on longitudinal planes according to the classical concept of a wave propagating in a rigid elastic medium.

But whereas the *Lorenz gauge* represents them in **Figure 2** as *timewise* reaching *simultaneously* their maximum amplitude, Maxwell initially conceived them as alternately reaching their maximum amplitude while being *timewise* dephased by 180° as represented in **Figure 3**, by introducing the concept of a *displacement current* linked to the ***E***-field as the *mechanical cause* of the induction of the ***B***-field, which, when reaching maximum amplitude, reduces the ***E***-field to zero, as in the well-known LC relation, at which point the ***B***-field being symmetrically out of balance, will re-induce the ***E***-field and re-establish the *displacement current* while the ***B***-field falls in turn to zero, thus establishing one complete cycle of the frequency corresponding to the energy of the propagating pulse.

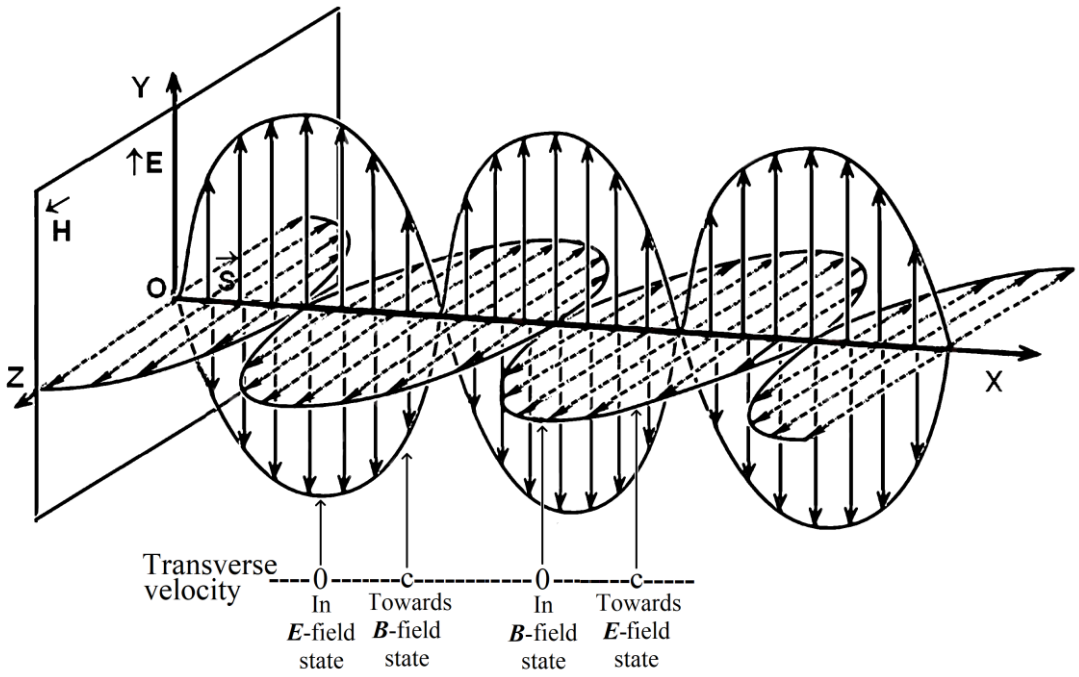


Figure 3: Orthogonal ***E***- and ***B***-fields transverse oscillation representation of an electromagnetic pulse propagating in an underlying rigid elastic medium – defined as the aether by Maxwell – *spacewise* offset by 90° and *timewise* dephased by 180°, mutually inducing each other and involving the assumed existence of a *displacement current* as conceived by Maxwell.

It was indeed the fact that the concept of *displacement current* made little sense in the absence of any possibility to conceptualize moving charges in Maxwell's theory of continuous electromagnetic waves that caused the *Lorenz gauge* perspective to initially dominate. It was only after de Broglie came up in the 1930's with the idea that *localized photons* according to Einstein's first 1905 article conception [47] could make sense only if they involved two corpuscles (or half photons) – the sought after "charges" not to be found from the continuous wave perspective – that the *displacement current* found logical anchoring again in electromagnetic theory, as analyzed in References [3][4].

The analysis carried out in Reference [46]then allowed confirming that maximum transverse velocity will be reached 4 times during each cycle of the sine wave representation of the cyclic motion of electromagnetic energy, that is at 1/8th, 3/8th, 5/8th and 7/8th of each cycle. In physical reality however, only two such velocity peaks are possible since 3/8th and 5/8th will coincide, as well as 7/8th and 1/8th of the next cycle due to an *incompressibility* characteristic that the fundamental *energy substance* must have to oscillate in this manner. This velocity is calculated from first principles in Reference [48].

The *incompressibility* of the *energy substance* is what causes the speed of light to be the only possible maximum velocity at mid-transfer of the *energy substance* that it can accelerate to that would allow this transverse velocity of the *energy substance* to decelerate down zero at maximum transverse amplitude of the oscillation in both perpendicular directions of the electromagnetic oscillation, as represented with **Figure 5** further down.

So, under the assumption that the *energy substance* of which the electron mass is made would be electromagnetic in nature, this is the reason why the product of these two maximum transverse velocities (c^2) in directions perpendicular to each other, can be related to the electron mass in Equation (11).

Reference [46] also analyzes the reason why the product of the wavelength λ by frequency ν is a constant, known to be the speed of light:

$$\lambda\nu = c \tag{12}$$

So understanding that the stabilized rest mass energy of the electron is *electromagnetic* in nature allows understanding why the electron Compton oscillation frequency of ν_c=1.235589976E20 Hz and the well established electron Compton wavelength of λ_c=2.426310215E-12 m could be associated with it.

This is why, with reference to the famous equation $E=m_0c^2$, the squared velocity c^2 turns out to be, in the configuration spaces, the product of the maximum velocity c of half the electron's rest mass energy during its transit towards the unit vector axis ***I***//***j***/***-j***, and of its maximum velocity c during its return transit towards the perpendicular unit vector volume ***K***//***ijk***/***-i-j-k*** of **Figure 4b**, as represented by Equation (18) and illustrated in **Figure 9** below, allowing the equation $E=m_0c^2$ as perceived in normal X-space to be represented as $E=m_0c_Ic_K$ in

the configuration spaces I and Z described below.

But of course, electromagnetic energy moving at the speed of light according to the continuous wave concept established by Maxwell cannot spontaneously spawn quantized electrons practically immobile in space, that is, moving at velocities $v \ll c$, without some logical mechanical conversion process being involved that would allow bridging the gap between Maxwell's continuous wave concept and quantized energy states such as that of permanently localized photons as considered by Einstein and de Broglie described by Einstein in his 1910 paper [49][50] quoted further down, that were still conceptualized as singularities or wavelets in the underlying continuous electromagnetic field conceived by Maxwell, and of course as that of the electron still perceived as a deformable solid mass as defined by Lorentz in his 1904 article [19].

VI. THE TRANSITION FROM THE CONTINUOUS ELECTROMAGNETIC ENERGY WAVE PERCEPTION TO THE PERMANENTLY QUANTIZED AND LOCALIZED ELECTROMAGNETIC ENERGY QUANTA PERCEPTION

The first inroad towards understanding the quantized nature of electromagnetic energy happened when Wilhelm Wien discovered that electromagnetic energy was quantized upon emission from the data collected during his blackbody experiments [51]. From his analysis of Wien's data, Max Planck discovered that each cycle of the frequencies of all energy quanta coming out of the black body were exactly equal to a very small energy amount of h=6.62606876E-34 joules, that was named *Planck's constant* to honor his discovery [52].

A few years later, after having considered the growing number of telltale observations favoring the idea of electromagnetic energy quantization, Einstein proposed in his first 1905 paper [47] the hypothesis that it would make more sense to conclude that discontinuous light quanta would remain localized as they move away from their point sources without spherically spreading out wavelike as in Maxwell's wave theory, to then be individually absorbed somewhere else only as a whole:

> *"Es scheint mir nun in der Tat, daß die Beobachtungen über die 'schwarze Strahlung', Photolumineszenz, die Erzeugung von Kathodenstrahlen durch ultraviolettes Licht und andere die Erzeugung bez. Verwandlung des Lichtes betreffende Erscheinungsgruppen besser verständlich erscheinen unter der Annahme, daß die Energie des Lichtes diskontinuierlich im Raume verteilt sei. Nach der hier ins Auge zu fassenden Annahme ist bei Ausbreitung eines von einem Punkte ausgehenden Lichtstrahles die Energie nicht kontinuierlich auf größer und größer werdende Räume verteilt, sondern es besteht dieselbe aus einer endlichen Zahl von in Raumpunkten lokalisierten Energiequanten, welche sich bewegen, ohne sich zu teilen und nur als Ganze absorbiert und erzeugt werden können."* Albert Einstein, 1905 ([47], p. 133)
>
> *"In fact, it seems to me that the observations on 'black-body radiation', photoluminescence, the production of cathode rays by ultraviolet light and other phenomena involving the emission or conversion of light can be better understood on the assumption that the energy of light is distributed discontinuously in space. According to the assumption considered here, when a light ray starting from a point is propagated, the energy is not continuously distributed over an ever increasing volume, but it consists of a finite number of energy quanta, localized in space, which move without being divided and which can be absorbed or emitted only as a whole."*

A further step was taken in 1937 [2], when Louis de Broglie identified the fundamental condition that localized electromagnetic photons postulated by Einstein must obey in order to perfectly explain the photoelectric effect, while remaining fully consistent with the properties of Dirac's theory of complementary corpuscles symmetry, and obey the Bose-Einstein statistic as required by the precision of Planck's blackbody law.

This fundamental condition for the existence of localized electromagnetic photons to be explainable is that the photon must involve two corpuscles, or half-photons of spin 1/2, that would be complementary to each other in the same way as the positive electron ["the positron"] is complementary to the negative electron in Dirac's hole theory. His final conclusion was that this photon model made it possible to define an electromagnetic field linked to the photon's annihilation probability, a field that obeyed Maxwell's equations and has all the characteristics of electromagnetic light waves, immediately bringing back into the picture Maxwell's notion of a *displacement current*, which could now be understood as being in action between these two half-photons, to establish locally a stationary mode of oscillation of the photon's $\boldsymbol{E}$-field energy, i.e. a stationary mode of oscillation of the energy quantum essential to explain the permanent localization in space of the photon's energy quantum as envisioned by Einstein.

But he also mentions that he found no satisfactory way to describe localized photons in this manner within the restricted frame of 3D space and even of 4D spacetime, and concluded that it did not seem possible to

accurately represent elementary particles in this too restricted space geometry, and that he was expecting that this issue would be resolved eventually.

> *"... la non-individualité des particules, le principe d'exclusion et l'énergie d'échange sont trois mystères intimement reliés : ils se rattachent tous trois à l'impossibilité de représenter exactement les entités physiques élémentaires dans le cadre de l'espace continu à trois dimensions (ou plus généralement de l'espace-temps continu à quatre dimensions). Peut-être un jour, en nous évadant hors de ce cadre, parviendrons-nous à mieux pénétrer le sens, encore bien obscur aujourd'hui, de ces grands principes directeurs de la nouvelle physique."* Louis de Broglie 1937 ([2], p. 273)
>
> *"... the non-individuality of particles, the exclusion principle and exchange energy are three closely related enigmas: they are all linked to the impossibility of accurately representing elementary physical entities within the framework of three-dimensional continuous space (or, more generally formulated, four-dimensional continuous space-time). Perhaps one day, by escaping from this framework, we'll be able to better penetrate the meaning, still quite obscure today, of these great guiding principles of the new physics."*

It was these conclusions by de Broglie that the description of quantized photons depends on involving 2 half-photons, and that our conception of space needed to be expanded beyond the well established 3D space and 4D spacetime concepts, that led to a deep analysis of what possibilities were available to expand the space geometry beyond the limits of the established conception of space over which the 3D Cartesian coordinate system is traditionally mapped, particularly in electromagnetism, as represented with **Figure 4a**, in a manner that would remain consistent with this long established coordinates system without which electromagnetic theory would make no sense.

The solution turned out in expanding the three mutually perpendicular Cartesian linear unit vectors into becoming fully expanded 3D spaces as in **Figures 4b** and **4c** each identified by major unit vectors ***IJK***, and major coordinates **XYZ**, and each maintaining its own set of minor ***ijk*** unit vectors and minor **xyz** coordinates, to allow the volume of the oscillating *continuous energy substance* quantum to be representable all through its oscillation process in the three spaces, now interconnected at *the center-of-presence* of the quantum, that now allows the three spaces to behave *as communicating vessels* within which the energy quantum substance can move freely to finds its least action equilibrium.

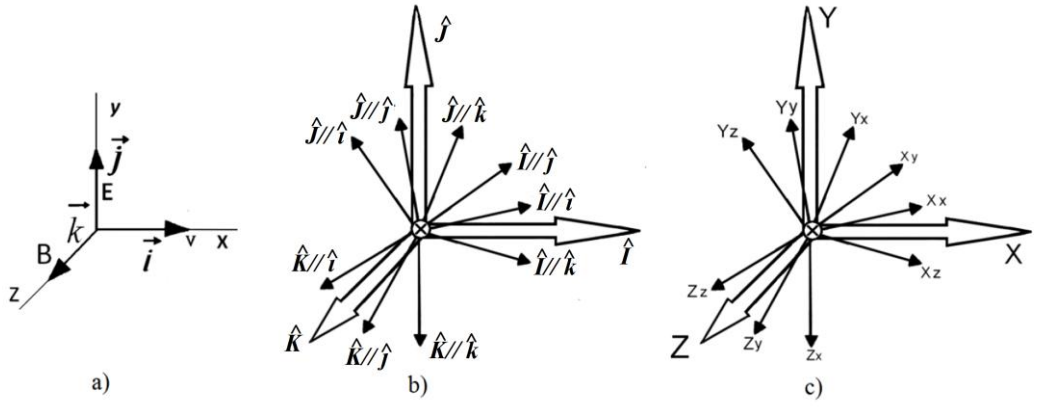


Figure 4: The classical unit vector structure of the Cartesian coordinate system versus the expanded coordinate structure of major and minor unit vectors of the trispatial geometry.

Further symmetry considerations seemed required to be applied to the manner in which these two half-photons had to move and interact to involve both the ***E***-field state and the ***B***-field state of Maxwell's electromagnetic theory, which eventually led to understand that the energy of a moving photon first had to be split into two symmetric parts, one of which would be the *momentum energy* required to propel the photon at its known speed of light, that is, a velocity which is mathematically proven in Reference [21] to be obtained if this partition results in exactly two equal parts; while the "propelled part" would symmetrically oscillate in standing mode between an ***E***-field state and a ***B***-field state on a plane transverse to the direction of motion provided by the vectorial orientation of the *momentum* energy half of the photon energy (**Figure 5**).

As established in References [3][4], such a two-part separation of the photon's energy implied the characteristic of *incompressibility* already mentioned of the *energy substance*, so that its volume does not vary during the separation, to which characteristics of *elasticity* and *fluidity* now need to also characterize the energy substance for this separation to be possible.

The propelled part must now be able to stretch to form the two parts that will be associated to the *displacement current* that will cause them to oscillate. It is here that the *elasticity* and *fluidity* properties must be associated with the *continuous energy substance* quantum to enable its now transversely oriented half to stretch symmetrically like an elastic material on either side of its junction point with the *momentum* part.

However, such a stretching cannot occur on its own without an additional property being assigned to the *energy substance*, namely *a-tendency-to-always-remain-in-motion*, which would explain both why its momentum energy tends to propel the transversely oriented part, and also why this propelled part will tend to stretch on its own to form the two parts that would elastically move away from each other, and whose elastic tension would then establish the *displacement current*, whose intensity would increase with the separation distance, until the substance's *incompressibility* property then determines the maximum extension of the stretch, with the restoring tension due to the substance's *elasticity* tending to constantly bring the two components back towards the neutral position. This is where Hooke's restoring force, applicable to all elastic media, can now be related to the process, as we'll see further on, as put in perspective in References [8][9].

