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Construction of Unknown Input Reduced Order Observer using Generalized Matrix Inverse and Application to Missile Autopilot

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Abstract:-In this paper a design procedure for construction of reduced order unknown input observer, for linear time invariant system subjected to unknown input is presented based on generalized matrix inverse. Das & Ghosal observer is extended and used for this purpose. A condition is proposed and to be satisfied to develop such an observer. Finally an illustrative numerical example of a class of guided missile (two-loop) autopilot (in pitch plane) with simulation results is presented.

Keywords:-Linear time invariant system (LTI), Reduced Order Observer, Unknown Input Observer (UIO), Generalized Matrix Inverse, Das & Ghosal Observer, State feedback.

I. INTRODUCTION

The problem of estimating state of an LTI system subject to both known and unknown inputs is of special importance in practice, since in many cases input of the system is partially accessible or disturbance present in the system itself. Under such circumstances conventional observers that assume all the inputs to the system are accessible cannot be used. The unknown input observer (UIO) was developed for the reconstruction of states of a system under the influence of unknown input. Thus UIO has received considerable attention from many researches (1-5). Shao-Kung Chang et al. [1] & [2] developed an UIO with the framework of General Structure Observer. A projection operator approach for state observation is used in [3]. Both [4] & [5] constructed UIO in line of the conventional Luenbarger reduced order observer where a special structure of measurement matrix is assumed. Elimination of unknown input from a part of the system by suitable co-ordinate transformation is adopted in [4], whereas [5] provides a design for observer gain using Generalized Matrix Inverse which leads to an UIO. In [6] G. Das and T.K. Ghosal proposed a construction method of reduced order Observer which possesses some certain advantages over the well known and well established Luenberger observer. A comparative study between the reduced order Luenbarger observer and the Das & Ghosal observer[6] is given in [7]. Previously this observer [6] has been implemented for various LTI systems without incorporating any disturbances. But if there are any unknown input or input disturbances then the observed states given by Das & Ghosal observer, may not converge to the system states. However in this paper, it will be established how Das & Ghosal Observer can be modified to incorporate the disturbances and can estimate the system states without any prior knowledge of the unknown input.
The following notation will be used in this paper. R denotes the field of real numbers; $m \times n$ will be used to represent the dimension of a matrix with m rows and n columns; A^g denotes the Moore-Penrose generalised inverse of the matrix A; A^T indicates the transpose of the matrix A , and I denotes the identity matrix of appropriate dimension.

A. Introduction to Generalized Matrix Inverse

If $A \in R^{m\times n}$ is a matrix and a matrix $A^g \in R^{n\times m}$ exists that satisfies the four conditions below

$$AA^g = (AA^g)^T \quad (1)$$
$$A^gA = (A^gA)^T \quad (2)$$
$$AA^gA = A \quad (3)$$
$$A^gAA^g = A^g \quad (4)$$

Then the matrix A^g is called the Moore-Penrose Generalized Matrix Inverse of A and is unique for each A.
If a system of linear equation is given by

$$Ax = y \quad (5)$$

where $A \in R^{m\times n}$ is a known matrix, $y \in R^{m\times 1}$ is a known vector and $x \in R^{n\times 1}$ is an unknown vector. Then eqn. (5) is consistent if and only if

$$AA^gy = y \quad (6)$$

And if eqn. (5) is consistent then its general solution is given by

$$x = A^gy + (I - A^gA)v \quad (7)$$

Where $v \in R^{n\times 1}$ denotes an arbitrary vector having elements as arbitrary function of time. Similarly for a system of linear equation given by

$$xB = y \quad (8)$$

If eqn(8) is dimensionally compatible and consistent, then consistency condition of the equation (8) is given by,

$$yB^gB = y \quad (9)$$

and the the general solution of the eqn (8) for x is given by

$$x = yB^g + v(I - BB^g) \text{ [9] (Graybill 1969 p.104).} \quad (10)$$

B. Lemma

If two matrixes $C \in R^{m\times n}$ and $L \in R^{n\times k}$ are such that the linear space spanned by the columns of L is equal to the linear space spanned by the columns of $(I - C^g C)$, then LL^g is equal to $(I - C^g C)$. This is symbolically written as

$$\{\ell(L) = \ell(I - C^g C)\} \Leftrightarrow \{LL^g = I - C^g C\} \quad (11)$$

Where the symbol $\ell(X)$, here, indicates the linear space spanned by the columns of any matrix X. (proposed by Das and Ghosal [6], 1981).

