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A Mathematical Model on the Two Phase Renal Systolic Blood Flow in Renal Arterioles with Special Reference to Kidney Infection (UTI)

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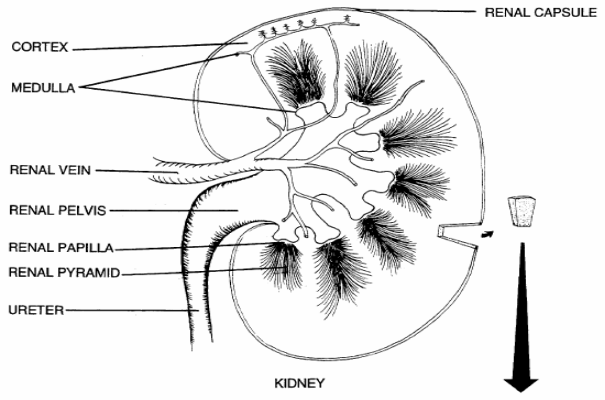
Abstract:- In the present paper we have formulated the renal Systolic blood flow in arteries. Keeping in view the nature of renal circulatory system in human body. The viscosity increases in the arterioles due to formation of roulex along axis by red blood cells, as we know the arteries are remote from heart and proximate to the kidney. P.N. Pandey and V. Upadhyay have considered the blood flow has two phased , one of which is that of red blood cells and other is Plasma .They have also applied the Herschel Bulkley non –Newtonian Model in bio-fluid mechanical set-up .We have collected a clinical data in case of Kidney Infection(UTI) for Hematocrit v/s Blood Pressure. The graphical presentation for particular parametric value is much closed to the clinical observation. The overall presentation is in tensorial form and solution technique adapted is analytical as well as numerical. The role of Hematocrit is explicit in the determination of blood pressure in case of renal disease –Kidney Infection (UTI).

Keywords:- UTI- Urinary Tract infection, E. Coli- Escherichia Coli, rouleaux-structure formed by RBC in tough situation, non Newtonian-fluid, renal.

I. INTRODUCTION (DESCRIPTION OF BIO-PHYSICAL PROBLEM)

The kidneys are part of a regulating system that performs a variety of functions. They ensure that electrolytes, such as sodium, potassium, calcium, phosphorous, and other chemicals are in balance. Kidneys produce several essential hormones and change vitamin D into its active form. The kidneys also help regulate the pH of body fluids and filter and excrete waste products. Kidneys filter blood plasma and produce urine, by which waste products are eliminated from the body. When kidney function is impaired, these waste products cannot be eliminated from the body. This causes a toxic condition known as uremic poisoning.

Each person is normally born with two kidneys, which are located in the back of the body, on each side of the spine and positioned under the rib cage. According to Moëll,H.(1956), It is important to note that the left kidney is usually slightly larger than the right, which results in increased volume and dynamic flow values. According to Columbia University (2003). These organs are truly remarkable, as a single kidney can function at 20% of its capacity while still providing all required renal filtration and regulation. Kidneys are normally about five- an-a-half inches long, three inches wide, two inches thick, and usually weigh ten to twelve ounces. It is interesting to note that the left kidney is usually larger than the right. [4]