The two half-photons, now concerning only the transversely oriented half of the quantum and whose requirement was established by de Broglie, can then be understood as a pair of opposite electric charges whose recall intensity would constitute precisely these charges, that would increase during the oscillation process with the increasing distance between them as the energy half-quantum stretches within electrostatic Y-space, progressively inducing the ***E***-field state of the photon as the *displacement current* increases in strength, a recall intensity that would then decrease with the also decreasing *displacement current* after having peaked to a maximum, as the energy now enters magnetostatic Z-space, inducing the ***B***-field state in this other space until peaking in this space, thus completing the first cycle of the standing electromagnetic oscillation, to then re-enters Y-space to initiate the following cycle.

As developed in Reference [53], from the vectorial perspective, this pair of opposite charges can be represented by a pair of oppositely oriented ***E***-field state vectors, oriented so as to map their tendency to move towards each other, with their kinetic energy and velocity progressively decreasing as a function of the inverse of the decreasing distance separating them, while this energy transits to a perpendicular ***B***-field state (**Figure 5a**), also represented by a pair of oppositely oriented vectors acting perpendicularly with respect to the ***E***-field state vector pair for symmetry to be maintained.

When all of the ***E***-field energy has transited to ***B***-field state, to be momentarily immobilized at velocity zero, symmetric equilibrium considerations mandate that the energy now displaying ***B***-field state characteristics starts moving back, returning to the ***E***-field state through the *center-of-presence* of the quantum to complete the first cycle of a standing stationary oscillatory motion at the frequency corresponding to the amount of energy of the quantum as represented in **Figure 5b**.

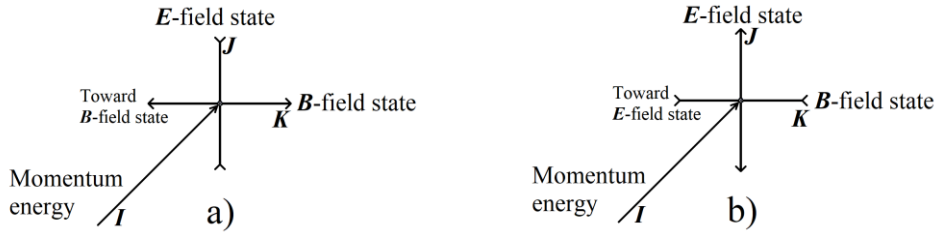


Figure 5: Vectorial representation of the standing oscillation of half a photon's energy between the ***E***-field state and the ***B***-field state in its configuration spaces, establishing the symmetric maximum velocity *c* in each orthogonal direction corresponding to the $c_J \times c_K = c^2$ velocities product, which is in half-half symmetric equilibrium with the other half of the photon's energy corresponding to its momentum energy located in normal X-space propelling the photon at velocity *c* along the ***I***//***i***/-*i* unit vector axis of **Figure 4b**.

What is of particular interest about these two opposite vector pairs inducing each other within the point-like behaving localized *center-of-presence* of the electromagnetic photon as represented with **Figure 5**, is that since they represent *a physically existing energy substance* cyclically moving from one maximum intensity to another oriented perpendicularly to the first, is that by structure, we are dealing with two acceleration sequences perpendicular to each other, whose maximum velocity will reach *c* but cannot exceed it if the velocity of the *substance* if to fall back to the required zero velocity when at maximum extent in either perpendicular orientations during the deceleration legs of the oscillation [47].

Given that the energy of the photon is symmetrically split into two equal parts between the propelling momentum part and the transversely oscillating part, that oscillates in standing mode between the **E**- and ***B***-field states, the vector cross product of the **E** and ***B*** vectors perpendicular to each other, will automatically result in exactly accounting for the momentum velocity vector of the photon, which is by structure perpendicular to both fields, in perfect conformity with Maxwell's electromagnetic theory as represented with Equation (14) further on. So, briefly summarized, the solution emerged from the long established invariant triple orthogonality of the

vector cross-product of the ***E***- and ***B***-fields vectors which is so fundamental in electromagnetism (**Figure 4a**).

The issue with the unit vectors ***ijk*** related to this traditional vectorial cross product $\boldsymbol{jE} \times \boldsymbol{-kB} \rightarrow \boldsymbol{-iv}$ related to the 3D Cartesian coordinate system represented with **Figure 4a** is that each unit vector ***i***, ***j*** and ***k*** are linear and can be made to describe a volume only when all three are involved to account for the 3 axes required to represent a volume, and that if the *energy substance* involved in the dynamic electromagnetic structure just described really has *physical existence*, it also has a *physical volume* that needs to be accounted for all through each phase of its separation into two halves, and also all through its standing oscillation sequence between the electromagnetic ***E*** and ***B*** phases of its oscillation, which is impossible to represent within the confines of the traditional 3D coordinate system of **Figure 4a.**

A first step in relating an actual volume to a photon's *energy substance* was the establishment of the *smallest possible spherical isotropic volume* that a localized photon's energy could be bundled into if its oscillation is theoretically stopped, during the first wave of derivations from Marmet's discovery published in Reference [32], and whose equation is:

$$V = \frac{\alpha^5 \lambda^3}{2\pi^2} \tag{13}$$

When the origin of the trispatial vector complex illustrated with **Figure 4b** is conceptually located at the *center-of-presence* of the energy quantum of an elementary charged particle or photon. This origin, represented by symbol $\otimes$ can be conceived of as an infinitesimal d*V* volume joining all 3 spaces, through which the particle's energy substance can be conceived as freely circulating as if within communicating vessels, to establish its symmetric equilibrium state as mentioned previously and analyzed in References [3][4].

The question now is: *A dV volume of what underlying medium?*

Inspired by Fourier's brilliant and so successful line of thinking according to which the rate of flow of heat energy through a homogenous macroscopic object should also be true of each infinitesimal *dV* volume of this object, it can be considered that the speed of electromagnetic energy through every infinitesimal *dV* volume of the vacuum of space at the subatomic level should be the same as confirmed in vacuum at our macroscopic and astronomical levels, which would mean that the well established impedance of vacuum $Z_0=\sqrt{(\mu_0/\varepsilon_0)}=376.7303135\ \Omega$ would also apply within such a conceptual infinitesimal *dV* volumes and across its infinitesimal *ds* surface.

Further replacing in **Figure 4b** the newly defined major unit vector set $\boldsymbol{I}=\boldsymbol{J}=\boldsymbol{K}=1$, each accounting for one of the mutually orthogonal 3D-spaces, each major unit vector being equal to 1 *by definition*, by a major unit vector set $\boldsymbol{I}=\boldsymbol{J}=\boldsymbol{K}=\sqrt{-1}$, that is, *the major unit vector set version* of Hamilton's quaternion, each equal to $1\angle 90°$ *by structure*, according to Caspar Wessel's definition [55], as analyzed in Reference [54], will set the stage to separate *real* normal X-space from the *complex* electromagnetic configuration spaces Y and Z (**Figure 4c**), that will be discussed further on.

For readers familiar with Hamilton's quaternions, the trispatial geometry expansion mathematically proceeded in the following manner. Setting *a* to zero in the standard definition of Hamilton's quaternion equation $H=a\cdot 1+b\boldsymbol{i}+c\boldsymbol{j}+d\boldsymbol{k}$ actually removes the real axis from the quaternion structure, leaving behind the three *rotated* orthogonal unit vectors of the *quaternion coordinate system* $H_0=H_{(a=0)}=b\boldsymbol{i}+c\boldsymbol{j}+d\boldsymbol{k}$.

For readers unfamiliar with the concepts of the imaginary axis of the complex plane, of the quaternion coordinate system, and of the geometric meaning of unit vectors defined as $\sqrt{-1}=1\angle 90°$, that is, a vector of length one rotated by 90° from a formerly defined direction, these concepts are completely put in perspective in Reference [54], but note that understanding these concepts is required only to gain understanding of why the *vectorial cross-product* of the two complex Y- and Z-spaces major unit vectors ***J*** and ***K*** results in the X-space major unit vector ***I*** to come out as mathematically "real" to reconverts the ***I*** vector space into a normal 3D Cartesian coordinate system mapping over normal 3D space, and why the orientation of the momentum energy vector residing in normal X-space is reversed to apply its related pressure against the infinitesimal *ds* surface of volume *dV* located at origin **O** of the *center-of-presence* of each quantized energy quantum.

Complete understanding of the trispatial model can be established all the same even if the reader is not curious to understand why $\boldsymbol{I}=\boldsymbol{J}=\boldsymbol{K}=\boldsymbol{i}=\boldsymbol{j}=\boldsymbol{k}=\sqrt{-1}=1\angle 90°$, which can be separately understood by reading the wonderfully well written book *The Story of* $\sqrt{-1}$ by Paul J. Nahin [55].

VII. SEPARATION OF REAL NORMAL X-SPACE FROM THE COMPLEX COMPLEMENTARY Y AND Z ELECTROMAGNETIC CONFIGURATION SPACES

The logical separation of *real 3D X-space* from its complex electromagnetic configuration spaces is already established by replacing the newly established $\boldsymbol{IJK}=1$ *major Cartesian unit vector set* by a corresponding $\boldsymbol{I}=\boldsymbol{J}=\boldsymbol{K}=\sqrt{-1}=1\angle 90°$ *major complex unit vector set*, and by actualizing the *vectorial cross-product* of major vectors ***J*** and ***K*** as analyzed in Reference [54], given that the quaternion vector space related to the minor complex unit vector set $\boldsymbol{i}=\boldsymbol{j}=\boldsymbol{k}=\sqrt{-1}=1\angle 90°$ already stands by definition out of real normal space, which

conceptually causes both Y and Z complex spaces of the trispatial geometry to also stand out of *real normal 3D X-space*.

Consequently, when centering the origin **O** of a trispatial vector complex to the *center-of-presence* of the energy quantum of a localized particle such as a photon or an electron, the complex ***I*** major unit vector resulting from the cross-product of the complex ***J*** and ***K*** major unit vectors now accounting for their ***E***- and ***B***-fields ends up converting to ***I***=-1, because ***J***×***K***=√-1×√-1=-1, which reverses the direction of application of the momentum energy residing in X-space to represent it as applying its pressure towards the *center-of-presence* ⊗ of the particle, on the other side of which the other half of the particle's energy oscillates between the ***E***-field state and the ***B***-field state, a *center-of-presence* corresponding to the trispatial junction ⊗ through which the particle's energy symmetrically maintains its stationary electromagnetic equilibrium as established in Reference [54] as represented with **Figure 6**, i.e. a momentum energy that accounts for an elementary particle velocity, which is *c* for free moving electromagnetic photons, and less than *c* for charged and massive particles, whose carrier-photon also accounts for their forward inertia.

As derived in Reference [32], we know that for free moving electromagnetic photons:

$$\boldsymbol{E}=\frac{\pi e}{\varepsilon_0\alpha^3\lambda^2},\ \boldsymbol{B}=\frac{\mu_0\pi ec}{\alpha^3\lambda^2},\ \frac{\boldsymbol{E}}{\boldsymbol{B}}=c \tag{14}$$

And for free moving charged and massive electrons:

$$\boldsymbol{E}=\frac{\pi e}{\varepsilon_0\alpha^3}\frac{\left(\lambda^2+{\lambda_C}^2\right)\sqrt{\lambda_C\left(4\lambda+\lambda_C\right)}}{\lambda^2{\lambda_C}^2\quad\left(2\lambda+\lambda_C\right)},\ \boldsymbol{B}=\frac{\pi\mu_0 ec}{\alpha^3}\frac{\left(\lambda^2+{\lambda_C}^2\right)}{\lambda^2{\lambda_C}^2},\ \frac{\boldsymbol{E}}{\boldsymbol{B}}=v\langle\langle c \tag{15}$$

So applying the major unit vector set ***I***=***J***=***K***=√-1=1∠90° to the major XYZ coordinate system of **Figure 4c** allows the following operation. Given that the product of ***J***=√-***1*** and ***K***=√-1 amounts to squaring √-1, then:

$$\boldsymbol{J}\mathrm{Y}\times\boldsymbol{K}\mathrm{Z}=\boldsymbol{I}\mathrm{X}\quad\rightarrow\quad\boldsymbol{J}\mathrm{Y}\times\boldsymbol{K}\mathrm{Z}=\left(\sqrt{-1}\cdot\sqrt{-1}\right)\left(\vec{\mathrm{Y}}\times\vec{\mathrm{Z}}\right)=-1\cdot\mathrm{X}_{\boldsymbol{I}} \tag{16}$$

And as put in perspective in Reference [54], given that the affected third major unit vector ***I***=√-1=1∠90° was established between the origin **O** and distance 1 along the major X-axis identifying the normal X-space, the reversal of the direction of application of the major unit vector will occur between the origin **O** and distance 1, causing whatever directed quantity is related to major unit vector ***I***to apply in direction of the origin of the major XYZ coordinate system, at which origin is located the infinitesimal *dV* volume ⊗ against whose *ds* surface the momentum energy located in X-space will now be shown to apply its pressure, as represented with **Figure 6**, on the complex side of which the other half of the energy of the particle is electromagnetically oscillating between electrostatic Y-space and magnetostatic Z-space, alternating between the ***E***-field state and the ***B***-field state, as represented with trispatial Equation (17) and **Figure 7** for the photon, as analyzed in Reference [54].

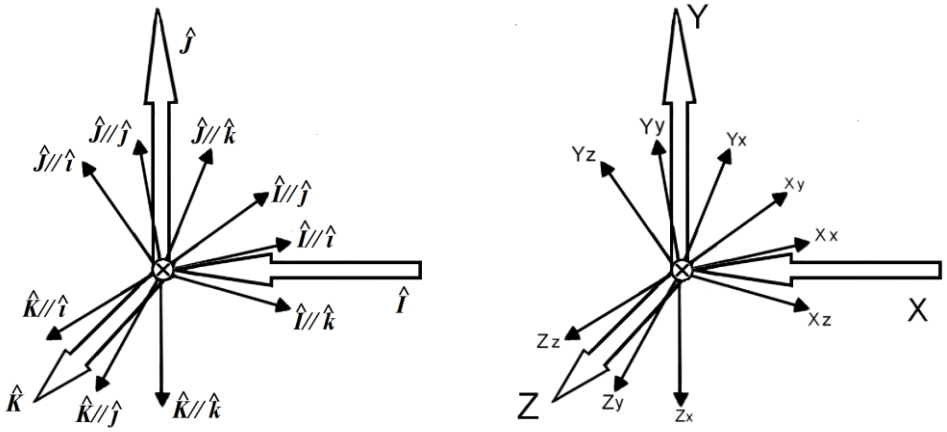


Figure 6: Coordinates of the trispatial geometry with *real* normal X-space major unit vector ***I***=-1 applying pressure against the origin **O** of the coordinate system where the cross-producted *complex* electromagnetic vectors ***J***×***K*** meet.

$$\vec{E}\,\vec{I}\,\vec{i} = \left(0, \left(-\boldsymbol{I//i}\right)\frac{hc}{2\lambda}\right)_X + \left[\begin{array}{l} \left\{\begin{array}{l} \left(0, \left(\boldsymbol{J//j}\right)\frac{e^2}{4C}\right) \\ +\left(0, \left(\boldsymbol{J// - j}\right)\frac{e^2}{4C}\right) \end{array}\right\}_Y \cos^2(\omega t) + \\ \left\{\begin{array}{l} \left(0, \left(\boldsymbol{K//ijk}\right)\frac{L\,i^2}{2}\right) \\ +\left(0, \left(\boldsymbol{K// - i - j - k}\right)\frac{L\,i^2}{2}\right) \end{array}\right\}_Z \sin^2(\omega t) \end{array}\right] \tag{17}$$

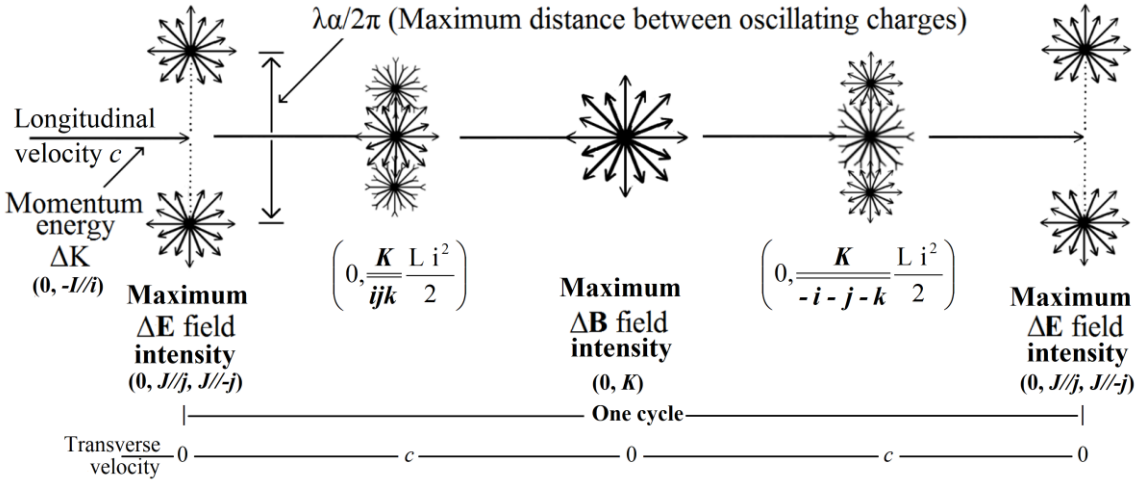


Figure 7: Representation of the stationary transverse oscillation cycle of the electromagnetic half-quantum of a free moving photon or of the carrier-photon of an electron.

