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Translates of (Anti) Fuzzy Submodules

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Abstract:- As an abstraction of the geometric notion of translation, author in [8] has introduced two operators $T_{\alpha+}$ and $T_{\alpha-}$ called the fuzzy translation operators on the fuzzy set and studied their properties and also investigated the action of these operators on the (Anti) fuzzy subgroups of a group in [8] and (Anti) fuzzy subring and ideal of a ring in [9]. The notion of anti fuzzy submodule of a module has also been introduced by the author in [6, 7]. In this paper, we investigate the action of these two operators on (Anti) fuzzy submodules of a module and prove that they are invariant under these translations. But converse of this is not true. We also obtain the conditions, when the converse is also true.

Keywords:- Fuzzy submodule (FSM), Anti- fuzzy submodule (AFSM), Translate operator.

I. INTRODUCTION

The pioneering work of Zadeh on the fuzzy subset of a set in [10] and Rosenfeld on fuzzy subgroups of a group in [5] led to the fuzzification of algebraic structures. For example, Biswas in [1] gave the idea of Anti fuzzy subgroups. Palaniappan and Arjunan [4] defined the homomorphism of fuzzy and anti fuzzy ideals. Sharma in [6, 7] introduced the notion of Anti fuzzy modules. The author has defined the notion of translates of fuzzy sets in [8], introduced two operators $T_{\alpha+}$ and $T_{\alpha-}$ called α- up and α- down fuzzy operators and investigated the action of these operators on the fuzzy and anti fuzzy groups in [8] and on fuzzy and anti fuzzy subring and ideals in [9].

In this paper, we study the action of these fuzzy operators on the fuzzy and anti fuzzy submodules. Some related results have been obtained.

II. PRELIMINARIES

In this section, we list some basic concepts and well-known results on fuzzy sets theory. Throughout this paper, R will be a commutative ring with unity.

Definition (2.1)[6] A fuzzy set of a nonempty set M is a mapping $\mu : M \rightarrow [0,1]$. For $t \in [0,1]$, the sets $U(\mu , t) = \{ x \in M : \mu(x) \geq t \}$ and $L(\mu , t) = \{ x \in M : \mu(x) \leq t \}$ are respectively called the upper t-level cut and lower t-level cut of μ .

Then following results are easy to verify

Results (2.2) (i) $U(\mu , t) \cup L(\mu , t) = M$, for every $t \in [0,1]$ and
(ii) if $t_1 < t_2$, then $L(\mu , t_1) \subseteq L(\mu , t_2)$ and $U(\mu , t_2) \subseteq U(\mu , t_1)$
(iii) $L(\mu , t) = U(\mu^c , 1-t)$, for all $t \in [0,1]$, where $\mu^c = 1 - \mu$.

Definition (2.3)[3 ,6] A fuzzy set μ of an R-module M is called a fuzzy submodule (FSM) of M if for all x , y $\in$ M and $r \in R$, we have
(i) $\mu(0) = 1$
(ii) $\mu(x + y) \geq \min\{ \mu(x) , \mu(y)\}$
(iii) $\mu(rx) \geq \mu(x)$

Definition (2.4)[6] A fuzzy set μ of an R-module M is called an anti fuzzy submodule (AFSM) of M if for all x , y $\in$ M and $r \in R$, we have
(i) $\mu(0) = 0$
(ii) $\mu(x + y) \leq \max\{ \mu(x) , \mu(y) \}$
(iii) $\mu(rx) \leq \mu(x)$

Proposition (**2.5**) For any anti fuzzy submodule μ of an R-module M , where R is a commutative ring with unity , we have
(i) $\mu(0) \leq \mu(x)$, for $\forall$ $x \in M$
(ii) $\mu(-x) = \mu(x)$, for $\forall$ $x \in M$
Proof. (i) Since , $\mu(0) = \mu(0x) \leq \mu(x)$, for $\forall$ $x \in M$
(ii) Since , $\mu(-x) = \mu((-1) x) \leq \mu(x)$, for $\forall$ $x \in M$

Example (2.6)[6] An example of an anti fuzzy R-module M with R = Z, M = Z_6 , is

$$\mu(x)=\begin{cases}0 & if \ \ x=0\\ 1/4 & if \ \ x=2,4\\ 1/3 & if \ \ x=1,3,5\end{cases}$$

Theorem (2.7) [3] A fuzzy set μ of an R-module M is called a fuzzy submodule (FSM) of M if and only if $U(\mu , t)$ is a submodule of M, for all $t \in [0,1]$.

