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Matrix Games with Trapezoidal Fuzzy Pay Offs

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Abstract:- An approach for solving a matrix game whose pay off elements are symmetric trapezoidal fuzzy numbers is proposed. P.Grzegorzewski's theorem[1]has been used to transform symmetric trapezoidal fuzzy numbers to interval fuzzy number such that length of all intervals are different.Acceptability index has been defined and ordering of fuzzy numbers has been established.

Key words:- Interval Number,Symmetric trapezoidal fuzzy number,Acceptability index.

I. INTRODUCTION

Fuzziness in matrix games can appear in so many ways but two classes of fuzziness seem to be very natural. These two classes of fuzzy matrix games are referred as matrix games with fuzzy goal and matrix games with fuzzy pay off[2]. It has been considered by Nayak and Pal[3] the pay off as interval number and symmetric triangular fuzzy number. Here the trapezoidal numbers are considered where the pay off elements are fuzzy but the parameters considered are crisp and P.Grzgyski's[1]concept has been used to approximate the interval fuzzy numbers corresponding to a trapezoidal fuzzy numbers. N.Mahdavi -Amiri and S.H Nasseri[4] have solved L.P Problem with trapezoidal fuzzy variable and applied linear ranking function to order trapezoidal fuzzy number. Here it has been considered that the entries of pay off matrix as symmetric trapezoidal fuzzy number and interval number. Our inspiration behind defining the symmetric trapezoidal fuzzy number is Adrian Ban[5]but here the work has been made in some different manner. A set of real lines is defined and then a trapezoidal membership function. Here the universe of fuzzy pay off matrix with elements as trapezoidal fuzzy numbers is also defined. Here 3 parameters are used to define a trapezoidal number , out of which one parameter is still reducible but we have used it for the sake of solving problems with reduced constraints. A solution method is proposed for both type of problems with saddle point and without saddle point. Numerical example has also been provided. We have defined acceptability index to compare two intervals. The main feature of our work is that it is simple and at the same time it represents a better model for real life problems.
The paper is organised as : In section II, we have discussed about interval numbers.In section III, trapezoidal fuzzy numbers have been defined and discussed. In section IV, trapezoidal fuzzy numbers have been transformed into interval numbers and in section V, basic interval arithmetic have been given. In section VI to IX, ranking order of the interval numbers are given, pay-off matrix and pure strategy have been defined, game without saddle point has been explained. In section X, numerical example is given and in XI, conclusion has been drawn.

II. INTERVAL NUMBERS

An interval number proposed by Moore [6], is considered as an extension of a real number and as a real subset of the real line $\Re$.

Definition 1 An interval number $\overline{A}$ is a closed interval defined by

$$\overline{A} = [a_L, a_R] = \{x \in \Re : a_L \le x \le a_R;\ \Re \textit{ be the set of all real numbers}\}. \quad (1)$$

The numbers a_L, a_R are called respectively the lower and upper limits of the interval $\overline{A}$. An interval number $\overline{A}$ alternatively represented in mean-width or center-radius form as

$$\overline{A} = \langle m_1(\overline{A}), m_2(\overline{A}) w(\overline{A})\rangle = \{x \in \Re : m_1(\overline{A}) - \frac{w(\overline{A})}{2} \le x \le m_2(\overline{A}) + \frac{w(\overline{A})}{2}\},$$

where $m_1(\overline{A}) = a_L + \frac{1}{2}w$, $m_2(\overline{A}) = a_R - \frac{1}{2}w$ and $w(\overline{A}) = \frac{1}{3}(a_R - a_L)$ are respectively the mean points and one-third width of the interval $\overline{A}$. Actually, each real number can be regarded as an interval, such as, $\forall x \in \Re$ an interval can be written as $[x, x]$, which has zero length.

The set of all interval numbers in $\Re$ is denoted by $I(\Re)$.

