

Study of Turbo Coded OFDM over Fading Channel

Suchita Chatterjee¹, Mangal Singh²
^{1,2} Chhatrapati Shivaji Institute of Technology, Durg (C.G.)

Abstract—The main problems of reliable data communication in the wireless environment are the distorting multipath fading channel and Additive White Gaussian Noise (AWGN) noise. These impairments can distort the transmitted signal severely and thus leading to Inter Symbol Interference (ISI). So the reception becomes erroneous and the Bit Error Rate (BER) increases. Orthogonal Frequency Division Multiplexing (OFDM) or multicarrier communication is a recent technique used to mitigate ISI introduced by the distorting frequency selective fading channel. The earlier approaches used to combat ISI are based on Equalization and Maximum Likelihood Sequence Estimation (MLSE). Though MLSE is the optimum detector, its complexity grows exponentially with the channel length. Equalization has a low complexity but is suboptimal. OFDM essentially bridges the performance gap between MLSE and Equalization at a reasonable complexity. In this paper, we attempt to study the performance of uncoded and turbo coded OFDM signal transmitted through frequency selective Rayleigh fading channels having uniform power delay profile. The channel is assumed to be static for one OFDM symbol and varies randomly from one symbol to the next. Simulation results are presented for rate 1/3 and rate 1/2 turbo code.

Keywords—Turbo codes, turbo encoder, fading channel, turbo decoder, MAP algorithm, likelihood ratio.

I. INTRODUCTION

Due to rapid growth in mobile and wireless communication, high speed data rate transmission is required with high reliability. However as the data rate increases, symbol duration decreases. In the single carrier modulation, if this symbol duration is less than the delay spread of the channel, then Inter Symbol Interference (ISI) occurs. So the maximum data rate is constrained by the channel's delay spread if single carrier modulation is used. Hence to reduce the effect of ISI, the symbol duration must be greater than the channel's delay spread. Hence to increase the data rate as well as avoiding ISI, multicarrier communication is evolved. Starting around 1990 multicarrier modulation has been used in many diverse wired and wireless applications. The concept of multicarrier transmission was first explicitly proposed by Chang in 1966. Orthogonal Frequency Division Multiplexing (OFDM) is the discrete time implementation of the multicarrier communication which is primarily used in wireless communication.

II. MULTICARRIER COMMUNICATION

Multicarrier communication is a special form of frequency division multiplexing (FDM). Whereas FDM is used for the transmission of the data of many users simultaneously, multicarrier communication involves transmitting the data of one user over a severely distorting channel. The basic idea of multicarrier communication comes from the competing desires of high data rates and ISI free transmission. But as the data rate goes up, the symbol time reduces. Hence ISI becomes severe for single carrier modulation. So in order to have ISI free communication along with high data rate, multicarrier communication is preferred. The main concept behind multicarrier communication is to divide the spectrum of a non-ideal frequency selective fading channel into small non-overlapping subchannels such that characteristics of each subchannel can be considered to be ideal (at magnitude response and a linear phase response). Thus, any communication that takes place in a sub channel is essentially distortion less. In the other words, multicarrier communication divides high data rate bit stream into N lower rate substreams (where N = number of subchannels). So in each substream symbol time is greater than channel delay spread. So each substream transmission becomes ISI free. These individual substreams can be sent over N parallel subchannels maintaining the overall data rate fixed. The bandwidth of each subchannel also becomes less than the coherence bandwidth of the channel making the subchannels to be at fading subchannels. Hence a frequency selective fading channel is now converted into N number of ISI free and frequency at fading channels. As the subchannels are non-overlapping in frequency domain, it can be said that the subchannels are orthogonal to each other.

III. METHODOLOGY

A. Turbo Coding

The discovery of turbo codes represents major milestones in the field of channel coding. This coding scheme can achieve realistic BERs between 10^{-6} and 10^{-12} with signal to noise ratios (SNRs) that are only slightly above the minimum SNR. For a given channel and code rate established by Shannon's original capacity theorems. Hence Turbo code is said to be "near capacity achieving" codes and are sometimes considered to have solved the coding problem for the additive white Gaussian noise (AWGN) channel and its derivative channels, at least in the practical sense. Turbo encoder and decoder is described in the following sections.

