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Controllability Resultsfor Damped Second-OrderImpulsive Neutral Functional Integrodifferential SystemwithInfinite Delayin Banach Spaces

F.Paul Samuel[1], T.Gunasekar[2], M.MallikaArjunan[3]
[1]DepartmentofMathematicsand Physics, UniversityofEasternAfrica, Baraton, Eldoret2500-30100, Kenya.
[2]Correspondingauthor: Research Scholar, Department of Mathematics,
KarunyaUniversity, Coimbatore-641114, India.
PresentAddress: Schoolof Advanced Sciences, Fluid Dynamics Division, VIT University, Vellore-632014,India.
[3]DepartmentofMathematics, C.B.M. College, Kovaipudur, Coimbatore -641114, India.

Abstract:- Inthis paper, thecontrollabilityproblem isdiscussed forthe damped second-order impulsive neutral functional integro-differentialsystems with infinite delay inBanach spaces.Sufficientconditions for controllability results are derived by means ofthe Sadovskii's fixedpointtheorem combined with anoncompactcondition onthe cosine familyofoperators.Anexample isprovidedtoillustratethetheory.

Keywords:-Controllability;Dampedsecond-orderdifferentialequations;Impulsive neutralintegrodifferentialequations; Mildsolutions; Infinite delay.
2000 SubjectClassification:34A37,34K30,34K40,47D09.

I. INTRODUCTION

The controllabilityofsecond-order systemwith localand nonlocal conditions are also veryinteresting and researchers areengaged init. Manytimes,itisadvantageoustotreatthesecond-order abstractdifferentialequations directly ratherthan toconvert themtofirst-ordersystem. Balachandranand Marshal Anthoni[5, 6]discussed the controllabilityof second-orderordinary anddelay,differentialandintegro-differentialsystemswiththeproper illustrations,withoutconverting them tofirst-orderbyusingthecosineoperators andLeraySchauderalternative.Thedevelopmentofthetheoryoffunctional differentialequationswithinfinite delayheavilydependsonachoiceofaphase space. Infact,various phasespaces havebeenconsideredandeachdifferentphasespacehasrequired aseparatedevelopmentofthetheory [17].Whenthedelayisinfinite,theselectionofthestate (i.e. phasespace)animportantroleinthe study ofboth qualitativeand quantitativetheory. Ausual choiceisanormed spacesatisfying suitable axioms, whichwasintroducedbyHaleand Kato [18]seealsoKappeland Schappacher[23].For adetailed discussion onthis topic, wereferthereader tothebookbyHinoetal. [19].Systemswith infinitedelaydeservestudy becausethey describeakindofsystems presentinthe realworld.Forexample, inapredator-preysystem the predationdecreasestheaveragegrowth rate ofthe preyspecies,tomature for particulardurationoftime(whichforsimplicityinmathematicalanalysishasbeenassumedto beinfinite) beforetheyare capable of decreasing the average growth rate ofthe preyspecies.

Theimpulsiveconditionisthecombinationoftraditional initialvalueproblemandshort-termperturbationswhosedurationcanbenegligibleincomparison withthedurationofthe process. Theyhaveadvantages overtraditionalinitial valueproblems because they canbeusedtomodelphenomenathatcannot bemodelledbytraditionalinitialvalueproblems. Forthe general aspects of impulsive differential equations, wereferthe reader tothe classicalmonographs [8,25,33]. Nowadays, there hasbeenincreasing interest inthe analysis andsynthesisofimpulsivesystems,orimpulsivecontrolsystems,duetotheirsignificanceinboththeory and applications;see[10,11,14,15,22]and the referencetherein. Itiswellknownthatthe issueofcontrollabilityplaysanimportantroleincontrol theory and engineering[29,34,38]becausetheyhavecloseconnectionstopoleassignment, structuraldecomposition,quadraticoptimalcontrol,observerdesignetc. Thetheoryofimpulsivedifferentialequationsasmuch asneutral differential equations hasbecomeanimportantarea ofinvestigationinrecentyear stimulatedby their numerous applications to problems arisingin mechanics,electricalengineering,medicine,etc.Partialneutralintegrodifferentialequationswithinfinitedelayhav ebeenusedformodellingtheevolution ofphysicalsystems inwhichtheresponseof the systems depends not onlyonthe currentstate, but alsoonthe pasthistoryofthe systems, forinstance,inthe theory developmentinGurtinand Pipkin[16]and Nunziato[31]forthedescriptionofheat conduction inmaterials withfadingmemory.

Hernendezetal. [20]studied the existence results forabstractimpulsive second-order neutralfunctional differentia equations with infinite delay. Indynamical systems dampingisanotherimportantissue;itmaybemathematicallymodelledasaforcesynchronous with the velocity oftheobjectbut opposite indirection to it. Motivation fordamped second-order differentialequations canbefoundin[21,26,37]. Inthe pastdecades, the problem ofcontrollabilityforvariouskindsofdifferentialandimpulsivedifferentia system havebeenextensivelystudied bymanyauthors[2,3,4,9,13,27,28]usingdifferentapproaches. Parketal.[5]investigatedthecontrollabilityof impulsiveneutralintegrodifferentialsystemswithinfinitedelayinBanach spacebyutilizingtheSchauderfixedpointtheorem.

Mostofthe abovementioned works, the authorsimposedsomestrictcompactness as- sumptionsonthe cosinefunction which implies thatthe underlying space isoffinite di- mensions.There isarealneedtodiscussfunctional differential systemswithanoncompactcondition onthecosinefamilyofoperators. Tothebestofourknowledge,there isnoworkreported onthe controllabilityofdamped second-order impulsiveneutral functional integrodifferentialsystemswithinfinitedelayinaphasespace,andtheaimofthispaperistoclosethegap. Theresultsobtained inthispaperaregeneralizationsoftheresults givenbyArthiandBalachandran[1].

