International Journal of Engineering Research and Development
e- ISSN: 2278-067X, p-ISSN: 2278-800X, www.ijerd.com
Volume 21, Issue 6 (June 2025), PP 134-144

Water Resources Allocation to Multipurpose Projects in Anambra-Imo River Basin using Game Theory

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Abstract
The Anambra-Imo River Basin covers the landscape of the eastern region of Nigeria. Water resources development in the basin is the responsibility of the Anambra-Imo River Basin Development Agency (AIRBDA). The underutilization of the basin resources and inherent challenges of AIRBDA pose setbacks to the development of water resources in the region. By modeling stakeholders' multi-purposes and multi-objectives as players and analyzing their interactions, the author's field survey identified five basic needs of the people living within the Anambra-Imo River Basin to include the need for Irrigation Agriculture, Hydropower Generation, Water Supply, Flood Control and Reservoirs, and Erosion Control in the region. Planning and providing engineering solutions as well as innovative and logical resource allocation to these multipurpose projects were expected to satisfy the objectives/benefits of Economic Efficiency, Regional Economic Redistribution, Youth Empowerment, and Environmental Quality improvement of the Anambra-Imo River Basin. A five-year strategic water resources development plan for AIRBDA determined the benefits accruing to each purpose. Data for AIRBDA revealed that N25.0 billion is to be spent on the multi-purpose/multi-objective water resources development. To simultaneously optimize the objectives, under the worst-case scenario, game theory was applied. The model allocated N7.975B, N10.65B, 0, N6.375B, and 0 for Irrigation, hydropower, water supply, Flood Control and Erosion Control respectively. Also, it is important to note that if the fund is allocated as demonstrated above, a net benefit of N53.1925 billion can be achieved under the worst condition of the conflicting objectives. This clearly showed that Game Theory approach offers benefits, return on investment, and sustainability potential for the multipurpose projects. Reliability and contingency tests were carried out; the value of chi-square (X^2) was 3.929, and the coefficient of correlation was 48%, indicating the presence of a positive degree of linear association between the observed and expected values, this also validates the hypothesis.
Keywords: *Water resources, AIRB, Game Theory, resources allocation, multipurpose projects.*

Date of Submission: 15-06-2025 Date of acceptance: 30-06-2025

I. Introduction

River basins are the natural units for managing water resources, as they encompass the entire watershed and all the water bodies within it. Planning and managing water resources is essential for sustainable development, particularly in areas with a growing demand for limited resources. More than 10 million people rely on the Anambra Imo River Basin (AIRB), a major watershed in southeastern Nigeria, as their main source of water (Adelanaand MacDonald, 2014). Numerous conflicting demands, including urbanization, agricultural growth, industrial expansion, and environmental protection, are posing issues for the basin. These issues are predicted to get worse as the region's population grows and climate change accelerates, necessitating the development of efficient, sustainable water management plans. To improve the quality of life in the area, the Anambra-Imo River Basin Development Authority (AIRBDA) was created in 1976 by the Federal Government of Nigeria. The agency was tasked with developing and managing surface and groundwater resources within its jurisdiction and ensuring that people have access to safe and sufficient water for domestic, industrial, and agricultural uses as well as flood control. This institution is battling with issues of insufficient funding, the distribution and appropriation of funds, overlap or duplication of functions among government agencies, and the organization's inability to hire and retain highly skilled employees. Thissituation necessitated careful planning and management, which includes both engineering solutions and optimization of resources. The application of game theory, a mathematical framework that simulates strategic interactions among logical decision-makers (players), was applied in this context for the logical allocation of resources to Anambra-Imo River Basin multi-objective projects to improve on their deliverables, provide valuable insights for policymakers with best practices in water resources management.

1.1 Background of the Study

Globally, increased population, climate change, and lack of good management practices posed a lot of water stress. Whereas other continents have systems in place to combat the aftermath of these natural phenomena, Africa is blessed with abundant water resources, but the means and techniques to tap these nature's gifts are not adequately available.Previous studies revealed that Africa suffers from full-scale policy implementation, monitoring, and maintenance of basin projects; uncompleted and/or abandoned water infrastructure projects; and lack of market strategy for cost recovery of water resources projects resulting in low return on investment and eventual collapse or failure of installed water projects. The consequences of these include inadequate water supply, limited hydropower generation, the incidence of flood control and erosion, environmental degradation, etc. which are drivers for poverty, unemployment, youth restiveness, poor living standards, poor economy, etc.

The Anambra-Imo River Basin is endowed with a network of rivers and water bodies that are vital for sustaining human life, supporting ecosystems, and driving economic activities. This region is highly susceptible to soil erosion, which is aggravated by deforestation, improper land use, and poor agricultural practices. The Anambra-Imo Basin is prone to seasonal flooding exacerbated by climate change actions. Industrial activities, agricultural runoff, and urban waste disposal have contributed to water pollution in the basin. Poor infrastructure for water storage, distribution, and drainage contributes to inefficient water use, particularly in agriculture and domestic water supply. This leads to water shortages during dry seasons and over-reliance on groundwater. Rapid population growth and urbanization in the region increase the demand for water and agricultural land, placing additional strain on the river basin's resources and leading to unsustainable exploitation. Conflicts over land ownership and use, coupled with inconsistent policies and weak regulatory enforcement, hinder effective management of the river basin. These challenges require careful planning and management strategies that involve engineering solutions as well as optimization strategies to drive sustainable development.

