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An Multi Resolution Using Discrete Wavelet Transforms and Fractals Transforms

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Abstract:- The proposed multi resolution fractals coders are image compression schemes that combine wavelets and fractals transforms. The main idea behind all fractal algorithms is to exploit the similarities present within many natural images: one block image is represented by an affine transform of larger block taken from the image it self .The characteristics property of fractal coders is to exploit similarities between scales. Wavelets transforms perform Multiresolution decompositions of images ,i.e decomposition of the originals images into sub images at different scales. Standard DWT (Discrete Wavelet Transform), being non-redundant, is a very powerful tool for many non-stationary Signal Processing applications. It presents a detailed review of Wavelet Transforms (WT) including standard DWT and its extensions. Denoising and Edge detection applications are investigated with DT-DWTs. Promising results are compared with other DWT extensions, and with the classical approaches..Multiresolution fractals coders introduces degrees of freedom on these constraints The research is includes other extensions of the Multiresolution fractal coders are wavelet transform and fractals are wavelet transform

Keywords:- Multi resolution, wavelets transforms, fractals transforms

I. INTRODUCTION

The main idea behind all fractal coding algorithms is to exploit the similarities present within many natural images: one block of an image is represented by an affine transform of another larger block taken from the image itself .The characteristic property of fractal coders is to exploit similarities between different scales. Wavelet transforms perform multiresolution decompositions of images, i.e, decompositions of the original images into sub images at different scales. The translation of the fractal property in the wavelet transform domain is straightforward: multiresolution decompositions through wavelet transforms of fractal coded images reveal strong relationships between sub images at different scales. These relationships limit the frequency content. Multiresolution fractal coders introduce degrees of freedom on these constraints

A 'wavelet' is a small wave which has its energy concentrated in time. It has an oscillating wavelike characteristic but also has the ability to allow simultaneous time and frequency analysis and it is a suitable tool for transient, non-stationary or time-varying phenomena .Wavelets are used to characterize a complex pattern as a series of simple patterns and coefficients that, when multiplied and summed, reproduce the original pattern.

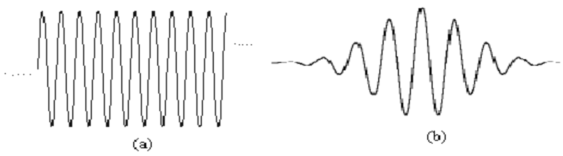

Fig Representation of wave and wavelet

Waves are smooth, predictable and everlasting, whereas wavelets are of limited duration, irregular and may be asymmetric. Waves are used as deterministic basis functions in Fourier analysis for the expansion of functions (signals), which are time-invariant, or stationary. The important characteristic of wavelets is that they can serve as deterministic or non-deterministic basis for generation and analysis of the most natural signals to provide better time-frequency representation, which is not possible with waves using conventional Fourier analysis

The wavelet analysis procedure is to adopt a wavelet prototype function, called an 'analysing wavelet' or 'mother wavelet'. Temporal analysis is performed with a contracted, high frequency version of the prototype wavelet, while frequency analysis is performed with a dilated, low frequency version of the same wavelet .

Mathematical formulation of signal expansion using wavelets gives Wavelet Transform (WT) pair, which is analogous to the Fourier Transform (FT) pair. Discrete-time and discrete-parameter version of WT is termed as Discrete Wavelet Transform (DWT). DWT can be viewed in a similar framework of Discrete Fourier Transform (DFT) with its efficient implementation through fast filterbank algorithms similar to Fast Fourier Transform (FFT) algorithms.