II. PRELIMINARIES

Consider thedamped second-order neutral impulsive neutral functional integrodifferentialequationswithinfinitedelaytheform

$$\frac{d}{dt}\left[x'(t)-g\left(t,x_t,\int_0^t a(t,s,x_s)ds\right)\right]=Ax(t)+\mathcal{D}x'(t)+Bu(t)+f\left(t,x_t,\int_0^t b(t,s,x_s)ds\right),t\in J=[0,T],$$
$$t\neq t_i,\qquad i=1,2,\dots,n, \qquad (2.1)$$
$$x_0=\varphi\in\mathcal{B},\ \ x'(0)=\xi\in X, \qquad (2.2)$$
$$\Delta x(t_i)=I_i\left(x_{t_i}\right),\ i=1,2,\dots,n, \qquad (2.3)$$
$$\Delta x'(t_i)=J_i\left(x_{t_i}\right),\ i=1,2,\dots,n, \qquad (2.4)$$

where A is the infinitesimal generator of a strongly continuous cosine family of bounded linear operators$\left(C(t)\right)_{t\in\mathbb{R}}$ defined on a Banach space X. The control function $u(.)$ is given in $L^2(J,U)$, a Banachspace of admissible control functions with U as a Banach space $B:U\to X$ as bounded linear operator; D is a bounded linear operator on a Banach space X with $D(\mathcal{D})\subset D(A)$. For $t\in J, x_t$ represents the function $x_t:]-\infty,0]\to X$ defined by $x_t(\theta)=x(t+\theta),-\infty<\theta\leq 0$ which belongs to some abstract phase space $\mathcal{B}$ defined axiomatically, $g:J\times\mathcal{B}\times X\to X, f:J\times\mathcal{B}\times X\to X, a:J\times J\times\mathcal{B}\to X$ are appropriate functions and will be specified later. The impulsive moments $\{t_i\}$ are given such that $0=t_0<t_1<\cdots<t_n<t_{n+1}=T, I_i:\mathcal{B}\to X, J_i:\mathcal{B}\to X, \Delta\xi(t)$ represents the jump of a function ξ at t, which is defined by $\Delta\xi(t)=\xi(t^+)-\xi(t^-)$, where $\xi(t^+)$ and $\xi(t^-)$ respectively the right and left limits of ξ at t .

In what follows, we recall some definitions, notations, lemmas and results that we need in the sequel. Throughout this paper, A is the infinitesimal generator of a strongly continuous cosine family$\left(C(t)\right)_{t\in\mathbb{R}}$ of bounded linear operators on a Banach space $(X,\|.\|)$. We refer the reader to [12] for the necessary concepts about cosine functions. Next we only mention a few results and notations about this matter needed to establish out results.

Definition 2.1A one-parameter family $\left(C(t)\right)_{t\in\mathbb{R}}$ of bounded linear operator mapping the Banach space X into itself is called a strongly continuous cosine family iff

(i) $C(s+t)+C(s-t)=2C(s)C(t)$for all $s,t\in\mathbb{R}$;
$(ii)C(0)=I$;
$(iii)C(t)x$is continuous in t on R for each fixed $x\in X$.

Wedenote by $\left(S(t)\right)_{t\in\mathbb{R}}$ the sine function associated with $\left(C(t)\right)_{t\in\mathbb{R}}$ which is defined by $S(t)x=\int_0^t C(s)xds, x\in X, t\in R$ and we always assume that M and N are positive constants such that $\|C(t)\|\le M$ and $\|S(t)\|\le N$ for every $t\in J$. The infinitesimal generator of strongly continuous cosine family $\left(C(t)\right)_{t\in\mathbb{R}}$ is the operator$A: X\to X$defined by

$$Ax=\frac{d^2}{dt^2}C(t)x|_{t=0}, x\in D(A),$$

where, $D(A)=\{x\in X: C(t) x is twice continuously differentiale in t\}$.
Define $E=\{x\in X: C(t) x is twice continuously differentiale in t\}$.
The following properties are well known [35]:

(i) If $x\in X$ then $S(t)x\in E$ for every $t\in\mathbb{R}$.
(ii) If $x\in E$ then $S(t)x\in D(A), (d/dt)C(t)x=AS(t)x$ and $(d^2/dt^2)S(t)x=AS(t)x$ for every $t\in\mathbb{R}$.
(iii) If $x\in D(A)$then $C(t)x\in D(A)$, and $(d^2/dt^2)C(t)x=AC(t)x=C(t)Ax$ for every $t\in\mathbb{R}$.
(iv) If $x\in D(A)$then $S(t)x\in D(A)$, and $(d^2/dt^2)S(t)x=AS(t)x=S(t)Ax$ for every $t\in\mathbb{R}$.

In this paper,$[D(A)]$ is the domain of A endowed with the graph norm $\|x\|_A=\|x\|+\|Ax\|, x\in D(A)$. The notation E represents the space formed by the vectors $x\in X$ for which $C(.)x$ is of class C^1on $\mathbb{R}$. We know from Kisynski [24] that E endowed with the norm $\|x\|_E=\|x\|+\sup_{0\le t\le 1}\|AS(t)x\|, x\in E$,is a Banach space. The operator-valued function

$$\mathcal{G}(t)=\begin{bmatrix} C(t) & S(t)\\ AS(t) & C(t)\end{bmatrix}$$

is strongly continuous group of bounded linear operators on the space $E\times X$ generated by the operator$\mathcal{A}=\begin{bmatrix}0 & 1\\ A & 0\end{bmatrix}$ defined on $D(A)\times E$. From this, it follows that $AS(t): E\to X$ is bounded linear operator and that $(t)x\to 0, t\to 0$, for each $x\in E$. Furthermore, if $x:[0,\infty[\to X$ is a locally integrable function, then the function $y(t)=\int_0^t S(t-s)x(s)\,ds$ defines an E-valued continuous function. This assertion is a consequence of the fact that

$$\int_0^t \mathcal{G}(t-s)\begin{bmatrix}0\\ x(s)\end{bmatrix}ds=\left[\int_0^t S(t-s)x(s)ds, \int_0^t C(t-s)x(s)ds\right]^T$$

defines an $E\times X$-valued continuous function.

