International Journal of Engineering Research and Development
e-ISSN: 2278-067X, **p**-ISSN : 2278-800X, www.ijerd.com
Volume 5, Issue 8 (January 2013), PP. 40-44

An Optimal Replacement Problem for Three–State Repairable Systems Using Arithmetico-Geometric Process

Dr.C.Manmatheswara reddy[1], Dr.B.Venkata Ramudu[2]
[1]Principal, Sri Balaji P.G.Collge, Anantapur-515001.A.P (India)
[2]Assistant Professor, Dept. of Statistics, S.S.B.N Degree & P.G. College, Anantapur-515001.A.P (India)

Abstract:- This paper demonstrates a Deteriorating Simple Repairable System with three states. They include two failure states and one working state. Assuming that the up times and down times are exposing to Arithmetico Geometric Process an attempt is made to obtain replacement policy N under which we replace the system when the number failures of the system reaches N. An explicit expression of the average cost rate is derived and corresponding optimal policy N* is obtained. Finally it provides numerical results.

I. INTRODUCTION

The replacement problem for a repairable system has aroused great attention since it was proposed by Lotka in 1939[6]. The field of maintenance (including inspection, replacement, condition-based, over haul) models was founded in the 1960s with the pioneering work of Barlow and Proschan[2], among others. This period is characterized by relatively simple, but lucid models. In the 1970s and early 1980s, researchers built on these models, and considered many extensions. The results of these more complicated models were mainly analytical and, except for some special cases, which yielded on explicit mathematical solution, the solutions obtained were computationally intractable. Then, after a few quiet years, renewed interest in maintenance emerged in the mid 1980s. Apart from further extensions, such as multi-component systems and interaction with production, more attention has been paid to computational tractability and practical usefulness. The results of models are often accompanied by numerical examples, algorithms and computer programs. Further advances in computer science and information technology support this development.

The earliest replacement models consider one component repairable systems with one repairman (called simple repairable systems). It is assumed that the system after repair is ' as good as new ' and this kind of repair is called a perfect repair. The replacement of a piece of equipment with a new one can be considered to be a perfect repair. However, almost repairs in practice are not perfect. consequently, the system after repair cannot be ' as good as new '. Barlow and Hunter[1] introduced a minimal repair model in which the repair activities do not change the failure rate of the system. There after Brown and Proschan[3] considered the imperfect repair model, which is the combination of the perfect repair and minimal repair models.

Consider a machine which operates for random amount of time before being repaired and then operates again until the next repair is required, continuing to alternative between periods of operation and repair. Under many circumstances, the times between repair (uptimes) would seem to most appropriately be modeled as a stochastically decreasing sequence of random variables (e.g.. Ross[7]). Since the machine may not always be repaired as good as new. For similar reasons the sequence of periods during which the machine being repaired (down times) may best be modeled as a stochastically increasing sequence of random variables.

In practice, because of the ageing effect and accumulated wearing, most systems are deteriorating so that the successive operating times are stochastically decreasing, while the consecutive operating times are stochastically increasing. It seems reasonable that an alternative approach to the maintenance problem of a deteriorating system is to study a monotone process model. For this purpose Lam [4] introduced the geometric process.

In most reported repair/replacement models including the geometric process repair model, it is usually assumed that a system may experience only two possible states. One working state and the other failure state. In practice, a system may have more than two states, for example both a relay, and a diode have a short circuit failure, and an open circuit failure some equipment or man – machine systems, such as those in the chemical industry, nuclear power stations, and soon, have a safety failure and a danger failure, or a slight failure and a serious failure, or some similar combination. A radio or microwave transmitter may be working with full transmission range, working with degraded transmission range, or completely failed. The health condition of an automobile may be considered excellent, good or poor. A review of research on systems with dual failure model is provided by Lesanovsky[5]. In these situations, a repair model for three – state repairable system in this chapter is more appropriate.