Theorem (2.8)[6] The fuzzy set μ of an R-module M is an AFSM of M if and only if μ^c is a FSM of M.
Proof. Let μ be AFSM of M , then for each $x , y \in M$ and $r \in R$, we have
$\mu^c(0) = 1 - \mu(0) = 1 – 0 = 1$
$\mu^c (x + y) = 1 - \mu(x + y)$
$\geq 1 – \max\{ \mu(x) , \mu(y)\}$
$= \min \{ 1 - \mu(x) , 1 - \mu(y)\}$
$= \min \{ \mu^c(x) , \mu^c(y)\}$ and
$\mu^c (rx) = 1 - \mu (rx) \geq 1 - \mu(x) = \mu^c (x)$

Hence μ^c is FSM of M. The converse is proved similarly.

Proposition (2.9)[6] A fuzzy set μ of an R-module M is an AFSM of M if and only if the lower t-level cut $L(\mu , t)$ is submodule of M, for all $t\in[\mu(0), 1]$.

Theorem (2.10)[6] Let μ be a fuzzy set of an R-module M. Then μ is an AFSM of M if and only if
(i) $\mu(0) = 0$
(ii) $\mu(rx + sy) \leq \max\{ \mu(x) , \mu(y) \}$, for all $r , s \in R$ and $x , y \in M$

Theorem (2.11) Let μ be a fuzzy set of an R-module M. Then μ is an FSM of M if and only if
(i) $\mu(0) = 1$
(ii) $\mu(rx + sy) \geq \min\{ \mu(x) , \mu(y) \}$, for all $r , s \in R$ and $x , y \in M$
Proof. Follows from Theorem (2.8) and Theorem (2.10)

III. TRANSLATES OF (ANTI) FUZZY SUBMODULES

In this section, we define two operators $T_{\alpha+}$ and $T_{\alpha-}$ on fuzzy sets and study their properties and investigate the action of these operators on fuzzy submodules and anti fuzzy submodules and prove that they are invariant under these translations. But converse of this is not true. We also obtain the conditions, when the converse is also true.
Definition(3.1) Let μ be fuzzy subset of an R-module M and let $\alpha \in [0, 1]$ and $x \in M$. We define $T_{\alpha +}(\mu)(x) = \text{Min}\{ \mu(x) + \alpha , 1\}$ and $T_{\alpha -}(\mu)(x) = \text{Max} \{ \mu(x) - \alpha , 0\}$
$T_{\alpha+}(\mu)$ and $T_{\alpha-}(\mu)$ are respectively called the α- up and α- down fuzzy operators of μ. We shall call $T_{\alpha +}$ and $T_{\alpha -}$ as the fuzzy operators.
Example (3.2) Let μ be a fuzzy set of an R-module M, where R = Z, $M = Z_6$, defined by

$$\mu(x)=\begin{cases}0 & if \ \ x=0\\ 1/4 & if \ \ x=2,4\\ 1/3 & if \ \ x=1,3,5\end{cases}$$

. Take $\alpha = 1/12$, we get

$$T_{\alpha+}(\mu)(x)=\begin{cases}1/12 & if \ \ x=0\\ 1/3 & if \ \ x=2,4\\ 5/12 & if \ \ x=1,3,5\end{cases} \quad \text{and} \quad T_{\alpha-}(\mu)(x)=\begin{cases}0 & if \ \ x=0\\ 1/6 & if \ \ x=2,4\\ 1/4 & if \ \ x=1,3,5\end{cases}$$

Remark (3.3) If μ is fuzzy set of an R-module M , then both $T_{\alpha +}(\mu)$ and $T_{\alpha -}(\mu)$ are fuzzy sets of an R-module M.
Results (3.4): The following results can be easily verified from definition