The existence of solutions of the second-order abstract Cauchy problem

$$x''(t)=Ax(t)+g(t), 0\le t\le T,$$
$$x(0)=y, x'(0)=z, \qquad (2.5)$$

where$g\in L^1([0,T],X)$ is studied in [35]. On the other hand, the semilinear case has been treated in [36]. We only mention here that the function $x(.)$is given by

$$x(t)=C(t)y+S(t)z+\int_0^t S(t-s)g(s)\,ds, 0\le t\le T, \qquad (2.6)$$

is called a mild solution of (2.5) and that when $y\in E$,the function $x(.)$ is of class C^1 and

$$x'(t)=AS(t)y+C(t)z+\int_0^t C(t-s)g(s)\,ds, 0\le t\le T, \qquad (2.7)$$

To consider the impulsive conditions (2.3) and (2.4), it is convenient to introduce some additional concepts and notations.

A function $x: [\mu, \tau] \to X$ is said to be a normalized piecewise continuous function on $[\mu, \tau]$ iff x is piecewise continuous and left continuous on $]\mu, \tau]$. We denote by $PC([\mu, \tau], X)$ the space of normalized piecewise continuous function from $[\mu, \tau]$ into . In particular, we introduce the space PC formed by all normalized piecewise continuous functions $x: [0, T] \to X$ such that $x(.)$ is continuous at $t \neq t_i$, $x(t_i^{-}) = x(t_i)$ and $x(t_i^{+})$ exists, for $i = 1,2, \dots, n$. It is clear that PC endowed with the norm of uniform convergence is a Banach space.

In what follows, we put $t_0 = 0, t_{n+1} = T$ and for $x \in PC$, we denote by $\tilde{x}_i$, for $i = 0,1,2, \dots, n$, the function $\tilde{x}_i \in C([t_i, t_{i+1}]; X)$ given by $\tilde{x}_i(y) = x(t)$ for $t \in]t_i, t_{i+1}]$ and $\tilde{x}_i(t_i) = \lim_{t \to t_i^+} x(t)$. Moreover, for a set $B \subseteq PC$, we denote $\tilde{B}_i$ for $i = 0,1,2, \dots, n$, the set $\tilde{B}_i = \{\tilde{x}_i : x \in B\}$.

We will here in define the phase space $\mathcal{B}$ axiomatically; using ideas and notations developed in [19] and suitably modify to treat retarded impulsive differential equations. More precisely, $\mathcal{B}$ will denote the vector space of functions defined from $]-\infty, 0]$ into X endowed with the seminorm denoted by $\|.\|_{\mathcal{B}}$ and such that the following axioms hold:

(A) If $x:]-\infty, \mu + b] \to X$, $b > 0$, is such that $x_\mu \in \mathcal{B}$ and $x|_{[\mu, \mu+b]} \in PC([\mu, \mu + b], X)$, then for every $t \in [\mu, \mu + b[$, the following condition hold:

$(i) x_t$ is in $\mathcal{B}$

(ii) $\|x(t)\| \leq H\|x_t\|_{\mathcal{B}}$,

$(iii) \|x_t\|_{\mathcal{B}} \leq K(t - \mu) \sup\{\|x(s)\| : \mu \leq s \leq t\} + M(t - \mu)\|x_\mu\|_{\mathcal{B}}$,

where $H > 0$ is a constant; $K, M : [0, \infty[\to [1, \infty[$, K is continuous, M is locally bounded and H, K, M are independent of $x(.)$.

(B) The space $\mathcal{B}$ is complete.

Remark 2.1: In impulsive functional differential systems, the map $[\mu, \mu + b] \to \mathcal{B}$, $t \to x_t$, is in general discontinuous. For this reason, this property has been omitted from our description of the phase space $\mathcal{B}$.

In [19] some examples of phase space $\mathcal{B}$ are given. We introduce the following notations and terminology. Let $(Z, \|.\|_Z), (Y, \|.\|_Y)$ be the Banach spaces, the notation $\mathcal{L}(Z, Y)$ stands for the Banach space of bounded linear operators form Z into Y endowed with the operator from and we abbreviate this notation to $\mathcal{L}(Z)$ when $Z = Y$. Moreover $B_r(x: Z)$ denotes the closed ball with centre at x and radius $r > 0$ in Z. Additionally, for a bounded function $\xi: I \to Z$ and $0 \leq t \leq T$, we employ the notation $\|\xi\|_t$ for $\|\xi\|_t = \sup\{\|\xi(s)\| : s \in [0, t]\}$.

The proof is based on the following fixed point theorem.

Theorem 2.1.([32], Sadovskii's Fixed Point Theorem]). Let F be a condensing operator on a Banach space X. If $F(S) \subset S$ for a convex, closed and bounded set $S$of$X$, then F has a fixed point in S.

III. CONTROLLABILITYRESULTS

Beforeprovingthemainresult, wepresentthedefinition ofthemildsolution tothesystem(2.1)-(2.4).

Definition 3.2 A function $x: (-\infty, T) \to X$ is called a mild solution of the abstract Cauchy problem (2.1)-(2.4) if $x_0 = \varphi \in \mathcal{B}, x|_J \in \mathcal{P}$C, the impulsive conditions $\Delta x(t_i) = I_i(x_{t_i}), \Delta x'(t_i) = J_i(x_{t_i})$, $i = 1,2, \dots, n$, are satisfied and

$$
\begin{aligned}
x(t) = {} & C(t)\varphi(0) + S(t)[\xi - g(0, \varphi, 0)] + \int_0^t C(t-s) g\left(s, x_s, \int_0^s a(s, \tau, x_\tau) d\tau\right) ds \\
& + \sum_{i=0}^{j-1} \left[S(t - t_{i+1})\mathcal{D}x(t_{i+1}^-) - S(t - t_i)\mathcal{D}x(t_i^+)\right] - S(t - t_j)\mathcal{D}x(t_j^+) + \int_0^t C(t-s)\mathcal{D}x(s)\, ds \\
& + \int_0^t S(t-s)\left[Bu(s) + f\left(s, x_s, \int_0^s b(s, \tau, x_\tau) d\tau\right)\right] ds + \sum_{0<t_i<t} C(t - t_i) I_i(x_{t_i}) \\
& + \sum_{0<t_i<t} S(t - t_i) J_i(x_{t_i}) \ .
\end{aligned}
$$

For all $t \in [t_j, t_{j+1}]$ and every $j = 1,2, \dots, n$ (3.1)

Remark 3.2 The above equation can also be written as

$$x(t) = C(t)\varphi(0) + S(t)[\xi - g(0,\varphi,0)] + \int_0^t C(t-s)g\left(s, x_s, \int_0^s a(s,\tau,x_\tau)d\tau\right)ds + \int_0^t S(t-s)\mathcal{D}x'(s)ds$$
$$+ \int_0^t S(t-s)\left[Bu(s) + f\left(s, x_s, \int_0^s b(s,\tau,x_\tau)d\tau\right)\right]ds + \sum_{0<t_i<t} C(t-t_i)I_i\left(x_{t_i}\right)$$
$$+ \sum_{0<t_i<t} S(t-t_i)J_i\left(x_{t_i}\right), t \in J.$$

Now an integration by parts permit us to infer that $x(\cdot)$ is a mild solution of (2.1)-(2.4).