III. TRAPEZOIDAL FUZZY NUMBERS

Our aim is to define a matrix with fuzzy symmetric trapezoidal pay-off in space of matrices. Nayak and Pal[3] has defined symmetric triangular fuzzy number over a real line . Here we define the symmetric trapezoidal fuzzy number over the real line L in some different manner as

$$L = \{a : x \in \Re \backslash a = a_0 + (3x-1)\alpha, x \in [0,1]\}$$

Here at $x = 0, a = a_0 - \alpha$

At $x = 1/3, a = a_0$

At $x = 2/3, a = a_0 + \alpha = a_1$ (say)

at $x = 1, a = a_0 + 2\alpha = a_1 + \alpha$

Where a_0 is the 1st arithmetic mean value(for $x = 1/3$), a_1 is the 2nd arithmetic mean (for x $= 2/3$) , $\underline{a}$ is the lower bound (for $x = 0$) , $\overline{a}$ is the upper bound(for $x = 1$). Similarly for a fuzzy matrix with symmetric trapezoidal membership function, we define its universe as the following line in the space of matrices

$$D = \{A : x \in \Re \, such \; that A = A_0 + (3x-1)\alpha, x \in [0,1]\}$$

Where $A_0 = [a_{ij}]_{m\times n}$ is a matrix with real elements and $= [\alpha]_{m\times n}$ is a constant matrix. Now mean values of the matrix is now given at $x = 1/3$ and $x = 2/3,$ the lower bound $\underline{A}$ and upper bounds $\overline{A}$ are respectively given at $x = 0$ and $x = 1.$ To define a trapezoidal fuzzy number four numbers are required and we define upper and lower bounds to use it as a mean to define a trapezoidal fuzzy number with four numbers, the other two being A_0 and A_1 . Using the universe D and four matrices $\underline{A}, A_0, A_1$ and $\overline{A}$ as defined above we define the fuzzy matrix $A = (\underline{A}, A_0, A_1, \overline{A})$ with symmetric trapezoidal membership function as a fuzzy matrix with the following membership function

$$\prod_A(x) = \begin{cases} 0; & When \; x \le 0 \\ 3x; & When \; 0 \le x \le \frac{1}{3} \\ 1; & When \; \frac{1}{3} \le x \le \frac{2}{3} \\ 3(1-x); & When \; \frac{2}{3} \le x \le 1 \\ 0; & When \; x \ge 1 \end{cases}$$

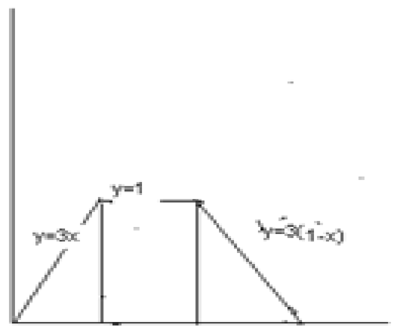

Figure 1: Trapezoidal fuzzy number

This is exactly the membership function of symmetric trapezoidal fuzzy number $a = (0,1/3,2/3,1) = (1/3,1/3,1/3)$. Thus the membership function of the matrix A is identified with the membership function of the fuzzy number a. The advantage of a notation of fuzzy number is that it is simple and at the same time it is a better model to represent the pay off values in practical situations. Here one observation should be made that we use three numbers to represent a symmetric fuzzy number though four numbers are required to represent a fuzzy number. Here we see that the symmetric trapezoidal fuzzy number arise from the fact that all the values of x are in arithmetic progression. To define the symmetric fuzzy number only two numbers a_0 and α are sufficient but we also use a_1 for the sake of computation.

IV. TRANSFORMATION OF TRAPEZOIDAL FUZZY NUMBERS

The membership function of any trapezoidal fuzzy number A can be described[9] in the following manner:-

$$\mu_A(x) = \begin{cases} 0; & if\ x < a_1 \\ \dfrac{x-a_1}{a_2-a_1}; & if\ a_1 \le x < a_2 \\ 1; & if\ a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3}; & if\ a_3 < x \le a_4 \\ 0; & if\ a_4 < x \end{cases}$$

Now let us consider the α cuts as

$$[A_L(\alpha), A_U(\alpha)] = [a_1 + (a_2-a_1)\alpha, a_4 - (a_4-a_3)\alpha]\forall \alpha \in [0,1]$$