B. Turbo Encoder

In general case, the code consists of three parts: the uncoded information bits, one set of parity sequences generated by passing the information bits through first convolutional encoder and one set of parity sequence generated by passing interleaved versions of the information bits through second convolutional encoders. Typically the encoders used are Recursive Systematic Convolutional encoders; also, in most turbo codes the encoders used are the same. So the turbo code rate is 1/3. Additionally the parity bits can be punctured in order to raise the code rate to 1/2. The data sequence may or may not be terminated, usually depending on the kind of interleaver used .

Turbo code is constructed as shown below.

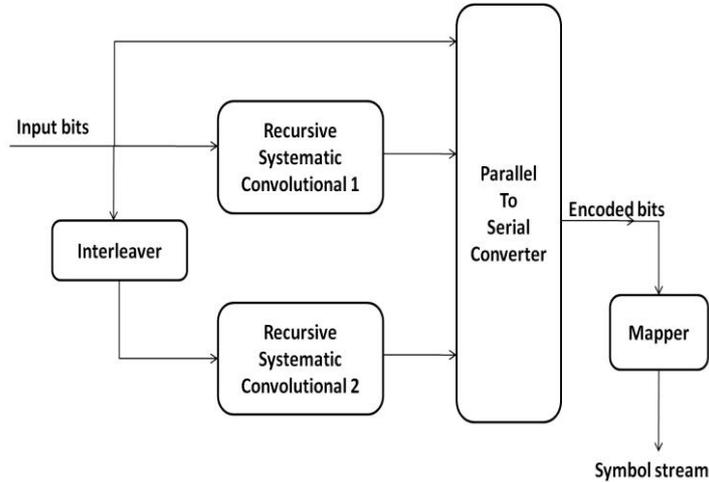


Fig. 1 Turbo Encoder

C. Turbo Decoder

For the typical two component turbo encoder shown in the above diagram, the standard iterative decoder is shown below.

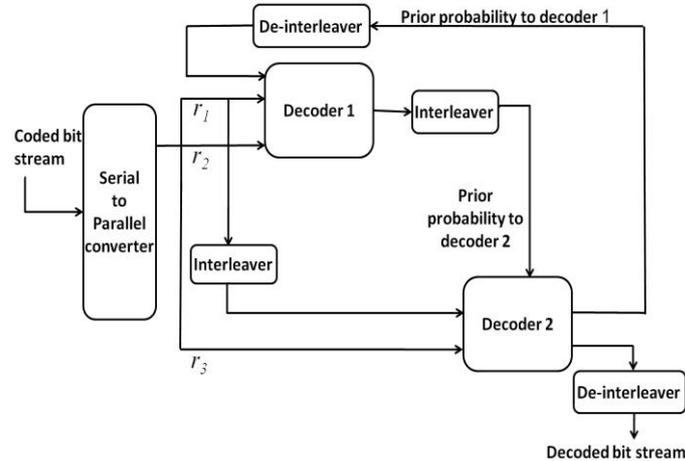


Fig 2. Turbo Decoder

The a priori information is usually set to be equiprobable before starting to decode a frame (unless we know that the information bits have a particular probability pattern). The decoder blocks usually use a modified Bahl-Cocke-Jelinek-Raviv(BCJR) algorithm. If the turbo code consists of more components, it is just a matter of inserting the relevant de-interleaver → decoder → interleaver blocks. The final decoder in the chain also has a hard-decision block associated with it to output the decoded bits. Successive iterations will use the extrinsic information from previous iterations as a priori information. Extrinsic information is that information generated by the decoders of this iteration and is computed by removing the a priori information and direct channel information from the iteration's a posteriori information. It is essential that only extrinsic information is passed between decoders and between iterations for correct operation of the turbo decoder.

D. Interleaver

Interleaving is a process of rearranging the ordering of a data sequence in a one to one deterministic format. The inverse of this process is called de-interleaving which restores the received sequence to its original order. Interleaving is a practical technique to enhance the error correcting capability of coding. In turbo coding, interleaving is used before the information data is encoded by the second component encoder. The basic role of an interleaver is to construct a long block

code from small memory convolutional codes, as long codes can approach the Shannon capacity limit. Secondly, it spreads out burst errors.

E. System Model

The block diagram of the digital communication system under consideration is shown below.