Zhang [8] studied a deteriorating simple repairable system with three states, including two failure states and one working state. Assume that the system after repair cannot be 'as good as new', and the deterioration of the system is stochastic. A replacement policy N based on the failure number of the system is adopted under which the system will be replaced at the time of N^{th} failure. He also derived an explicit expression for an optimal replacement policy N* by minimizing the average cost rate.

In many practical situations, a system may experience many possible states. For example, a relay circuit may experience two different failure modes, called dual failure modes, in addition to the working mode. When it is energized and thus required to close, it may fail to do so due to the presence of dust and other insulating media. When it is de-energized and thus required to open, it may fail to do so because the contacts are stuck together due to overheating. A flow control chart valve may also experience such dual failure modes. Another example is home security system. It may fail to detect a break-in due to mechanical or electrical circuit failures. It may also create a false alarm due to the presence of a pet. Systems with dual failure modes are studied.

In the next section, we develop the model and optimal solution for a deteriorating repairable system with three states and (K+1) states. Among these states, K of them are failure states and the other one is failure state with exposing to arithmetico- geometric process.

For case of reference, we provided the definitions of stochastic ordering and arithmetico- geometric process as follows.(see Lam[4]).

1 Definition

Given two random variables X and Y

if $p(x > \alpha) \geq p(y>\alpha)$ for all real α,

then we say that X is stochastically larger than Y and written $X \geq_{st} Y$ or Y is stochastically less than X and written $Y \leq_{st} \mathbf{X}$.

We say that a sequence of random variables $\{Z_n, n=1, 2,\ldots\}$ is stochastically decreasing if $Z_n \geq_{st} Z_{n+1}$ for all n=1,2,….; similarly, we say that a sequence of random variables $\{Z_n, n=1, 2,3,\ldots\ldots\}$ is stochastically increasing if $Z_n \leq_{st} Z_{n+1}$ for all n=1,2,……

Now, we give the definition of the arithmetico -geometric process(AGP).

2 Definition :

If a>1 and $d \in (0, \mu / (n-1)a^{n-1})$, where n=2,3,…….. and μ is the mean of the first random variable H_1, then the AGP is called a decreasing AGP. If d<0 and 0<a<1, then the AGP is called as increasing AGP. If d=0 and a=1, then the AGP reduces to a Renewal process.

II. MODEL

In this section, we develop a model for three states and (K+1) states of the systems exposing to arithmetico -geometric process by maximizing the long-run expected reward per unit time with the following assumptions.

Assumptions:

1. At time o, a new system is installed, this system will eventually be replaced by a new and identical one.
2. (i) The system may experience three states including one working state and two failure states. State 0 represents the working state and state 1 and 2 represents the failure states of the system.
   (ii) The system may experience (K+1) different states including one working state and K different failure states. State 0 represents the working state while state i (for $1 \leq i \leq K$) represents the i^{th} type of failure state of the system.
   These failure states are exhaustive, mutually exclusive and stochastic.
3. Whenever the system was failed, a repairman starts to repair it right away. The system after repair is not as good as new. Let us denote Xn and Yn to represent the consecutive working time after the $(n-1)^{th}$ repair, the repair time after the n^{th} failure respectively.
4. Assume that Xn, Yn ; n = 1,2,…. are independent random variables.
5. (i) We denote Cr, Cw, C to represent respectively, the repair cost of the system per unit time, working reward of the system per unit time and replacement cost of the system.
   (ii) Let r be the reward rate of the system when it is operating, and let C be the repair cost rate of the system. Assume further that the replacement cost comprises two parts. One part is the basic replacement cost R, the other is proportional to the replacement time Z at rate $C_{p.}$

III. OPTIMAL SOLUTIONS

We use replacement policy N based on the failure number of the system. Our aim is to determine an optimal replacement policy N* such that the average cost rate is minimum. Let T_1 be the first replacement time of the system under policy N. Let n (n≥2) be the time between the $(n-1)^{Th}$ replacement, and the n^{Th} replacement of the system under policy N. Obviously { T_1, T_2, ….} forms a renewal process, while the inter arrival time between two consecutive replacements is called renewal cycle.