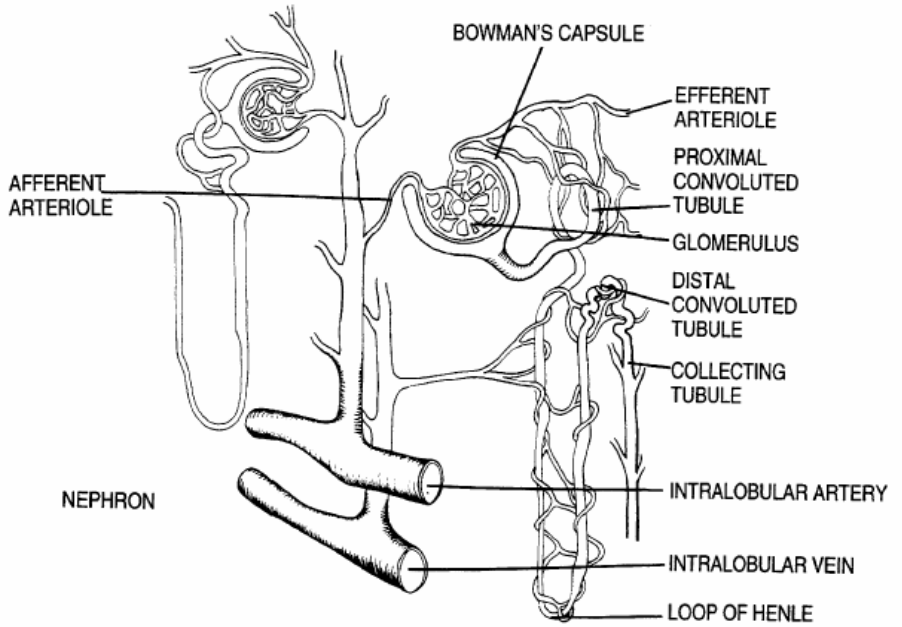


Fig 1.: Kidney anatomy and enhanced view of nephron. [5]

Blood is a complex fluid consisting of particulate curpuscles suspended in a non-Newtonian fluid. The particulate solids are red blood cells (RBCs), white blood cells (WBCs) and platelets. The fluid is plasma, which itself is a complex mixture of proteins and other intergradient in an aqueous base. 50% of the plasma and 45% of the blood cells in a whole blood and approximately 98% of RBCs in 45% of blood cells and there are a few parts (approximately 2%) of the other cells. Which are ignorable, so one phase of the blood is plasma and 2^{nd} phase of the blood is RBCs. Two phase renal blood flow is a study of measuring the blood pressure if hemoglobin known. The percentage of volume covered by blood cells in the whole blood is called hematocrit.

This work is important for human health. There are several researches, who examined the blood flow in the artery and veins. This work will focus on two phase renal blood flow in arterioles with special reference to kidney infection (UTI). A lot of work is available, but P.N.Pandey and V. Upadhyay (2001) discussed a some phenomena in two phase blood flow gave an idea on the two phase renal blood flow in arterioles with a renal disease kidney infection (UTI). The work of P.N. Pandey and V.Upadhyay in whole circulatory system but this work will focus on renal circulatory system, and renal circulatory system is a sub system of whole circulatory system. In this work, applied the Herschel Bulkley non-Newtonian model.

We present an improvement on the previous work in the field and this is discussed separately below. The ultimate use of this model is to predict normal reference levels of two phase blood flow in arterioles for individual patients under going kidney infection (UTI) disease. According to Dan Med Bull kidney infection belongs to the family of infection of the Urinary system called Urinary tract infection (UTI's) (2011). A UTI is a serious health problem that affects millions of people each year. UTIs are the second most common type of infection in the body. Dan Med Bull studied on bacterial characteristics of importance for recurrent Urinary tract infection caused by Escherichia Coli (2011).[10]

II. BASIC BIO-FLUID EQUATION FOR TWO PHASE BLOOD FLOW

Let us the problem of blood flow in renal circulatory system is different from the problems in cylindrical tube and select generalized three dimensional orthogonal curvilinear coordinate system. Briefly described as E^3 called as Euclidean space. According to mishra the biophysical laws thus expressed fully hold good in any co-ordinate system which is a compulsion for the truthfulness of the laws (1990).[17]

According to Sherman I.W. and Sherman V.G. Blood is mixed fluid. Mainly there are two phases in blood. The first phase is plasma, while the other phase is that of blood cells are enclosed with a semi-permeable membrance whose density is greater than that of plasma. These blood cells are uniformly distributed in plasma. Thus, blood can be considered as a homogeneous mixture of two phases (1989).[23]

(2.1) Equation of Continuity for two phase blood flow-

According to Singh P. and Upadhyay K.S. The flow of blood is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells [12]. Let the volume portion covered by blood cells in unit volume be X , this X is replaced by H/100, where H is the Hematocrit the volume

percentage of blood cells .Then the volume portion covered by the plasma will be 1-X . If the mass ratio of blood cells to plasma is r then clearly

$$r = \frac{X\rho_c}{(1-X)\rho_p} \qquad (2.1)$$

where ρ_c and ρ_p are densities of blood cells and blood plasma respectively .Usually this mass ratio is not a constant , even then this may be supposed to constant in present context (1986) [13]

The both phase of blood, I. e. blood cells and plasma move with the common velocity .Campbell and Pitcher has presented a model for two phase of blood separately (1958). Hence equation of continuity for two phases according to the principle of conservation of mass defined by J.N and Gupta R.C. as follow

$$\frac{\partial(X\rho_c)}{\partial t} + (X\rho_c v^i)_{,i} = 0$$

(2.2)

And $$\frac{\partial(1-X)\rho_p}{\partial z} + ((1-X)\rho_p v^i)_{,i} = 0$$

(2.3)

Where, v is the common velocity of two phase blood cells and plasma.