1.2 Statement of the Problems

The basis of water resources management lies in the use of dams to impound water to satisfy various needs for water supply, Irrigation Agriculture, Flood control, Hydropower generation, etc. These purposes are jettisoned when dams are either not in place or not functional. The five states under the Anambra Imo River basin have dams, many of which are not functional or abandoned. Anambra State is home to the IfiteOgwari Pumping Station, the Omor Dam in the Ayamelum Local Government Area, the Ogboji Dam in the Orumba South Local Government Area, and the IfiteOgwari Dam in theAyamelum Local Government Area. In Abia state, we have the Akanu dam at Ohafia for water supply. In Ebonyi state, we have the Ufiobodo Dam that supplies water for irrigation and domestic use, the Ebonyi River Dam that supplies water for irrigation and domestic use also, the Mpu/Ishiagu dam, a project that was abandoned by the contractor, and a section of the dam collapsed which led to flooding in many communities in Ishiagu. In Imo state, there is an Ezealakpaka Dam for water supply to communities in the area, Amauzaari Dam which is yet to be completed, so is Inyishi Dam and Nworie River Dam which is expected to be used for hydropower. In Enugu State, we have the Adada River Dam at UzoUwani local government area, the AmechiAwkunanaw Multi-Purpose Dam, and the Ivo River Dam which had silted, thus often spilling over to cause flooding downstream. These challenges culminated into decline in wealth creation, economic efficiency, regional economic redistribution, social well-being, youth employment, and environmental quality improvement of persons in the southeastern region.

II. Area of the study

The area of study was the Anambra-Imo River Basin managed by the Anambra-Imo River Basin Development Authority (AIRBDA). The Anambra-Imo River Basin (AIRB) covers an area of about 18,441 km², which includes the landmass of the five eastern states of Nigeria: Anambra, Imo, Enugu, Ebonyi, and Abia. The basin has a typical wet and dry savanna climate. The climate between the states drained by the basin is relative. The Anambra region experiences an average annual rainfall of 212.36mm, an annual temperature of 28.990c, and humidity up to 73.34%, while the Imo region has an annual precipitation of 1500-2200mm, with an average temperature of 200°C and humidity of 75% and higher at the peak of the rainy season. The rainy season begins in March and lasts till October or early November. Rainfall is often at its peak in September, which often leads to large volumes of runoff.

III. Materials and Methods

The study involved a field survey to derive the state of the art of the Anambra-Imo River Basin, determination of benefits accruing to design schemes for irrigation agriculture, hydropower demand, water supply, and flood and erosion control projects; and logical allocation of resources to these projects using a game theory model.

Irrigation Agriculture: This involved the benefit accruing from 30 hectares of arable land, considering the Omor reservoir in Anambra state with a sufficient quantity of water, and also evaluated channels for withdrawal for a rice farm.

Hydropower: The study estimated objectives derivable from hydropower generation to serve urban Enugu, considering a water source, penstock, turbine type, and generator required for the operation.

Water supply: A water scheme involving groundwater abstraction, pump sizing, raw water storage tank sizing, treatment process, and distribution to service reservoirs for the Nnewi metropolis of Anambra state was considered. The benefits herein were evaluated.

Flood control/reservoir: This involved evaluation of the benefits of water containment in a reservoir for use in the dry season and prevention of flood damage in the rainy season, with adequate channeling that safely regulates overflow.

Erosion control: The evaluation of management practices and determination of structural measures to control runoff in an erosion-prone area in the Anambra-Imo basin were considered.

3.1 Application of Game Theory in Water Resources Management

A novel and practical approach to water resource management is provided by game theory, which studies mathematical depictions of strategic interactions between rational actors.By representing stakeholders as rational beings that want to maximize their own utility while realizing the interdependence of their activities, game theory offers a framework for understanding the dynamicsof competition and cooperation in water management (Tisdell, 2017).It helps predict the outcomes of different management strategies while taking stakeholder tensions and motives into account. Applying game theory to the Anambra Imo River Basin assist in determining the best resource allocation that take into account the demands of several stakeholders. Because water resources are shared among different parties have conflicting interests, game theory models how these interest are satisfied (Zhu & Wang, 2018). According to Wang et al. (2017), this method is especially helpful since it aids in the creation of policies that encourage collaboration and encourage more effective water use. (Tisdell, 2017). It helps predict how different management strategies will perform while taking stakeholder motives and disputes into consideration.

IV. Data Analysis Using Game Theory

The summary of net benefits (payoff) for the five purposes and objectives (strategies) is presented in the table below.