Let C(N) be the long-run expected reward per unit time under policy N. Thus, according to the renewal reward theorem(Ross), we have:

$$C(N) = \frac{\text{The expected cost incurred in a renewal cycle}}{\text{The expected length of the renewal cycle}} \quad (3.1)$$

$$= \frac{C_r E\left[\sum_{n=1}^{N-1} y_n\right] + C - C_w E\left[\sum_{n=1}^{N} x_n\right]}{E\left[\sum_{n=1}^{N-1} y_n\right] + E\left[\sum_{n=1}^{N} x_n\right]}$$

$$C(N) = \frac{\frac{C_r}{\mu}\sum_{n=1}^{N-1}\left[\frac{1}{b^{n-1}} - (n-1)d_2\right] + C - \frac{C_w}{\lambda}\left[\frac{1}{a^{n-1}} - (n-1)d_1\right]}{\frac{1}{\mu}\sum_{n=1}^{N-1}\left[\frac{\lambda}{b^{n-1}} - (n-1)d_2\right] + \frac{1}{\lambda}\sum_{n=1}^{N}\left[\frac{1}{a^{n-1}} - (n-1)d_1\right]} \quad (3.2)$$

Where a = $\left(\frac{p_1}{a_1} + \frac{p_2}{a_2}\right)^{-1}; b = \left(\frac{p_1}{b_1} + \frac{p_2}{b_2}\right)^{-1}$

Clearly $p_1 + p_2 = 1$

Suppose that a replacement policy N is adopted. To derive the long-run average cost per unit time, first we should evaluate the expected values of X_n and Y_n. To do this, let $\int_0^\infty t dU(t) = \lambda$ and $\int_0^\infty t dV(t) = \mu$.

Then $E(X_1) = \lambda$ and $E(Y_1) = \mu$. In general, we have:

$$E(X_n) = \sum_{\sum_0^{k-1} i_j = n-1} \frac{(n-1)!}{i_0!\ldots.i_{k-1}!} q_0^{i_0}\ldots.q_{k-1}^{i_{k-1}} \text{ X } \int_0^\infty t dU(a_0^{i_0}\ldots.a_{k-1}^{i_{k-1}} t)$$

$$= \sum_{\sum_{i=0}^{k-1} i_j = n-1} \frac{(n-1)!}{i_0!\ldots.i_{k-1}!}\left(\frac{q_0}{a_0}\right)^{i_0}\ldots.\left(\frac{q_{k-1}}{a_{k-1}}\right)^{i_{k-1}} \lambda$$

$$= \lambda\left(\frac{q_0}{a_0} + \ldots. + \frac{q_{k-1}}{a_{k-1}}\right)^{n-1}$$

Similarly we have

$$E(Y_n) = \mu\left(\frac{q_0}{b_0} + \ldots\ldots + \frac{q_{k-1}}{b_{k-1}}\right)^{n-1}$$

Now, we say a cycle is completed if a replacement is completed. In other words, a cycle is actually a time interval between the installation and the first replacement or two successive replacements. Thus the successive cycles and the costs incurred in each cycle will form a renewal reward process. By applying the standard result in renewal reward process, the long-run average cost per unit time (or simply average cost) is given by:

$$C(N) = \frac{\text{The expected cost incurred in a renewal cycle}}{\text{The expected length of the renewal cycle}}$$

$$= \frac{cE\left(\sum_{n=1}^{N-1} Y_n\right) + R + c_p E(Z) - rE\left(\sum_{n=1}^{N} X_n\right)}{E\left(\sum_{n=1}^{N} X_n\right) + E\left(\sum_{n=1}^{N-1} Y_n\right) + E(Z)}$$

$$= \frac{1}{\sum_{n=1}^{N}\left(\sum_{i=0}^{k-1} q_i \cdot \frac{\lambda}{a_i^{n-1}} - (n-1)d_1\right)^{n-1} + \sum_{n=1}^{N-1}\left(\sum_{i=0}^{k-1} \frac{q_i \mu}{b_i^{n-1}} - (n-1)d_2\right)^{n-1} + \tau x} \tag{3.3}$$

$$x\left[C\sum_{n=1}^{N-1}\left(\sum_{i=0}^{k-1} qi.\frac{\mu}{b_i^{n-1}} - (n-1)d_2\right)^{n-1} + (R + C_p\tau) - \gamma\sum_{n=1}^{N}\left(\sum_{i=0}^{k-1} q_i \cdot \left(\frac{\lambda}{a_i^{n-1}} - (n-1)d_1\right)^{n-1}\right)\right]$$

Where r = E(Z) is the expected replacement time.