Remark 3.3 In what follows , it is convenient to introduce the function $\bar{\phi}: (-\infty, T) \to X$ defined by

$$\bar{\phi}(t) = \begin{cases} \phi(t), & if t \in (-\infty, 0], \\ C(t)\phi(0), & if t \in J. \end{cases}$$

We introduce the following assumptions:

(H1) There exists a constant $N_1 > 0$ such that
$\left\|\int_0^t [a(t,s,x) - a(t,s,y)]ds\right\| \le N_1\|x - y\|_{\mathcal{B}}$, for$t, s \in J, x, y \in \mathcal{B}$.and$L_2 = T\sup_{t,s\in J\times J}\|a(t,s,0)\|$.

(H2) There exists a constant L_g such that

$$\|g(t, v_1, w_1) - g(t, v_2, w_2)\| \le L_g[\|v_1 - v_2\|_{\mathcal{B}} + \|w_1 - w_2\|]$$

where $0 < L_g < 1$, $(t, v_i, w_i) \in J \times \mathcal{B} \times X$, $i = 1,2$ and
$\|g(t,u,v)\| \le L_g(\|u\|_{\mathcal{B}} + \|v\|) + L_1andL_1 = \max_{t\in J}\|g(t,0,0)\|$.

(H3) The function $f: J \times \mathcal{B} \times X \to X$ satisfies the following conditions:

(i) Let $x: (-\infty, T) \to X$ be such that $x_0 = \varphi$and $x|_J \in \mathcal{PC}$. For each $t \in J, f(t,.): J \times \mathcal{B} \to X$ is continuous and the function $t \to f\left(t, x_t + \int_0^t b(t,s,x_s)ds\right)$ is strongly measurable.

(ii) The function $f: J \times \mathcal{B} \times X \to X$ is completely continuous.

(iii) There exist an integrable function $m: J \to (0, \infty)$ and a continuous non-decreasing function $\Omega: [0, \infty) \to (0, \infty)$, such that,

$$\|f(t,v,w)\| \le m(t)\Omega(\|v\|_{\mathcal{B}} + \|w\|), \quad \liminf_{\xi\to\infty}\left(\frac{\xi + L_0\phi(\xi)}{\xi}\right) = \Lambda < \infty, \text{where} t \in J, (v,w) \in \mathcal{B} \times X.$$

(iv) For every positive constant r, there exists an $\alpha_r \in L^1(J)$ such that

$$\sup_{\|w\|\le r}\|f(t,v,w)\| \le \alpha_r(t).$$

(H4)B is continuous operator from U to X and the linear operator $W: L^2(J,U) \to X$, is defined by

$$Wu = \int_0^T S(T-s)\,Bu(s)ds$$

has a bounded invertible operator W^{-1}, which takes values in $L^2(J,U)/\ker W$ such that $\|B\| \le M_1$and $\|W^{-1}\| \le M_2$, for some positive constants M_1, M_2.

(H5) The impulsive functions satisfy the following conditions:

(i) The maps $I_i, J_i: \mathcal{B} \to X, i = 1,2,\dots,n$ are completely continuous and there exist continuous non-decreasing functions $\lambda_i, \mu_i: [0,\infty) \to (0,\infty), i = 1,2,\dots,n$, such that

$$\|I_i(\psi)\| \le \lambda_i(\|\psi\|_{\mathcal{B}}), \quad \|J_i(\psi)\| \le \mu_i(\|\psi\|_{\mathcal{B}}), \quad \psi \in \mathcal{B}.$$

(ii) There are positive constants K_1, K_2 such that

$$\|I_i(\psi_1) - I_i(\psi_2)\| \le K_1\|\psi_1 - \psi_2\|,$$
$$\|J_i(\psi_1) - J_i(\psi_2)\| \le K_2\|\psi_1 - \psi_2\|, \quad \psi_1, \psi_2 \in \mathcal{B}, i = 1,2,\dots,n.$$

Definition 3.3 The system (1.1)-(1.4) is said to be controllable on the interval $[0,T]$ iff for every $x_0 = \varphi \in \mathcal{B}$, $x'(0) = \xi \in X$and $z_1 \in X$, there exists a control $u \in L^2(J,U)$ such that the mild solution $x(.)$ of (2.1)-(2.4) satisfies $x(T) = z_1$.

Theorem 3.1 If the notation (H1)-(H6) are satisfied, then the system (2.1)-(2.4) is controllable on Jprovided

$$(1+TNM_1M_2)\left[K_T\left(TM\left(L_g+N_1\right)+\frac{1}{K_d}(3N+TM)\|\mathcal{D}\| \qquad + N \wedge\int_0^T m(s)ds+\sum_{i=1}^{n}(MK_1+NK_2)\right)\right]<1.$$