Table 1: Summary Table of Net Benefits of all the objectives under consideration in this study

PURPOSE (A_{1-5})	Objective (B_{1-5})				
	Economic Efficiency (B_1)	Regional Economic Redistribution (B_2)	Social Well-being (B_3)	Youth Empowerment (B_4)	Environmental Quality Improvement (B_5)
Irrigation Agriculture (A_1)	2.763	2.081	0.943	2.211	3.120
Hydropower (A_2)	2.776	1.362	1.997	2.667	2.419
Water Supply (A_3)	2.733	2.146	2.991	1.529	1.822
Flood Control (A_4)	1.690	2.549	1.606	2.146	1.515
Erosion Control (A_5)	3.247	1.763	2.938	1.574	0.936

The entries presented in Table 1 was used to form the payoff matrix of the game problem involving Player A and Player B as shown below:

Table 2: Table of the payoff Matrix

	B_1	B_2	B_3	B_4	B_5	Minimax
A_1	2.763	2.081	0.943	2.211	3.120	**0.943**
A_2	2.776	1.362	1.997	2.667	2.419	**1.362**
A_3	2.733	2.146	2.991	1.529	1.822	**1.529**
A_4	1.690	2.549	1.606	2.146	1.515	**1.515**

A_5	3.247	1.763	2.938	1.574	0.936	**0.936**
Maximin	**3.247**	**2.549**	**2.991**	**2.667**	**3.120**	

The analysis of the payoff matrix revealed the absence of a saddle point within the game. Instead, the value of the game was determined to fall within the range of 1.529 to 2.549. This implies that there was no clear-cut dominance of one row or column over another in the matrix. As a result, in the context of game theory, the use of more complex strategies like linear programming is required due to the lack of evident supremacy.

Table 3. Table of players and their probabilities

	B_1	B_2	B_3	B_4	B_5	Probability
A_1	2.763	2.081	0.943	2.211	3.120	p_1
A_2	2.776	1.362	1.997	2.667	2.419	p_2
A_3	2.733	2.146	2.991	1.529	1.822	p_3
A_4	1.690	2.549	1.606	2.146	1.515	p_4
A_5	3.247	1.763	2.938	1.574	0.936	p_5
Probability	q_1	q_2	q_3	q_4	q_5	

4.1 Model Construction

Suppose we let V to be the value of the game, where, p_1, p_2, p_3, p_4 and p_5 = the probabilities of selecting strategies A_1, A_2, A_3, A_4, and A_5 respectively; and q_1, q_2, q_3, q_4 and q_5 = the probabilities of selecting strategies B_1, B_2, B_3, B_4, and B_5 respectively.

The expected gain for player A is stated below:

$2.763p_1 + 2.081p_2+0.943p_3+2.211p_4+3.120p_5\geq V$ (if B uses strategy B_1)

$2.776p_1 + 1.361p_2+1.997p_3+2.667p_4+2.141p_5\geq V$ (if B uses strategy B_2)

$2.733p_1 + 2.146p_2+2.991p_3+1.529p_4+1.822p_5\geq V$ (if B uses strategy B_3)

$1.690p_1 + 2.549p_2+1.606p_3+2.146p_4+1.515p_5\geq V$ (if B uses strategy B_4)

$3.247p_1 + 1.763p_2+2.938p_3+1.574p_4+0.936p_5\geq V$ (if B uses strategy B_5)

$P_1+ p_2 + p_3 + p_4 + p_5 = 1$; (probability condition); and

$p_1, p_2, p_3, p_4, p_5 \geq 0$ (non-negativity condition)

Dividing each inequality by V, we have

$(2.763p_1 + 2.081p_2+0.943p_3+2.211p_4+3.120p_5)/V\geq 1$ (if B uses strategy B_1)

$(2.776p_1 + 1.361p_2+1.997p_3+2.667p_4+2.141p_5)/V \geq 1$ (if B uses strategy B_2)

$(2.733p_1 + 2.146p_2+2.991p_3+1.529p_4+1.822p_5)/V \geq 1$ (if B uses strategy B_3)

$(1.690p_1 + 2.549p_2+1.606p_3+2.146p_4+1.515p_5)/V\geq 1$ (if B uses strategy B_4)

$(3.247p_1 + 1.763p_2+2.938p_3+1.574p_4+0.936p_5)/V\geq 1$ (if B uses strategy B_5)

$p_1/V + p_2 /V +p_3/V+p_4/V + p_5/V = 1/V$

Let $p_1/V= x_1$, $p_2/V= x_2$, $p_3/V= x_3$, $p_4/V= x_4$, and $p_5/V= x_5$

Therefore, the problem for player A becomes as stated below.

Minimize $Z_p= x_1 + x_2 + x_3 + x_4 +x_5$

Subject to constraints:

$2.763x_1 + 2.081x_2+0.943x_3+2.211x_4+3.120x_5\geq 1$

$2.776x_1 + 1.361x_2+1.997x_3+2.667x_4+2.141x_5\geq 1$

$2.733x_1 + 2.146x_2+2.991x_3+1.529x_4+1.822x_5\geq 1$

$1.690x_1 + 2.549x_2+1.606x_3+2.146x_4+1.515x_5\geq 1$

$3.247x_1 + 1.763x_2+2.938x_3+1.574x_4+0.936x_5\geq 1$

$x_1, x_2, x_3, x_4, x_5\geq 0$ (non-negativity condition)

The objective of player's B is to minimize his expected losses which can be reduced to minimizing the value of the game, V. Hence, the expected loss for player B will be as follows:

$2.763q_1 + 2.776q_2+2.733q_3+1.690q_4+3.247q_5 \leq V$ (if A uses strategy A_1)

$2.081q_1 + 1.362q_2+2.146q_3+2.549q_4+1.763q_5\leq V$ (if A uses strategy A_2)

$0.943q_1 + 1.997q_2+2.991y_3+1.606q_4+2.938q_5\leq V$ (if A uses strategy A_3)

$2.211q_1 + 2.667q_2+1.529y_3+2.146q_4+1.574q_5\leq V$ (if A uses strategy A_4)

$3.120q_1 + 2.419q_2+1.822y_3+1.515q_4+0.936q_5 \leq V$ (if A uses strategy A_5)

$q_1 + q_2 + q_3 + q_4 + q_5 = 1$; (probability condition); and

$q_1, q_2, q_3, q_4, q_5 \geq 0$ (non-negativity condition)

Dividing each inequality by V, we have

$(2.763q_1 + 2.776q_2+2.733q_3+1.690q_4+3.247q_5)/V \leq 1$ (if A uses strategy A_1)

$(2.081q_1 + 1.362q_2+2.146q_3+2.549q_4+1.763q_5)/V \leq 1$ (if A uses strategy A_2)

$(0.943q_1 + 1.997q_2+2.991y_3+1.606q_4+2.938q_5)/V \leq 1$ (if A uses strategy A_3)

$(2.211q_1 + 2.667q_2+1.529y_3+2.146q_4+1.574q_5)/V \leq 1$ (if A uses strategy A_4)

$(3.120q_1 + 2.419q_2+1.822y_3+1.515q_4+0.936q_5)/V \leq 1$ (if A uses strategy A_5)

Where $y_1, y_2, y_3, y_4, y_5 \geq 0$

$q_1/V + q_2/V + q_3/V + q_4/V + q_5/V = 1/V$
Let $q_1/V = y_1$, $q_2/V = y_2$, $q_3/V = y_3$, $q_4/V = y_4$, and $q_5/V = y_5$
Therefore, the problem for player B becomes;
Minimize $Z_p = y_1 + y_2 + y_3 + y_4 + y_5$
Subject to constraints:
$2.763y_1 + 2.776Y_2 + 2.733y_3 + 1.690y_4 + 3.247y_5 \leq 1$
$2.081y_1 + 1.362y_2 + 2.146y_3 + 2.549y_4 + 1.763y_5 \leq 1$
$0.943y_1 + 1.997y_2 + 2.991y_3 + 1.606y_4 + 2.938y_5 \leq 1$
$2.211y_1 + 2.667y_2 + 1.529y_3 + 2.146y_4 + 1.574y_5 \leq 1$
$3.120y_1 + 2.419y_2 + 1.822y_3 + 1.515y_4 + 0.936y_5 \leq 1$
Where $y_1, y_2, y_3, y_4, y_5, s_1, s_2, s_3, s_4, s_5 \geq 0$

The problem of player A is dual of the problem of player B, hence, the dual problem can be solved using the optimal simplex table of the primal. To solve the problem of player B, slack variables were introduced to convert the inequalities to equalities. The problem therefore becomes:

Minimize $Z_p = y_1 + y_2 + y_3 + y_4 + y_5 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5$
Subject to constraints:
$2.763y_1 + 2.776Y_2 + 2.733y_3 + 1.690y_4 + 3.247y_5 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 = 1$
$2.081y_1 + 1.362y_2 + 2.146y_3 + 2.549y_4 + 1.763y_5 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 = 1$
$0.943y_1 + 1.997y_2 + 2.991y_3 + 1.606y_4 + 2.938y_5 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 = 1$
$2.211y_1 + 2.667y_2 + 1.529y_3 + 2.146y_4 + 1.574y_5 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 = 1$
$3.120y_1 + 2.419y_2 + 1.822y_3 + 1.515y_4 + 0.936y_5 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 = 1$
Where $y_1, y_2, y_3, y_4, y_5 \geq 0$ (non-negativity condition)

4.2 Model Solution

The table below shows successive iteration to achieve optimal solution of the model using linear programming solver (LPE)

1st Iteration

Basic variable	y_1	y_2	y_3	y_4	y_5	S_1	S_2	S_3	S_4	S_5	Soln value (b)	Ratio (b/y_i)
S1	2.763	2.776	2.733	1.690	3.247	1	0	0	0	0	1	0.36
S2	2.0811	1.362	2.146	2.549	1.763	0	1	0	0	0	1	0.48
S3	0.943	1.997	2.991	1.606	2.938	0	0	1	0	0	1	1.06
S4	2.211	2.667	1.529	2.146	1.574	0	0	0	1	0	1	0.45
S5	3.120	2.419	1.822	1.515	0.936	0	0	0	0	1	1	0.32
Z	-1	-1	-1	-1	-1	0	0	0	0	0	0	

