

# Using Adaptive Output Control Method to Investigate the Lateral Stability of Aircraft In The Presence Of Interference

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## ABSTRACT

The manuscript presents a mathematical framework that elucidates the dynamics of the lateral channel of an unmanned aerial vehicle (UAV), considering both the inclusion and exclusion of the electric actuator parameters controlling the rudder, as well as the effects of aerodynamic noise. In this study, the author employs an adaptive control methodology based on series compensation of the output. Results obtained from computer-based simulations using Simulink tools demonstrate that the system retains stability in the presence of both internal noise (electrical characteristics) and external noise (aerodynamic disturbances).

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## I. INTRODUCTION

Unmanned aerial vehicles are widely used in the military and civil fields due to their convenience. Therefore, the development of new control methods that are both effective and easy to implement is required.

Adaptive control for uncertain objects in conditions of internal and external disturbances is an urgent and practical problem of modern control theory. The assumption that the controlled object is linear, stationary, capable of measuring all state variables, and has no gradual impact disturbances will no longer be used in modern control. To synthesize controllers when the derivative of the output variable cannot be determined or all state variables of the object cannot be measured, the adaptive control method based on the output signal is currently used [2]-[5]. Control based on the output signal allows to reduce the cost of designing and using measuring sensors, and at the same time allows to reduce the measurement error caused by measuring instruments. In addition, for the real system (UAV-70V model), it is not possible to directly measure all the state variables of the controlled object. There have been many methods for synthesizing adaptive control systems according to the output signal [2]-[5]. However, the known methods for synthesizing adaptive control systems according to the output signal have complex synthesis steps and structures.

In this paper, we investigate an adaptive control algorithm based on the output signal, utilizing the series compensation method to design a controller for the UAV lateral channel. The study considers both the inclusion and exclusion of variations in the electric actuator parameters and wind disturbances. This method is straightforward, easy to implement, compact in design, and does not require knowledge of the derivative of the output signal.

## II. MATERIAL AND METHODS

### 1. Mathematical model of lateral channel motion of UAV

The system of differential equations of lateral channel motion of UAV under the influence of electromagnetic ( $L_u \neq 0$ ) force has the form [2, 3]:

$$\begin{cases} \dot{\beta} = -\frac{C_{z\beta} qS}{mV_k} \beta - \omega_y; \\ \dot{\omega}_y = \frac{m_{y\beta} qSb_a}{J_y} \beta + \frac{m_{y\omega_y} qSb_a^2}{V_k J_y} \omega_y + \frac{m_{y\delta_h} qSb_a}{J_y} \delta_h; \\ \dot{\delta}_h = \omega_{\delta_h}; \dot{\omega}_{\delta_h} = J_{\Sigma}^{-1} k_m I_u; \\ \dot{I}_u = -L_u^{-1} k_y k_{\pi} \beta_t \beta_c \beta_{\pi} \delta_h - L_u^{-1} (k_y k_c \beta_t + k_e) \omega_{\delta_h} - L_u^{-1} (k_y k_t \beta_t + R_u) I_u + L_u^{-1} k_y \beta_t \beta_c \beta_{\pi} u_{\Sigma 1} \end{cases} \quad (1)$$