If we define the uniform density of the blood ρ_m as follow $\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p}$ [20]

(2.4)

Then equation (2.2) and (2.3) can be combined together as follow,

$$\frac{\partial\rho_m}{\partial t} + (\rho_m v^i)_{,i} = 0$$

(2.5)

(2.2) Equation of Motion for two phase blood flow-

According to Ruch, T.C. and H.D. The hydro dynamical pressure p between the two phases of blood can be supposed to be uniform because the both phases i.e. blood cells and plasma are always in equilibrium state in blood (1973) [21]. Taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum, we get the equation of motion for the phase of blood cells as follows:

$$X\rho_c \frac{\partial v^i}{\partial t} + (X\rho_c v^j)v^i_{,j} = -Xp_{,j}g^{ij} + X\eta_c(g^{jk}v^i_{,k})_{,j}$$

(2.6)

Similarly, taking the viscosity coefficient of plasma to be. The equation of motion for plasma will be as follows:

$$(1-X)\rho_p \frac{\partial v^i}{\partial t} + \left\{(1-X)\rho_p v^i\right\} v^i_{,j} = -(1-X)p_{,j}g^{ij} + (1-X)\eta_c\left(g^{jk}v^i_{,k}\right)_{,j}$$

(2.7)

Now adding equation (2.6) and (2.7) and using relation (2.4), the equation of motion for blood flow with the both phases will be as follows:

$$\rho_m \frac{\partial v^i}{\partial t} + \left(\rho_m v^j\right)v^i_{,j} = -p_{,j} + \eta_m\left(g^{jk}v^i_{,k}\right)_{,j}$$

(2.8)

Where $\eta_m = X\eta_c + (1-X)\eta_p$ is the viscosity coefficient of blood as a mixture of two phases.

III. MATHEMATICAL MODELING

As the velocity of Blood flow decreases, the viscosity of blood increases. The velocity of blood decreases successively. The Herschel Bulkley law holds good on the two phase blood flow through veins arterioles, veinules and whose constitutive equation is as follows:

$T' = \eta_m e^n + T_p\left(T' \geq T_p\right)$ and $e = 0(T' < T_p)$ where, T_p is the yield stress.

When strain rate $e = 0\left(T' < T_p\right)$ a core region is formed which flows just like a plug. Let the radius of the plug be r_p . The stress acting on the surface of plug will be T_p . Equating the forces acting on the plug, we get,

$$P\pi r_p^2 = T_p 2\pi r_p \Rightarrow r_p = 2\frac{T_p}{P}$$

(3.1)

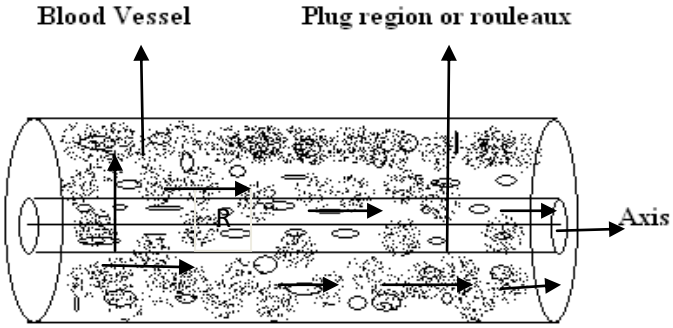


Fig .2 : Herschel Bulkley blood flow

The Constitutive equation for test part of the blood vessel is
$T' = \eta_m e^n + T_p \text{ or } T' - T_p = \eta_m e^n = T_e$ Where, T_e = effective stress , Whose generalized form will be as follows

$T^{ij} = -Pg^{ij} + T_e^{ij}$ where, $T_e^{ij} = \eta_m \left(e^{ij}\right)^n$ While $e^{ij} = g^{jk} V_k^i$

Where, the symbols have their usual meanings.
Now we describe the basic equations for Herschel Bulkley blood flow as follows:
(3.1) Equation of Continuity-

$$\frac{1}{\sqrt{g\sqrt{(gV^i)_{,i}}}} = 0$$

(3.2) Equation of Motion-

$$\rho_m \frac{\partial v^i}{\partial t} + \rho_m V^j V^i_{,j} = -T^{ij}_{e,j} \qquad (3.2)$$