y_1 enters the basis, S_5 leaves from the basis

2nd Iteration

Basic variable	y_1	y_2	y_3	y_4	y_5	S_1	S_2	S_3	S_4	S_5	Soln value(b)
S1	0	3.77	0.47	0.15	1	0.42	0	0	0	-0.36	0.05
S2	0	-0.48	0.41	1.36	0	-0.47	1	0	0	-0.24	0.28
S3	0	0.39	1.22	0.77	0	-1.09	0	1	0	0.68	0.58
S4	0	0.93	-0.18	0.95	0	-0.37	0	0	1	-0.37	0.25
X1	1	0.59	0.45	0.45	0	-0.12	0	0	0	0.44	0.31
Z	0	-0.05	-0.09	-0.41	0	0.29	0	0	0	0.07	0.39

y_5 enters the basis, S_1 leaves from the basis

3rd Iteration

Basic variable	y_1	y_2	y_3	y_4	y_5	S_1	S_2	S_3	S_4	S_5	Soln value(b)
S1	0	0.42	0.43	0	1	0.47	-0.1	0	0	-0.34	0.02
S2	0	-0.35	0.3	1	0	-0.34	0.73	0	0	-0.18	0.21
S3	0	0.66	0.99	0	0	-0.83	-0.55	1	0	0.81	0.42
S4	0	1.26	-0.46	0	0	-0.05	-0.68	0	1	-0.2	0.06
X1	1	0.74	0.32	0	0	0.03	-0.32	0	0	0.52	0.22
Z	0	-0.2	0.03	0	0	0.15	0.31	0	0	-0.01	0.44

y_4 enters the basis, S_2 leaves from the basis

4th Iteration

Basic variable	y_1	y_2	y_3	y_4	y_5	S_1	S_2	S_3	S_4	S_5	Soln value(b)
X5	0	1	1.03	0	2.43	1.13	-0.25	0	0	-0.82	0.05
X4	0	0	0.66	1	0.86	0.06	0.64	0	0	-0.47	0.22
S3	0	0	0.32	0	-1.60	-1.57	-0.38	1	0	1.36	0.39
S4	0	0	-1.74	0	-3.04	-1.46	-0.36	0	1	0.84	0.01

X1	1	0	-0.43	0	-1.78	-0.8	-0.13	0	0	1.12	0.19
Z	0	0	0.24	0	0.5	0.38	0.25	0	0	-0.17	0.45

y_2 enters the basis, x_5 leaves from the basis

5th Iteration

Basic variable	y_1	y_2	y_3	y_4	y_5	S_1	S_2	S_3	S_4	S_5	Soln value(b)
X2	0	1	-0.7	0	-0.59	-0.32	-0.61	0	1	0	0.05
X4	0	0	-0.33	1	-0.87	-0.77	0.44	0	0.57	0	0.22
S3	0	0	3.15	0	3.36	0.81	0.21	1	-1.62	0	0.39
S4	0	0	-2.09	0	-3.66	-1.75	-0.43	0	1.21	1	0.01
X1	1	0	1.91	0	2.31	1.17	0.36	0	-1.34	0	0.19
Z	0	0	-0.13	0	-0.16	0.07	0.18	0	0.22	0	0.45

S_5 enters the basis, S_4 leaves from the basis

6th Iteration

Basic variable	y_1	y_2	y_3	y_4	y_5	S_1	S_2	S_3	S_4	S_5	Soln value(b)
X2	0.26	1	-0.21	0	0	-0.02	-0.52	0	0.65	0	0.1
X4	0.38	0	0.39	1	0	-0.33	0.57	0	0.06	0	0.29
S3	-1.45	0	0.38	0	0	-0.89	-0.31	1	0.33	0	**0.13**
S4	1.59	0	0.93	0	0	0.1	0.13	0	-0.93	1	0.3
X1	0.44	0	0.83	0	1	0.51	0.16	0	-0.58	0	0.08
Z	0.08	0	-0.1	0	0	0.15	0.2	0	0.13	0	0.46

y_5 enters the basis, y_1 leaves from the basis

7TH Iteration

Basic variable	y_1	y_2	y_3	y_4	y_5	S_1	S_2	S_3	S_4	S_5	Soln value(b)
X2	0.38	1	0	0	0.27	0.11	-0.48	0	0.5	0	0.12
X4	0.18	0	0	1	-0.46	-0.57	0.5	0	0.34	0	0.26
S3	-1.65	0	0	0	-0.45	-1.12	-0.38	1	0.6	0	0.09
S4	1.1	0	0	0	-1.12	-0.47	-0.04	0	-0.27	1	0.21
X5	0.53	0	1	0	1.22	0.62	0.19	0	-.07	0	0.1
Z	0.08	0	0	0	0.01	0.15	0.2	0	0.12	0	0.47

y_3 enters the basis, y_5 leaves from the basis

8TH Iteration

Basic variable	y_1	y_2	y_3	y_4	y_5	S_1	S_2	S_3	S_4	S_5	Soln value(b)
X2	0.38	1	0	0	0.27	0.11	-0.48	0	0.5	0	0.12
X4	0.18	0	0	1	-0.46	-0.57	0.5	0	0.34	0	0.26
S3	-1.65	0	0	0	-0.45	-1.12	-0.38	1	0.6	0	0.09
S4	1.1	0	0	0	-1.12	-0.47	-0.04	0	-0.27	1	0.21
X5	0.53	0	1	0	1.22	0.62	0.19	0	-.07	0	0.1
Z	0.08	0	0	0	0.01	0.15	0.2	0	0.12	0	0.47

The result of optimality has been achieved since the coefficients for the decision variables y1 to y5 are all non-negative, indicating that the current solution is optimal. The objective function value cannot be further improved by increasing any of the decision variables within the current feasible region. The solution value in the last column is 0.47, indicating the optimal value of the objective function at the current solution.