(i) $T_{0+}(\mu) = T_{0 -}(\mu) = \mu$ (ii) $T_{1 +}(\mu) = \tilde{1}$ (iii) $T_{1 -}(\mu) = \tilde{0}$
Proposition (3.5) For any fuzzy set μ of an R-module M and $\alpha \in [0,1]$, we have
(i) $T_{\alpha +}(\mu^c) = (T_{\alpha -}(\mu))^c$ (ii) $T_{\alpha -} (\mu^c) = (T_{\alpha +}(\mu))^c$
Proof. Let $x \in M$ be any element and and $\alpha \in [0,1]$. Then
(i) $T_{\alpha +}(\mu^c)(x) = \text{Min}\{ \mu^c(x) + \alpha , 1\}= \text{Min}\{ 1- \mu(x) + \alpha , 1\}= 1 – \text{Max}\{\mu(x) - \alpha , 0\}$
$= 1 - (T_{\alpha -}(\mu))(x) = (T_{\alpha -}(\mu))^c(x)$
Thus $T_{\alpha +}(\mu^c)(x) = (T_{\alpha -}(\mu))^c(x)$ holds for all $x \in M$ and so $T_{\alpha +}(\mu^c) = (T_{\alpha -}(\mu))^c$
(ii) $T_{\alpha -} (\mu^c)(x) = \text{Max} \{ \mu^c(x) - \alpha , 0\}= \text{Max} \{ 1- \mu(x) - \alpha , 0\} = 1 – \text{Min}\{\mu(x) + \alpha, 1\}$
$= 1 - (T_{\alpha +}(\mu))(x) = (T_{\alpha +}(\mu))^c(x)$

Thus $T_{\alpha -}(\mu^c)(x) = (T_{\alpha +}(\mu))^c(x)$ holds for all $x \in M$ and so $T_{\alpha -}(\mu^c) = (T_{\alpha +}(\mu))^c$

Theorem(3.6) If μ is FSM of an R-module M , then $T_{\alpha+}(\mu)$ and $T_{\alpha-}(\mu)$ are also FSM of M

Proof. Let μ be FSM of an R-module M , then for any $x \in M$, we have

$T_{\alpha +}(\mu)(x) = \text{Min}\{ \mu(x) + \alpha , 1\}$ and $T_{\alpha -}(\mu)(x) = \text{Max}\{ \mu(x) - \alpha , 0 \}$

Now $T_{\alpha +}(\mu)(0) = \text{Min}\{ \mu(0) + \alpha , 1\} = \text{Min}\{ 1 + \alpha , 1\} = 1$ ………………….(1)

Further, let $x, y \in M$ and $r , s \in R$ be any element , then we have

$T_{\alpha +}(\mu)(rx + sy) = \text{Min}\{ \mu(rx + sy) + \alpha , 1\}$

$\geq \text{Min}\{ \text{Min}\{\mu(x) , \mu(y)\} + \alpha , 1 \}$

$= \text{Min}\{ \text{Min}\{ \mu(x) + \alpha , 1\} , \text{Min} \{\mu(y) + \alpha , 1\}\}$

$= \text{Min} \{ T_{\alpha +}(\mu)(x) , T_{\alpha +}(\mu)(y)\}$

Thus $T_{\alpha +}(\mu)(rx + sy) \geq \text{Min} \{ T_{\alpha +}(\mu)(x) , T_{\alpha +}(\mu)(y)\}$ ………..…………………(2)

From (1) , (2) and Theorem (2.11) , we get $T_{\alpha +}(\mu)$ is FSM of M

Similarly, we can show that $T_{\alpha -}(\mu)$ is also FSM of M .

Remark (3.7) The converse of above theorem (3.6) also holds, for if $T_{\alpha +}(\mu)$ and $T_{\alpha -}(\mu)$ are FSM of M , for all $\alpha \in [0,1]$, then on taking $\alpha = 0$, we get $T_{0 +}(\mu) = \mu$ and $T_{0 -}(\mu) = \mu$ and hence μ is a FSM of M.

Remark(3.8) If $T_{\alpha +}(\mu)$ or $T_{\alpha -}(\mu)$ is FSM of an R-module M, for a particular value of $\alpha \in [0,1]$, then it cannot be deduced that μ is FSM of M .