Wavelets are functions defined over a finite interval. The basic idea of the wavelet transform is to represent an arbitrary function $f(x)$ as a linear combination of a set of such wavelets or basis functions. These basis functions are obtained from a single prototype wavelet called the mother wavelet by dilations (scaling) and translations (shifts). The purpose of wavelet transform is to change the data from time-space domain to time-frequency domain which makes better compression results. The simplest form of wavelets, the Haar wavelet function .As discussed earlier, for image compression, loss of some information is acceptable. The fundamental idea behind wavelets is to analyze the signal at different scales or resolutions, which is called multi resolution. Wavelets are a class of functions used to localize a given signal in both space and scaling domains. A family of wavelets can be constructed from a mother wavelet. Compared to Windowed Fourier analysis, a mother wavelet is stretched or compressed to change the size of the window. In this way, big wavelets give an approximate image of the signal, while smaller and smaller wavelets zoom in on details. Therefore, wavelets automatically adapt to both the high-frequency and the low-frequency components of a signal by different sizes of windows. Any small change in the wavelet representation produces a correspondingly small change in the original signal, which means local mistakes will not influence the entire transform. The wavelet transform is suited for non stationary signals, such as very brief signals and signals with interesting components at different scales.The need of simultaneous representation and localisation of both time and frequency for non-stationary signals (e.g. music, speech, images) led toward the evolution of wavelet transform from the popular Fourier transform. Different 'time-frequency representations' (TFR) are very informative in understanding and modelling of WT

II. DISCRETE WAVELET TRANSFORM

Discrete wavelet transform is a multi resolution decomposition of a signal. Considering an image, 1 level .DWT involves applying a low pass and a high pass filters along the columns and then the rows respectively. The low pass filter applied along a certain direction extracts the low frequency (approximation) coefficients of a signal. On the other hand, the high pass filter extracts the high frequency (detail) coefficients of a signal. In order to compare wavelet methods, a MinImage was originally created to test one type of wavelet and the additional functionality was added to Image to support other wavelet types, and the EZW coding algorithm was implemented to achieve better compression results.
The wavelet image compressor, MinImage, is designed for compressing either 24-bit true color or 8-bit gray scale digital images. It was originally created to test Haar wavelet using sub band coding. To compare different wavelet types, other wavelet types, including Daubechies and birothogonal spine wavelets were implemented.. A very useful property of MinImage is that different degrees of compression and quality of the image can be obtained by adjusting the compression parameters through the interface. The user can trade off between the compressed image file size and the image quality. The user can also apply different wavelets to different kind of images to achieve the best compression results.

For transform based compression, JPEG compression schemes based on DCT (Discrete Cosine Transform) have some advantages such as simplicity, satisfactory performance, and availability of special purpose hardware for implementation. However, because the input image is blocked, correlation across the block boundaries cannot be eliminated. This results in noticeable and annoying “blocking artifacts”" particularly at low bit rates .wavelet-based schemes achieve better performance than other coding schemes like the one based on DCT. Since there is no need to block the input image and its basis functions have variable length, wavelet based coding schemes can avoid blocking artifacts. Wavelet based coding also facilitates progressive transmission of images.

In mathematics, a wavelet series is a representation of a square-integral(real-or complex-valued) function by a certain orthonormal series generated by a wavelet. This article provides a formal, mathematical definition of an orthonormal wavelet and of the integral wavelet transform.

A function $\psi \in L^2(\mathbb{R})$ is called an orthonormal wavelet if it can be used to define a Hilbert basis, that is a complete orthonormal system, for the Hilbert space $L^2(\mathbb{R})$ of square integrable functions. The Hilbert basis is constructed as the family of functions $\{\psi_{jk} : j, k \in \mathbb{Z}\}$ by means of dyadic translations and dilations of ψ,

$$\psi_{jk}(x) = 2^{j/2}\psi(2^j x - k)$$

for integers $j, k \in \mathbb{Z}$. This family is an orthonormal system if it is orthonormal under the inner product

$$\langle \psi_{jk}, \psi_{lm} \rangle = \delta_{jl}\delta_{km}$$

where δ_{jl} is the Kronecker delta and $\langle f, g \rangle$ is standard inner product $\langle f, g \rangle = \int_{-\infty}^{\infty} f(x)\overline{g(x)}dx$ on $L^2(\mathbb{R})$. The requirement of completeness is that every function $h \in L^2(\mathbb{R})$ may be expanded in the basis as

$$h(x) = \sum_{j,k=-\infty}^{\infty} c_{jk}\psi_{jk}(x)$$

with convergence of the series understood to be convergence in norm. Such a representation of a function *f* is known as a wavelet series. This implies that an orthonormal wavelet is self-dual.