Thus, $y_1 = 0$, $y_2 = 0.12$, $y_3 = 0$, $y_4 = 0.26$, and $y_5 = 0.1$. The values $S_3 = 0.09$ and $S_4 = 0.21$ represent unused resources at the optimal solution. Therefore, the solution value Z = 0.47 and the expected value of the game, V, is obtained from the relation Zq = 1/V.

Therefore, $V= 1/Z_q= 1/0.47 = 2.1277$

Converting these solution values back into the original variables, we have

From $y_1 = q_1/V$;

$q_1 = y_1 \times V = \mathbf{0}$

$q_2 = y_2 \times V = 0.12 \times 2.1277 = \mathbf{0.255}$

$q_3 = y_3 \times V = \mathbf{0}$

$q_4 = y_4 \times V = 0.26 \times 2.1277 = \mathbf{0.553}$

$q_5 = y_5 \times V = 0.1 \times 2.1277 = \mathbf{0.213}$

For player A,

The optimal strategies for player A are as read off from the reduced cost row (i.e. the Z_j-C_j row) of the final table.

$x_1 = s_1 = 0.15$

$x_2 = s_2 = 0.2$

$x_3 = s_3 = 0$
$x_4 = s_4 = 0.12$
$x_5 = s_5 = 0$
From $x_1 = p_1/V$;
$p_1 = x_1 \times V = 0.15 \times 2.1277 = \mathbf{0.319}$
$p_2 = x_2 \times V = 0.2 \times 2.1277 = \mathbf{0.426}$
$p_3 = x_3 \times V = 0 \times 2.1277 = \mathbf{0}$
$p_4 = x_4 \times V = 0.12 \times 2.1277 = \mathbf{0.255}$
$p_5 = x_5 \times V = 0 \times 2.1277 = \mathbf{0}$
Hence, the probabilities of using strategies by both players are:
Player A = (0.319, 0.426, 0, 0.255, 0)
Player B = (0, 0.255, 0, 0.553, 0.213)
From the following the strategy of player A is superior to that of player B, when the values of the probabilities are placed side by side, hence the strategy of player A is adopted for resources allocation.

4.3Allocation of Cost to the Various Purposes
Fund allocation by the Federal Government to Anambra Imo River Basin multipurpose projects is shown in the table below.

Table 4. Anambra Imo River Basin Development Authority Budget Implementation Status for (2019 -2023)

S/N	YEAR	APPRIOPRIATION	RELEASE
1	2019	9,302,610,010.94	3,721,065,865.32
2	2020	3,840,559,700.00	3,840,559,700.00
3	2021	8,005,903,730.00	8,005,903,730.00
4	2022	7,381,126,249.93	6,650,180,108.04
5	2023	4,861,299,404.00	3,236,005,295.58

Source: Anambra Imo River Basin Development Authority, Imo State, Nigeria

From the foregoing, N25.0 billion is to be spent on the multi-purpose/multi-objective water resources development, to simultaneously optimize the objectives even the worst case scenario; the allocation is presented in Table 5 below:

Table 5. Cost allocation to the various purposes using Game Theory method

S/No.	PURPOSE	Probability	Allocation (in Billion Naira)
1.	Irrigation	p_1= 0.319	$p_1 \times 25.0$ = N7.975
2.	Hydropower	p_2= 0.426	$p_2 \times 25.0$ =N10.65
3.	Water Supply	P_3= 0	0
4.	Flood Control	p_4=0.255	$p_4 \times 25.0$ = N6.375
5.	Erosion Control	P_5= 0	0

4.4. Discussion of Results

The findings outlined above indicated that the Game Theory approach designated a larger share of funding to hydropower initiatives (N10.65 B), accounting for 43% of the total (fig. 1). This reflects a greater focus on hydropower within the basin to meet the increasing energy demands of its growing population for multiple applications. The Game model assigned N7.975 B, or 32%, to irrigation initiatives. Furthermore, the model allocated N6.375 B, which represents 25%, to flood control efforts, highlighting it as the next significant budget allocation for basin projects following hydropower. This situation underscores a critical challenge within the basin that has resulted in extensive loss of life and property over the years.