Example(3.9) Consider the $\mathbb{Z}$- module $\mathbb{Z}_4$, where $\mathbb{Z}_4 = \{ 0 , 1 , 2 , 3 \}$ and define the fuzzy set μ of $\mathbb{Z}_4$ as $\mu = \{ < 0 , 0.7 > , < 1, 0.4 >, < 2 , 0.6 > , < 3 , 0.5 > \}$. Take $\alpha = 0.8$, then we get $T_{\alpha +}(\mu) = \{ < 0 , 1 > , < 1, 1 >, < 2 , 1 > , < 3 , 1 > \} = 1$

which is a FSM of $\mathbb{Z}_4$, but μ is not FSM of $\mathbb{Z}_4$ as $U(\mu ,0.5) = \{ 0, 2, 3\}$ is not a submodule of $\mathbb{Z}$-module $\mathbb{Z}_4$.

Theorem (3.10) If μ is an AFSM of a R- module M, then $T_{\alpha +}(\mu)$ and $T_{\alpha -}(\mu)$ are also AFSM of M, for all $\alpha \in [0,1]$.

Proof. Since μ is an AFSR of a R-module M $\Rightarrow \mu^c$ is a FSM of M [By Proposition (2.8)]

Then by Theorem (3.6) $T_{\alpha +}(\mu^c)$ and $T_{\alpha -}(\mu^c)$ are FSM of M.

So $(T_{\alpha -}(\mu))^c$ and $(T_{\alpha +}(\mu))^c$ are FSM of M [By Proposition (3.5)]

$\Rightarrow$ $T_{\alpha +}(\mu)$ and $T_{\alpha -}(\mu)$ are AFSM of M [By Proposition (2.8)]

Proposition (3.11) Let μ be any FSM of an R-module M, then the set $M_\mu = \{ x \in M ; \mu(x) = \mu(0)\}$ is a submodule of M.

Proof. Clearly, $M_\mu \neq \varnothing$ for $0 \in M_\mu$. So let $x , y \in M_\mu$ and $r , s \in R$, then we have

$\mu(rx + sy) \geq \text{Min}\{\mu(x) , \mu(y)\} = \text{Min}\{\mu(0) , \mu(0)\} = \mu(0)$ and

But $\mu(0) \geq \mu(rx + sy)$ always. Therefore $\mu_A(rx + sy) = \mu_A(0)$. Thus $rx + sy \in M_\mu$. Hence M_μ is a submodule of M .

Corollary (3.12) Let μ be any AFSM of an R-module M , then the set $M_\mu = \{ x \in M ; \mu(x) = \mu(0)\}$ is a submodule of M.

Proposition (3.13) Let μ be a fuzzy subset of an R-module M with $\mu(0) = 1$ such that $T_{\alpha +}(\mu)$ be a FSM of M, for some $\alpha \in [0,1]$ with $\alpha < 1- p$, then μ is FSM of M, where $p= \text{Max}\{ \mu(x) : x \in M - M_\mu\}$

Proof. Let $T_{\alpha +}(\mu)$ be FSM of an R-module M , for some $\alpha \in [0,1]$ with $\alpha < 1- p$

Therefore , $T_{\alpha +}(\mu)(0) = 1$

Let $x, y \in M$, $r , s \in R$ with $\alpha < 1- p$, then we have

$T_{\alpha +}(\mu)(rx +sy) \geq \text{Min}\{ T_{\alpha +}(\mu)(x) , T_{\alpha +}(\mu)(y)\}$ ………………………………………….......(*)

Case (i) When $T_{\alpha +}(\mu)(x) = 1$ and $T_{\alpha +}(\mu)(y) = 1$

$\text{Min} \{ \mu(x) + \alpha , 1 \} = 1$ and $\text{Min} \{ \mu(y) + \alpha , 1 \} = 1$

$\Rightarrow$ $\mu(x) + \alpha \geq 1$ and $\mu(y) + \alpha \geq 1$

$\Rightarrow$ $\mu(x) \geq 1 - \alpha$ and $\mu(y) \geq 1 - \alpha$ …..…………………………… ……………….(1)