Where:  $\beta$  - slip angle;  
 $C_{z\beta}$  - coefficient of drift force according to slip angle;  
 $q$  - dynamic pressure;  
 $s$  - wing area of UAV;  
 $m$  - mass of UAV;  
 $V_k$  - UAV speed in the body-fixed coordinate system;  
 $\omega_y$  - angular velocity of UAV rotation around the axis  $Oy$  in the body-fixed coordinate system;  
 $m_{y\beta}$  - component of the aerodynamic moment coefficient  $m_y$  due to the slip angle component;  
 $b_a$  - average aerodynamic chord;  
 $J_y$  - UAV moment of inertia relative to the  $Oy$  axis in the body-fixed coordinate system;  
 $m_{y\omega_y}$  - a derivative of torque coefficient concerning angular velocity  $\omega_y$  ;  
 $m_{y\delta_h}$  - a derivative of the moment coefficient concerning the rudder angle;  
 $\delta_h$  - rudder angle;  
 $\omega_{\delta_h}$  - rudder angular speed;  
 $J_\Sigma$  - total moment of inertia, corresponding to the armature moment of inertia  $J_u$ , the transmission moment of inertia connected to the engine armature  $J_{TP}$  and the rudder  $J_{\delta_h}$   
 $(J_\Sigma = J_u i^2 + J_{TP} + J_{\delta_h})$ ;  
 $i$  - decline coefficient;  
 $k_e, k_m$  - constant flux coefficient;  
 $I_u$  - motor armature current;  
 $\beta_t, \beta_c, \beta_\pi$  - transfer function (gain) of closed-loop regulator;  
 $k_t, k_c, k_\pi$  - transfer coefficient of current  $I_u$ , angular velocity  $\omega_u$  and position  $q_u$ ;  $L_u, R_u$  - inductance and armature resistance;  
 $u_{\Sigma 1} = u^0 + u_{m1}$  - lateral channel synthetic control signal;  
 $u^0$  - program control signal;  
 $u_{m1}$  - adaptive control signal with electromagnetic properties.

From equation (1) set the coefficients:

$$a_1 = -\frac{C_{z\beta} qS}{mV_k}; a_2 = -\frac{m_{y\beta} qSb_a}{J_y}; a_3 = -\frac{m_{y\omega_y} qSb_a^2}{V_k J_y}; a_4 = -\frac{m_{y\delta_h} qSb_a}{J_y}; a_5 = J_\Sigma^{-1} k_m;$$

$$a_6 = -L_u^{-1} k_y k_\pi \beta_t \beta_c \beta_\pi; a_7 = -L_u^{-1} (k_y k_c \beta_t + k_e);$$

$$a_8 = -L_u^{-1} (k_y k_t \beta_t + R_u); b = -L_u^{-1} k_y \beta_t \beta_c \beta_\pi.$$

Description (1) according to the state equation [3]:

$$\dot{\mathbf{y}} = \mathbf{A}_1 \mathbf{y} + \mathbf{B}_1 u_{\Sigma 1}, \quad (2)$$

Where:  $\mathbf{A}_1 = \{a_{ij}\}$ ,  $ij = \overline{1,5}$ ;  $\mathbf{B}_1 = (0 \ 0 \ 0 \ 0 \ b)^T$ ;  $\mathbf{y} = (\beta \ \omega_y \ \delta_h \ \omega_{\delta_h} \ I_u)^T$ .

The system of differential equations of motion of a small UAV lateral channel without considering the influence of electromagnetic force ( $L_u = 0$ ) has the form:

$$\begin{cases} \dot{\beta} = -\frac{C_{z\beta} qS}{mV_k} \beta - \omega_y; \dot{\omega}_y = \frac{m_{y\beta} qSb_a}{J_y} \beta + \frac{m_{y\omega_y} qSb_a^2}{V_k J_y} \omega_y + \frac{m_{y\delta_h} qSb_a}{J_y} \delta_h; \dot{\delta}_h = \omega_{\delta_h}; \\ \dot{\omega}_{\delta_h} = J_\Sigma^{-1} \left[ -(R_u^{-1} k_m k_y k_\pi \beta_c \beta_\pi) \delta_h - R_u^{-1} k_m (k_y k_c \beta_c + k_e) \omega_{\delta_h} + (R_u^{-1} k_m k_y \beta_c \beta_\pi) u_{\Sigma 2} \right]. \end{cases} \quad (3)$$

Where:  $u_{\Sigma 2} = u^0 + u_{m2}$  - lateral channel synthetic control signal;

$u_{m2}$  - adaptive control signal without electromagnetic force.