2.2 Wavelet transform:

The integral wavelet transform is the integral transform defined as

$$\left[W_\psi f\right](a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} \overline{\psi\left(\frac{x-b}{a}\right)} f(x)dx$$

The wavelet coefficients c_{jk} are then given by

$$c_{jk} = \left[W_\psi f\right](2^{-j}, k2^{-j})$$

Here, $a = 2^{-j}$ is called the binary dilation or dyadic dilation, and $b = k2^{-j}$ is the binary or dyadic position.

2.3 Wavelet compression:

Wavelet compression a form of data compression well suited for image compression (sometimes also video compression and audio compression). Notable implementations are JPEG 2000, DjVu and ECW for still images, REDCODE, CineForm, the BBC's Dirac, and Ogg Tarkin for video. The goal is to store image data in as little space as possible in a file. Wavelet compression can be either lossless or lossy. Using a wavelet transform, the wavelet compression methods are adequate for representing transients, such as percussion sounds in audio, or high-frequency components in two-dimensional images, for example an image of stars on a night sky. This means that the transient elements of a data signal can be represented by a smaller amount of information than would be the case if some other transform, such as the more widespread discrete cosine transform, had been used. Wavelet compression is not good for all kinds of data: transient signal characteristics mean good wavelet compression, while smooth, periodic signals are better compressed by other methods, particularly traditional harmonic compression (frequency domain, as by Fourier transforms and related).

First a wavelet transform is applied. This produces as many coefficients as there are pixels in the image (i.e.: there is no compression yet since it is only a transform). These coefficients can then be compressed more easily because the information is statistically concentrated in just a few coefficients. This principle is called transform coding. After that, the coefficients are quantized and the quantized values are entropy encoded and/or run length encoded. A few 1D and 2D applications of wavelet compression use a technique called "wavelet footprints The wavelet transform can provide us with the frequency of the signals and the time associated to those frequencies, making it very convenient for its application in numerous fields. For instance, signal processing of accelerations for gait analysis, and for fault detection

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III. FRACTAL COMPRESSION

Fractal compression is a lossy compression method for digital images, based on fractals. The method is best suited for textures and natural images, relying on the fact that parts of an image often resemble other parts of the same image Fractal algorithms convert these parts into mathematical data called "fractal codes" which are used to recreate the encoded image. Fractal image representation can be described mathematically as an iterated function system (IFS).

3.1 For Binary Images:

We begin with the representation of a binary image, where the image may be thought of as a subset of $\mathbb{R}^2$. An IFS is a set of contraction mappings $f_1,...,f_N$,

$$f_i : \mathbb{R}^2 \to \mathbb{R}^2.$$

According to these mapping functions, the IFS describes a two-dimensional set S as the fixed point of the Hutchinson operator

$$H(A) = \bigcup_{i=1}^{N} f_i(A), \quad A \subset \mathbb{R}^2.$$

That is, H is an operator mapping sets to sets, and S is the unique set satisfying $H(S) = S$. The idea is to construct the IFS such that this set S is the input binary image. The set S can be recovered from the IFS by fixed point iteration: for any nonempty compact initial set A_0, the iteration $A_{k+1} = H(A_k)$ converges to S. The set S is self-similar because $H(S) = S$ implies that S is a union of mapped copies of itself:

$$S = f_1(S) \cup f_2(S) \cup \cdots \cup f_N(S)$$

So we see the IFS is a fractal representation of S.

3.2 Extension to Grayscale:

IFS representation can be extended to a grayscale image by considering the image's graph as a subset of $\mathbb{R}^3$. For a grayscale image $u(x,y)$, consider the set $S = \{(x,y,u(x,y))\}$. Then similar to the binary case, S is described by an IFS using a set of contraction mappings $f_1,...,f_N$, but in $\mathbb{R}^3$,

$$f_i : \mathbb{R}^3 \to \mathbb{R}^3.$$

3.3 Encoding

A challenging problem of ongoing research in fractal image representation is how to choose the $f_1,...,f_N$ such that its fixed point approximates the input image, and how to do this efficiently. A simple approach[1] for doing so is the following:

1. Partition the image domain into blocks R_i of size $s{\times}s$.
2. For each R_i, search the image to find a block D_i of size $2s{\times}2s$ that is very similar to R_i.
3. Select the mapping functions such that $H(D_i) = R_i$ for each i.