Since $\alpha < 1- p$ $\Rightarrow$ $p < 1 - \alpha$

$\Rightarrow$ $\text{Max} \{ \mu(x) : x \in M - M_\mu \} < 1 - \alpha$ and $\text{Max} \{ \mu(y) : y \in M - M_\mu \} < 1 - \alpha$

Therefore from (1) , we get $x \in M_\mu$ and $y \in M_\mu$, but M_μ is a submodule of M

Therefore , $rx + sy \in M_\mu$ and so $\mu(rx + sy) = \mu(0)$

Thus $\mu(rx + sy) = \mu(0) \geq \mu(x)$ or $\mu(y)$

$\Rightarrow$ $\mu (rx + sy) \geq \text{Min}\{ \mu(x) , \mu(y)\}$

$\Rightarrow \mu$ is FSM of M in this case

Case (ii) When $T_{\alpha +}(\mu)(x) = 1$ and $T_{\alpha +}(\mu)(y) < 1$

As in case (i) , we get $x \in M_\mu$ and so $\mu(x) = \mu(0)$

From (*) , we get

$T_{\alpha +}(\mu)(rx +sy) \geq \text{Min}\{ T_{\alpha +}(\mu)(x), T_{\alpha +}(\mu)(y)\} = \text{Min} \{1, T_{\alpha +}(\mu)(y)\}= T_{\alpha +}(\mu)(y)$

$\text{Min} \{ \mu(rx + sy) + \alpha , 1 \} \geq \text{Min} \{ \mu(y) + \alpha , 1 \}$

$\Rightarrow$ $\mu(rx + sy) + \alpha \geq \mu(y) + \alpha$ i.e. $\mu(rx + sy) \geq \mu(y)$

Also, $T_{\alpha +}(\mu)(rx +sy) \geq \text{Min}\{ T_{\alpha +}(\mu)(x), T_{\alpha +}(\mu)(y)\}$

$\Rightarrow$ $T_{\alpha +}(\mu)(rx +sy) \geq T_{\alpha +}(\mu)(x)$ or $T_{\alpha +}(\mu)(rx +sy) \geq T_{\alpha +}(\mu)(y)$

$\Rightarrow$ $\text{Min} \{ \mu(rx + sy) + \alpha , 1 \} \geq \text{Min} \{ \mu(x) + \alpha , 1 \}$ or

$\text{Min} \{ \mu(rx + sy) + \alpha , 1 \} \geq \text{Min} \{ \mu(y) + \alpha , 1\}$

$\Rightarrow$ $\mu(rx + sy) \geq \mu(x)$ or $\mu(rx + sy) \geq \mu(y)$

$\Rightarrow \mu(rx + sy) \geq \text{Min} \{ \mu(x) , \mu(y)\}$.