II. PROBLEM FORMULATION

Consider an LTI system described by

$$\dot{x} = Ax + Bu + E_d w,\ x_0 = x(0) \quad (12)$$

$$y = Cx \quad (13)$$

where $x\in R^{n\times 1}$ is the unknown state vector; $u \in R^{n\times 1}$ and $w \in R^{n\times 1}$ are known and unknown input vector respectively; and $y\in R^{m\times 1}$ denotes the output vector. The matrixes A, B, C and E_d are known and have appropriate dimension. We assume that the pair {A, C} is completely observable which implies the simultaneous solution for eqn.(12) and eqn.(13) for x is unique when x_0, u and y are given.

The Reduced order Das & Ghosal observer in presence of unknown input w can be derived in similar way [6] and is governed by following equations.

$$x = C^g y + Lh \quad (14)$$

$$\dot{h} = L^g ALh + L^g AC^g y + L^g Bu + L^g E_d w \quad (15)$$

$$\dot{y} = CALh + CAC^g y + CBu + CE_d w \quad (16)$$

$$\dot{\hat{q}} = (L^g AL - KCAL)\hat{q} + \{(L^g AL - KCAL)K + (L^g AC^g - KCAC^g)\}y + (L^g B - KCB)u + (L^g E_d - KCE_d)w \quad (17)$$

$$\hat{q} = h - Ky \quad (18)$$

$$\hat{x} = L\hat{q} + (C^g + LK)y \quad \text{(P.374 of [6])} \quad (19)$$

Where $h \in R^{n\times 1}$ denotes an arbitrary vector; $K \in R^{(n-r)\times m}$ is an arbitrary matrix denotes the observer gain. The matrix L has to be chosen on the basis of the linearly independent columns of $(I - C^g C)$ matrix so that it satisfies the Lemma $(I - C^g C = LL^g)$, proposed by Das & Ghosal [6].

A. Condition for Unknown Input Observer

The unknown input w is explicitly present in observer dynamic equation (refer to eqn(17). To nullify the effect of unknown input from the observer dynamics, arbitrary observer gain parameter K can be designed such that

$$L^g E_d - KCE_d = 0 \quad (20)$$

The equation (20) is consistent and can be solved for K, if and only if

$$L^g E_d - L^g E_d (CE_d)(CE_d)^g = 0 \quad (21)$$

The eqn (21), is the consistency condition of eqn(20) and also the condition for existence of the UIO. The general solution for K can be expressed as

$$K = (L^g E_d)(CE_d)^g + H(I - (CE_d)(CE_d)^g) \quad (22)$$

where H is any arbitrary matrix and its dimension is same as dimension of K.

Such design of the observer gain K leads the unknown input w effectively non-functioning in the observer dynamics. Hence, if the unknown input is present in system or not (w=0 or $w \neq 0$), the observer will be able to estimate the system states effectively without having any prior information about the unknown input.

Hence, the observer dynamics (eqn(17)) becomes,

$$\dot{\hat{q}} = (L^g AL - KCAL)\hat{q} + \{(L^g AL - KCAL)K + (L^g AC^g - KCAC^g)\}y + (L^g B - KCB)u \quad (23)$$

Now eqn(23) and eqn(19) with arbitrary initial condition and proper design of K using eqn (22), represents an reduced order observer for the original system given by eqn(12) and eqn(13). Here $\hat{x}$ represents the observed state.

B. State Feedback Design

Till now the observer has been designed to be used in open loop mode but to implement state feedback control, the observer has to be converted to closed loop mode by incorporating the control law given by

$$u = r - G\hat{x} \quad (24)$$

where G denotes the state feedback gain matrix.