VIII. THE DECOUPLING OF A 1.022 MEV ELECTROMAGNETIC PHOTON INTO AN ELECTRON-POSITRON PAIR

The nature of the electromagnetic *energy substance* of which the electron mass is made became clearer in the 1930's when Anderson discovered that electron-positron pairs were produced when gamma photons exceeding the 1.022 MeV energy threshold could be destabilized into converting to such a pair during his bubble chamber experiments [22], that is, an amount of energy minimally twice the 0.511 MeV of which the invariant electron rest mass is made, which is what confirmed that the electron rest mass was indeed made of the same electromagnetic *energy substance* as electromagnetic photons.

Now that normal X-space has clearly been separated from its two complementary configuration spaces, the stage is set to proceed to an overview – deeply analyzed in Reference [54] – of the decoupling mechanics of electromagnetic photons into massive electron-positron pairs, because this decoupling does not occur within normal X-space as would be expected, but within electrostatic Y-space as represented with **Figure 8**, which, like the complex plane and the quaternion vector space, is conceptualized as residing out of normal 3DX-space.

To illustrate the mechanics of this conversion, **Figure 8** does not represent the photon's magnetic field energy $\Delta\boldsymbol{B}$, since this energy will be considered at the moment when it has completely converted to the twin oscillating charges of the photon, represented at their maximum value in Y-space.

References [8][9] analyze the parallel relation between the Coulomb force and Hooke's recall force as applicable to the decoupling of such a photon, establishing the possible origin of the Coulomb force and identification of the unit charge of the electron as being the maximum charge intensity reachable in the universe, which sets it as the *fundamental elastic recall intensity constant in the universe*. Reference [5] on its part analyzes in detail the electron-positron pair decoupling mechanics in the trispatial geometry.

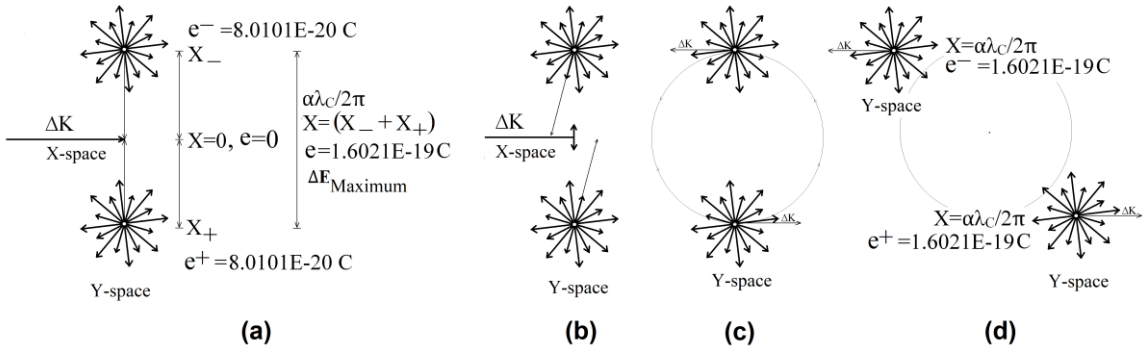


Figure 8: A 1.022+ MeV photon decoupling into an electron-positron pair.

Figure 5b illustrates the fact that the only local source of reserve energy to power the Y-space pair decoupling process is the momentum energy of the photon in process of decoupling. Since the pair of previously oscillating half-photons of the decoupling photon each initially had only an amount of energy of 0.2555 MeV at the beginning of the decoupling circular trajectory, the missing energy required for them to each build up to the 0.511 MeV required for escape velocity c to be reached within Y-space and for the actual separation to occur had to be burrowed from the only available source available, which is the momentum energy of the photon located within X-space, as represented with **Figure 5b**.

Whatever amount of energy of the photon in process of decoupling that remains in excess of the exact amount of 1.022 MeV, that went into establishing the rest masses of both particles, is known to be symmetrically shared between the two escaping massive particles, because the equal velocities of both particles in opposite directions was measured as corresponding to half this remaining energy.

For example, from the capture of such an event in a FERMILAB bubble chamber photograph of Experiment E632 previously shown as **Figure 1**, given that the energy/velocity of the positron that hit an electron head on at position **A**, converting both particles to a single photon of energy in excess of 1.022 MeV, that was observed as decoupling a few fractions of a second later at point **B**, as it reconverted to a pair of electron-positron moving away from each other with equal velocities, an event that was luckily captured in the same photograph.

These two quantities of energy now separated between both particles will not however remain unstructured. Thanks to Marmet's discovery we know that in addition to the momentum energy that propels the particles, this carrying energy also adds its own contribution to the invariant magnetic ***B*** field of the rest mass of the electron – corresponding to half the electron's rest mass – in the form of a magnetic field increment $\Delta\boldsymbol{B}$ [31][32]. We therefore know that this energy shared between the two particles *self-organizes* to establish this velocity-related magnetic field increment $\Delta\boldsymbol{B}$ in addition to the momentum energy $\Delta\boldsymbol{K}$ that establishes this velocity.

Due to the properties that the fundamental *energy substance* must have of *incompressibility*, *fluidity*, *elasticity* and of its *tendency-to-always-remain-in-motion* identified in Reference [3], we know that in order for this newly shared energy to obey the set of conditions that de Broglie determined as being required for localized photons to exist and obey Maxwell's equations, the only way for this energy to develop the $\Delta\boldsymbol{B}$ magnetic field increment identified by Marmet, as well as its mandatory $\Delta\boldsymbol{E}$ complement, in addition to their momentum energy, is for these amounts of kinetic energy to *self-organize* according to the electromagnetic structure illustrated with **Figure 5**. The mechanical process of establishment of this electromagnetic structure in the trispatial geometry is described in Reference [4], leading to Equation (17) illustrated with **Figure 7**.

The same self-structuring process will occur for the two separating half-photons whose energy increased from 0.2555 MeV to the required separation level of 0.511 MeV, now becoming a separately moving pair of massive and charged electron and positron as illustrated with **Figure 8d**, causing the energy that initially built up within Y-space for each particle to also reorganize according to the pattern illustrated with **Figure 5**, which can now be represented with Equation (18) and **Figure 9**, as initially developed in Reference [42].

$$m_e c^2\ \vec{I}//\vec{0} = E\ \vec{I}//\vec{0} = \left(0,(-J//i)\frac{hc}{2\lambda_C}\right)_Y + \left[\begin{array}{l}\left\{\begin{array}{l}\left(0,(I//j)\dfrac{v^2}{4C_{\lambda_C}}\right)\\ +\left(0,(I//-j)\dfrac{v^2}{4C_{\lambda_C}}\right)\end{array}\right\}_X \cos^2(\omega t) + \\ \left\{\begin{array}{l}\left(0,(K//ijk)\dfrac{(L\,i^2)_{\lambda_C}}{2}\right)\\ +\left(0,(K//-i-j-k)\dfrac{(L\,i^2)_{\lambda_C}}{2}\right)\end{array}\right\}_Z \sin^2(\omega t)\end{array}\right] \quad (18)$$

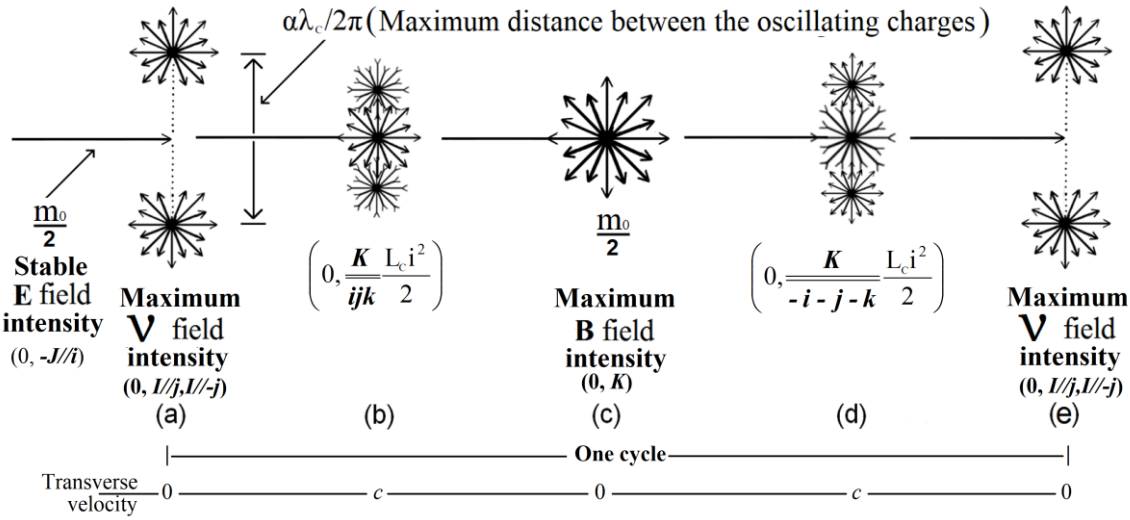


Figure 9: Representation of the cyclic oscillation of half the electron's rest mass energy between its magnetic ***B*** state and its neutrinic double-charge ***ν*** state, while the other half constitutes the invariant energy of its ***E***-field that establishes its invariant charge vectorially applying its pressure in direction of its *center-of-presence* ⊗ that separates it from X-space.

As developed in Reference [5], Equation (18) describes the inner electromagnetic structure of the electron as well as that of the positron, except for the vectorial direction of their ***E***-field energy within Y-space. Whereas this direction is (0,(-***J***//***i***)) for the electron in the $()_Y$ term of Equation (18), it is (0,(-***J***//-***i***)) for the positron, which accounts for the fact that their ***E***-field energies apply their *pressure* in opposite directions along the Y-x axis from inside Y-space against the *ds* surface of the *dV* volume of their *center-of-presence* ⊗ through which the energy of the quantum establishes its dynamic equilibrium between the three spaces, that is, between *real* X-space and the *complex* Y and Z configuration spaces, which pressure is what represents the invariant intensity of the opposite electric unit charges of the electron and of the positron and of the intensity of the ***E***-field of the electron and of the positron as perceived from within normal X-space.

The manner in which the carrier-photon that has now developed its own infinitesimal *dV* volume at its newly established *center-of-presence* ⊗ between the 3 spaces of its own trispatial coordinate complex will interact with the equally newly established *center-of-presence* ⊗ of the electron or positron, each newly established between the 3 spaces of their own trispatial coordinate complexes, will be analyzed in **Section XII** below.

IX. REPRESENTATION OF NORMAL SPACE AND OF THE TWO COMPLEX ELECTROMAGNETIC CONFIGURATION SPACES IN THE TRISPATIAL GEOMETRY

In the first decade of the 20th century, the unit charge of the electron was considered a secondary characteristic of the electron, that was still understood as a very small rigid body with no internal structure from the classical mechanics perspective, in the same manner as in macroscopic masses, with their total masses still

being seen as the characteristic of ponderable bodies on which gravitation was acting. The understanding of the internal structures of atoms was still too vague at the time to allow understanding macroscopic masses otherwise, that is, without any clear internal structure, except that they were accumulations of atoms whose possible inner structure was still unknown

So, from the perspective of classical mechanics as defined by Newton, the only known way to set a mass in motion was either to give it an *initial impulse*, or to lift it off the ground to a certain height, supposedly causing it to accumulate a certain amount of *potential energy* and to then let it fall to the ground. In the latter case, the force of gravitation as defined by Newton came into play, causing the body to accelerate according to the fundamental acceleration formula *F=ma*, supposedly transforming the accumulated *potential energy* into *kinetic energy*, which was then released on impact with the ground.

The problem with the theory of gravitation as defined by Newton was that the cause of the planets of the solar system setting themselves in motion and stabilizing in their obviously stable orbits was not explained by his theory, despite the fact that it allowed a fairly accurate calculation of their orbits. Identifying this cause became a major preoccupation of the physics community at the beginning of the 20th century.

But given that the too limited extent of the knowledge available about quantized electromagnetic energy at the time prevented a clearer analysis of the Kaufmann data, that would have allowed understanding that the continuous induction of energy in charged and massive elementary particles that does allow to set them in motion and control their velocity, offered a promising alternative to be explored in search of a replacement to the only previously identified means of an *initial impulse* to set a ponderable body in motion that could also be applied to planetary motion. Einstein conceived the alternate solution of his General Relativity theory in the second decade of the 20th century that immediately found approval in the community by apparently bringing at long last a logical method by which planetary bodies could be set in motion and kept in motion on their orbits.

Einstein had come to the conclusion that all ponderable bodies in the universe must be in inertial motion relative to one another, and conceived of a curvature of the four-dimensional space-time fabric recently defined by Minkowski [56][57] – who conceived of time as being a fourth dimension complementary to the 3D dimensions of space by burrowing the $\boldsymbol{i}=\sqrt{-1}$ dimension from the complex plane [55] – as a function of each of their masses, which could then explain their observed tendency to move closer together, their acceleration on their inertial trajectories and related increase of their momentum energy then being caused and influenced by the curvature of a conceptualized *fabric* of space-time whose local curvatures would be caused by the presence of all of the other astronomical masses. The vector treatment devised by David Hilbert at the same time proved useful for describing these states of motion in the form of a vector field mapped onto the entire universe.

Inspired by the quaternion orthogonal coordinate system to expand yet further the space geometry according to de Broglie's line of thinking as previously put in perspective, the trispatial geometry naturally emerged as a method allowing the establishment of a vector field common to both kinematic and electromagnetic mechanics as generally described in References [8][9].

In a manner similar to that used by Einstein, the *center-of-presence* of each elementary charged particle or photon can be conceptualized as being located at the center of a trispatial vector complex. But, instead of being in inertial motion, each *center-of-presence* can be seen as in force interaction with all other *centers-of-presence* according to the inverse square of the distances separating them in agreement with the Coulomb law, and in mutual adiabatic energy induction[15][16] in each particle as a function of the simple inverse of these distances, both cases being covered by the Coulomb interaction law between the charges of each elementary particle, and consequently in interaction between each of their masses as we will see.

The $\Delta\boldsymbol{K}$ *momentum* half of this induced adiabatic energy in the carrier-photon of each charged particle is what explains the motion of each of them in the trispatial geometry instead of a space-time curvature for inertial motion in Einstein's General Relativity theory, while the other half of their induced adiabatic energy, which oscillates transversely, explains the simultaneous increase $\Delta\boldsymbol{B}$ in their magnetic field and the increase in mass Δm associated with them in relation to the velocity supported by the $\Delta\boldsymbol{K}$ energy of their momentum, and that neither Special Relativity nor General Relativity take into account.

Whereas traditional vector fields represent elementary massive particles as dimensionless mathematical points, often perceived in the community as physically existing singularities in an underlying electromagnetic/aether field, corresponding to each of their *centers-of-presence*, the trispatial vector field allows representing them not as singularities, but with their *centers-of-presence* conceived of as infinitesimal *dV* volumes symbolized by $\otimes$ in normal 3D X-space, through which the energy the particle finds its natural internal equilibrium by separating symmetrically between real *normal* X-space and *complex* Y and Z-spaces, and can freely oscillate between an electrostatic 3D Y-space and a magnetostatic 3D Z-space, both perpendicular to each other and to X-space, while their $\Delta\boldsymbol{K}$ momentum energy remains in normal X-space to define their velocity by applying pressure against the infinitesimal *ds* surface that can be conceptualized as corresponding to this infinitesimal *dV* volume, thus establishing the internal triply perpendicular electromagnetic equilibrium mandated to harmonize localized energy quanta with Maxwell's equations.