Thus μ is FSM of M in this case also
Case (iii) When $T_{\alpha+}(\mu)(x) < 1$ and $T_{\alpha+}(\mu)(y) < 1$
Min { $\mu(x) + \alpha, 1$ } < 1 and Min { $\mu(y) + \alpha, 1$ } < 1
$\Rightarrow$ $\mu(x) + \alpha < 1$ and $\mu(y) + \alpha < 1$
Therefore from (*) , we have
$T_{\alpha+}(\mu)(rx+sy) \geq \text{Min}\{ T_{\alpha+}(\mu)(x), T_{\alpha+}(\mu)(y)\}$
$\Rightarrow \text{Min}\{ \mu(rx+sy) + \alpha, 1\} \geq \text{Min}\{ \text{Min}\{ \mu(x) + \alpha, 1\}, \text{Min}\{ \mu(y) + \alpha, 1\}\}$
$= \text{Min}\{ \mu(x) + \alpha, \mu(y) + \alpha\} = \text{Min}\{ \mu(x), \mu(y)\} + \alpha$
Therefore $\mu(rx+sy) + \alpha \geq \text{Min}\{ \mu(x), \mu(y)\} + \alpha$
$\Rightarrow \mu(rx+sy) \geq \text{Min}\{ \mu(x), \mu(y)\}$.
Thus μ is FSM of M in this case also. Thus in all cases, we see that μ is a FSM of M .
Proposition (3.14) Let μ be a fuzzy subset of an R-module M with $\mu(0) = 1$, such that $T_{\alpha-}(\mu)$ be a FSM of M, for some $\alpha\in[0,1]$ with $\alpha < q$, then μ is a FSM of M, where $q = \text{Min}\{ \mu_A(x) : x \in M - M_\mu\}$
Proof. Similar to the proof of Proposition (3.13)
Proposition (3.15) Let μ be a fuzzy subset of an R-module M with $\mu(0) = 0$ such that $T_{\alpha-}(\mu)$ be an AFSM of M, for some $\alpha \in [0,1]$ with $\alpha < q$, then μ is an AFSM of M, where $q = \text{Min}\{ \mu(x) : x \in M - M_\mu \}$
Proof. Let $T_{\alpha-}(\mu)$ be an AFSM of R-module M , for some $\alpha \in [0,1]$ with $\alpha < q$
Therefore , $T_{\alpha-}(\mu)(0) = 0$
Let $x, y \in M$, $r, s \in R$ with $\alpha < q$, then we have
$T_{\alpha-}(\mu)(rx+sy) \leq \text{Max}\{ T_{\alpha-}(\mu)(x), T_{\alpha-}(\mu)(y)\}$……………………………………….......(*)
Case (i) When $T_{\alpha+}(\mu)(x) = 0$ and $T_{\alpha+}(\mu)(y) = 0$
Max { $\mu(x) - \alpha, 0$ } = 0 and Max { $\mu(y) - \alpha, 0$ } = 0
$\Rightarrow$ $\mu(x) - \alpha \leq 0$ and $\mu(y) - \alpha \leq 0$
$\Rightarrow$ $\mu(x) \leq \alpha$ and $\mu(y) \leq \alpha$ …………………………………… ………………….(1)
Since $\alpha < q \Rightarrow q > \alpha$
$\Rightarrow \text{Min}\{ \mu(x) : x \in M - M_\mu\} > \alpha$ and $\text{Min}\{ \mu(y) : y \in M - M_\mu\} > \alpha$
Therefore from (1) , we get $x \in M_\mu$ and $y \in M_\mu$, but M_μ is a submodule of M
Therefore , $rx + sy \in M_\mu$ and so $\mu(rx + sy) = \mu(0)$
Thus $\mu(rx + sy) = \mu(0) \leq \mu(x)$ or $\mu(y)$
$\Rightarrow$ $\mu(rx + sy) \leq \text{Max}\{ \mu(x), \mu(y)\}$
$\Rightarrow \mu$ is an AFSM of M in this case
Case (ii) When $T_{\alpha-}(\mu)(x) = 0$ and $T_{\alpha-}(\mu)(y) > 0$
As in case (i) , we get $x \in M_\mu$ and so $\mu(x) = \mu(0)$. Therefore from (*) , we get
$T_{\alpha-}(\mu)(rx+sy) \leq \text{Max}\{ T_{\alpha-}(\mu)(x), T_{\alpha-}(\mu)(y)\} = \text{Max}\{0, T_{\alpha-}(\mu)(y)\} = T_{\alpha-}(\mu)(y)$
$\text{Max}\{ \mu(rx+sy) - \alpha, 0\} \leq \text{Max}\{ \mu(y) - \alpha, 0\}$
$\Rightarrow \mu(rx+sy) - \alpha \leq \mu(y) - \alpha$ i.e. $\mu(rx+sy) \leq \mu(y)$
Also, $T_{\alpha-}(\mu)(rx+sy) \leq \text{Max}\{ T_{\alpha-}(\mu)(x), T_{\alpha-}(\mu)(y)\}$
$\Rightarrow$ $T_{\alpha-}(\mu)(rx+sy) \leq T_{\alpha-}(\mu)(x)$ or $T_{\alpha-}(\mu)(rx+sy) \leq T_{\alpha-}(\mu)(y)$
$\Rightarrow$ $\text{Max}\{ \mu(rx+sy) - \alpha, 0\} \leq \text{Max}\{ \mu(x) - \alpha, 0\}$ or
$\text{Max}\{ \mu(rx+sy) - \alpha, 0\} \leq \text{Max}\{ \mu(y) - \alpha, 0\}$
$\Rightarrow \mu(rx+sy) \leq \mu(x)$ or $\mu(rx+sy) \leq \mu(y)$
$\Rightarrow \mu(rx+sy) \leq \text{Max}\{ \mu(x), \mu(y)\}$.
Thus μ is an AFSM of M in this case also
Case (iii) When $T_{\alpha-}(\mu)(x) > 0$ and $T_{\alpha-}(\mu)(y) > 0$
Max { $\mu(x) - \alpha, 0$ } > 0 and Max { $\mu(y) - \alpha, 0$ } > 0
$\Rightarrow$ $\mu(x) - \alpha > 0$ and $\mu(y) - \alpha > 0$
Therefore from (*) , we have
$T_{\alpha-}(\mu)(rx+sy) \leq \text{Max}\{ T_{\alpha-}(\mu)(x), T_{\alpha-}(\mu)(y)\}$
$\Rightarrow \text{Max}\{ \mu(rx+sy) - \alpha, 0\} \leq \text{Max}\{ \text{Max}\{ \mu(x) - \alpha, 0\}, \text{Max}\{ \mu(y) - \alpha, 0\}\}$
$= \text{Max}\{ \mu(x) - \alpha, \mu(y) - \alpha\} = \text{Max}\{ \mu(x), \mu(y)\} - \alpha$
Therefore $\mu(rx+sy) - \alpha \leq \text{Max}\{ \mu(x), \mu(y)\} - \alpha$
$\Rightarrow \mu(rx+sy) \leq \text{Max}\{ \mu(x), \mu(y)\}$.
Thus μ is an AFSM of M in this case also. Thus in all cases we see that μ is an AFSM of M.