The close loop system dynamics with observer can be given by

$$\dot{x} = Ax + Br - BG\hat{x} + E_d w \quad (25)$$

Now putting eqn (19) in eqn (24) one can obtain the following equation

$$\dot{x} = Ax + Br - BG(C^g + LK)y - BGL\hat{q} + E_d w \quad (26)$$

where $\hat{q}$ is given by the following dynamics.

$$\dot{\hat{q}} = (L^g AL - KCAL)\hat{q} + \{(L^g AL - KCAL)K + (L^g AC^g - KCAC^g)\}y + (L^g B - KCB)(r - G\hat{x}) \quad (27)$$

Now putting eqn (19) in eqn(27) we obtain

$$\dot{\hat{q}} = (L^g AL - KCAL - (L^g B - KCB)GL)\hat{q} + \{(L^g AL - KCAL)K + (L^g AC^g - KCAC^g) - (L^g B - KCB)G(C^g + LK)\}y + (L^g B - KCB) \quad (28)$$

III. NUMERICAL EXAMPLE

As an illustrative example, in this paper, flight path rate demand Missile (two-loop) autopilot has been considered [8]. The state space model of two loop autopilot can easily be developed from the corresponding transfer function model given in literature [8]. Let the state variables of the system be chosen as $x_1 = \dot{\gamma}\ (Flight\ path\ rate\ demand)$;
$x_2 = q\ (pitch\ rate)$;
$x_3 = \eta\ (elevator\ deflection)$;
$x_4 = \dot{\eta}\ (rate\ of\ chance\ of\ elevator\ deflection$
out of which $x_1\ and\ x_2$ have been considered as outputs.
The state space model of the two-loop autopilot has been provided below, along with the simulation results for the same.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} -\frac{1}{T_a} & \frac{1+\sigma^2 w_b^2}{T_a} & -\frac{K_b\sigma^2 w_b^2}{T_a} & -K_b\sigma^2 w_b^2 \\ -\frac{1+w_b^2 T_a^2}{T_a(1+\sigma^2 w_b^2)} & \frac{1}{T_a} & \frac{(T_a^2-\sigma^2)K_b w_b^2}{T_a(1+\sigma^2 w_b^2)} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -w_a^2 & -2\zeta_a w_a \end{bmatrix} \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ K_q w_a^2\end{bmatrix} u + E_d w$$

$$and\ \ y = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$$

The following numerical values have been taken for a class of guided missile for MATLAB simulation:
$T_a = 0.36\ sec;\ \sigma^2 = 0.00029\ \text{sec}^2;\ w_b = 11.77\frac{rad}{sec};\ \zeta_a = 0.6;\ K_b = -9.91\ per\ sec;\ K_p = 5.8;\ K_q = -0.6;$
$w_a = 180\frac{rad}{sec}$

The unknown input (w) has been taken to be of step input type and the coefficient matrix is chosen as $E_d = \begin{bmatrix}1\\ 1\\ 0\\ 0\end{bmatrix}$

According to the proposed design the observer gain K for the above system is found to be
$K = H\begin{bmatrix}0.5 & -0.5\\ -0.5 & 0.5\end{bmatrix}$ where H is any arbitrary matrix of proper dimension. We have considered H= $\begin{bmatrix}0.8 & 1\\ 1 & 0,8\end{bmatrix}$ and the state feedback gain G has been taken as [5.86 0.99 0 0].

A. Simulation responses

Responses of individual system states (x) and corresponding observed states ($\hat{x}$) are plotted using MATLAB and shown in figure 4.1 – 4.4. In all of the bellow four graphs the blue lines indicate the two-loop autopilot responses whereas the dashed green lines indicate the corresponding observer responses.The system initial condition, x_0 has been chosen arbitrarily as in practice it is inaccessible and considered as $[2,3,-4,500]^T$, and. The responses indicate reduced order UIO dynamics is starting from zero initial condition and converges with system dynamics.