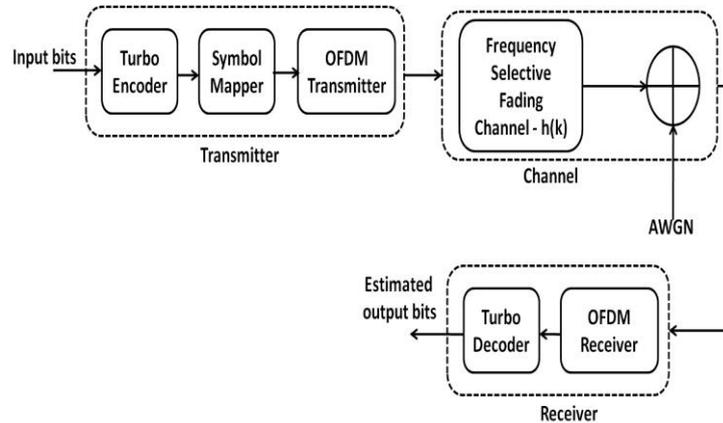


Fig 3. System Model

It consists of three blocks

1. The Turbo coded OFDM Transmitter
2. The Fading Channel
3. The Receiver

1. The transmitter considered here consists of a channel encoder which is turbo encoder and an OFDM transmitter. First the input bit stream is encoded using turbo encoder. Turbo encoder consists of parallel concatenation of two recursive systematic convolutional encoders with rate 1/2 each separated by interleaver.
2. First we consider the frequency selective fading channel. For a typical wireless environment, this frequency selective fading channel bandwidth is more than the coherence bandwidth. If the transmitted signal bandwidth is within coherence bandwidth, then we get a flat fading channel. So if we divide the total frequency selective fading channel bandwidth by the coherence bandwidth we get the required number of subchannels.
3. The receiver does exactly the opposite of the transmitter. The signal received from the channel is a noisy signal. This noisy signal is fed to the OFDM receiver. The OFDM receiver demodulates the symbols transmitted across various subchannels and makes them in a serial format to be fed to the turbo decoder. OFDM successfully removes the ISI by using the principle of multicarrier communication.

The analysis of multicarrier communication transmitter and receiver is given. The practical frequency selective fading channels are non ideal. Hence the frequency response of the channel is not at and the phase response is not linear. As a result of that, the channel is assumed to be an linear transversal filter which consists of finite memory. Because of this memory, ISI happens when the signal passed through the channel.

There are three methods to handle the ISI removal:

1. The complex time domain equalization.
2. The symbol sequence estimation by ML likelihood Viterbi decoding or the symbol by symbol detection using MAP algorithm.
3. The multicarrier communication especially OFDM.

We will focus in the multicarrier communication followed by OFDM. First we will derive the multicarrier communication transmitter and then multicarrier communication receiver.

The treatment of digital analysis of multicarrier communication is given. The matrix analysis can be found.

The multicarrier communication discussed above suffers from the following disadvantages:

1. The system uses N transmit filters and N matched filters. So for a large N, the system becomes too complex.
2. The use of non-overlapping frequency bands for various subchannels, the spectral efficiency decreases.
3. The assumption that the channel characteristic is ideal over each subchannel is only approximately true.

So to overcome these problems, discrete time implementation of multicarrier communication termed as OFDM was introduced which uses overlapping frequency bands for various subchannels along with digital implementation using

FFT and IFFT which invalidates the requirement of separate N transmit filters at transmitter and separate N receive filters at receiver.

IV. SIMULATION RESULTS

In this section, we provide computer simulation results to illustrate the performance of turbo coded signal (bpsk modulated) using OFDM and uncoded bpsk modulated signal using OFDM. The performance is demonstrated in terms of Bit Error Rate (BER) versus various SNRs. The SNR per bit is defined as,

$$\frac{SNR}{bit} \text{ in dB} = 10 \log_{10} \frac{E(|H_k|^2) E(|A_k|^2)}{N_o}$$

where $E(|H_k|^2)$ denotes the variance of the random fading coefficients and $E(|A_k|^2)$ is the average power of the symbol constellation. In the simulation we have considered the uniform power delay profile. So the term $E(|H_k|^2)$ is normalized to 1. $N_o/2$ is the two sided power spectral density of Gaussian Noise. Simulations were carried over 103 frames and each containing 1024 bits. Both the recursive systematic convolutional encoders in the turbo encoder have the generator values $(1; 27/31)_8$. So we need total number of memory elements equal to 4. Hence total 16 number of states are there. We have considered one OFDM symbol to be 1024 bpsk symbols. This means the number of parallel subchannels are 1024. We have used the cyclic prefix of length 16 on the OFDM simulation which is sufficient for the indoor wireless communication.