And $c_d(A) = c_{0.5}(A) = [\dfrac{a_1+a_2}{2}, \dfrac{a_3+a_4}{2}]$

But here we consider

$a_{ij} = a_2, b_{ij} = a_3, a_1 = a_{ij} - t_{ij}$ and $a_4 = b_{ij} + t_{ij}$

Hence, $c_d(A) = [a_{ij} - \dfrac{1}{2}t_{ij}, b_{ij} + \dfrac{1}{2}t_{ij}]$

$= [a - \dfrac{1}{2}t, b + \dfrac{1}{2}t]$

$= \{x \in \Re; m_1(\overline{A}) - \dfrac{1}{2}\omega \le x \le m_2(\overline{A}) + \dfrac{1}{2}\omega\}$ Where $\omega = \dfrac{1}{3}(a_R - a_L)$, $m_1 = a_L + \dfrac{1}{2}\omega$ and $m_2 = a_R - \dfrac{1}{2}\omega$ if the interval is considered to be $\overline{A} = [a_L, a_R]$. Then we may also write the interval $\overline{A}$ as $\overline{A} = \langle m_1, m_2, \omega\rangle$.

Here $m_1(\overline{A})$ and $m_2(\overline{A})$ are respectively 1st mean and 2nd mean and $\omega(\overline{A})$ is the one-third of the interval length of the interval $\overline{A}$ The set of all interval numbers in $\Re$ is denoted by $I(\Re)$.

V. BASIC INTERVAL ARITHMETIC

Let $\overline{A} = [a_1, a_2] = \langle m_1, \overline{m}_1, \omega_1\rangle$ and $\overline{B} = [b_1, b_2] = \langle m_2, \overline{m}_2, \omega_2\rangle \in I(\Re)$, then

$$\overline{A} + \overline{B} = [a_1 + b_1, a_2 + b_2];\ \overline{A} + \overline{B} = \langle m_1 + m_2, \overline{m}_1 + \overline{m}_2, \omega_1 + \omega_2\rangle. \quad (2)$$

The multiplication of an interval by a real number $c \ne 0$ is defined as

$$c\overline{A} = [ca_1, ca_2];\quad if\ c \ge 0\ and\ c\overline{A} = [ca_2, ca_1];\quad if\ c < 0.$$

$$c\overline{A} = c\langle m_1, \overline{m}_1, \omega_1\rangle = \langle cm_1, c\overline{m}_1, |c|\omega_1\rangle. \quad (3)$$

The difference of these two interval numbers is

$$\overline{A} - \overline{B} = [a_1 - b_2, a_2 - b_1]. \quad (4)$$

The product of these two distinct interval numbers is given by

$$\overline{A}\overline{B} = \left[\min\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}, \max\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}\right]. \quad (5)$$

The division of these two interval numbers with $0 \notin B$ is given by

$$\overline{A}/\overline{B} = \left[\min\left\{\frac{a_1}{b_1}, \frac{a_1}{b_2}, \frac{a_2}{b_1}, \frac{a_2}{b_2}\right\}, \max\left\{\frac{a_1}{b_1}, \frac{a_1}{b_2}, \frac{a_2}{b_1}, \frac{a_2}{b_2}\right\}\right]. \qquad (6)$$

VI. COMPARISON BETWEEN INTERVAL NUMBERS

Let $\overline{A} = [a_L, a_R] = \langle m_1, \overline{m}_1, \omega_1 \rangle, \overline{B} = [b_L, b_R] = \langle m_2, \overline{m}_2, w_2 \rangle$ be two interval numbers within $I(\Re)$. These two intervals may be one of the following types:

1. Two intervals are completely disjoint (non-overlapping).
2. Two intervals are nested, (fully overlapping).
3. Intervals are partially overlapping.

A brief comparison on different interval orders is given in [7, 3].

Case 1 (Disjoint subintervals): Moore [6] defined transitive order relations over intervals as :

______[______]__________________[_______]___
a L   a R   b L   b R

Figure 2: Disjoint subintervals

$\overline{A}$ is strictly less than $\overline{B}$ if and only if $a_R < b_L$ and this is denoted by $\overline{A} < \overline{B}$. This relation is an extension of `<' on the real line. This relation seems to be strict order relation that $\overline{A}$ is smaller than $\overline{B}$.