This *ds* surface located at the *center-of-presence* of each particle, that is, at the very center of equilibrium of the particle's symmetric energy distribution within the three spaces of the trispatial complex, against which the $\Delta\boldsymbol{K}$ momentum energy of a free moving photon or of the carrier-photon of a massive particle would encounter an insuperable resistance and against which it would apply the pressure that causes the motion of the whole quantum, comes into existence from the simple fact that the three spaces structure acts as communicating vessels through this *dV* volume, and that the energy of the photon or carrier-photon cannot be separated by symmetry between the 3 spaces otherwise than in equal parts between its $\Delta\boldsymbol{K}$ momentum energy in X-space and its other half, which is oscillating between the Y- and Z-spaces located on the other side of this intervening *ds* surface, perpendicularly to the direction of application of the momentum energy other half, as represented with **Figure 5**, which is a symmetric equilibrium state that will systematically resist more energy crossing over into the Y- and Z-spaces side of the trispatial junction.

Like all previous vector fields, this idealized vector field emerged from Gauss's ontological definition of a *potential* ***E***-field by removing one of the two point-charges from the Coulomb equation:

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 d^2} \rightarrow \boldsymbol{E} = \frac{F}{q_1} = \frac{q_2}{4\pi\varepsilon_0 d^2} \tag{19}$$

In which q_1 and q_2 were initially defined as *mathematical point-charges* by French physicist Augustin de Coulomb in 1784 as he was studying the force of attraction and repulsion between two *point-charges*, that is, charged bodies whose dimensions were made relatively very small compared to the distance *d* separating them, and whose resulting relative parallax angle made them behave as satisfactory *point-charges* for calculation purposes [58].

This type of macroscopic experimental set up meant to draw conclusions applicable to infinitesimal point-like behaving particles was historically proven valid by Newton [59], who confirmed that a spherically symmetric mass distribution behaves in the same manner as a point particle located at its center.

9.1 Traditional conception of the unit charge

Incidentally, Coulomb was experimentally confirming in 1784 the same interaction law of a force proportional to the inverse square of the distance separating point-like behaving *charges* that Newton previously confirmed one hundred years earlier in 1687 in his famous work *Philosophiæ Naturalis Principia Mathematica* as applying between *astronomical masses* from his analysis of Johannes Kepler's third law that "*the cube of the mean radius of a planetary orbit in the solar system is proportional to the square of the orbiting body's period*" [60], that Kepler empirically established in 1618 from the data carefully collected over many years by Tycho Brahe about the observable planetary orbits in the solar system.

So, Newton's mathematical proof of the inverse square force law with distance between astronomical masses as well as Coulomb's mathematical proof of the same interaction force law with distance between point-like behaving electric charges were both grounded on the study of carefully collected experimental data, but no relation was yet established between point-like behaving electric *charges* and the *masses* of astronomical bodies.

The first relationship between a *true charge* behaving like a point and a *true mass* behaving like a point as *two characteristics of the same ponderable body* was established much later, when J.J. Thompson discovered the electron in 1897, whose *invariant unit charge* and *invariant rest mass* were confirmed experimentally in 1913 by Millikan, as previously mentioned.

The same technique used by Coulomb that was previously validated by Newton was also used in 1998 to experimentally confirm the inverse cube interaction law with distance between two magnetic fields both poles of each of which coincide with the geometric center of each magnet, and whose details were formally published in 2013 [61], to simulate the hypothesized identical behavior of the magnetic fields of 2 point-like behaving electrons, whose two magnetic poles of each of which can only be located by structure at their point-like behaving *centers-of-presence*, as experimentally proven one year later by the Kotler et al.'s experiment in 2014 with real electrons [62].

According to Reference [58], as an outcome of the Coulomb experiments, the definition of the Coulomb law was then established as:

> *"The force of attraction or repulsion between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them"*

It was also determined that when two or more such point-charges simultaneously exert forces on a third point-charge, the total force experienced by this third charge is the *vector sum* of the forces that the first two point-charges would exert individually, which is the definition of the *Principle of superposition*, that allows application of the Coulomb law to arrays of charges of any degree of complexity.

This is what inspired Gauss to define the electric ***E***-field at the location of a given point-charge by dividing the Coulomb force by the magnitude of one of the point-charges defined by Coulomb in his equation $q_1=e$, thus leaving in the equation the *potential magnitude* ***E*** of the *vector sum* $q_2=Q_2=\mathrm{N}\cdot e$ of all other point-charges that would have been acting on the now absent q_1 test charge:

$$\boldsymbol{E}=\frac{F}{q_1}=\frac{q_2}{4\pi\varepsilon_0 R^2} \quad \rightarrow \quad \boldsymbol{E}=\frac{F}{q_1}=\frac{Q_2}{4\pi\varepsilon_0 R^2}=\frac{\mathrm{N}\cdot e}{4\pi\varepsilon_0 R^2} \tag{20}$$

Given that it is clearly established that the unit charge of the electron is universally quantized at the fixed value of e=1.602176462E-19 Coulomb, the Q_2element represents any number of unit charges N·e with N possibly varying from 1 as in Coulomb's original equation to the total number of other elementary charges in the universe:

$$\boldsymbol{F}=e\boldsymbol{E}=e\frac{\mathrm{N}\cdot e}{4\pi\varepsilon_0 R^2} \tag{21}$$

When N is set to 1 in Equation (21), this single remaining point-charge came to be defined as the "test-charge", representable as a mathematically dimensionless point from which the intensity of the related *potential* ***E***-field would diminish omnidirectionnally from a theoretical infinite intensity level at the location of the test charge, as an inverse square function of the increasing distance.

This mathematical conception of an idealized dimensionless point charge, however, introduced an element of confusion when applied to the electron, which possesses a charge with point-like behavior but a clearly non-infinite mass, both characteristics being localized at its *center-of-presence* ⊗, as Patrick Cornille remarks in his synthesis of the state of electromagnetic theory in 2003 [63]:

> *"If electrons are strictly point-like objects, their self energy is infinite and therefore they must have infinite mass because of the equivalence between energy and mass."* Patrick Cornille 2003 ([63], p. 261)

For visualization purposes, the exponential increase with diminishing distance of the *potential* force field that can be conceptualized as reaching maximum intensity at the location of each point-charge considered will be visually represented in **Figure 10** and following figures even if it is only *virtual*, so that the reader remains aware of the Coulomb force which is in permanent action between real charged particles in physical realty.

This conceptualization of point-like charges as represented with **Figure 10** is seen as problematic of course as mentioned by Cornille, and is traditionally dealt with by means of the renormalization method and is considered completely addressed if the electron is considered as a *wave packet* in Quantum Mechanics, even though the *wave packet* solution conceived by de Broglie in his 1924 thesis [29][30] was established from a wrong frequency associated to the electron *phase wave*, that is, half the frequency of the energy which is really induced by the Coulomb force in the unit charge of the electron at Bohr radius distance from the proton in the Bohr model of the hydrogen atom as analysed in depth in Reference [20].

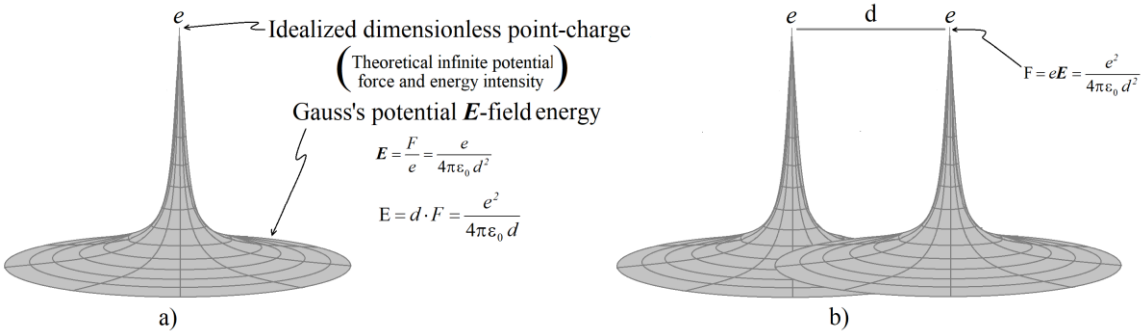


Figure 10: Test point-charge and *potential* ***E***-field representation as conceptualized by Gauss, and theoretical point-charges interaction as initially established by Coulomb.

Such dimensionless point representations of charges obviously amount to theoretical singularities. The reader must also keep in mind that **Figure 10** provides only a simplified representation of such singularities due to the difficulty of representing 3D concepts on a flat sheet or screen and that in reality, what should be mentally visualized is that the intensity variation would be *omnidirectional* from this idealized dimensionless point, meaning that the intensity would *spherically diminish in all directions* in space about this point as an inverse function of the square of the increasing distance for the *potential force* or *potential* ***E****-field*, and as a simple

inverse function of the increasing distance for the *potential energy* involved. So the **Figure 10** representation of the *potential* ***E***-field energy should be seen as just a small segment of the tridimensional spherical concept that cannot be directly illustrated on a flat surface.

Now, Gauss's definition of such a *potential* ***E***-field centered on a single localized test charge as represented with **Figure 10a**, *de facto* establishes that a similar *potential* ***E***-field can be conceptualized about any isolated elementary charge in the universe. Consequently, a second *potential* ***E***-field can also be conceptualized as centered on a second charge that would be introduced at any distance from the initial test charge, as represented in **Figure 10b** and the initial Coulomb force Equation (21) is re-established between the two charges involved according to the Coulomb law, and the actual force in action between both charges can be calculated as a function of the inverse square of the rectilinear distance separating them.

$$F = e\boldsymbol{E} = \frac{e^2}{4\pi\varepsilon_0 d^2} \tag{22}$$

9.2 The Atomic Units System

A note of caution is in order here regarding the squared charge value e^2 of the Coulomb equation as used in this work and in all engineering reference works such as the comprehensive Giancoli Reference [64] and also in some reputed reference theoretical works such as [65], which is not to be confused with the similarly named e^2 or q^2 "*unit*" from the very confusing *System of Atomic Units* used in some theoretical works such as [66], in which theoreticians equate all fundamental values to 1in Quantum Physics research such as:

$$c = \hbar = m_e = \frac{e^2}{4\pi\varepsilon_0} = 1 \quad \text{and also redefine } e^2 \text{ as } e^2 = q^2 = \frac{e^2}{4\pi\varepsilon_0} \tag{23}$$

This counter intuitive measurement system that some theoreticians apparently find convenient can be quite confusing to non-specialized readers, and can even confuse theoreticians themselves, such as John Wheeler himself, who, in order to "prove" that the electrostatic force could not possibly be the same as the gravitational force – which is blatantly false as mathematically demonstrated in Reference [60] – quite inappropriately *visually* compared in Reference ([66], page 391) a strangely formulated Coulomb equation with the gravitational equation as:

$$\mathrm{F_{electr}} = \mathrm{e_1 e_2 / r^2} \quad \text{and} \quad \mathrm{F_{grav}} = -\mathrm{G m_1 m_2 / r^2} \tag{24}$$

without clarifying that with e_1e_2, he was referring to the $e^2=q^2=e^2/4\pi\varepsilon_0$ "*unit*" of the *Atomic Units System*, indifferently worth "1", or "2.307077056E-28" which is its real numerical value, depending on the context, as provided with Equations (23). Or was he himself confused into thinking that the Coulomb force reduces to a pair of charges divided by the square of a distance without proportionality constant, as analyzed in References [67][68][69]? Or was he simply under the impression that only other theoreticians familiar with this unit system would read his book?

Now back to our main line of reasoning. If a third charge is introduced at any distance from the two charges represented in **Figure 10b** now conceptually populating the virtual space, a third *potential* ***E***-field can also be conceptualized as centered on this third charge, and two new occurrences of the linearly acting Coulomb force interaction come into being between this third charge and each of the two previously introduced charges as illustrated with **Figure 11a**, and so on for all additional elementary charges introduced in the virtual space until all existing elementary charges in the universe have theoretically been accounted for, as illustrated with **Figure 11b** as well as all Coulomb force interactions between all charges in the universe as a function of the network of all rectilinear relations now established between each charge and all other charges in the universe.

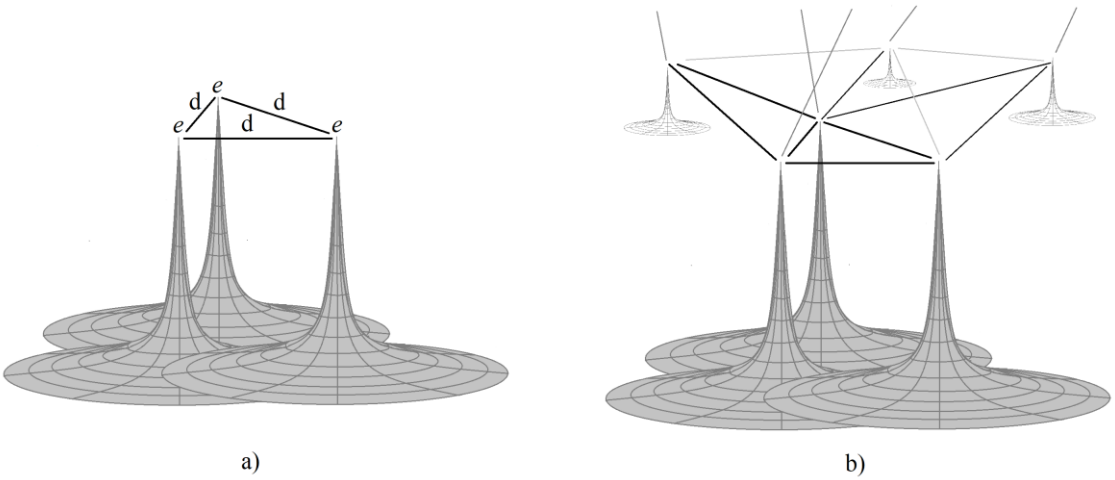


Figure 11: Punctual charges interaction network in the traditional vector field.

9.3 The concept of unit charge at the beginning of the twentieth century

It also was customary since Maxwell conceived of his electromagnetic fields theory in the 1860's, to represent the vacuum of space as being physically filled with *fields*, in which charged particles would emerge as *singularities*, even from the classical mechanics perspective, such as in Hilbert vector fields.

> *"...on s'habitua à considérer les champs électrique et magnétique comme des entités dont l'interprétation mécanique était superflue. On en vint ainsi à regarder ces champs dans le vide comme des états particuliers de l'éther, n'exigeant pas une analyse plus approfondie. Une particule chargée en mouvement par rapport à l'éther est assimilable à un élément de courant; les actions du champ électromagnétique sur la particule et les réactions de cette dernière sur le champ sont les seuls liens qui lient la matière à l'éther."* Albert Einstein (1910) [49][50]
>
> *"...the electric and magnetic fields eventually came to be considered to be entities whose mechanical interpretation was superfluous. This led to the view of these fields in vacuum as particular states of the aether that did not require further analysis. A charged particle in motion relative to the aether is like an element of current; the actions of the electromagnetic field on the particle and the reactions of the latter on the field are the only bonds that bind matter to the aether."*

A few years later, Einstein also represented ponderable masses as lying at the bottom of gravity wells in his coming General Relativity theory, which lead to the eventual hypothesis from other researchers that *infinite energy point-like black holes* would lay as *real singularities* at the very bottom of such gravity wells, a concept that then evolved into a whole set of further theories, as put in perspective in Reference [53].

So we observe that at the beginning of the 20th century, the original concept of the Coulomb force being in action between charges that was initially experimentally established by Coulomb as illustrated with **Figures 10** and **11** progressively evolved in the theoreticians community from the original Coulomb conception of charges being understood as point-like behaving "objects" interacting with each other according to an inverse square relation with mutual distances, into *supposedly real singularities* that would emerge as local energy concentrations in an underlying *energy field*, that was initially established as *conceptually virtual* by Gauss, now perceived as becoming a "*physically existing*" *electromagnetic/aether energy field* as represented with **Figure 12**, which caused the initial understanding that charges actually directly interact rectilinearly with each other to progressively fade from general awareness in the theoreticians community.