Case - 1 (Rate 1/2)

Turbo encoder of rate 1/2 will generate 2×1024 bits in one frame. For bpsk modulation, total number of symbols will also be 2×1024 . So we generate 2 OFDM symbols for each frame for 1/2 rate. The figure 5 demonstrates the BER of turbo coded (of rate 1/2) bpsk transmission using OFDM for various SNRs. At the decoder the turbo decoding is done for iterations 2, 4 and 8. Also in the graph the BER of uncoded bpsk transmission using OFDM is also drawn.

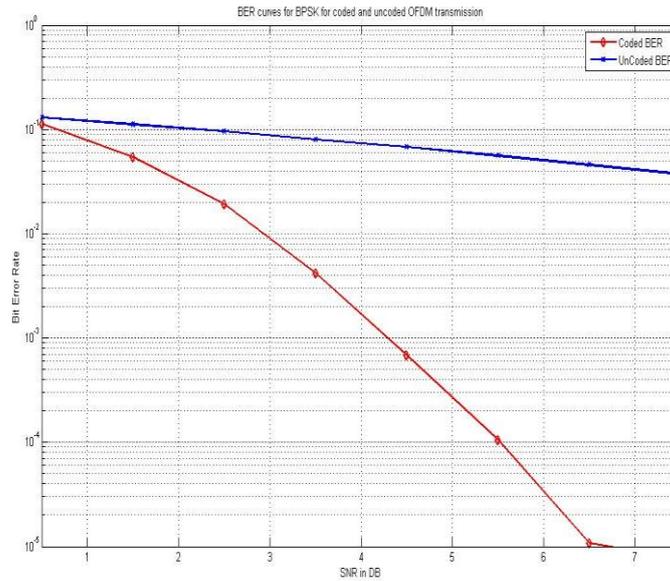


Fig 4. BER Curve for bpsk for coded & uncoded transmission

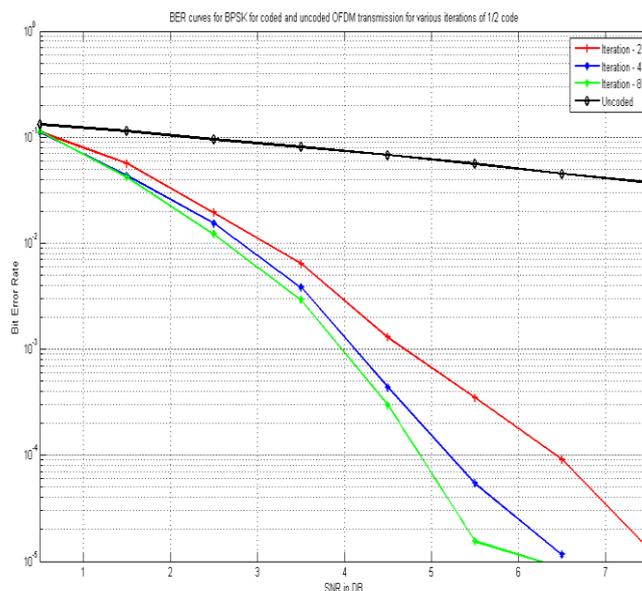


Fig 5. BER Curve for bpsk for OFDM transmission at various iteration at $\frac{1}{2}$ rate.

Hence from the figures we can see that the BER reduces significantly for the turbo coded OFDM in comparison to the uncoded bpsk at low SNRs. At high SNRs, the BER for turbo coded OFDM transmission is almost drops below 10^{-5} , which we can consider that BER is zero at high SNR.

V. CONCLUSION

In this paper we have analyzed the performance of turbo coded OFDM in a frequency selective fading channel along with AWGN. We have shown that in the non ideal channel that exhibit a non ideal frequency response (that can distort the transmitted signal significantly) along with AWGN, the combination of powerful turbo code and OFDM can mitigate ISI as well as the additive noise at low SNR (as low as SNR 3dB).

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