Case 2 (Nested subintervals) : Let $\overline{A} = [a_L, a_R], \overline{B} = [b_L, b_R] \in I(\Re)$

______[______[__________________]_______]___
a L   b L   b R   a R

Figure 3: Nested subintervals

be such that $a_L \le b_L < b_R \le a_R$. Then $\overline{B}$ is contained in $\overline{A}$ and it is denoted by $\overline{B} \subseteq \overline{A}$ which is the extension of the concept of the set inclusion [6]. The extension of the set inclusion here only describes the condition that, $\overline{B}$ is nested in $\overline{A}$ but it can not order $\overline{A}$ and $\overline{B}$ in terms of value.

Let $\overline{A}$ and $\overline{B}$ be two cost intervals and minimum cost interval is to be chosen. If the decision maker (DM) is optimistic then he/she will prefer the interval with maximum width along with the risk of more uncertainty giving less importance. Again, if the DM is pessimistic then he/she will pay more attention on more uncertainty i.e., on the right end points of the intervals and will choose the interval with minimum width. The case will be reverse when $\overline{A}$ and $\overline{B}$ represent profit intervals. In this case, we define the ranking order of $\overline{A}$ and $\overline{B}$ as

$$\overline{A} \vee \overline{B} = \begin{cases} \overline{A}, & \textit{if the player is optimistic} \\ \overline{B}, & \textit{if the player is pessimistic.} \end{cases}$$

The notation $\overline{A} \vee \overline{B}$ represents the maximum among the interval numbers $\overline{A}$ and $\overline{B}$. Similarly

$$\overline{A} \wedge \overline{B} = \begin{cases} \overline{B}, & \textit{if the player is optimistic} \\ \overline{A}, & \textit{if the player is pessimistic.} \end{cases}$$

The notation $\overline{A} \wedge \overline{B}$ represents the minimum among the interval numbers $\overline{A}$ and $\overline{B}$.

Case 3 (Partially overlapping subintervals) : The above mentioned order relations introduced by Moore [6] can not explain ranking between two overlapping closed intervals.

______[______[__________________]_______]___
a L   b L   a R   b R

Figure 4: Partially overlapping subintervals

We define an acceptability index to compare and order any two interval numbers on the real line in terms of value as in [7, 3], which are used throughout the paper.

Definition 2 For $m_1 + \overline{m}_1 \leq m_2 + \overline{m}_2$ and $\omega_1 + \omega_2 \neq 0$, the value judgement index or acceptability index (AI) of the premise $\overline{A} \prec \overline{B}$ is defined by

$$AI(\overline{A} \prec \overline{B}) = \frac{(m_2 + \overline{m}_2) - (m_1 + \overline{m}_1)}{2(\omega_1 + \omega_2)},$$

which is the value judgement by which $\overline{A}$ is inferior to $\overline{B}$ ($\overline{B}$ is superior to $\overline{A}$) in terms of value. Here 'inferior to', 'superior to', are analogous to 'less than', 'greater than', respectively.

The above observations can be put into a compact form as follows

$$\overline{A} \vee \overline{B} = \begin{cases} \overline{B}, & if \ AI(\overline{A} \prec \overline{B}) > 0 \\ \overline{A}, & if \ AI(\overline{A} \prec \overline{B}) = 0 \ and \ \omega_1 < \omega_2 \ and \ DM \ is \ pessimistic \\ \overline{B}, & if \ AI(\overline{A} \prec \overline{B}) = 0 \ and \ \omega_1 < \omega_2 \ and \ DM \ is \ optimistic. \end{cases}$$

Similarly, in the following we have given another function *max* w hich determines the maximum between two interval numbers.