Proof. The notation $\mathcal{H}(T)$ stands for the space

$$\mathcal{H}(T)=\{y:(-\infty,T]\to X: y|_J\in\mathcal{PC},\qquad y_0=0\}$$

endowed with the sup norm. Using the assumption (H4), for an arbitrary function $x(.)$, we define the control

$$u(t)=W^{-1}\Bigg[z_1-C(T)\varphi(0)-S(T)[\xi-g(0,\varphi,0)]-\int_0^T C(T-s)g\left(s,x_s,\int_0^s a(s,\tau,x_\tau)d\tau\right)ds$$
$$-\sum_{i=0}^{j-1}\left[S(T-t_{i+1})\mathcal{D}x(t_{i+1}^-)-S(T-t_i)\mathcal{D}x(t_i^+)\right]+S(T-t_j)\mathcal{D}x(t_j^+)-\int_0^T C(T-s)\mathcal{D}x(s)\,ds$$
$$-\int_0^T S(T-s)f\left(s,x_s,\int_0^s b(s,\tau,x_\tau)d\tau\right)ds-\sum_{i=1}^{n}C(T-t_i)I_i(x_{t_i})-\sum_{i=1}^{n}S(T-t_i)J_i(x_{t_i})\Bigg](t).$$

We shall now show that when using this control the operator ψ on the space $\mathcal{H}(T)$ defined by $(\psi y)_0=0$ and

$$\psi y(t)=S(t)[\xi-g(0,\varphi,0)]+\int_0^t C(t-s)g\left(s,y_s+\bar{\phi}_s,\int_0^s a(s,\tau,y_\tau+\bar{\phi}_\tau)d\tau\right)ds$$
$$+\sum_{i=0}^{j-1}\left[S(t-t_{i+1})\mathcal{D}\left(y(t_{i+1}^-)+\bar{\phi}(t_{i+1}^-)\right)-S(t-t_i)\mathcal{D}\left(y(t_i^+)+\bar{\phi}(t_i^+)\right)\right]$$
$$-S(t-t_j)\mathcal{D}\left(y(t_j^+)+\bar{\phi}(t_j^+)\right)+\int_0^t C(t-s)\mathcal{D}\left(y(s)+\bar{\phi}(s)\right)ds$$
$$+\int_0^t S(t-s)f\left(s,y_s+\bar{\phi}_s,\int_0^s b(s,\tau,y_\tau+\bar{\phi}_\tau)d\tau\right)ds$$
$$+\int_0^t S(t-\eta)BW^{-1}\Bigg[z_1-C(T)\varphi(0)-S(T)[\xi-g(0,\varphi,0)]$$
$$-\int_0^T C(T-s)g\left(s,y_s+\bar{\phi}_s,\int_0^s a(s,\tau,y_\tau+\bar{\phi}_\tau)d\tau\right)ds$$
$$-\sum_{i=0}^{j-1}\left[S(T-t_{i+1})\mathcal{D}\left(y(t_{i+1}^-)+\bar{\phi}(t_{i+1}^-)\right)-S(T-t_i)\mathcal{D}\left(y(t_i^+)+\bar{\phi}(t_i^+)\right)\right]$$
$$+S(T-t_j)\mathcal{D}\left(y(t_j^+)+\bar{\phi}(t_j^+)\right)-\int_0^T C(T-s)\mathcal{D}\left(y(s)+\bar{\phi}(s)\right)ds$$
$$-\int_0^T S(T-s)f\left(s,y_s+\bar{\phi}_s,\int_0^s b(s,\tau,y_\tau+\bar{\phi}_\tau)d\tau\right)ds-\sum_{i=1}^{n}C(T-t_i)I_i(y_{t_i}+\bar{\phi}_{t_i})$$
$$-\sum_{i=1}^{n}S(T-t_i)J_i(y_{t_i}+\bar{\phi}_{t_i})\Bigg](\eta)\,d\eta+\sum_{0<t_i<t}C(t-t_i)I_i(y_{t_i}+\bar{\phi}_{t_i})$$
$$+\sum_{0<t_i<t}S(t-t_i)J_i(y_{t_i}+\bar{\phi}_{t_i}),\quad t\in[t_j,t_{j+}],j=0,1,\ldots,n,$$

has a fixed point $x(.)$. This fixed point is then a mild solution of the system (2.1)-(2.4). Clearly $\psi x(T) = z_1$ which means that the control u steers the systems from the initial state φ to z_1 in time T, provided we can obtain a fixed point of the operator ψ which implies that the system is controllable. From the assumptions, it is easy to see that ψ is well defined and continuous.

Next we claim that there exists $r > 0$ such that $\psi\left(B_r\left(0, \mathrm{H}(T)\right)\right) \subseteq B_r\left(0, \mathrm{H}(T)\right)$. If we assume that this assertion is false, then for each $r > 0$, we can choose $x^r \in B_r\left(0, \mathrm{H}(T)\right)$, $j = \{0,1,\dots,n\}$ and $t^r \in [t_j, t_{j+1}]$ such that $\|\psi y^r(t^r)\| > r$.

Using the notation $\|y_t + \tilde{\varphi}_t\|_{\mathcal{B}} \le K_T\|y_t\| + \|\tilde{\varphi}_t\|_{\mathcal{B}}$, we observe that