From the system of equations (3) the values  $a_1, a_2, a_3, a_4$  are as presented ( $L_u \neq 0$ );

$$a_5 = -J_{\Sigma}^{-1} R_u^{-1} k_y k_m k_{\pi} \beta_c \beta_{\pi}; a_6 = -J_{\Sigma}^{-1} R_u^{-1} (k_y k_c \beta_c + k_e); b = -J_{\Sigma}^{-1} R_u^{-1} k_y k_m \beta_c \beta_{\pi}.$$

Describe (3) according to the state equation:

$$\dot{y} = \mathbf{A}_2 y + \mathbf{B}_2 u_{\Sigma 2} \quad (4)$$

Where:  $\mathbf{A}_2 = \{a_{ij}\}; ij = \overline{1,4}; \mathbf{B}_2 = (0 \ 0 \ 0 \ b)^T; y = (\beta \ \omega_y \ \delta_h \ \omega_{\delta_h})^T$ .

## 2. Adaptive control method according to output

Suppose the linear control object has a mathematical model in the form of a transfer function [4]:

$$y = \frac{b(p)}{a(p)} u \quad (5)$$

Where:  $y$  - The output signal is measured, but its derivative is not measured;

$p = d/dt$  - Differential Operator;

$u = u^0 + u_m$  - control signal;

$b(p) = b_m p^m + b_{m-1} p^{m-1} + b_1 p + b_0; a(p) = a_n p^n + b_{n-1} p^{n-1} + a_1 p + a_0$  - polynomials with corresponding parameters.

The relative degree of (5) is  $\rho = n - m$  known ( $m \leq n - 1$ )

The desired state of the signal  $y$  is  $y^*$  given by the reference model and satisfies the condition:

$$\left| \frac{d^i y^*}{dt^i} \right| \leq C_0 < \infty; \text{ khi } i = \overline{1, \rho_{td}}. \quad (6)$$

The control law using the series compensation method is as follows [5, 6]:

$$u_m = -\bar{\alpha}(p)(\mu + \kappa) \hat{e} \quad (7)$$

Where:  $\hat{e}$  - function formed from evaluation algorithm;

$e = y - y^*$  - the deviation signal between the output signal of the object and the desired output signal of the reference model;

$\mu$  - any positive coefficient;

$\bar{\alpha}(p)$  - Hurwit polynomial of degree  $\rho_{td} - 1$ ;

$\kappa$  - coefficient used to perform signal tracking accuracy  $y^*$ .

$$\begin{cases} \dot{\xi}_1 = \sigma \xi_2; \\ \dot{\xi}_2 = \sigma \xi_3; \\ \dots; \\ \dot{\xi}_{\rho-1} = \sigma (-k_1 \xi_1 - k_2 \xi_2 - \dots - k_{\rho-1} \xi_{\rho-1} + k_1 e) \end{cases} \quad (8)$$

$$\hat{e} = \xi_1 \quad (9)$$

Where:  $\sigma > \mu + \kappa$  - The coefficient  $k_i$  is calculated from the asymptotic stability condition of the system  $i = \overline{1, (\rho_{td} - 1)}$  (8).

When  $L_u \neq 0$  (with electromagnetic force parameters) the adaptive control algorithm using the series compensation method has the form:

$$u_{m1} = -(\mu + \kappa) \left( \hat{e}^{(4)} + 12\hat{e}^{(3)} + 54\hat{e}^{(2)} + 108\hat{e}^{(1)} + 81\hat{e} \right) \quad (10)$$

$$\begin{cases} \dot{\xi}_1 = \sigma \xi_2; \dot{\xi}_2 = \sigma \xi_3; \dot{\xi}_3 = \sigma \xi_4; \\ \dot{\xi}_4 = \sigma (-k_1 \xi_1 - k_2 \xi_2 - k_3 \xi_3 - k_4 \xi_4 + k_1 e) \end{cases} \quad (11)$$