Function *max* $(\overline{A}, \overline{B})$
if $\overline{A} = \overline{B}$ then maximum $= \overline{A}$;
else
if $\overline{A} = \langle m_1, \overline{m}_1, \omega_1 \rangle$ and $\overline{B} = \langle m_2, \overline{m}_2, \omega_2 \rangle$ are not
non-dominating then
$((\overline{A} \prec \overline{B})$ if or $(\overline{A} \prec_P \overline{B}))$ then
maximum $= \overline{B}$;
else
maximum $= \overline{A}$;
endif;
else
if $(\omega_1 > \omega_2)$ then
if the decision maker is optimistic maximum $= \overline{B}$;
if the decision maker is pessimistic maximum $= \overline{A}$;
endif;
endif;
endif;
return(maximum);
End Function.

Similarly, if $m_1 + \overline{m}_1 \geq m_2 + \overline{m}_2$ and $\omega_1 \geq \omega_2$, then there also exist a strict preference relation between $\overline{A}$ and $\overline{B}$. Thus similar observations can be put into a compact form as

$$\overline{A} \wedge \overline{B} = \begin{cases} \overline{B}, & if \ AI(\overline{B} \prec \overline{A}) > 0 \\ \overline{A}, & if \ AI(\overline{B} \prec \overline{A}) = 0 \ and \ \omega_1 > \omega_2 \ and \ DM \ is \ pessimistic \\ \overline{B}, & if \ AI(\overline{B} \prec \overline{A}) = 0 \ and \ \omega_1 > \omega_2 \ and \ DM \ is \ optimistic. \end{cases}$$

The following function computes the minimum between two interval numbers.

Function *min* $(\overline{A}, \overline{B})$
if $\overline{A} = \overline{B}$ then minimum $= \overline{A}$;
else
if $\overline{A} = \langle m_1, \overline{m}_1, \omega_1 \rangle$ and $\overline{B} = \langle m_2, \overline{m}_2, \omega_2 \rangle$ are not
non-dominating then
$((\overline{A} \prec \overline{B})$ if or $(\overline{A} \prec_P \overline{B}))$ then

```
    minimum = ;
  else
    minimum = B̄ ;
  endif;
 else
  if (ω1 < ω2) then
    if the decision maker is optimistic minimum = B̄ ;
    if the decision maker is pessimistic minimum = Ā ;
  endif;
 endif;
endif;
return(minimum);
 End Function.
```

This matrix has been considered over symmetric trapezoidal numbers. It can be made equivalent to interval fuzzy numbers. Here a_{ij} and b_{ij} are respectively the first and second mean. Here we define the first mean as ' comparison mean' and second mean as the ' promotional mean'. Here a particular pattern is considered where first mean is an independent variable for all entries and the second mean and interval length are obtained as follows.

We find promotional mean b_{ij}' of another entry by adding an amount $\dfrac{(a_{ij}'-a_{11})b_{11}}{a_{11}}$ and the interval length as $t_{ij}'=b_{ij}'-a_{ij}'$. Any interval can be made as

$$\left[a_{ij}'-\frac{t_{ij}'}{2}, b_{ij}'+\frac{t_{ij}'}{2}\right].$$

Here we assume that $a_{11}\neq 0.$ Thus we get the following theorem.

Theorem 1 *If* $\left[a_{ij}-\dfrac{t_{ij}}{2}, b_{ij}+\dfrac{t_{ij}}{2}\right]$ *and* $\left[b_{ij}'-\dfrac{t_{ij}'}{2},\ b_{ij}'+\dfrac{t_{ij}'}{2}\right]$ *are two intervals with* $a_{ij}'>a_{ij}$ *, then the intervals are partial overlapping or non overlapping when* $b_{11}\leq 3a_{11}$ *and overlapping when* $b_{11}>3a_{11}$.