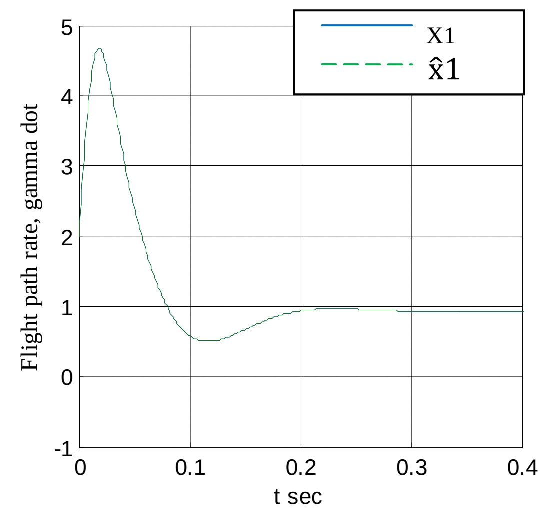

Figure 4.1 **Flight Path Rate**

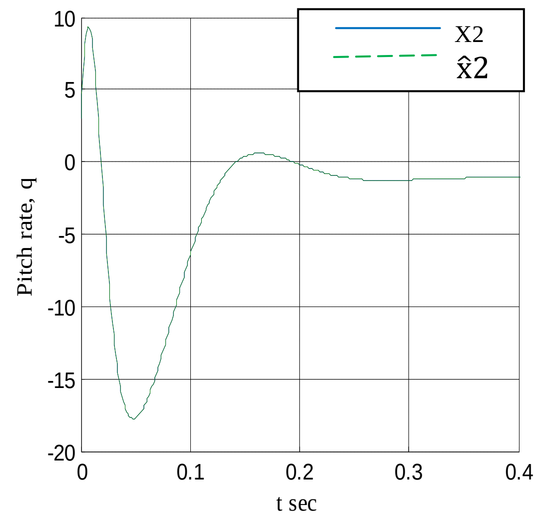

Figure 4.2 **Body Rate (Pitch Rate)**

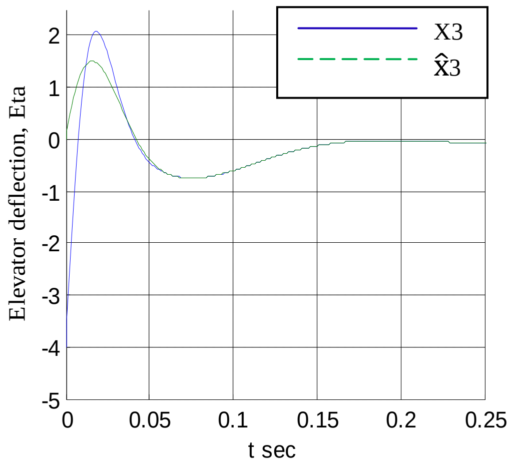

Figure 4.3: Elevator Deflection

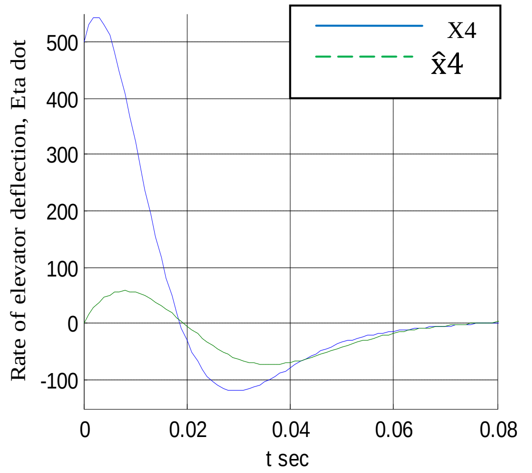

Figure 4.4: Rate of Elevator Deflection

IV. CONCLUSION

This paper considers reduced order UIO problem in light of Das & Ghosal observer based on Generalised Matrix Inverse. The unknown inputs are active in the system and are made perfectly ineffective in the observer dynamics by specific design of the observer gain parameter. The states of the system influenced by unknown input as additional input signal are reconstructed by observer without any knowledge of unknown input. To develop such an UIO a condition is set and solved using simple generalized matrix inverse. Simulation is carried out for a flight path rate demand Missile (two-loop) autopilot (pitch plane) model and results are presented to illustrate the proposed UIO.

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