When  $L_u = 0$  (excluding the electromagnetic force parameter), the adaptive control algorithm using the series compensation method has the form:

$$u_{m_2} = -(\mu + \kappa) \left( \dot{e}^{(3)} + 9\dot{e}^{(2)} + 27\dot{e}^{(1)} + 27\dot{e} \right) \quad (12)$$

$$\begin{cases} \dot{\xi}_1 = \sigma \xi_2; \dot{\xi}_2 = \sigma \xi_3; \\ \dot{\xi}_3 = \sigma (-k_1 \xi_1 - k_2 \xi_2 - k_3 \xi_3 + k_1 e). \end{cases} \quad (13)$$

The algorithm for setting the adaptive coefficients according to (7)  $\tilde{k}(t) = \mu + \kappa$  has the following form:

$$\tilde{k}(t) = \int_{t_0}^t \lambda(\tau) d\tau; \quad \lambda(t) = \begin{cases} \lambda_0 & \text{khi } |e(t)| > \varepsilon_0 \ (\lambda_0 > 0); \\ 0 & \text{khi } |e(t)| \leq \varepsilon_0. \end{cases} \quad (14)$$

Where:  $\varepsilon_0$  - small positive number, characterizing the accuracy of the controller, this number can vary depending on the control law.

Select the coefficient  $\sigma$  as follows:

$$\sigma = \sigma_0 \tilde{k}^2 \quad \text{v\o i} \quad \sigma_0 > 0 \quad (15)$$

Thus, the expressions (10)-(11), (12)-(13) are the output adaptive control laws using the UAV lateral channel motion compensation method when and without taking into account the change of electromagnetic force parameters.

### 3. Wind turbulence model

In this paper, the authors consider the stepwise wind disturbance with the direction of impact perpendicular to the UAV and impact from left to right. The stepwise wind disturbance model in the lateral plane [7] is shown in Figure 1:

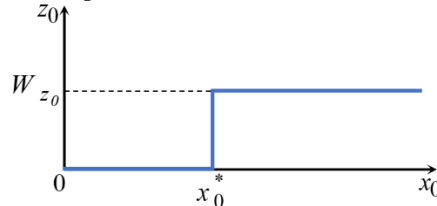


Figure 1: Stepped Wind Turbulence Model

$$W_z = \begin{cases} 0 & \text{khi } x_0 < x_0^* \\ W_{z_0} & \text{khi } x_0 \geq x_0^* \end{cases} \quad (16)$$

Where:  $x_0$  - Axis coordinates  $Ox$ ;

$x_0^*$  - coordinates of the starting point of the wind (in the wind disturbance survey model at the 15th second);

$W_{z_0}$  - crosswind amplitude, m/s .

### 4. Simulation and survey parameters

In the paper, the small UAV-70V model is used [1].

+ UAV-70V parameters used for the survey:  $V_k = 40\text{m/s}$ ;  $m = 56\text{kg}$ ;  $b_a = 0,35\text{m}$ ;  $S = 1,05\text{m}^2$ ;

$$J_y = 33,5\text{kg.m}^2; \quad m_{y_\beta} = -1,1674; \quad m_{y_{\delta_h}} = -0,8875; \quad m_{y_{\omega_y}} = -9,5373; \quad C_{z_\beta} = -2,865.$$

+ Electric drive parameters:  $k_e = 1,025$ ;  $k_m = 0,7$ ;  $k_t = 1$ ;  $k_c = 0,246$ ;  $k_\pi = 0,04$ ;  $R_u = 11$ .

+ Amplitude stepped wind disturbance:  $W_{z_0} = 5\text{m/s}$ .