Proof: From the definition of promotional mean, we get,

$$b_{ij}'=b_{11}+\frac{(a_{ij}'-a_{11})b_{11}}{a_{11}}=\frac{a_{ij}'b_{11}}{a_{11}}$$

$$\Rightarrow b_{ij}'-b_{ij}=\frac{(a_{ij}'-a_{ij})b_{11}}{a_{11}}>0$$

$$\Rightarrow b_{ij}'>b_{ij}. \qquad (7)$$

$$t_{ij}'-t_{ij}=\left(b_{ij}'-a_{ij}'\right)-\left(b_{ij}-a_{ij}\right)$$

$$=\frac{a_{ij}'(b_{11}-a_{11})}{a_{11}}-\frac{a_{ij}(b_{11}-a_{11})}{a_{11}}$$

$$=\frac{(a_{ij}'-a_{ij})(b_{11}-a_{11})}{a_{11}}>0;\ \ as\ a_{ij}'>a_{ij} \qquad (8)$$

$$\Rightarrow t_{ij}'>t_{ij}. \qquad (9)$$

From (7) and (9) we get,

$$b_{ij}'+\frac{t_{ij}'}{2} > b_{ij}+\frac{t_{ij}}{2}. \qquad (10)$$

If $b_{11} \le 3a_{11}$, then from (8) we get

$$\frac{t_{ij}'-t_{ij}}{2} \le a_{ij}'-a_{ij}; \quad i.e., \ a_{ij}-\frac{t_{ij}}{2} \le a_{ij}'-\frac{t_{ij}'}{2}. \qquad (11)$$

If $b_{11} > 3a_{11}$, then,

$$\frac{t_{ij}'-t_{ij}}{2} > a_{ij}'-a_{ij}; \quad i.e., \ a_{ij}'-\frac{t_{ij}'}{2} < a_{ij}-\frac{t_{ij}}{2}. \qquad (12)$$

(10) and (11) gives that the intervals are partial overlapping or non overlapping and (10) and (12) gives that the intervals are nested intervals. Hence proved.

VII. PAY-OFF MATRIX

If the player A has m strategies available to him and the player B has n strategies available to him, then the pay-off for various strategies is represented by $m \times n$ pay-off matrix whose entries are trapezoidal fuzzy numbers as

$$\begin{array}{cc} & \begin{array}{ccc} B_1 \ .... B_2 & \cdots & B_n \end{array} \\ \overline{A} = \begin{array}{c} A_1 \\ A_2 \\ \cdots \\ A_m \end{array} & \begin{pmatrix} \langle a_{11}, b_{11}, t_{11} \rangle & \langle a_{12}, b_{12}, t_{12} \rangle & \cdots \langle a_{1n}, b_{1n}, t_{1n} \rangle \\ \langle a_{21}, b_{21}, t_{21} \rangle & \langle a_{22}, b_{22}, t_{22} \rangle & \cdots \langle a_{2n}, b_{2n}, t_{2n} \rangle \\ \cdots & \cdots & \cdots \\ \langle a_{m1}, b_{m1}, t_{m1} \rangle & \langle a_{m2}, b_{m2}, t_{12} \rangle & \cdots \langle a_{mn}, b_{mn}, t_{mn} \rangle \end{pmatrix}. \end{array}$$

This matrix has been considered over symmetric trapezoidal fuzzy numbers. It can be made equivalent to interval fuzzy numbers. Here a_{ij} and b_{ij}, are respectively the first mean and second mean. Here we define the first mean as comparison mean and the second mean as the promotional mean. Here we consider that the first mean as an independent variable for all entries and the second mean and interval length are obtained as follows

We find promotional mean b_{ij}' of another entry by adding an amount $\frac{(a_{ij}'-a_{11})b_{11}}{a_{11}}$ and the interval length as $t_{ij}' = b_{ij}'-a_{ij}'$. Any interval can be made as $[a_{ij}'-\frac{t_{ij}}{2}', b_{ij}'+\frac{t_{ij}}{2}']$. Here we assume that $a_{11} \neq 0$.

Here it is assumed that when the player A plays a strategy A_i and B plays the strategy B_j it results in a pay-off to the player A which is symmetric trapezoidal fuzzy number. We make the following assumptions:

1. the player is only interested in the behavior of the matrix $\overline{A}$
2. The player assumes that $\overline{A}$ is a fuzzy matrix with symmetric trapezoidal fuzzy number $\langle a_{ij}, b_{ij}, t_{ij} \rangle$.