Where all the symbols have their usual meanings, since, the blood vessels are cylindrical; the above governing equations have to be transformed into cylindrical co-ordinates. As we know earlier: $X^1 = r, X^2 = \theta, X^3 = Z$ Matrix of metric tensor in cylindrical co-ordinates is $\left[g_{ij}\right]$ and matrix of conjugate metric tensor is $\left[g^{ij}\right]$ whereas the chritoffel's symbols of 2nd kind are as follows:

$\left\{ \begin{matrix} 1 \\ 2\ 2 \end{matrix} \right\} = -r, \left\{ \begin{matrix} 1 \\ 2\ 2 \end{matrix} \right\} = \left\{ \begin{matrix} 1 \\ 2\ 2 \end{matrix} \right\} = \frac{1}{r}$ Remaining others are zero.

The governing tensorial equations can be transformed into cylindrical forms which are follows: the equation of Continuity-: $\frac{\partial v}{\partial z} = 0$

The equation of Motion-

r-component: $-\frac{\partial p}{\partial z} = 0, \theta - \text{component} : 0 = 0$

z-component: $0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r}\left[r\left(\frac{\partial v_z}{\partial r} \right)^n \right]$

Here, this fact has been taken in view that the blood flow is axially symmetric in arteries concerned, i.e. $v_\theta = 0$ and v_r and v_z and p do not depend upon θ.

We get $v_z = v(r)$ and $dp = p(z)$ and $0 = -\frac{dp}{dz} + \frac{\eta_m}{r}\left[r\left(\frac{dv}{dz} \right)^n \right]$

(3.3)

Since, pressure gradient $-\frac{dp}{dz} = P$ $r\left(\frac{dv}{dz} \right)^n = -\frac{pr^2}{2\eta_m} + A$, we apply boundary condition: at r=0. $V = V_0$

then $\Rightarrow -\frac{dv}{dr} = \left(\frac{pr}{2\eta_m} \right)^{\frac{1}{n}}$ Replace r from $r - r_p$

$$-\frac{dv}{dr} = \left(\frac{\frac{1}{2}pr - \frac{1}{2}pr_p}{\eta_m} \right)^{\frac{1}{n}} \qquad \Rightarrow \frac{dv}{dr} = -\left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} (r - r_p)^{\frac{1}{n}}$$

(3.4)

Integrating above equation (12) under the no slip boundary condition: v=0 at r = R so as to get:

$$V = \left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} \frac{n}{n+1}\left[(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1} \right]$$

(3.5)

This is the formula for velocity of blood flow in arterioles, veinules and veins.

Putting $r = r_p$ to get the velocity V_p of plug flow as follows:

$$V_p = \frac{n}{n+1}\left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1}$$

(3.6)

Where the value of r_p is taken from (2.7)

IV. RESULT (BIO-PHYSICAL INTERPRETATION)

Observations: Hematocrit Vs Blood pressure from an authorized Jabalpur Hospital & Research Centre by **Dr. Anil Jain**

Patient Name: - Mr. Shyambihari Singh

Diagnosis: - Hepatitis/ARF/Anemia/UTI

Date	HB(Hemoglobin)	B.P.(Systolic blood Pressure in mm hg)	Hematocrit	B.P(in Pascal))
19/7/11	7.4	110	22.2	0.82507
20/7/11	7.0	120	21.0	0.90008
21/7/11	6.7	100	20.1	0.75006
22/7/11	6.9	120	20.7	0.90008
25/9/11	6.9	130	20.7	0.97508
29/7/11	7.5	120	22.5	0.90008

According to Berkow, Robert The hematocrit (expressed as percentage points) is normally about three times the hemoglobin concentration (reported as grams per deciliter). [16]
The flow flux of two phased blood flow in arterioles, veinules and veins is

$$Q=\int_0^{r_p}2\pi r V_p dr+\int_{r_p}^{R}2\pi r V dr$$

$$=\int_0^{r_p}2\pi r\frac{n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}\left(R-r_p\right)^{\frac{1}{n}+1}dr+\int_0^{r_p}2\pi r\frac{n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}\left[\left(R-r_p\right)^{\frac{1}{n}+1}-\left(r-r_p\right)^{\frac{1}{n}+1}\right]dr$$

$$=\frac{\pi n}{\left(n+1\right)}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}R^{\frac{1}{n}+3}\left[\frac{r_p^2}{R^2}\left(1-\frac{r_p^2}{R}\right)^{\frac{1}{n}+1}+\left(1+\frac{r_p}{R}\right)\left(1-\frac{r_p}{R}\right)^{\frac{1}{n}+2}-\frac{2\left(1-\frac{r_p}{R}\right)^{\frac{1}{n}+2}}{\left(\frac{1}{n}+2\right)}+\frac{2\left(1-\frac{r_p}{R}\right)^{\frac{1}{n}+3}}{\left(\frac{1}{n}+2\right)\left(\frac{1}{n}+3\right)}\right]$$