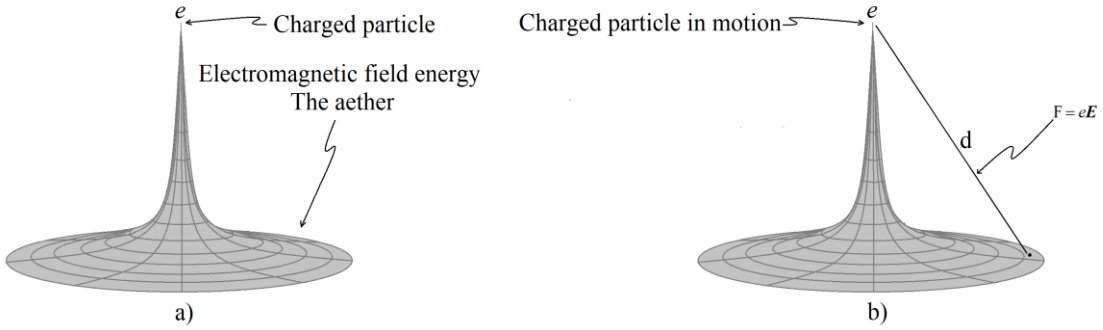


Figure 12: Representation of the *electromagnetic-aether energy field* understood as physically existing from which charged particles would emerge as local higher energy singularities interacting with the field as reported by Einstein in 1910 as being the then accepted view at the beginning of the 20th century.

Eventually, even the original Gauss definition of the ***E***-field as presented in Equation (22) ceased being related to the Coulomb equation in numerous formal sources such as References [59][70], to end up often presented only with nondescript formulations such as $F=e\boldsymbol{E}$ or $F=d\boldsymbol{A}$ that do not refer at all to the Coulomb law, but fortunately not in other references such as [65], although the latter nevertheless actively encourages readers *to think of the field as a "real" physical entity* ([65], p. 61). All engineering reference works however on their part such as Reference [64] kept on correctly defining the ***E***-field as defined by Gauss.

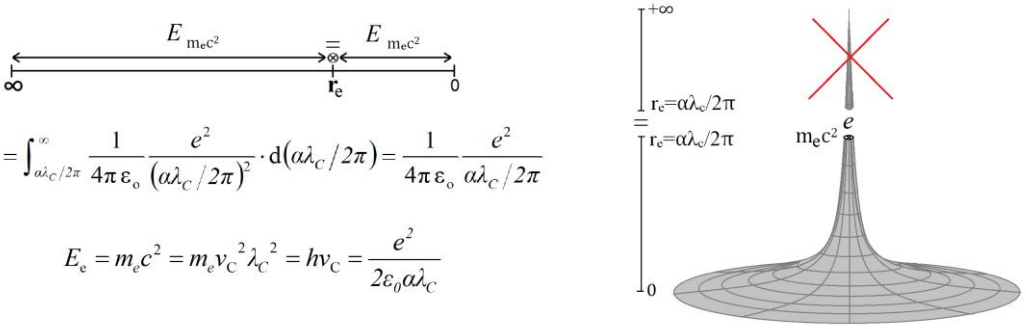
But as mentioned by Patrick Cornille in his above mentioned quote and further comments ([63], p. 261) the representations of **Figures10**, **11** and **12** of charged particles as being strictly dimensionless point objects is problematic in many respects, mainly because it involves that their energy would be infinite and so would their mass, which makes no experimental sense, since it is clearly established that the rest mass of all electrons is universally invariant at the very precise value of m_0=9.10938188E-31 kg.

So, we know that such fields are only convenient mental representations, and that in physical reality no elementary charged particle can be such dimensionless points, and that at best, these point-representations represent only each of their *centers-of-presence* in space, even if, as mentioned by Einstein, the related idealized potential fields involved have come to be seen as having a physical existence.

9.4 The trispatial concept of the unit charge

It is however possible to symbolically represent the physical energy intensity limit that we know that these charged particles reach in stable stationary action manner, by conceptually stopping the representation of their increase in energy at levels that experimental measurements now allows us to clearly establish, on their way to these physically unreachable dimensionless point-like states, as represented with **Figure 13**.

It is then possible to represent the charge of an electron as being located at the same level as the infinitesimal *dV* volume that represents the energy level of 8.187104135E-14 joules related to the *center-of-presence* ⊗ of the electron as in **Figure 13**, which lies way below this dimensionless singularity point representation of **Figures 9**, **11** and **12** which is so problematic.



$$E_e = \int_{\alpha\lambda_C/2\pi}^{\infty} \frac{1}{4\pi\varepsilon_o}\frac{e^2}{(\alpha\lambda_C/2\pi)^2}\cdot d(\alpha\lambda_C/2\pi) = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{\alpha\lambda_C/2\pi}$$

$$E_e = m_e c^2 = m_e v_C^{\,2}\lambda_C^{\,2} = hv_C = \frac{e^2}{2\varepsilon_0\alpha\lambda_C}$$

Figure 13: In the trispatial vector field, the invariant energy, invariant charge and invariant rest mass of the electron are represented as residing at the *center-of-presence* ⊗ of the electron.

This more accurate level of the *center-of-presence* of the electron in the increasing *potential* Gaussian ***E***-field of **Figure 10a** can be established by integrating the energy of the field from infinity ∞ to the distance from zero established by the lower limit of integration of the known rest mass energy of the electron that Marmet used in his Equation (16) [31] and which is the constant that was inappropriately named the *electron classical radius* r_e at a time when the electron was still perceived as a small solid mass as understood from the

classical mechanics perspective, but that turns out in reality to be both the *lower limit of integration* of the known rest mass energy of the electron and the *transverse amplitude of oscillation* of the electron rest mass energy, as determined in Reference [32]:

$$r_e = \frac{\alpha\lambda_c}{2\pi} = 2.817940285\text{E}-15\,\text{m} \tag{25}$$

In which λ_C=2.426310215E-12 m is the constant established as the *electron Compton wavelength*, i.e. the known invariant wavelength of the electron rest mass energy, and α=7.297352533E-3 the fine structure ratio constant. Let us confirm that r_e really is the lower limit of integration of the electron rest mass energy by obtaining the known energy of this rest mass from infinity by means of integrating the Coulomb equation with reference to its transverse amplitude r_e as calculated with Equation (25):

$$\text{E}_\text{e} = \int_{\text{r}_\text{e}}^{\infty} \frac{1}{4\pi\,\varepsilon_\text{o}} \frac{\text{e}^2}{\text{r}_\text{e}^{\,2}} \cdot \text{dr}_\text{e} = 0 - \frac{1}{4\pi\,\varepsilon_\text{o}} \frac{\text{e}^2}{\text{r}_\text{e}} = \text{-8.187104135E -14 J} \tag{26}$$

Now, by means of the known relation:

$$E = h\nu = \frac{hc}{\lambda} \tag{27}$$

in which wavelength λ can be isolated and with the electron rest mass energy E_e calculated with Equation (26), let us recuperate the electron Compton wavelength λ_C, that confirms the relation between r_e and λ_c, established with Equation (25) as developed in Reference [32]:

$$\lambda_\text{C} = \frac{hc}{E_\text{e}} = \frac{6.62606876\text{E}-34\times 299792458}{8.187104135\text{E}-14} = 2.426310216\text{E}-12\,\text{m} \tag{28}$$

Isolating frequency ν in Equation (27), and again with the electron rest mass energy E_e calculated with Equation (26), let us also recuperate the invariant electron Compton frequency ν_c:

$$\nu_\text{C} = \frac{E_\text{e}}{h} = \frac{8.187104135\text{E}-14}{6.62606876\text{E}-34} = 1.235589975\text{E}20\,\text{Hz} \tag{29}$$

A little documented peculiarity about integrating energy from zero to infinity is that at whatever distance the lower limit of integration is set from zero, as calculated with Equation (26), the amount of energy integrated from infinity to this distance from zero will always be equal by structure to the energy that can be calculated from zero to this lower limit, as illustrated in **Figure 13**.

This peculiarity came to light during the analysis carried out in Reference [32] by replacing r_e by its *transverse amplitude of oscillation* established from its electron Compton wavelength as used in Equation (26), after having been established with Equation (25):

$$\text{E}_\text{e} = \int_{\text{r}_\text{e}}^{\infty} \frac{1}{4\pi\,\varepsilon_\text{o}} \frac{\text{e}^2}{\left(\alpha\lambda_\text{C}/2\pi\right)^2} \cdot \text{dr}_\text{e} = 0 - \frac{1}{4\pi\,\varepsilon_\text{o}} \frac{\text{e}^2}{\alpha\lambda_\text{C}/2\pi} = \text{-8.187104135E -14 J} \tag{30}$$

Further simplifying Equation (30) led to this version of the Coulomb equation for calculating the amount of energy related to this amplitude of this transverse oscillation of the double-components of the electron ***ν***-field as in Equation (31) of any photon ***E***-field from its specific wavelength:

$$\text{E}_\text{e} = \frac{1}{4\pi\,\varepsilon_\text{o}} \frac{\text{e}^2}{\alpha\lambda_\text{C}/2\pi} = \frac{\text{e}^2}{2\,\varepsilon_\text{o}\alpha\lambda_\text{C}} = 8.187104135\text{E -14 J} \tag{31}$$

This form then turned out to be a general energy calculation equation developed in Reference [32], which is directly equivalent to Planck's equation $E=h\nu$, allowing to calculate the infinitesimally progressive blue- and red-shift variation of the energy of electromagnetic photons as they circulate in the universe after emission, without any need to use Planck's constant, as a complement to Planck's equation required to account for the quantized nature of electromagnetic photon energy emission by de-exciting electrons as they return to their rest orbitals in atoms after having been excited away from these orbitals when over-energized by convection, conduction or having been hit by, and having absorbed part or all of the energy of incoming electromagnetic photons, as more clearly put in perspective in References [43][71]:

$$E = h\nu = \frac{hc}{\lambda} = \frac{\text{e}^2}{2\,\varepsilon_\text{o}\alpha\lambda} \tag{32}$$

While the standard Coulomb force Equation (19) applied to the transverse amplitude $\alpha\lambda_C/2\pi$ of oscillation of the two neutrinic energy charges of the electron rest mass [44], as integrated with Equation (30), converts to the *Hooke force version* of the *Coulomb transverse force equation* that allows calculating the Hooke recall force that will force both neutrinic energy charges to oscillate back towards the *center-of-presence* of the electron, as analyze in References [8][9]:

$$\boldsymbol{F} = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{(\alpha\lambda_C/2\pi)^2} = \frac{\pi e^2}{\varepsilon_o\alpha^2{\lambda_C}^2} = -k\cdot\frac{\alpha\lambda_C}{2\pi} = -29.05350472\,\mathrm{N} \tag{33}$$

in which *k* is the *elastic recall constant* of the Hooke force equation initially calculated in Reference [46] as applying at the decoupling distance of an electron-positron pair illustrated with **Figure 8**.

As previously mentioned, this peculiarity of the integration process of energy from zero to infinity is what allowed replacing the traditional representation of point-charges of **Figure 10a** leading to a theoretical singularity related infinite mass and energy, by the more realistic infinitesimal *dV* volume representation of **Figure 13** in which the charge of the electron is represented at the same level as the *center-of-presence* of its rest mass energy, and allowed doing away with the need to use the concept of a dimensionless point charge leading to a theoretical but physically impossible infinite energy singularity to be used with the Coulomb equation.

So, comparing **Figure 14**, which reconfigures **Figure 10** according to the invariant charge and invariant rest mass characteristics of the electron represented with **Figure 13**, now allows observing the difference between the traditional representation of **Figure 10** of charges as idealized dimensionless singularities implying infinite mass and their trispatial representation of **Figure 14** at their more realistic invariant energy level, invariant rest mass and invariant charge both being symbolized as being located in normal X-space at the same *center-of-presence* of the electron.

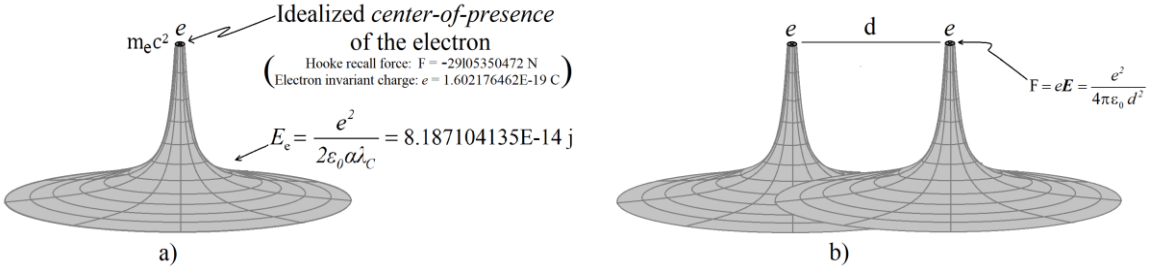


Figure 14: The Coulomb force interaction in the trispatial vector field between electrons represented at their known invariant energy, invariant charge and invariant rest mass energy levels.

Similarly reconfiguring the representation of **Figure 11** according to the same invariant electron characteristics represented with **Figure 13** now allows observing a more realistic representation of the Coulomb force interaction between charged particles in the universal vector field with **Figure 15** when compared to **Figure 11**.

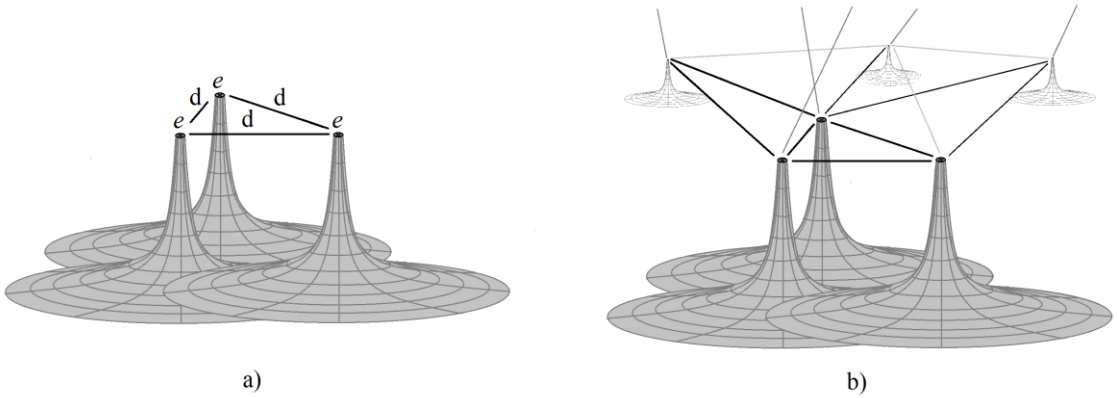


Figure 15: Coulomb force interaction between unit charges in the trispatial vector field.

X. DISTRIBUTION OF THE ELECTRON REST MASS ENERGY BETWEEN THE TWO COMPLEX SPACES PERPENDICULAR TO NORMAL SPACE

Figure 16 represents the conceptual separation between the normal 3D X-space in which the momentum energy of all elementary electromagnetic particles resides, and the two complementary complex configuration spaces Y and Z into which the electromagnetic energy of these particle reside and oscillate, with the infinitesimal *dV* volume and its *ds* surface corresponding to the *center-of-presence* ⊗ of each elementary particle against which all other elementary particles can physically collide in normal 3D X-space, and which is conceptually located at the junction between X-space on the one hand and Y- and Z-spaces on the other.