Let $a^{-1} = \sum_{i=0}^{k-1} \frac{q_i}{a_i}$ and $b^{-1} = \sum_{i=0}^{k-1} \frac{q_i}{b_i}$ then above equation becomes

$$C(N) = \frac{C\sum_{n=1}^{N-1}\left[\frac{\mu}{b^{n-1}} - (n-1)d_2\right] + (R + C_P\tau) - \sum_{n=1}^{N}\left[\frac{\lambda}{a^{n-1}} - (n-1)d_1\right]}{\sum_{n=1}^{N}\left[\frac{\lambda}{a^{n-1}} - (n-1)d_1\right] + \sum_{n=1}^{N-1}\left[\frac{\mu}{b^{n-1}} - (n-1)d_2\right] + \tau} \tag{3.4}$$

Obviously, we can determine the optimal replacement policy N* by analytical or empirical methods such that C(N*) is minimized.

IV. EMPIRICAL RESULTS AND CONCLUSIONS

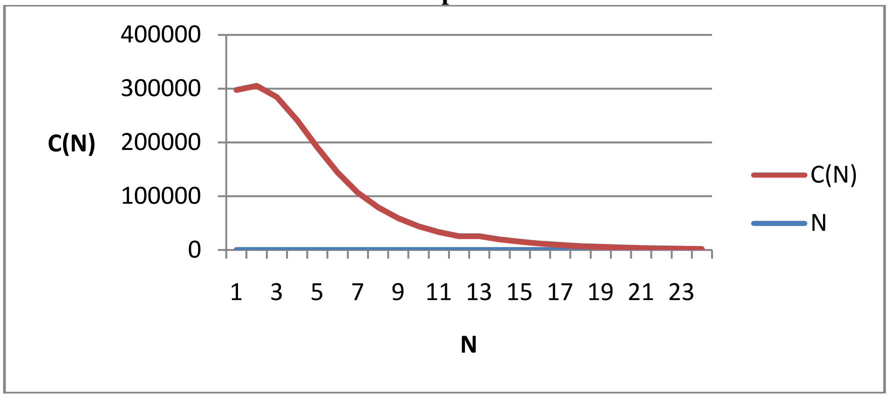
For given fixed values of λ, μ, C, C_r, C_w the optimal replacement policy N* is calculated as follows:

Let a=0.85, b=0.8, d_1=1.05,d_2=-0.9, λ = 50, μ=70, C=5000, C_r=20, C_w=250

Table 1 : N Vs C(N)

N	**C(N)**	N	**C(N)**
2	297330.718750	13	25638.978516
3	**305209.250000**	14	19773.625000
4	284539.406250	15	15364.040039
5	241149.187500	16	12015.141602
6	190165.375000	17	9448.106445
7	143803.171875	18	7464.026367
8	106829.929688	19	5919.299316
9	79141.687500	20	4708.994629
10	58922.847656	21	3755.533936
11	44237.757812	22	3000.923096
12	33524.457031	23	2401.357666
13	25638.978516	24	1923.437744

Graph:1



V. CONCLUSIONS

(i) From the table and graph (1), it can observed that the expected long-run average cost is minimum at the 3^{rd} failure..Thus N*=3.That is, the system should be replaced at the time of third failure.
(ii) It was observed that the optimal number of failure N* is positively changes as the values of 'b' changes.
(iii) It was observed that the optimal number of failure N* is positively changes as the values of 'a' changes.
(iv) This system can also be modeled as an improving system by an appropriate changes in the values of 'a' & 'b'.

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