+ Matrix results  $A, B$  :

When ( $L_u \neq 0$ ):

$$\mathbf{A}_1 = \begin{pmatrix} -1,28 & -1 & 0 & 0 & 0 \\ 12,27 & 0,877 & 9,327 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0,1138 \\ 0 & 0 & 5,491e+0,4 & -2206 & -150 \end{pmatrix}; \quad \mathbf{B}_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1,373e+06 \end{pmatrix}.$$

When ( $L_u = 0$ ):

$$\mathbf{A}_2 = \begin{pmatrix} -1,28 & -1 & 0 & 0 \\ 12,27 & 0,877 & 9,327 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 31,59 & -1,274 \end{pmatrix}; \mathbf{B}_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -789,8 \end{pmatrix}.$$

### 5. Simulation and survey results

By simulating and analyzing the lateral channel motion of the hypothetical UAV using Simulink tools, the following results were obtained:

- The results of evaluating the stability of the adaptive control according to the output using the serial compensation method [5], are shown in Figure 2:

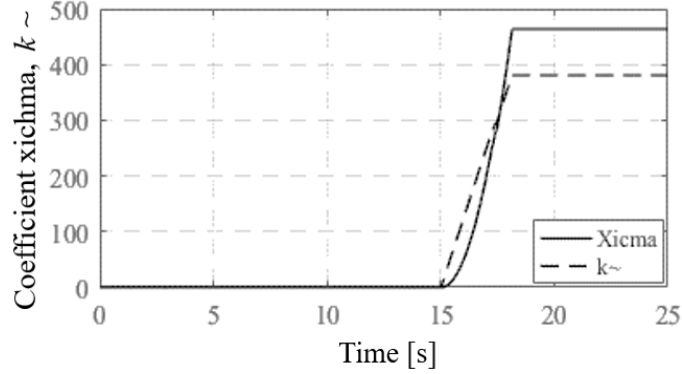


Figure 2: Coefficient  $k$  and  $\sigma$

Figure 2 shows the coefficients  $\tilde{k}$ ,  $\sigma$  of the adaptive control system according to the output using the series compensation method (14), (15). The results prove that, after the disturbance (15<sup>th</sup> second), the coefficients  $\tilde{k}$ ,  $\sigma$  quickly stabilize at the 18<sup>th</sup> second.

- Simulation results without and with electromagnetic parameters and wind noise:

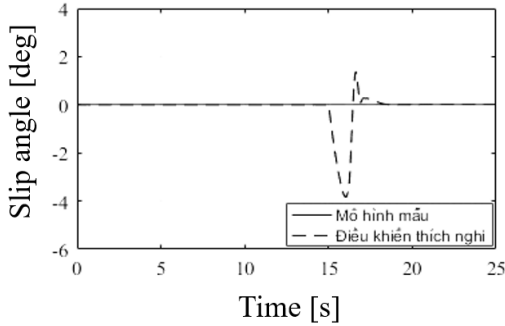


Figure 3: Slip angle without electromagnetic parameter change and wind disturbance

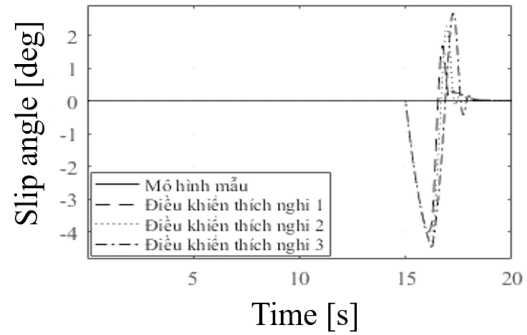


Figure 4: Slip angle when there is a change in electromagnetic parameters at 3 levels and there is wind noise

Figure 3 shows the slip angle  $\beta$ , when there is no change in electromagnetic parameters but there is wind noise. Figure 4 shows the slip angle  $\beta$  when the inductance changes at three levels:  $L_u = 1,5L_u^0$ ;  $L_u = 2,5L_u^0$ ;  $L_u = 3,5L_u^0$  and wind noise. The results shown in Figures 3 and 4 demonstrate that, with the output adaptive controller using the series compensation method, the system is stable when considering both wind noise and electromagnetic force (in the 3s period from the 15<sup>th</sup> to the 18<sup>th</sup> second).

### III. DISCUSSION AND CONCLUSION

When using an output-based adaptive control system using the serial compensation method, the lateral channel motion of the small UAV is always stable when considering both wind disturbances (external disturbances) and electromagnetic forces (internal disturbances) affecting the system.

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