(4.1)

Q=900 ml. /min R =1, $r_p=\frac{1}{3}$ [8] According to Gustafson, Daniel R. (1980)

η_p = 0.0015 (Pascal-sec.) [19] According to Glenn Elert (2010)

η_m = 0.035 (Pascal-sec.) H = 21.2

By using relation $\eta_m=\eta_c X+\eta_p\left(1-X\right)$ where, $X=\frac{H}{100}$ We get,

η_c = 0.1595 and again using same above relation we get, η_m = 0.0015 + 0.00158 H
Substituting the values of, r_p and R in equation (4.1), we get

$$Q=\pi\left(\frac{2P}{6\eta_m}\right)^{\frac{1}{n}}\left(\frac{2}{27}\right)\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$

$$\frac{Q\times 27}{2\pi}=\left(\frac{P}{3\eta_m}\right)^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right] \qquad (4.2)$$

Let $A=\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}$

$$\frac{P}{3\eta_m}=\left(\frac{27Q}{2\pi A}\right)^n \quad \Rightarrow P=\left(\frac{27Q}{2\pi A}\right)^n\times 3\eta_m$$

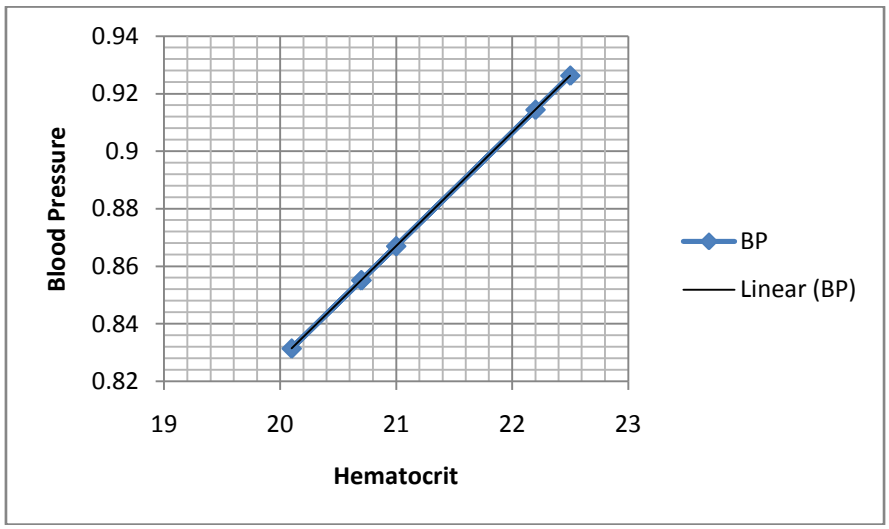
$$\int dp=\int_0^1\left(\frac{27Q}{2\pi A}\right)^n\times 3\eta_m dz \quad \Rightarrow \; p=\left(\frac{27Q}{2\pi A}\right)^n\times 3\eta_m$$

Substituting the values of Q, P, η_m in equation (4.2), and solve by Numerical method we get, **n = -0.86091** again from equation (4.2)

$\Rightarrow 3\eta_m=\left(\frac{2\pi A}{27Q}\right)^n p$ and substitute the value of η_m , η_c ,X and n we get

H (Hematocrit)	20.1	20.7	21.0	22.2	22.5
p(Blood Pressure)	0.83145	0.85515	0.867	0.9144	0.92625

$\Rightarrow p=0.0395H+0.0375$



Graph 1

V. CONCLUSION

A simple survey of the graph (1) between blood pressure and hematocrit in Urinary Tract Infection patient shows that when hematocrit increased then Blood pressure also increased. That is Hematocrit proportional to Systolic blood pressure.

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Remark

If this would have been possible to get blood Pressure on the particular tissue (Kidney) then the relation between blood pressure and hemoglobin has been measured more accurately.

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