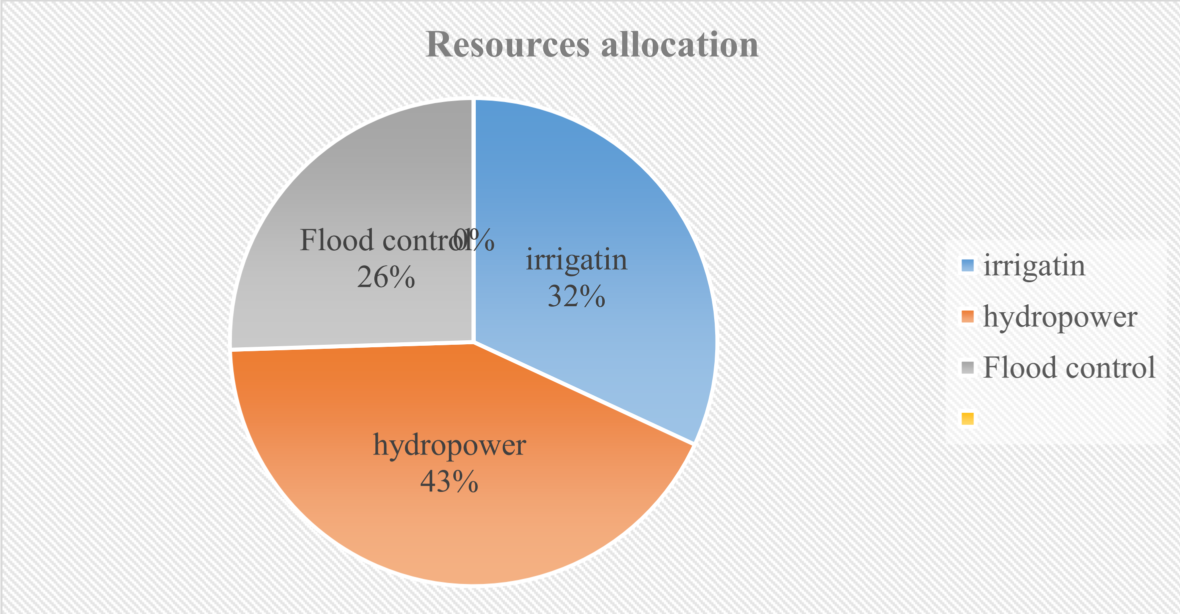

Fig 1. A chart showing percentage allocation of resources to projects

Also, it is important to note that if the fund is allocated as demonstrated above, a minimum of N25.0 billion x 2.0325 (the value of the game) can be achieved under the worst condition of the conflicting objectives while using the method of Game theory. Hence, the financial benefit achievable under the worst condition = N25.0 billion x 2.1277= N53.1925 billion can be achieved under the worst condition of the conflicting objectives. This clearly showed that Game theory approach offers benefit and return on investment

4.5 Contingency and Reliability Test

Table 6. Observe contingency table

	Economic Efficiency	Regional Economic Redistribution	Social Well-being	Youth Empowerment	Environmental Quality Improvement	Row Total
Irrigation Agriculture	2.763	2.081	0.943	2.211	3.120	11.118
Hydropower	2.776	1.362	1.997	2.667	2.419	11.221
Water Supply	2.733	2.146	2.991	1.529	1.822	11.221
Flood Control	1.690	2.549	1.606	2.146	1.515	9.506
Erosion Control	3.247	1.763	2.938	1.574	0.936	10.458
Column Total	13.209	9.901	10.475	10.127	9.812	53.524

The steps involve is as follows;

$$\text{Cell ij} = \frac{ith\ row\ total\ \times jth\ column\ total}{Grand\ total}$$

Chi-square $x^2 = \sum \frac{(O_{ij}-E_{ij})}{E_{ii}}$

Table 7. Expected contingency Table

	Economic Efficiency	Regional Economic Redistribution	Social Well-Being	Youth Empowerment	Environmental Quality Improvement	Row Total
Irrigation Agriculture	2.743772177	2.056634743	2.175865967	2.103579441	2.038147672	11.118
Hydropower	2.769191185	2.075687934	2.196023746	2.12306754	2.057029594	11.221
Water Supply	2.769191185	2.075687934	2.196023746	2.12306754	2.057029594	11.221
Flood Control	2.345952358	1.758443054	1.860386929	1.798581235	1.742636425	9.506
Erosion Control	2.580893095	1.934546334	2.046699611	1.978704245	1.917156715	10.458
Column Total	13.209	9.901	10.475	10.127	9.812	53.524

Table 8.Computation of chi square

Observed (O)	Expected (E)	O-E	(O-E)^2	$X^2 = \frac{(O-E)^2}{E}$
2.763	2.743772177	0.019227823	0.000369709	0.00013474
2.776	2.769191185	0.006808815	4.636E-05	1.6741E-05
2.733	2.769191185	-0.036191185	0.001309802	0.00047299
1.69	2.345952358	-0.655952358	0.430273496	0.18341101
3.247	2.580893095	0.666106905	0.443698409	0.17191662
2.081	2.056634743	0.024365257	0.000593666	0.00028866
1.362	2.075687934	-0.713687934	0.509350467	0.24538875
2.146	2.075687934	0.070312066	0.004943787	0.00238176
2.549	1.758443054	0.790556946	0.624980285	0.35541685
1.763	1.934546334	-0.171546334	0.029428145	0.01521191
0.943	2.175865967	-1.232865967	1.519958493	0.69855336
1.997	2.196023746	-0.199023746	0.039610451	0.01803735
2.991	2.196023746	0.794976254	0.631987244	0.28778707
1.606	1.860386929	-0.254386929	0.06471271	0.03478454
2.938	2.046699611	0.891300389	0.794416383	0.38814508
2.211	2.12306754	0.08793246	0.007732118	0.00364196
2.667	2.12306754	0.54393246	0.295862521	0.13935615
1.529	1.798581235	-0.269581235	0.072674042	0.04040632
2.146	1.978704245	0.167295755	0.02798787	0.01414454
1.574	2.103579441	-0.529579441	0.280454384	0.13332246
3.12	2.038147672	1.081852328	1.17040446	0.5742491
2.419	2.057029594	0.361970406	0.131022575	0.06369504
1.822	2.057029594	-0.235029594	0.05523891	0.02685373
1.515	1.742636425	-0.227636425	0.051818342	0.0297356
0.936	1.917156715	-0.981156715	0.962668499	0.50213344
53.524	53.524	1E-09	8.151543127	3.92948577