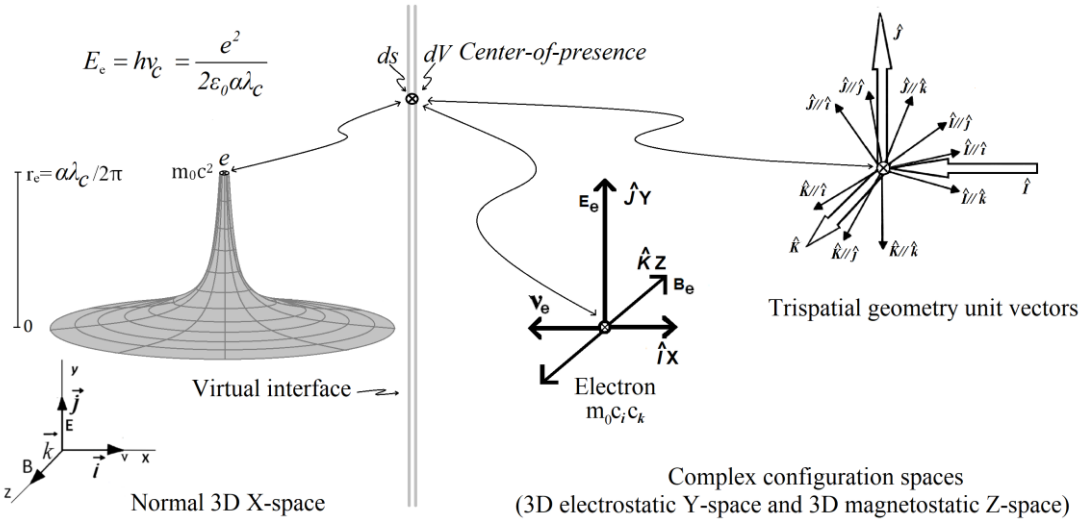


Figure 16: Idealized representation in the trispatial geometry of the electron stable charge intensity related to half of its rest mass energy residing in Y-space and of the stable stationary oscillation of the other half of its rest mass energy between X-space and Z-space.

The reader will note that, according to the perspective of the trispatial geometry, none of the energy of which the electron's rest mass is made up resides in normal 3D X-space, with the exception of the fleeting visits of half of its energy during its oscillation at the frequency v_c=1.235589976E20 Hz per second of the two opposite-signs ***v***-components on the 2D ***I//jk*** vectorial plane of X-space, perpendicularly to the X//x-axis of ***I//i*** vectorial orientation, i.e. an energy that oscillates between this dual-particle ***v***-field state and its spherically expanding and regressing ***B***-field state that resides within Z-space according to its 3D ***K//ijk*** vectorial volume.

It must be noted that this oscillating energy of the electron's rest mass can in no way contribute to the particle's motion, since it oscillates in X-space on a plane perpendicular to the X//x axis of vector orientation ***I//i*** along which unidirectional momentum energy must be aligned for a pressure to be exerted against the electron's *center-of-presence* ⊗, in order for electron motion along this axis to be possible in the trispatial geometry.

This oscillating energy of the ***v***-field state cyclically present within X-space that cyclically returns to Z-space in ***B***-field state, i.e. a Z-space which is perpendicular by structure to X-space, was however accounted for as half the rest mass of the electron that Marmet derived from the Biot-Savart equation, identified by him as the invariant magnetic mass $\boldsymbol{M}_e/2$ of the electron at rest, previously reproduced as the second Equation (9) – his Equations 23 [31] – while its first Equation (9) provides the exact amount of total magnetic energy of the moving electron, including its velocity related increment $\Delta\boldsymbol{B}$, which is also part of the momentary variable ***B***-field of the moving electron at any given velocity $\boldsymbol{M}_e v^2/2c^2$.

For the electron to move in space, its unidirectional momentum energy in clear alignment with the X//x-axis of X-space, in vectorial orientation ***I//i*** must consequently be provides by energy from another source.

Since the electron's rest mass energy is invariant, this magnetic $\Delta\boldsymbol{B}$-field energy increase with velocity in excess of the electron's invariant $\boldsymbol{B}_e$ field energy derived by Marmet from the Biot-Savart equation as being the same amount of energy that was measured transversely as a mass increase Δm in Kaufmann's data by means of Lorentz's Equation (30) for transverse mass m_2 of the moving electron is the experimental proof that confirms the actual physical existence of this added energy.

Deep analysis of Marmet's derivation carried out in References [3][4] allowed understanding that both the $\Delta\boldsymbol{K}$ momentum energy along the vectorial axis ***I//i*** of X-space to determine the velocity of the electron, as well as the $\Delta\boldsymbol{B}$-field energy increment that increases its effective mass in relation with this velocity, can only be provided by a carrier-photon that has the very same electromagnetic structure as de Broglie's free moving double-particle electromagnetic photon [3][4], whose velocity is reduced to the measured velocity of the electron due to its having to "carry", so to speak, the rest mass energy of the electron, which is inert in normal X-space, on top of having to propel its own transversely oriented $\Delta\boldsymbol{B}$-field energy, by applying its $\Delta\boldsymbol{K}$ momentum energy pressure against its own *center-of-presence* ⊗, then dragging along the *center-of-presence* ⊗ of the carried electron, as analyzed in Reference [21], and that we will now examine. At least, this is how this relation can be geometrically represented and mathematically dealt with in the trispatial coordinate system.

But before addressing further this relation, let us examine how the energy of a free moving double-particle de Broglie photon or carrier-photon is distributed between normal X-space and its complex configuration Y- and Z-spaces.

XI. DISTRIBUTION OF ENERGY OF A PHOTON OR OF A CARRIER-PHOTON BETWEEN NORMAL SPACE AND ITS TWO COMPLEMENTARY CONFIGURATION SPACES

In a manner similar to the representation of **Figure 16**, that was meant to make obvious that only the electron infinitesimal *dV* volume located at its *center-of-presence* ⊗ represented by mapping over it the origin **O** of a trispatial coordinate complex, is really present within normal X-space, where it is subject to interact or to even collide with the infinitesimal *dV* volumes of other elementary particles when the right force and energy circumstances are met, **Figure 17** is meant to make obvious that in the case of free moving photons – or carrier-photons – both their Δ***K*** momentum energy and their infinitesimal *dV* volumes located at each their own *centers-of-presence* ⊗, represented by mapping over them the origin **O** of each their copy of a trispatial coordinate complex, are present within normal X-space, making them all susceptible to interact or even collide with the *centers-of-presence* of other elementary particles.

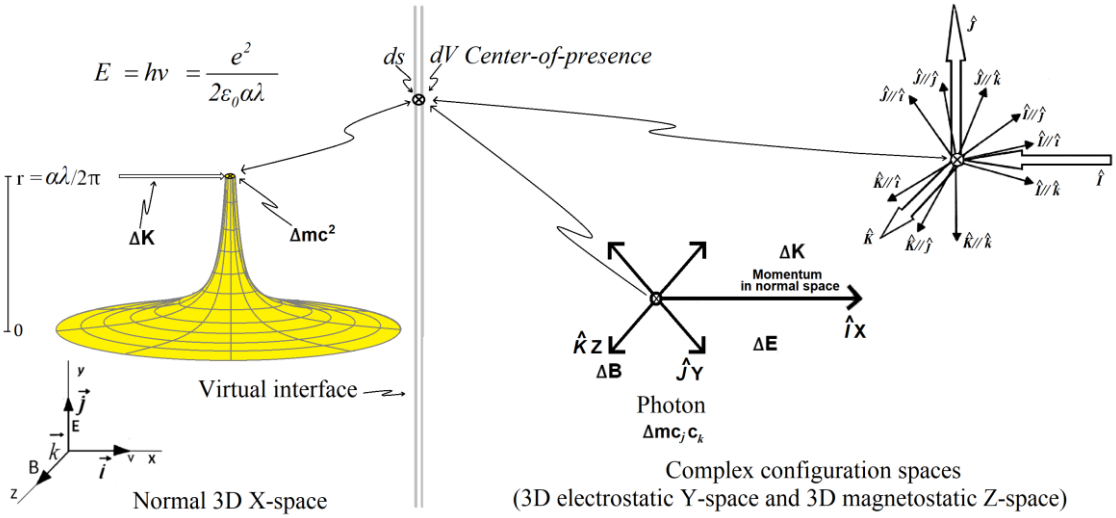


Figure 17: Idealized representation in the trispatial geometry of an electromagnetic photon Δ***K*** momentum energy related to half of its energy quantum residing in X-space and of the stable stationary oscillation of the other half of its energy between configuration spaces Y and Z.

Carefully comparing **Figure 17** with **Figure 16** will allow the reader to observe that while the transverse amplitude of oscillation of the rest mass energy of the electron is invariant at the fixed value of $\alpha\lambda_c/2\pi$=2.817940285E-15 m, that of the energy of photons or carrier-photons is set to the more general formulation $\alpha\lambda/2\pi$ in which wavelength λ is variable and can take any value in the whole range covered by the electromagnetic frequencies spectrum, from the longest radio wavelengths to the shortest possible gamma wavelengths.

In the case of free moving photons, the default and invariant velocity of light in vacuum is invariantly set at *c*, due to the simple fact that the energy of its quantum is mandatorily split in exactly two equal halves, which means that its Δ***K*** momentum energy component propels *an always exactly equal amount of energy* that electromagnetically oscillates on a plane transverse to the direction of application of the Δ***K*** momentum energy against its *center-of-presence* ⊗ along the ***I//i*** vectorial axis of normal X-space.

In the case of the carrier-photon of an electron, the variable velocity of the electron can only be lower than *c* because the Δ***K*** momentum energy component of its carrier-photon propels *two amounts of energy that electromagnetic oscillate on a plane transverse* to the direction of application of the Δ***K*** momentum energy of the carrier-photon along the ***I//i*** vectorial axis of normal X-space, and whose sum is always larger than the Δ***K*** momentum energy component of the carrier-photon.

The lower the total amount of energy of the carrier-photon, the lower the electron velocity and its effective mass will consequently be, and the larger the total amount of energy of the carrier-photon, the higher will be the velocity of the electron towards the asymptotic limit velocity *c*, because it is impossible by structure that the sum of the inert transverse energy of the electron plus the inert transverse energy Δ***B*** of the carrier-photon can ever become equal to the amount of energy of the Δ***K*** momentum energy component of the carrier-photon, which is always equal by structure to only its transversely oscillating Δ***B*** amount of energy, as analyzed in Reference [21].

XII. INTERACTION BETWEEN AN ELECTRON'S REST MASS ENERGY AND THAT OF ITS CARRIER-PHOTON

Table 1 established in Reference [5] and analyzed in References [33][34] puts in perspective the fact

that the electron in motion involves two different energy quanta, that not only electromagnetically oscillate at different frequencies, but whose *centers-of-presence* ⊗ through which this oscillation separately occurs for each of them can only be physically separated by structure on a plane transverse to the direction of motion of the system in space (See **Figure 18**).

Table 1: Combined fields equations of the moving electron and its carrier-photon

	Momentum kinetic energy in X-Space (normal space)	Energy located in Y- and Z-spaces making up the translationally inert mass of the electron in motion
Electron rest mass energy (m_oc^2)		$\left\{\left(\frac{\varepsilon_0\mathbf{E}^2}{2}\right)_Y \vec{\mathbf{J}}\,\vec{\mathbf{i}} + \left(\frac{\mathbf{B}^2}{2\mu_0}\right)_Z \overleftrightarrow{\mathbf{K}}\right\} V_{m_e}$
Carrier-photon energy $\Delta\boldsymbol{K} + \Delta m_m c^2$	$\left[\frac{hc}{2\lambda}\right]_X \vec{\mathbf{I}}\,\vec{\mathbf{i}}$	$\left[\left(\frac{\mathbf{B}_K^{\,2}}{2\mu_0}\right)_Z \overleftrightarrow{\mathbf{K}}\right] V_K$
Electron Total Relativistic mass energy (mc^2)		$\left\{\left(\frac{\varepsilon_0\mathbf{E}^2}{2}\right)_Y \vec{\mathbf{J}}\,\vec{\mathbf{i}} + \left(\frac{\mathbf{B}^2}{2\mu_0}\right)_Z \overleftrightarrow{\mathbf{K}}\right\} V_{m_e} + V_K\left(\frac{\mathbf{B}_K^{\,2}}{2\mu_0}\right)_Z \overleftrightarrow{\mathbf{K}}$

Comparing the electron rest mass Equation (18) with its carrier-photon's Equation (17) indeed shows that each quantum possesses its own trispatial junction ⊗, which are separated by structure by the simple fact that their energy oscillates between different pairs of spaces in the trispatial complex; that of the electron oscillating between Z-space and X-space, while that of its carrier-photon oscillates between Z-space and Y-space, besides oscillating at different frequencies.

This means that except for the case in which the electron carrier-photon would possess exactly 0.511 MeV of energy, both components of the electron in motion are physically unable to associate in exactly synchronized attractive least action relative anti-parallel magnetic spin alignment, which highlights the contrast between these predictable and measurable asynchronous resonance interactions generating identifiable regular beat frequencies at any given velocity of the electron, and their interpretation as unpredictable spontaneous stochastic fluctuations about the zero-point energy level of Quantum Field Theory (QFT) currently assumed to be responsible for zitterbewegung ΔZ.

In reality, any difference in frequency between both components can only force both *trispatial junctions* ⊗ to follow oscillating longitudinal trajectories that may appear erratic transversely with respect to the direction of motion of the twin component system, due to the uninterrupted asynchronous sequence of cyclic alternance between attractive anti-parallel spin alignment states and repulsive parallel spin alignment states, which can only generate a beat resonance state that came to be identified as the zitterbewegung ΔZ of the moving electron.

As a matter of fact, the relative motion freedom of both *tri-spatial junctions* ⊗ with respect to each other can only be perpendicular to the direction of motion of the system, since the stability by structure of the amount of translational energy of the carrier-photon at any given moment depends uniquely on the Coulomb interaction between the carried electron's *center-of-presence* ⊗ and those of other charged particles. From the trispatial geometry perspective, this constraint prevents any longitudinal deceleration or acceleration relative to each other of the two *centers-of-presence* ⊗ of the set from being involved in their motion.

The only remaining possible direction of motion available for the two *trispatial junctions* ⊗ with respect to each other is thus transverse to the direction of motion of the system, which implies that at any given moment, both *trispatial junctions* ⊗ will be at varying ΔZ distances (zitterbewegung distances) from each other (see **Figure 18**), computable as a function of the state of the electromagnetic harmonic oscillations parameters of both quanta at this moment, as analyzed in References [33][34].

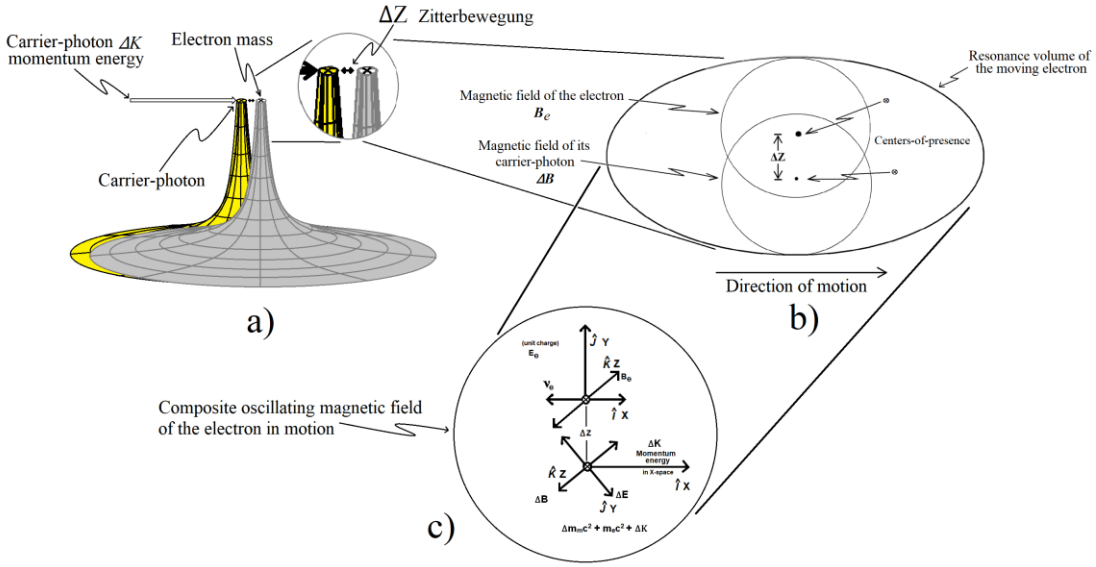


Figure 18: Idealized representation of the *center-of-presence* ⊗ of the electron in normal X-space and of that of its carrier-photon, whose ratio between the latter's $\Delta\boldsymbol{K}$ momentum energy and the total energy $\boldsymbol{B}_{\text{Total}}=\boldsymbol{B}_{\text{Electron}}+\Delta\boldsymbol{B}_{\text{Carrier-photon}}$ oscillating transversely defines a velocity of the electron lower than c in X-space, and whose magnetic fields interaction between $\boldsymbol{B}_{\text{Electron}}$ at a fixed frequency and $\Delta\boldsymbol{B}_{\text{Carrier-photon}}$ at variable frequencies establishes the beat frequency of the system identified as the zitterbewegung ΔZ of the system along the electron trajectory in normal X-space.