$$
\begin{aligned}
r < \|\psi y^r(t^r)\| \le\ & N[\|\xi\| + \|g(0,\varphi,0)\|] \\
&+ M\int_0^{t^r}\left\| g\left(s, y_s^r + \tilde{\varphi}_s, \int_0^s a(s,\tau,y_\tau^r + \tilde{\varphi}_\tau)d\tau\right) - g\left(s, \tilde{\varphi}_s, \int_0^s a(s,\tau,\tilde{\varphi}_\tau)d\tau\right)\right\| ds \\
&+ M\int_0^{t^r}\left\| g\left(s, \tilde{\varphi}_s, \int_0^s a(s,\tau,\tilde{\varphi}_\tau)d\tau\right)\right\| ds + (3N + TM)\|\mathcal{D}\|(r + \|\tilde{\varphi}\|_a) \\
&+ NM_1M_2\int_0^{t^r}\Bigg[\|z_1\| + M\varphi(0) + N[\|\xi\| + \|g(0,\varphi,0)\|] \\
&+ M\int_0^{T}\left\| g\left(s, y_s^r + \tilde{\varphi}_s, \int_0^s a(s,\tau,y_\tau^r + \tilde{\varphi}_\tau)d\tau\right) - g\left(s, \tilde{\varphi}_s, \int_0^s a(s,\tau,\tilde{\varphi}_\tau)d\tau\right)\right\| ds \\
&+ \int_0^{T}\left\| g\left(s, \tilde{\varphi}_s, \int_0^s a(s,\tau,\tilde{\varphi}_\tau)d\tau\right)\right\| ds + (3N + TM)\|\mathcal{D}\|(r + \|\tilde{\varphi}\|_T) \\
&+ N\int_0^T m(s)\Omega\left(\|y_s^r + \tilde{\varphi}_s\|_{\mathcal{B}} + \left\|\int_0^s b(s,\tau,y_\tau^r + \tilde{\varphi}_\tau)d\tau\right\|\right) ds \\
&+ M\sum_{i=1}^{n}[\|I_i(y_{t_i}^r + \tilde{\varphi}_{t_i}) - I_i(\tilde{\varphi}_{t_i})\| + \|I_i(\tilde{\varphi}_{t_i})\|] \\
&+ N\sum_{i=1}^{n}[\|J_i(y_{t_i}^r + \tilde{\varphi}_{t_i}) - J_i(\tilde{\varphi}_{t_i})\| + \|J_i(\tilde{\varphi}_{t_i})\|]\Bigg] d\eta \\
&+ N\int_0^T m(s)\Omega\left(\|y_s^r + \tilde{\varphi}_s\|_{\mathcal{B}} + \left\|\int_0^s b(s,\tau,y_\tau^r + \tilde{\varphi}_\tau)d\tau\right\|\right) ds \\
&+ M\sum_{i=1}^{n}[\|I_i(y_{t_i}^r + \tilde{\varphi}_{t_i}) - I_i(\tilde{\varphi}_{t_i})\| + \|I_i(\tilde{\varphi}_{t_i})\|] \\
&+ N\sum_{i=1}^{n}[\|J_i(y_{t_i}^r + \tilde{\varphi}_{t_i}) - J_i(\tilde{\varphi}_{t_i})\| + \|J_i(\tilde{\varphi}_{t_i})\|]
\end{aligned}
$$

$$
\begin{aligned}
r < \|\psi y^r(t^r)\| \le\ & N[\|\xi\| + \|g(0,\varphi,0)\|] \\
&+ M\int_0^{t^r}\left\| g\left(s, y_s^r + \tilde{\varphi}_s, \int_0^s a(s,\tau,y_\tau^r + \tilde{\varphi}_\tau)d\tau\right) - g\left(s, \tilde{\varphi}_s, \int_0^s a(s,\tau,\tilde{\varphi}_\tau)d\tau\right)\right\| ds \\
&+ M\int_0^{t^r}\left\| g\left(s, \tilde{\varphi}_s, \int_0^s a(s,\tau,\tilde{\varphi}_\tau)d\tau\right)\right\| ds + (3N + TM)\|\mathcal{D}\|(r + \|\tilde{\varphi}\|_a) \\
&+ NM_1M_2\int_0^{t^r}\Bigg[\|z_1\| + M\varphi(0) + N[\|\xi\| + \|g(0,\varphi,0)\|] \\
&+ M\int_0^{T}\left\| g\left(s, y_s^r + \tilde{\varphi}_s, \int_0^s a(s,\tau,y_\tau^r + \tilde{\varphi}_\tau)d\tau\right) - g\left(s, \tilde{\varphi}_s, \int_0^s a(s,\tau,\tilde{\varphi}_\tau)d\tau\right)\right\| ds \\
&+ \int_0^{T}\left\| g\left(s, \tilde{\varphi}_s, \int_0^s a(s,\tau,\tilde{\varphi}_\tau)d\tau\right)\right\| ds + (3N + TM)\|\mathcal{D}\|(r + \|\tilde{\varphi}\|_T) \\
&+ N\int_0^T m(s)\Omega\left(\|y_s^r + \tilde{\varphi}_s\|_{\mathcal{B}} + \left\|\int_0^s b(s,\tau,y_\tau^r + \tilde{\varphi}_\tau)d\tau\right\|\right) ds \\
&+ M\sum_{i=1}^{n}[\|I_i(y_{t_i}^r + \tilde{\varphi}_{t_i}) - I_i(\tilde{\varphi}_{t_i})\| + \|I_i(\tilde{\varphi}_{t_i})\|] \\
&+ N\sum_{i=1}^{n}[\|J_i(y_{t_i}^r + \tilde{\varphi}_{t_i}) - J_i(\tilde{\varphi}_{t_i})\| + \|J_i(\tilde{\varphi}_{t_i})\|]\Bigg] d\eta \\
&+ N\int_0^T m(s)\Omega\left(\|y_s^r + \tilde{\varphi}_s\|_{\mathcal{B}} + \left\|\int_0^s b(s,\tau,y_\tau^r + \tilde{\varphi}_\tau)d\tau\right\|\right) ds \\
&+ M\sum_{i=1}^{n}[\|I_i(y_{t_i}^r + \tilde{\varphi}_{t_i}) - I_i(\tilde{\varphi}_{t_i})\| + \|I_i(\tilde{\varphi}_{t_i})\|] \\
&+ N\sum_{i=1}^{n}[\|J_i(y_{t_i}^r + \tilde{\varphi}_{t_i}) - J_i(\tilde{\varphi}_{t_i})\| + \|J_i(\tilde{\varphi}_{t_i})\|]
\end{aligned}
$$

and hence

$$1 \le (1+TNM_1M_2)\left[K_T\left(TML_g(1+N_1)+\frac{1}{K_T}(3N+TM)\|\mathcal{D}\| \quad + N \wedge \int_0^T m(s)ds + \sum_{i=1}^{n}(MK_1+NK_2)\right)\right],$$

which contradicts our assumption.

Let $r>0$ be such that $\psi\left(B_r\big(0,Ꜧ(T)\big)\right) \subseteq B_r\big(0,Ꜧ(T)\big)$. In order to prove that ψ is condensing map on $B_r\big(0,Ꜧ(T)\big)$into $B_r\big(0,Ꜧ(T)\big)$.