Chi-square $X^2 = \frac{(O-E)^2}{E} = 3.92948577$

Contingency coefficient, $C = \frac{X^2}{N+X^2} = \frac{3.92948577}{53.524+3.92948577} = 0.5184149639$

The maximum value of C_{max} for a 5×5 table = $\sqrt{\frac{5-1}{5}} = 0.894$

Since the calculated value of C < C_{max}, the result is ok.

The degree of freedom d_f= (r-1) (c-1) = (5-1) (5-1) = 16

From the chi- square distribution table, the critical value of chi-square at 0.1 level of significance = 26.54. Therefore, since the calculated value (x^2 = 3.929248) is less than the table value $X^2_{critical}$ = 26.54, it implies that there was no significance difference between the observed and expected values of the study, in other words the observed data closely matched the expected values.

4.6. The Pearson's Product Moment Correlation Coefficient

Table 4.43 Table showing values for Pearson Correlation computation

Observed (X)	Expected (Y)	XY	X^2	Y^2
2.763	2.743772177	7.581042525	7.634169	7.528285759
2.776	2.769191185	7.68727473	7.706176	7.668419819
2.733	2.769191185	7.568199509	7.469289	7.668419819
1.69	2.345952358	3.964659485	2.8561	5.503492466
3.247	2.580893095	8.380159879	10.543009	6.661009168
2.081	2.056634743	4.2798569	4.330561	4.229746466
1.362	2.075687934	2.827086966	1.855044	4.308480399
2.146	2.075687934	4.454426306	4.605316	4.308480399
2.549	1.758443054	4.482271345	6.497401	3.092121974
1.763	1.934546334	3.410605187	3.108169	3.742469518
0.943	2.175865967	2.051841607	0.889249	4.734392706
1.997	2.196023746	4.385459421	3.988009	4.822520293
2.991	2.196023746	6.568307024	8.946081	4.822520293
1.606	1.860386929	2.987781408	2.579236	3.461039526
2.938	2.046699611	6.013203457	8.631844	4.188979298
2.211	2.12306754	4.694102331	4.888521	4.507415779
2.667	2.12306754	5.662221129	7.112889	4.507415779
1.529	1.798581235	2.750030708	2.337841	3.234894459
2.146	1.978704245	4.24629931	4.605316	3.915270489
1.574	2.103579441	3.31103404	2.477476	4.425046465
3.12	2.038147672	6.359020737	9.7344	4.154045933
2.419	2.057029594	4.975954588	5.851561	4.231370751
1.822	2.057029594	3.74790792	3.319684	4.231370751

1.515	1.742636425	2.640094184	2.295225	3.03678171
0.936	1.917156715	1.794458685	0.876096	3.67548987
53.524	**53.524**	**116.8232994**	**125.138662**	**116.6594799**

The Correlation Coefficient (r) is given as:

$$r = \frac{n\sum XY - (\sum X)(\sum Y)}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}$$

$$= \frac{25 \times 116.8232994 - 53.524 \times 53.524}{\sqrt{[25 \times 125.138662 - 53.524^2][25 \times 116.6594799 - 53.524^2]}} = 0.48$$

The correlation coefficient of 48% was found, indicating the presence of a positive degree of linear association between the observed and expected value.

At 0.05 level of significance and degree of freedom, df = (r-1)(c-1)= (25-1)(2-1) = 24, the critical value (r_c) from statistical tables is 0.388. Therefore since the calculated value (r = 0.48) is higher than the table value,(0.388, i.e. ($r > r_c$), we reject the null hypothesis

V. Conclusion

The study identified five basic areas of need in the basin that required engineering solutions; these include irrigation agriculture, hydropower generation, water supply, flood control, and erosion control. Based on the analysis of a five-year strategic development fund of ₦25.0B, cost allocation based on game theory to the various purposes is as follows: ₦7.975B was allocated to irrigated agriculture, ₦10.65B to hydropower, ₦0 to water supply, ₦6.375B to flood control, and ₦0 was allocated to erosion control. Results from game theory revealed that the basin can make a huge financial benefit of up to ₦53.1925 billion (return on investment is ₦28.1925 billion) if the probabilities of the various purposes determine what each purpose should get in the cost allocation. The result of the Pearson's correlation (r = 0.48) implies a positive linear relationship between the observed and expected values of the study.

It is recommended that the arbitrary allocation of funds for different purposes is a significant mistake and a misappropriation of limited resources that will not produce returns and should be eliminated immediately All fund allocations should be based on logical and mathematical justification.

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