XIII. LINEAR COULOMB INTERACTION BETWEEN THE INVARIANT CHARGES OF DECOUPLED MASSIVE ELEMENTARY PARTICLES

Ever since Newton established from Kepler's conclusions about the data collected by Tycho Brahe about the orbits of the planets of the solar system that the gravitational force seems to act in attraction on the massive bodies orbiting the Sun as a function of the inverse square of the distances separating them, and that Coulomb experimentally established that the same inverse square relation with distance is acting in attraction between point-like behaving electric charges of opposite electric signs, and in repulsion between same sign point-like behaving charges, a long standing misconception about an assumed astronomical difference in intensity between these two forces, revealed by a careful numerical analysis carried out in Reference [60], prevented the community from realizing the implications of the fact that the *invariant charge* of the electron and its *invariant rest mass* are two inseparable properties of the same massive particle, which is that electrons can only obey by structure with the same intensity to both Newton's gravitational law for masses and to the Coulomb law for electric charges.

Even a direct mention of this state of fact by Einstein in his 1910 paper [49], that strangely was available only in French for more than a century due to the German original having been lost, until it was finally formally translated to English in 2021 by the *Minkowski Institute* [50] did not attract attention:

> *"On peut, par exemple, obtenir de cette façon les équations du mouvement d'un point matériel de masse m portant une charge électrique e (par exemple un électron) et soumis à l'action d'un champ électromagnétique. On connaît, en effet, les équations du mouvement d'un point matériel à l'instant où sa vitesse est nulle. D'après les équations de Newton et la définition de l'intensité du champ électrique, on a:"*

$$m\frac{d^2x}{dt^2}=e\boldsymbol{E}_x \tag{34}$$

Comment excerpted from Reference [49] and associated Equation (2), p. 143)

> *"We can, for example, obtain in this way the equations of motion of a material point of mass m carrying an electric charge e (for example an electron) and subjected to the action of an electromagnetic field. We know, in fact, the equations of motion of a material point at the moment when its velocity is zero. According to Newton's equations and to the definition of the electric field strength, we have:"* Related to ([50], Equation (2), p. 95)

Comment excerpted from Reference [50] and associated Equation (2), p. 95)

Once the physical existence of the positron was established in 1933 [22], whose characteristics of

invariant rest mass and *invariant charge* were established as identical to those of the electron *except for the sign of its charge*, the stage was set to realize that both the Coulomb force and Newton's gravitational force were the same force, hadn't it been for the unfortunate erroneously assumed astronomical difference in intensity between these two forces previously mentioned, as analyzed in Reference [60].

The issue of the proton having a mass 1836 times that of the electron while displaying the same resultant unit charge with a sign opposite that of the electron did not help alleviating the confusion, because the same Coulomb equation applied to both a positronium metastable system – involving a negatively charged electron and a positively charged positron interacting in close quarter with exactly *the same invariant unit charge* and *the same invariant rest mass* –, and a stable hydrogen atom – involving a negatively charged electron and a positively charged proton interacting in a stable manner at a fixed mean distance of a_0=5.291772083E-11 m with *the same invariant unit charge* but *with widely different masses* can be accounted for with the very same formulation of the Coulomb equation, *giving exactly the same attractive force* at distance a_0 taken as a numerical example:

$$F = e\boldsymbol{E} = \frac{\left|e^-\right|\left|e^+\right|}{4\pi\varepsilon_0\, a_0{}^2} = 8.238721809\text{E}-8\,\text{N} \tag{35}$$

and inducing exactly the same amount of energy in both particles at this distance:

$$E_{\text{Iinduced}} = a_0 \cdot F = \frac{\left|e\right|^2}{4\pi\varepsilon_0\, a_0} = 4.359743806\text{E}-18\,\text{j} \tag{36}$$

The difference however between the Positronium metastable system and the stable hydrogen atom is that the two oppositely charged particles of the Positronium system only fleetingly cross the distance a_0=5.291772083E-11 m on their irrepressively shrinking inwards spiralling trajectory towards the point at which both of their equal rest masses will finally meet and convert to electromagnetic photons, leaving the premises at velocity *c*.

On their part, the two oppositely charged components of the hydrogen atom will remain stable at about this approximate distance as the system resonates in a stationary action state due the equilibrium established by their $\Delta\boldsymbol{K}$ momentum energy establishing a constant counteracting inwards oriented pressure against the constant outwards oriented pressure of their predominantly repelling oscillating magnetic fields as analyzed in References [33][34]. See also **Figure 21** further down on this issue.

The reason why the Positronium is unstable and that its orbit quickly decays until the particles meet and have their masses converted to electromagnetic photons is due to their magnetic fields oscillating at the exact same frequency of v_c=1.235589976E20 Hz, which, due to their magnetic fields quickly aligning in their default least action antiparallel magnetic spin alignment as they close in towards each other, contrary to the magnetic fields in the hydrogen atom [33][34], and consequently offering no resistance whatsoever to the pressure exerted by the momentum energy of both particles, vectorially oriented towards each other.

But whether a pair of interacting charged particles, of same sign or opposite signs unit charges are interacting, the very same amount of instantaneous force – Equation (35) – will be acting on them at any given distance, for example a_0=5.291772083E-11 m, and the very same instantaneous amount of energy – Equation (36) – will be induced in each of them at this specific distance as represented with **Figure 19**.

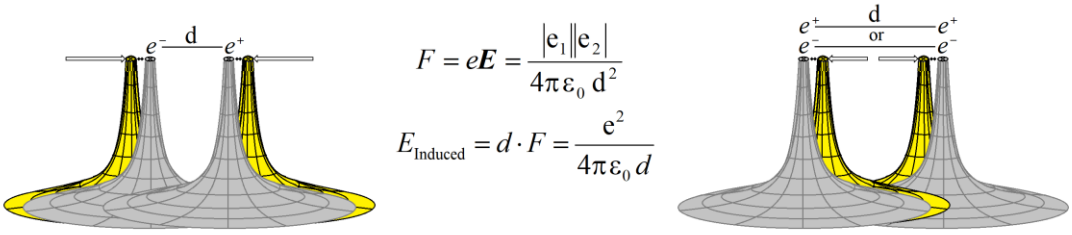


Figure 19: Coulomb force interaction as a function of the inverse square of the distance ($1/d^2$) and energy induction as a function of the inverse of the distance ($1/d$) between the invariant same sign unit charges of two decoupled elementary particles such as the electron, or between the opposite signs unit charges of an electron and a positron, or between the two opposite signs unit charges of an electron and a proton.

Indeed, this very equality of the instantaneous repelling or attracting Coulomb force at any given distance between charged particles – repelling between same sign charges and attracting between opposite signs charges – and induction of the very same amount of kinetic energy in each of them whether the charges attract or repel at this distance, has induced the perception that the Coulomb force is not a credible candidate to be equated with the gravitational force as established by Newton, which led to an intuitive assumption that

electrostatic repulsion can have an important effect at large distances just like electrostatic attraction, given that Newton's gravitational force is conceptually devoid of any repelling characteristic contrary to the Coulomb force.

However, a deep analysis carried out in Reference [72] revealed that since electrostatic repulsion decreases in intensity as a function of the inverse square of the increasing distance between any pair of like sign repelling particles, its effect quickly becomes infinitesimal as the distance increases between any pair of these particles; to the point of becoming barely detectable, if at all, even at millimetre range distances between two same sign particles.

In fact, we can directly observe that this electrostatic repulsion doesn't prevent us from getting as close as we like to all the massive bodies around us without feeling the slightest repulsion until "tactile" contact is established. In fact, this "tactile" contact that prevents the interpenetration of objects in our environment is the most intense perceptible manifestation of this electrostatic repulsion between the same-sign charged electronic escorts of all the atoms of which all massive bodies are made that can be perceived at our macroscopic level of perception, and it is easily verifiable that it occurs at submicroscopic distances, even with the massive quantities of same-sign charged electronic escorts that make up the surface of all the objects in our environment.

Contrary to electrostatic repulsion between same sign elementary particles, electrostatic attraction between elementary particles of opposite signs increases in intensity as a function of the inverse square of the decreasing distance that separates them, constantly adiabatically adding momentum energy vectorially oriented towards the other particle [15][16], and this, without even taking into account the same amount of energy oriented transversely being induced in each of them at the same time, that increases the magnetic field and measurable mass of each elementary particle.

So the mutual repelling aspect of the Coulomb force between same sign charged elementary particles loses all credibility as an objection to the Coulomb force attractive aspect between opposite signs elementary charged particle as being identical to Newton's inverse square attraction with distance between masses, since the effects of electrostatic repulsion between pairs of elementary charged particles of the same sign is no longer even perceptible at distances between them of the order of a millimetre.

In fact, Reference [18] proposes a method to calculate the force of gravity in the universe as an alternate method to Newton's method from the Coulomb equation approach as applied to the set of elementary charged particles of which all atoms are made that constitute all macroscopic masses in the universe, with the added benefit that the asymptotic velocity limit of light is taken account of by structure for all masses.

And there is an intriguing plus to the Coulomb force that was revealed by the Kaufmann data, which is that it is not the Coulomb force as such that caused electrons to move, but the kinetic energy oriented longitudinally – the momentum energy – that the force adiabatically induces in each electron according to Equations (37) and (38). In reality, it appears that the Coulomb force does not itself propel the particles, but only induces kinetic energy in charged particle, whose momentum component vectorially oriented towards opposite signs charged elementary particles in the environment is actually what propels the particles, a momentum energy component that will be oriented away from same sign charged elementary particles, as analyzed in Reference [73].

Summarily described, Kaufmann accelerated these electrons on curved trajectories in his bubble chamber by means of $\boldsymbol{E}$- and $\boldsymbol{B}$-fields calculated according to the Lorentz force equation:

$$\mathrm{F} = \mathrm{e}(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \tag{37}$$

whose first term is of course the Coulomb force equation that adiabatically induces kinetic energy in all charged particles as a function of the inverse of the distances that separates each elementary charged particle from all other elementary charged particles:

$$F = e\boldsymbol{E} = \frac{\mathrm{e}Q}{4\pi\varepsilon_0 R^2} \rightarrow E_{\mathrm{linduced}} = R \cdot F = \frac{\mathrm{e}Q}{4\pi\varepsilon_0 R} \tag{38}$$

in which Q represents the resultant of all elementary charges in the environment that interact with an electron and defines the intensity of their common relative $\boldsymbol{E}$-field, and R represents the mean distance at which the charges represented by Q are from the electron.

XIV. TRANSVERSE COULOMB INTERACTION BETWEEN THE VARYING CHARGES OF THE PAIR OF COMPONENTS OSCILLATING WITHIN EACH ELEMENTARY PARTICLE

As reported in Reference [58] previously quoted, the Coulomb equation was historically established by Coulomb as applying between point-charges separated in space. During the analysis in References [3][4] of Louis de Broglie's condition that he identified for quantized photons to obey Maxwell's equations, which was that each localize photon must involve two particles, or half-photons of spin ½, "... *that must be complementary with respect to each other in the same manner that the positive electron* [the positron] *is complementary to the negative electron in the Dirac Hole Theory*" ([2], p.277), the question obviously came up as to whether the

Coulomb equation was involved or not within localized photons.

From the *Lorenz gauge* perspective on which electromagnetic theory was grounded at the time, and still is, according to which the intensity of both continuous ***E*** and ***B*** fields of Maxwell's theory peak simultaneously to maximum as represented in **Figure 2**, such an involvement of the Coulomb force between two such half-photons as envisioned by de Broglie seemed so problematic that the option was not even considered by de Broglie, as confirmed to me by his lifelong friend and colleague Georges Lochak, in correspondence initiated by me with the *Fondation Louis de Broglie* [74], precisely to clarify this issue.

But contrariwise, from Maxwell's initial concept of both fields inducing each other in alternance with the involvement of the *displacement current* on the ***E***-field side of the relation that he had conceived and that was at the origin of the development of his electromagnetic theory as represented with **Figure 3**, such a relation became possible to consider, given that a current involves the motion or oscillation of interacting charges, that the de Broglie's condition was now bringing into the picture.

These developments allowed the conceptual transposition of the longitudinal oscillation of the continuous fields conceived by Maxwell as oscillating on planes parallel to the direction of motion of the energy, to oscillation in standing mode on planes perpendicular to the direction of motion of quantized amounts of electromagnetic energy, whose oscillation between a local ***E***-field state and the related local ***B***-field state was driven by the Coulomb related *displacement current* conceived by Maxwell, now causing the two particles conceived by de Broglie to cyclically generate the related local ***B***-field according to the LC relation developed in References [3][4].

The extent of the amplitude of this oscillation of the two de Broglie particles within the localized photon on this plane transverse to the direction of motion was identified in the first wave of derivations from Marmet's discovery in Reference [32] published in 2007 as related to the ratio of the *fine structure constant* α in the following manner.

Whereas the traditional amplitude of oscillation of all electromagnetic frequencies on the longitudinal planes parallel to the direction of motion of the continuous fields energy conceived by Maxwell corresponds to $\lambda/2\pi$, it was understood as Reference [32] was in process of development, that the actual maximum extent of the transverse oscillation of both de Broglie particles within each localized electromagnetic quantum was in reality $\alpha\lambda/2\pi$ – see Equation (25).

See Equation (18) and **Figure 9** for an illustration of the case of the electron rest mass energy transverse oscillation amplitude in opposite directions of its two neutrinic components on the ***I//jk*** vectorial plane, and see Equation (17) and **Figure 7**, for an illustration of the oscillation of the two electric components of the free moving photon or carrier-photon on the ***J//jk*** vectorial plane.

This led to the development in Reference [32] of a version of the Coulomb equation requiring only the wavelength of any free moving electromagnetic photons to calculate its energy without any need of Planck's constant, as presented with Equation (32):

$$E = \frac{e^2}{2\varepsilon_o \alpha \lambda} \tag{39}$$

And to a recall force equation whose maximum intensity when both particles of any localized electromagnetic quantum are at maximum transverse amplitude separation in agreement with Hooke's law, as first analyzed in Reference [46] and fully developed in References [8][9]:

$$\boldsymbol{F} = \frac{\pi e^2}{\varepsilon_o \alpha^2 \lambda^2} = -k \cdot \frac{\alpha\lambda}{2\pi} \tag{40}$$

XV. CORRELATING THE THREE FORCE LAWS THAT INTERACT AS A FUNCTION OF THE INVERSE SQUARE OF DISTANCES

So, we observe that the three known force laws that interact as a function of the inverse square of distances have been known for hundreds of years, Newton's gravitational law and Hooke's recall force law dating back to the seventeenth century, and Coulomb's electrostatic law dating back to the eighteenth century.

Moreover, it is Newton's proof that a spherically symmetric mass distributions of an object behaves in the same manner as a point particle located at its center, that allowed calculation of all trajectories of any ponderable body in motion as if all of its mass was concentrated at its center of mass – that is, at its *center-of-presence* ⊗ in the universe.

This same proof was used by Coulomb to establish the interaction law of the inverse square of the distances separating electric charges behaving point-like, a behavior later confirmed as physically applying between electrons and other charged elementary particles whose *centers-of-presence* ⊗ systematically behave point-like in any collisions between these elementary particles.

This proof was also used in 1998 to experimentally establish the interaction law of the inverse cube of

distances separating magnetic fields whose two magnetic poles geometrically coincide with their own *center-of-presence* ⊗, published in 2013 [61], which was experimentally confirmed in 2014 as applying between the magnetic fields of real electrons [62], which can be understood as oscillating through the localized infinitesimal volume *dV* that can be conceptualized as being mapped over the *center-of-presence* ⊗ of each elementary electromagnetic particle.

15.1 Hooke's restoring force

As mentioned and analyzed in References [8][9], as unexpected as it may seem, it turns out that the classical equation of Hooke's law also applies to the electromagnetic oscillating motion of the *energy substance* of which photons and electrons are made, as established in **Section XXIII** of Reference [46], given the property of *elasticity* that each continuous quantum of this fundamental *energy substance* must possess for its behavior to remain in agreement with Maxwell's equations.