Consider the decomposition$\psi = \psi_1 + \psi_2$, where

$$\psi_1 x(t) = S(t)[\xi - g(0,\varphi,0)] + \int_0^t C(t-s)g\left(s, y_s+\bar{\phi}_s, \int_0^s a(s,\tau,y_\tau+\bar{\phi}_\tau)d\tau\right)ds$$

$$+\sum_{i=0}^{j-1}\left[S(t-t_{i+1})\mathcal{D}\big(y(t_{i+1}^-)+\bar{\phi}(t_{i+1}^-)\big) - S(t-t_i)\mathcal{D}\big(y(t_i^+)+\bar{\phi}(t_i^+)\big)\right]$$

$$-S(t-t_j)\mathcal{D}\left(y(t_j^+)+\bar{\phi}(t_j^+)\right) + \int_0^t C(t-s)D\left(y(s)+\bar{\phi}(s)\right)ds$$

$$+\sum_{0<t_i<t} C(t-t_i)I_i\big(y_{t_i}+\bar{\phi}_{t_i}\big) + \sum_{0<t_i<t} S(t-t_i)J_i\big(y_{t_i}+\bar{\phi}_{t_i}\big),$$

$$\psi_2 x(t) = \int_0^t S(t-s)\left[f\left(s,y_s+\bar{\phi}_s,\int_0^s b(s,\tau,y_\tau+\bar{\phi}_\tau)d\tau\right)+Bu(s)\right]ds.$$

Now

$$\|Bu(s)\| \le M_1M_2\Bigg[\|z_1\| + M\varphi(0) + N\big[\|\xi\| + L_g\|\varphi\|_{\mathcal{B}} + L_1\big]$$

$$+\int_0^T\left(L_g\left(\|y_s+\tilde{\varphi}_s\| + \left\|\int_0^s a(s,\tau,y_\tau+\tilde{\varphi}_\tau)d\tau\right\|\right)+L_1\right)ds + N\|\mathcal{D}\|\left(\left\|y(t_j^+)\right\| + \left\|\bar{\phi}(t_j^+)\right\|\right)$$

$$+N\|\mathcal{D}\|\sum_{i=0}^{j-1}\left[\left\|y(t_{i+1}^-)\right\| + \left\|\bar{\phi}(t_{i+1}^-)\right\| + \left\|y(t_i^+)\right\| + \left\|\bar{\phi}(t_i^+)\right\|\right] + M\|\mathcal{D}\|\int_0^T\left(\|y(s)\| + \left\|\bar{\phi}(s)\right\|\right)ds$$

$$+N\int_0^T \alpha_r(s)ds + M\sum_{i=1}^{n}\lambda_i\left[\left\|y_{t_i}\right\| + \left\|\bar{\phi}_{t_i}\right\|\right] + N\sum_{i=1}^{n}\mu_i\left[\left\|y_{t_i}\right\| + \left\|\bar{\phi}_{t_i}\right\|\right]\Bigg]$$

$$\le M_1M_2\Bigg[\|z_1\| + M\varphi(0) + N\big[\|\xi\| + L_g\|\varphi\| + L_1\big] + TMK_TL_g(1+N_1)r$$

$$+M\int_0^T\left[L_g\big((K_Tr + \|\tilde{\varphi}(s)\|)(1+N_1)+L_2\big)+L_1\right]ds + (3N+TM)\|\mathcal{D}\|(r+\|\tilde{\varphi}\|_T)$$

$$+N\int_0^T \alpha_r(s)ds + \sum_{i=1}^{n}(M\lambda_i + N\mu_i)\left[K_Tr + \left\|\bar{\phi}_{t_i}\right\|\right]\Bigg] = A_0.$$

From [[30], Lemma 3.1], we infer that ψ_2 is completely continuous. This fact and he estimate

$$\|\psi_1 v - \psi_2 w\| \le K_T\left[TML_g(1+N_1) + \frac{1}{K_T}(3N+TM)\|\mathcal{D}\| + \sum_{i=1}^{n}(MK_1+NK_2)\right]\|v-w\|_T,$$

Together imply that ψ is condensing operator on $B_r\big(0,Ꜧ(T)\big)$.

Finally from sadovskii's fixed point theorem we obtain a fixed point $y$and ψ. Clearly, $x = y + \bar{\phi}$ is a mild solution of the problem (2.1)-(2.4). This completes the proof.

Corollary 3.1 If all conditions of Theorem 3.1 hold except (H5) replaced by the following one,
(C1): there exist positive constants a_i, b_i, c_i, d_i and constants $\theta_i, \delta_i \in (0,1), i = 1,2,\ldots,n$ such that for each $\phi \in X$

$$\|I_i(\phi)\| \le a_i + b_i(\|\phi\|)^{\theta_i}, i = 1,2,\ldots,n$$

And

$$\|J_i(\phi)\| \le c_i + d_i(\|\phi\|)^{\delta_i}, i = 1,2,\ldots,n$$

Then the system (2.1)-(2.4) is controllable on J provided that

$$(1 + TNM_1M_2)\left[K_T\left(TML_g(1+N_1) + \frac{1}{K_T}(3N + TM)\|\mathcal{D}\| + N \wedge \int_0^T m(s)ds\right)\right] < 1.$$

IV. EXAMPLE

In this section, we consider an application of our abstract result. We choose the space $X = L^2([0,\pi]), \mathcal{B} = \mathcal{P}C_0xL^2(h,X)$ is the space introduced in [19]. Let A be an operator defined by $A\omega = \omega''$ with domain

$$D(A) = \{\omega \epsilon H^2\,]0,\pi[:\ \omega(0) = \omega(\pi) = 0\}.$$

It is well known that A is the infinite generator of a strongly continuous cosine function $(C(t))_{t\in\mathbb{R}} on X$. Moreover, A has a discrete spectrum with eigen values of the form $-n^2, n \in N$, and the corresponding normalized eigenfunctions given by $e_n(\xi) \coloneqq \left(\frac{2}{\pi}\right)^{\frac{1}{2}}\sin(n\xi)$. Also the following properties hold:

(a) The set of functions $\{e_n : n \in N\}$ forms an orthonormal basis of X.

(b) If $\omega \in D(A)$, then $A\omega = \sum_{n=1}^{\infty} -n^2 < \omega\ , e_n > e_n$.

(c) For $\omega \in X, C(t)\omega = \sum_{n=1}^{\infty}\cos(nt) < \omega, e_n > e_n$. The associated sine family is given by $S(t)\omega = \sum_{n=1}^{\infty}\frac{\sin(nt)}{n} < \omega, e_n > e_n, \omega \in X$.