Under these assumptions we obtain the solution with a fuzzy decision matrix. In this section we consider the case of a two-person game in which although the players have perfectly defined their sets of strategies they have however some lack of precision on the knowledge of the associated pay-offs.
Before we reach at the solution we must define some of the basic terminologies for symmetric trapezoidal fuzzy numbers.

VIII. PURE STRATEGY

Pure strategy is a decision making rule in which one particular course of action is selected. For fuzzy games the min-max principle is described by Nishizaki [2]. The course of the fuzzy game is determined by the desire of A to maximize his gain and that of restrict his loss to a minimum. Now for interval game,

$$max-min = \bigvee_i \{\bigwedge_j \{\langle a_{ij}, b_{ij}, t_{ij}\rangle\}\}; \quad min-max = \bigwedge_j \{\bigvee_i \{\langle a_{ij}, b_{ij}, t_{ij}\rangle\}\}. \tag{13}$$

Based on interval order, for such games, we define the concepts of minimax equilibrium strategies.

Definition 3 Saddle Point : The concept of saddle point in classical form is introduced by Neumann[9]. The $(k,r)th$ position of the pay-off matrix will be called a saddle point, if and only if,

$$\langle a_{kr}, b_{kr}, t_{kr}\rangle = \bigvee_i \{\bigwedge_j \langle a_{ij}, b_{ij}, t_{ij}\rangle\} = \bigwedge_j \{\bigvee_i \langle a_{ij}, b_{ij}, t_{ij}\rangle\}. \tag{14}$$

We call the position (k,r) of entry a saddle point, the entry itself $\langle a_{kr}, b_{kr}, t_{kr}\rangle$ the value of the game (denoted by $\overline{V}$) and the pair of pure strategies leading to it are optimal pure strategies. If the saddle point exists for the pay off matrix, then we will use max-min and min-max principle to find the saddle point and thus the entry represent the value of the game.

IX. GAME WITHOUT SADDLE POINT

Let us now consider the pay off matrix as

$$\begin{array}{cc} & \begin{array}{cc} B_1 & \quad B_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{pmatrix} \langle a_{11}, b_{11}, t_{11}\rangle & \langle a_{12}, b_{12}, t_{12}\rangle \\ \langle a_{21}, b_{21}, t_{21}\rangle & \langle a_{22}, b_{22}, t_{22}\rangle \end{pmatrix} \end{array}.$$

Suppose this is the game where saddle point does not exist. Let x_i and y_j be the probability that A will play the i^{th} strategy and B will play j^{th} strategy. Then the mixed strategies for A and B respectively (x_1, x_2) and (y_1, y_2) such that $x_1 + x_2 = 1;\ x_1, x_2 \ge 0$ and $y_1 + y_2 = 1;\ y_1, y_2 \ge 0$. Now A's gain for any choice of strategies of B are respectively

$$x_1\left[a_{11} - \frac{1}{2}t_{11}, b_{11} + \frac{1}{2}t_{11}\right] + (1-x_1)\left[a_{21} - \frac{1}{2}t_{21}, b_{21} + \frac{1}{2}t_{21}\right]$$

$$and\ \ x_1\left[a_{12} - \frac{1}{2}t_{12}, b_{12} + \frac{1}{2}t_{12}\right] + (1-x_1)\left[a_{22} - \frac{1}{2}t_{22}, b_{22} + \frac{1}{2}t_{22}\right]$$

respectively. Now A's gain should be same whatever be the strategy of B. So

$$x_1\left[a_{11} - \frac{1}{2}t_{11}, b_{11} + \frac{1}{2}t_{11}\right] + (1-x_1)\left[a_{21} - \frac{1}{2}t_{21}, b_{21} + \frac{1}{2}t_{21}\right]$$

$$= x_1\left[a_{12} - \frac{1}{2}t_{12}, b_{12} + \frac{1}{2}t_{12}\right] + (1-x_1)\left[a_{22} - \frac{1}{2}t_{22}, b_{22} + \frac{1}{2}t_{22}\right]$$

$$\Rightarrow x_1\left(a_{11} - \frac{t_{11}}{2}\right) + (1-x_1)\left(a_{21} - \frac{t_{21}}{2}\right) = x_1\left(a_{12} - \frac{t_{12}}{2}\right) + (1-x_1)\left(a_{22} - \frac{t_{22}}{2}\right) \tag{15}$$

$$and\ \ x_1\left(b_{11} + \frac{t_{11}}{2}\right) + (1-x_1)\left(b_{21} + \frac{t_{21}}{2}\right) = x_1\left(b_{12} + \frac{t_{12}}{2}\right) + (1-x_1)\left(b_{22} + \frac{t_{22}}{2}\right). \tag{16}$$