Indeed, it is this property of *elasticity* that allows the transversely oriented half-quantum of each quantum to stretch into two parts without breaking as required by de Broglie's conditions, constituting the two electric charges of opposite signs moving away from each other under the impetus of the substance's *tendency to-always-remain-in-motion*, inducing the *displacement current* required by Maxwell's theory and increasing the tension that constitute the opposite-signs charges, that reach their maximum intensity at the maximum amplitude distance allowed by the substance's property of *incompressibility*.

The return tension towards the neutral position located at the quantum's *center-of-presence* ⊗, that increases as the two charges move further apart, will then dominate and force the two charges – electric for the photon and neutrinic for the electron – back towards the neutral position, as illustrated in **Figure 4a**.

Each time the neutral tension position is reached, the restoring force ceases to exist and begins to exist again as soon as the next cycle of *elastic* separation is initiated. The Hooke restoring force thus has a *cyclic intermittent existence* throughout the existence of each quantum.

It simply ceases to exist locally in the case of any photon of energy equal to or greater than 1.022 MeV that decouples into a massive electron-positron pair charged in opposition and moving separately in space, since the *energy substance* of the initial quantum that decouples is then physically severed into two different quanta, at the moment when the charges of the separating particles simultaneously reach the maximum invariant return tension of e=1.602176462E-19 Coulomb and the escape velocity of light in Y-space, to be replaced by the Coulomb force, which then enters into permanent action between the oppositely-charged particles that have just separated in space.

It could well be then that the Hooke restoring force could be the most fundamental force in the universe, in the sense that if we consider that the existence of electromagnetic energy must logically have preceded the existence of the first massive particles of opposite signs, since they arise precisely from a decoupling of quanta of this fundamental energy, the Coulomb force could only have begun to exist when the first sufficiently energetic photon decoupled at the very beginning of the universe, as put in perspective in Reference [53].

15.2 Coulomb's electrostatic force

The Coulomb force then, often derogatorily dismissed as being a "*spooky action at a distance*", appears to rather be a force ambient in the universe, not actually directly causing same sign charged elementary particles to repel each other and opposite signs charged elementary particles to attract each other, but to rather permanently induce kinetic energy in each charged particle as a function of the inverse of the distances separating them, whose momentum component will be vectorially oriented towards oppositely charged elementary particles and vectorially oriented away from same sign elementary charged particles; somewhat as if Hooke's restoring force continued to exist even after the elastic bond of the initial photon's energy substance had been broken as it decoupled into two spatially separated particles, and continued to attempt to bring opposite charges closer together even after they had separated.

It is therefore the unidirectionally oriented momentum component $\Delta\boldsymbol{K}$ of the induced energy that allows elementary charged particles to move in space, or if the velocity allowed by this momentum energy $\Delta\boldsymbol{K}$ is hindered by local electromagnetic circumstances, it will exert an equivalent pressure in the same vectorial direction against any electromagnetic conditions that oppose the particle's motion; while the other component of the induced energy oscillates electromagnetically on a plane perpendicular to the direction of application of the momentum energy, and is measurable either as a $\Delta\boldsymbol{B}$ increase in the particle's magnetic field, or as a Δm increase in its effective mass.

References [8][9] have established that all charged and massive elementary particles in free fall in space are intrinsically inert in normal X-space because none of their stabilized energy is available to provide the momentum energy needed to cause their *centers-of-presence* ⊗ to move in normal space. This momentum energy must therefore be provided via a carrier-photon whose momentum component $\Delta\boldsymbol{K}$ then enables them to

move in normal space, as illustrated for the hydrogen atom with **Figures 18** and **19**.

In the case of larger stabilized structures directly made of elementary charged and massive particles up to the level of atoms and molecules, the momentum energy that causes these larger structures, that were named*level 3 trispatial vector complexes* in References [8][9], the nature of the Coulomb interaction mandates that it induces supplementary momentum energy in each elementary charged subcomponents of these structures, oriented towards all charged particles of opposite sign located outside these structures in the universe.

As it is not possible to represent in an intelligible manner such a complex network of vector momentum components oriented in all directions in space, it was chosen to summarize their whole set for each atom or molecule with *a theoretical carrier-photon*, as in **Figure 20** for the hydrogen atom, whose direction would be the vector resultant of the vector orientations of all these additional momentum components.

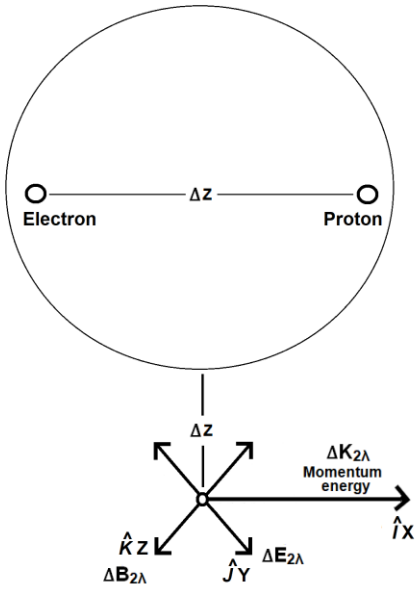


Figure 20: The hydrogen atom in motion – level 3 trispatial vector complex.

All *level 3 vector complexes* tend to assemble into *level 4 vector complexes* accumulations of larger masses in the trispatial vector field as described in References [8][9], due to the vectorial resultant of their supplementary momentum energies oriented so that they constantly seek the most stable least action state with respect to all other *level 4* accumulations in the universe.

The composite attractive charge of the Earth can then be calculated, leading to calculating the number of elementary charges of which its *level 4* mass is made as well as that of the Sun, and so on for the astronomical level, according to the method proposed in Reference [18].

15.3 Newton's gravitational force

What happened historically is that Tycho Brahe carefully collected during his lifetime a considerable amount of rather precise data about the orbits of all the major *level 4 matter accumulations* in the solar system, that is, the planets, from which Johannes Kepler established the three laws that this data experimentally confirms as governing the motion of the planets in the solar system:

1- All planets run elliptical orbits with the Sun located at one of its two foci.

2- A line segment joining a planet to the Sun sweeps equal areas between the orbit of the planet and the Sun during an equal time interval.

3- The cube of the mean radius of a planetary orbit is proportional to the square of the time taken by the planet to run one orbit.

Studying these conclusions by Kepler, Newton came up with the concept of "force" and confirmed the general soundness of his gravitational theory by deriving Kepler's three laws from his own gravitational equations, in a manner clearly explained by Georges Gamow, Nobel Prize winner for his contribution to relativistic theory, in his popularization work "*Gravity*" [75].

It was apparently obvious to him from analyzing Kepler's laws that the motion of any planet about the Sun can mathematically be simplified at the limit as if it was circular at a distance from the Sun equal to the mean radius of the elliptical orbit. This is what allowed Newton to associate the centripetal acceleration of circular motion v^2/r to orbital motion, where v is the velocity of an orbiting body of mass m and whose radius of the theoretical circular orbit is r.

His basic postulate was that each planet and the Sun must be attracted to each other with a force proportional to the product of the their masses and inversely proportional to the square of the distance separating them, a relation that mathematically can be represented by equation:

$$F = G\frac{Mm}{r^2} \tag{41}$$

As explained by Gamow, Newton's insight was that the centripetal acceleration multiplied by the mass of a planet should be equal to the gravitational force of attraction, which implied the following relation:

$$F = ma = \frac{mv^2}{r} = \frac{GMm}{r^2} \tag{42}$$

Finally a paper published in 2013 mathematically demonstrated that all classical force equations ultimately resolve to dealing with a mass or a charge being accelerated and that consequently, *only one force is at play for all of these equations* [60].

Regarding massive bodies resting at the surface of the Earth, the *weight* of an object can only be a measure of this *pressure* exerted by the sum of the individual momentum energies vectorially oriented towards its *centre-of-presence* ⊗, belonging to half of the whole set of separate charged particles of the atoms that constitute the measurable mass of this object. For example, when we climb on a bathroom scale to verify our *weight*, it is this pressure that the sum of the momentum energies that this half of the crowd of elementary charged particles, of which our body is made, exerts toward the ground that we are measuring. In other words, what we name "*the force of gravity*" can be considered as an *impeded velocity* expressed as a *pressure*, since the unidirectional momentum energy oriented towards the ground, which is induced in all charged and massive elementary particles constituting all bodies cannot be expressed as a motion, as clarified in Reference [18].

15.4 The magnetostatic inverse cube law

Although having nothing to do with gravitation as such, the interaction law of the inverse cube of the distances between the magnetic fields of elementary particles, whose two magnetic poles coincide geometrically by structure with their *center-of-presence* ⊗ is also important, because the physical impossibility for the three main magnetic fields of the triad of charged elementary sub-components of each nucleon to align in perfect least-action anti-parallel spin alignment is precisely the cause of the establishment of measurable atomic volumes, by preventing electrons from approaching atomic nuclei closer than certain distances, due to the counter-pressure established between the electron's magnetic field and that of the nucleus's complex magnetic field, always in a predominantly repelling parallel relationship with it by default, as put into perspective in Reference [61].

The reason is that when two of a nucleon's main magnetic fields fall into complete least action antiparallel magnetic spin alignment, the third field will always remain in parallel magnetic spin alignment with one of the other two.

Due to the large difference in oscillation frequencies between the magnetic fields of these nucleon subcomponents and the oscillation frequency of the magnetic fields of electrons, the predominantly repelling counter-pressure permanently existing between electronic escorts and atomic nuclei causes all electrons to fall in resonance equilibrium at some distances from nuclei at which their momentum energy oriented towards the nuclei exactly counter balances the mutual repulsion between their own magnetic field and the combination of the magnetic fields of the nucleons of the nuclei, as illustrated with **Figure 21** for the hydrogen atom, developed in Reference [33].

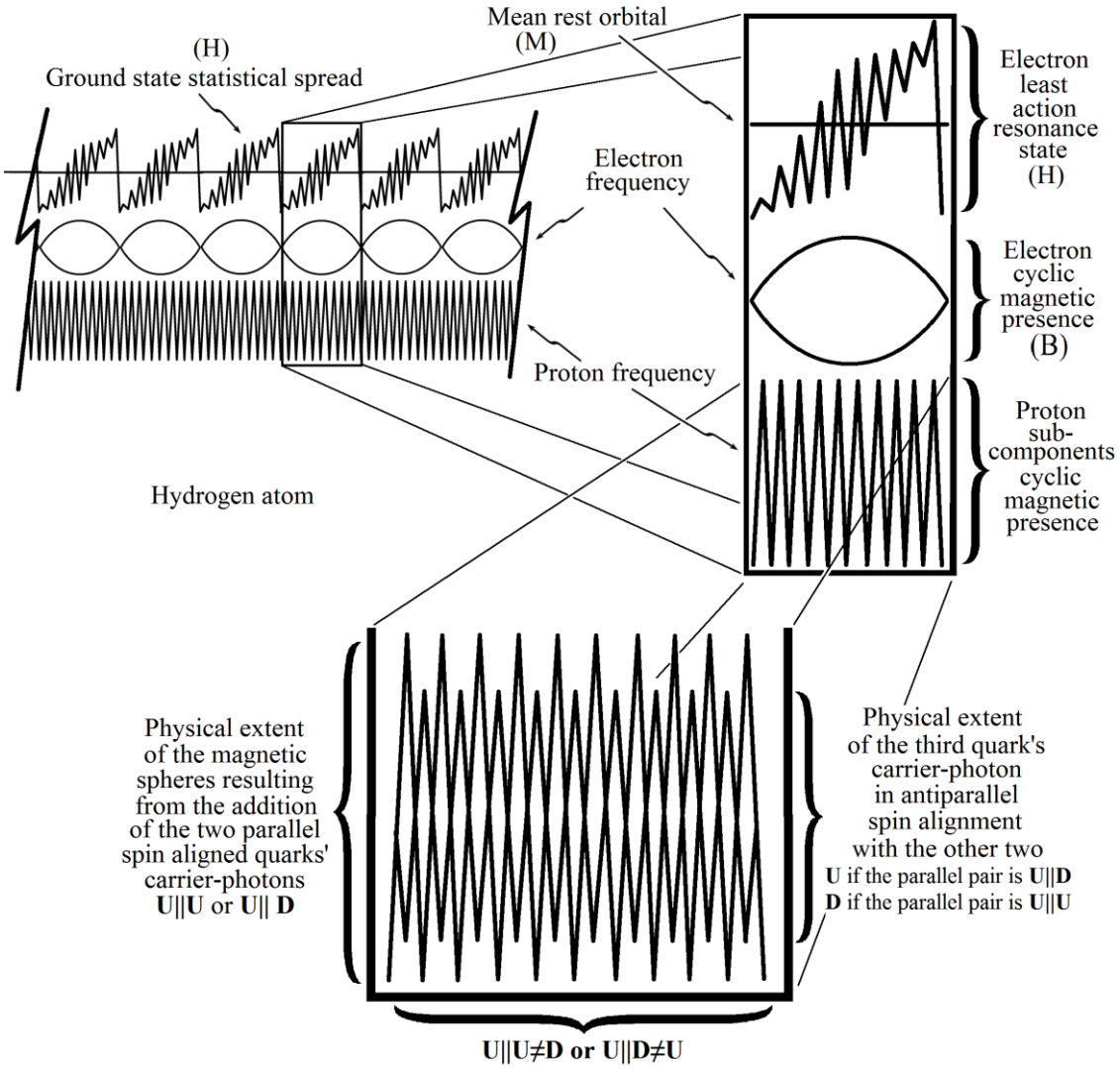


Figure 21: The stationary resonance state of the electron in the hydrogen atom

The inverse cube law with distance involved between such oscillating magnetic fields was experimentally confirmed by two experiments, one of which can easily be reproduced in laboratory with magnets magnetized so that both poles of each magnet coincide with the geometric center of each magnet [61], whose account was published in 2013, that is 1 year before Kotler et al published the account of their confirming experiments with real electrons [62].

XVI. Conclusion

This research project allowed identifying four major disconnects between the set of premises that underlies the current set of kinematic and electromagnetic mechanics theories and the pool of experimental data accumulated over the course of the past centuries.

The first disconnect concerns the long standing belief that Newton's gravitational force is 39 orders of magnitude weaker than the Coulomb force, due to a neglect in numerically verifying that despite that gravitational constant G embeds the mass of the Sun, the mean radius of Earth's orbit and the time taken – one year – for the Earth to run one orbit, it is nevertheless used to compare the force obtained with gravitational Equation (41) between the infinitesimal masses of an electron and a proton in the Bohr atom by means of this constant specifically adapted to deal with the astronomical masses in the solar system, to the force obtained with the Coulomb Equation (35) as applying between their equal charges in the Bohr atom, as analyzed in Reference [60].

The second disconnect concerns the decision taken in 1907 by the theoreticians community not to take into account in their theoretical developments the possibilities that seemed to emerge from the newly identified and confirmed electromagnetic properties of free-moving electrons established from Kaufmann's experimental data, and to not even have documented them in the reference works of the time, which was reported only

indirectly nearly 70 years later in 1982 on page 159 of Abraham Pais' biography of Einstein [13].

The third disconnect concerns the calculation error inadvertently made by Louis de Broglie in calculating *the velocity of the phase wave* of the electron on the ground orbit of the Bohr atom in his 1924 thesis, due to not having been informed for lack of proper referencing in the literature, of the fact that the Kaufmann data had revealed 20 years earlier that at any velocity induced in electrons by the Coulomb force, an amount of energy twice that of its momentum energy is induced in the electron, which caused him to obtain *a phase wave velocity only half of that of the particle that it controls*, which then was established as *an incorrect foundation on which were grounded all subsequent developments of Quantum Mechanics and of Quantum Theory in general*, as analyzed in Reference [20].

The forth disconnect concerns the long standing unjustified belief that electrostatic repulsion between same sign elementary charged particles has long range effects as important as electrostatic attraction between opposite signs elementary charged particles, which was unjustifiably considered as disqualifying the Coulomb interaction from being involved in gravitational attraction, as analyzed in Reference [72].

It is expected that the four major disconnects that were identified during this project in the progression of knowledge acquisition from experimental data collected over the course of history will also be identified and eventually integrated by the next generation of theoreticians and that complete reconnection will eventually be made with the 300 years of continuous experimental progress that culminated with the literal explosion of new discoveries that characterized the last decades of the 19th century and that resulted in the wave of successful technological applications that the engineering community graced us with over the course of the 20th century.

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