(d) If ψ is the group of translations on X defined by $\psi(t)x(\xi) = \tilde{x}(\xi + t)$, where $\tilde{x}(.)$ is the extension of $x(.)$ with period 2π, then $C(t) = \frac{1}{2}[\psi(t) + \psi(-t)]; A = B^2$where B is the infinitesimal generator of ψ and $\{x \in H^1\,]0,\pi[: x(0) = x(\pi) = 0\}$, see [32] for more details.

Consider the impulsive second-order partial neutral differential equation with control $\hat{\mu}(t,.)$

$$\frac{\partial}{\partial t}\left[\frac{\partial}{\partial t}\omega(t,\tau) - \int_{-\infty}^{t}\int_0^{\pi} b(t-s,\eta,\tau)\omega(s,\eta)d\eta ds\right]$$
$$= \frac{\partial^2}{\partial t^2}\omega(t,\tau) + \alpha\frac{\partial}{\partial t}\omega(t,\tau) + \int_0^{\pi}\beta(s)\frac{\partial}{\partial t}\omega(t,s)ds + \hat{\mu}(t,\tau)$$
$$+ \int_{-\infty}^{t} c(s-t)\omega(s,\tau)ds. \quad (4.1)$$

For $t \in J = [0,T], \tau \in [0,\pi]$, subject to the initial condition

$$\omega(t,0) = \omega(t,\pi) = 0,\ t \in J, \frac{\partial}{\partial t}\omega(0,\tau) = \tau(\pi),$$
$$\omega(\xi,\tau) = \varphi(\xi,\tau),\ \ \xi \in]-\infty,0], \qquad 0 \le \tau \le \pi,$$
$$\Delta\omega(t_i)(\tau) = \int_{-\infty}^{t_i}\gamma_i(t_i - s)\omega(s,\tau)ds, i = 1,2,\ldots,n,$$
$$\Delta\omega'(t_i)(\tau) = \int_{-\infty}^{t_i}\widehat{\gamma_i}(t_i - s)\omega(s,\tau)ds, i = 1,2,\ldots,n,$$

where that assume that $\varphi(s)\tau = \varphi(s,\tau), \varphi(0,.) \in H^1([0,\pi])$ and
$0 < t_1 < \cdots < t_n < T$. Here α is prefixed real number $\beta \in L^2([0,\pi])$.

We have to show that there exists a control $\hat{\mu}$ which steers (4.1) from any specified initial state to the final state in a Banach space X.

To do this, we assume that the functions $b, c, \gamma_i, \widehat{\gamma_i}$ satisfy the following condtions:

(i) The functions $b(s,\eta,\tau), \frac{\partial b(s,\eta,\tau)}{\partial t}$ are continuous and measurable , $b(s,\eta,\pi) = b(s,\eta,0) = 0$ for every $(s,\eta) \in]-\infty,0] \times J$ and

$$L_g = max\left\{\left(\int_0^{\pi}\int_{-\infty}^{0}\int_0^{\pi}\frac{1}{\rho(s)}\left(\frac{\partial^i b(s,\eta,\tau)}{\partial \tau^i}\right)d\eta ds d\tau\right)^{\frac{1}{2}} : i = 0,1\right\} < \infty.$$

(ii) The functions $c(.), \gamma_i, \widehat{\gamma_i}$ are continuous,

$$L_f = \left(\int_{-\infty}^{0}\frac{(c^2(-s))}{\rho(s)}ds\right)^{\frac{1}{2}}, L_{I_i} = \left(\int_{-\infty}^{0}\frac{({\gamma_i}^2(-s))}{\rho(s)}ds\right)^{\frac{1}{2}},$$

$$L_{J_i} = \left(\int_{-\infty}^{0}\frac{(\widehat{\gamma_i}^2(-s))}{\rho(s)}ds\right)^{\frac{1}{2}},\ i = 1, \dots, n, \text{ are finite.}$$

Assume that the bounded linear operator $B : U \subset J \to X$ defined by

$$(Bu)(t)(\tau) = \hat{\mu}(t,\tau), \tau \in [0,\pi].$$

Define on operator $\mathcal{D}: X \to X, g: J \times \mathcal{B} \times X \to X,\ f: J \times \mathcal{B} \times X \to X$ and $I_i,\ J_i :\ \mathcal{B} \to X$by

$$\mathcal{D}\psi(\tau) = \alpha\psi(t,\tau) + \int_0^{\pi}\beta(s)\psi(t,s)ds,$$

$$g(\psi)(\tau) = \int_{-\infty}^{0}\int_0^{\pi} b(-s,\eta,\tau)\psi(s,\eta)d\eta ds,$$

$$f(\psi)(\tau) = \int_{-\infty}^{t} c(-s)\psi(s,\tau)ds,$$

$$I_i(\psi)(\tau) = \int_{-\infty}^{0}\gamma_i(-s)\psi(s,\tau)ds,$$

$$J_i(\psi)(\tau) = \int_{-\infty}^{0}\widehat{\gamma_i}(-s)\psi(s,\tau)ds,$$

Further, the linear operator W is given by

$$(Wu)(\tau) = \sum_{n=1}^{\infty}\int_0^{\pi}\frac{1}{n}\sin ns(\hat{\mu}(s,\tau),e_n)e_n ds\ ,\ \ \tau \in [0,\pi].$$

Assume that this operator has a bounded inverse operator W^{-1} in $L^2([J,U])/\ker W$. With the choice of $A, \mathcal{D}, B, W, f, g, I_i$and J_i, (2.1)-(2.4) is the abstract formation of (4.1). Moreover the functions $, \mathcal{D}, f, g, I_i$ and $J_i, i = 1,2, \dots, n$ are bounded linear operators with$\|\mathcal{D}\|_{\mathcal{L}(X)} \le |\alpha| + \|\beta\|_{L^2(0,T)}$, $\|f\| \le L_f$, $\|g\| \le L_g$, $\|I_i\| \le L_{I_i}$ and $\|J_i\| \le L_{J_i}$. Hence the damped second-order impulsive neutral system (4.1) is controllable.

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