Proposition (3.16) Let μ be a fuzzy subset of an R-module M with $\mu(0) = 0$ such that $T_{\alpha+}(\mu)$ be an AFSM of M, for some $\alpha \in [0,1]$ with $\alpha < 1 - p$, then μ is an AFSM of M, where $p = \text{Max}\{ \mu(x) : x \in M - M_\mu \}$
Proof. Similar to the proof of Proposition (3.15)

IV. CONCLUSIONS

A study on the structure of the collection of (Anti) fuzzy submodules will be a prosperous venture. We have made a humble beginning in this direction. We have studied the action of some operators on certain sub-lattices of the complete lattice of all fuzzy submodules of a fixed R-module. Lattice properties of the collection of (Anti) fuzzy modules is a problem to be tackled further.

REFERENCES

[1]. R. Biswas, " Fuzzy subgroups and anti fuzzy subgroups", Fuzzy Sets and Systems 35 (1990), 512-517

[2]. K.H. Kim , Y.B . Jun and Y.H. Yon, "On anti fuzzy ideals in near rings", Iranian Journal of Fuzzy Systems , Vol. 2 , no.2, (2005) , pp. 71-80.

[3]. R. Kumar , S.K. Bhambri and Pritaba Kumar , " Fuzzy submodules: Some Analogues and Deviations", Fuzzy Sets and Systems, 70 (1995), 125-130

[4]. N. Palaniappan and K. Arjunan, The homomorphism, anti-homomorphism of a fuzzy and anti-fuzzy ideals, Varahmihir Journal of Mathematical Sciences, Vol.6 No.1 (2006), 181-188.

[5]. A. Rosenfeld , " Fuzzy Groups", Journal of mathematical analysis and application", 35(1971), 512-517

[6]. P.K. Sharma, " Anti fuzzy submodules of a module", Advances in Fuzzy Sets and Systems , Vol.12, No.1 , 2012, pp. 49-57

[7]. P.K. Sharma, "Lattices of Anti fuzzy submodules", International Journal of Fuzzy Mathematics and Systems, Vol. 2, No. 3 (2012), pp. 345-350

[8]. P.K. Sharma, "Translates of Anti fuzzy subgroups ", International Journal of Applied Mathematics and Applications (July-Dec 2012) , Vol, 4 , No. 2 , 175-182

[9]. P.K. Sharma, "Translates of Anti Fuzzy Subrings And Ideals", International Journal of Applied Mathematics and Physics" (July-Dec 2012) (Accepted)

[10]. L.A. Zadeh, "Fuzzy sets" , Information and control, 8 (1965), 338-353