Solving for x_1 and x_2 we get,

$$x_1 = \frac{2a_{22} - t_{22} - 2a_{21} + t_{21}}{2a_{11} - t_{11} - 2a_{21} + t_{21} - 2a_{12} + t_{12} + 2a_{22} - t_{22}}$$

$$and\ \ x_2 = \frac{2b_{12} - t_{12} - 2b_{11} - t_{11}}{2b_{21} + t_{21} - 2b_{11} - t_{11} + 2b_{12} + t_{12} - 2b_{22} - t_{22}}.$$

with the condition of the solution that,

$$\frac{2a_{22} - t_{22} - 2a_{21} + t_{21}}{2a_{11} - t_{11} - 2a_{21} + t_{21} - 2a_{12} + t_{12} + 2a_{22} - t_{22}}$$

$$= \frac{2b_{12} - t_{12} - 2b_{11} - t_{11}}{2b_{21} + t_{21} - 2b_{11} - t_{11} + 2b_{12} + t_{12} - 2b_{22} - t_{22}}.$$

Similarly, we get,

$$y_1 = \frac{2a_{22} - t_{22} - 2a_{21} + t_{21}}{2a_{11} - t_{11} - 2a_{21} + t_{21} - 2a_{12} + t_{12} + 2a_{22} - t_{22}}$$

$$and\ \ y_2 = \frac{2b_{12} - t_{12} - 2b_{11} - t_{11}}{2b_{21} + t_{21} - 2b_{11} - t_{11} + 2b_{12} + t_{12} - 2b_{22} - t_{22}}.$$

The value of the game is given by

$$\overline{V} = \langle a_{11}x_1 + a_{21}x_2, b_{11}x_1 + b_{21}x_2, t_{11}x_1 + t_{21}x_2 \rangle. \qquad (17)$$

where x_1 and x_2 are obtained from (15) and (16). Here x_1, x_2, a_{ij}, b_{ij} and t_{ij} are all crisp but $\overline{V}$ is fuzzy.

X. NUMERICAL EXAMPLE

Let us now consider the pay off matrix as

$$\begin{array}{cc} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{pmatrix} \langle 2,3,1 \rangle & \langle 5,7.5,2.5 \rangle \\ \langle 7,10.5,3.5 \rangle & \langle 3,4.5,1.5 \rangle \end{pmatrix} \end{array}.$$

Since, $\bigwedge_j \{\bigvee_i \langle a_{ij}, b_{ij}, t_{ij} \rangle\} = \langle 5,7.5,2.5 \rangle \neq \langle 3,4.5,1.5 \rangle = \bigvee_i \{\bigwedge_j \langle a_{ij}, b_{ij}, t_{ij} \rangle\}$, so saddle point does not exist. Calculating from the method above we get,

$$x_1 = \frac{4}{7}, x_2 = \frac{3}{7}, y_1 = \frac{2}{7}, y_2 = \frac{5}{7}\ and\ \overline{V} = \left\langle \frac{29}{7}, \frac{43.5}{7}, \frac{14.5}{7} \right\rangle.$$

XI. CONCLUSION

In this paper an approach for solution of fuzzy game is considered, where the elements are symmetric trapezoidal fuzzy number.These symmetric trapezoidal fuzzy numbers are converted to the interval numbers.Then a solution method has been given and a numerical example illustrate the technique. This approach has special future that we can consider the pay off elements as interval number with unequal length and hence there is a scope to discuss the uncertainty. Here is also a scope to discuss about the bimatrix games with same approach.

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