In the second step, it is important to find a similar block so that the IFS accurately represents the input image, so a sufficient number of candidate blocks for D_i need to be considered. On the other hand, a large search considering many blocks is computationally costly. This bottleneck of searching for similar blocks is why fractal encoding is much slower than for example DCT and wavelet based image representations.

3.4 Features:

With fractal compression, encoding is extremely computationally expensive because of the search used to find the self-similarities. Decoding, however is quite fast. While this asymmetry has so far made it impractical for real time applications, when video is archived for distribution from disk storage or file downloads fractal compression becomes more competitive. At common compression ratios, up to about 50:1, Fractal compression provides similar results to DCT-based algorithms such as JPEG.At high compression ratios fractal compression may offer superior quality. For satellite imagery, ratios of over 170: have been achieved with acceptable results. Fractal video compression ratios of 25:1-244:1 have been achieved in reasonable compression times (2.4 to 66 sec/frame). Compression efficiency increases with higher image complexity and color depth, compared to simple grayscale images.

In Fractal Image Coding the image to be encoded is partitioned into non-overlapping range blocks. For each of these range blocks a larger domain block of the same image has to be determined such that a contractive (geometrical and luminance) transformations of this block is a good approximation of the range block. At the decoder all transformations are iteratively applied to an arbitrary initial image which then converges to the fractal approximation of the image. Fractal Coding relies on the existence of self similarities within the image. As “fractal self similarity” can only be found in certain regions of natural images, fractal coding fails for non-fractal image contents. In its original form fractal coding suffered from low coding efficiency, difficulties to obtain high quality encoding of images and blocking artifacts at low bit rates and exhaustive inherent coding time. Blocking artifacts can be avoided, if fractal coding is performed in the wavelet domain. Fractal coding assumes that all wavelet sub trees that make up an image can be described by other wavelet sub trees from coarser scales. In the image compression field, high compression is needed without losing the quality of an image.

IV. RELATED WORK

Fractal image coding in the wavelet domain has quite different characteristics from the spatial domain coders and can be interpreted as the prediction of a set of wavelet coefficients in the higher frequency subbands from those in the lower ones. A contractive mapping associates a domain tree of wavelet coefficients with a range tree that it approximates.

Various structures have been used for the domain to range mappings. The potential of applying the theory of iterated function systems to the problem of image compression was recognized by Barnsley and Sloan . They patented their idea in 1990 . A method of fractal encoding that utilizes a system of domain and range sub image blocks was introduced . This approach is the basis for the most fractal encoders today. It has been enhanced by Fisher and a number of others. This block fractal encoding method partitions an image into disjoint range sub images and defines a system of overlapping domain sub images. For each range, the encoding process searches for the best domain and affine transformation that maps that domain onto the range. Image structure is mapped onto the system of ranges, domains and transformations. Much of the recent research in fractal image compression has focused on reducing long encoding times. Wavelet transform approaches to image compression exploit redundancies in scale. Wavelet transform data can be organized into a sub tree structure that can be efficiently coded. In Hybrid fractal-wavelet techniques domain-range transformation idea of fractal encoding is applied to the realm of wavelet sub trees. Improved compression and decoded image fidelity is obtained. A range tree is fractally encoded by a bigger domain tree. The approximating procedure is very similar to that in the spatial domain: sub sampling and determining the orientation and scaling factor. Note that one does not need an additive constant because the wavelet tree does not have a constant offset. Matching the size of a domain tree with that of a range tree by truncating all coefficients in the highest subbands of the domain tree is called Subsampling. The orientation operation consists of a combination of a 90 degree rotation and a flip, and it is done within each subband. A switch of HL (High-Low) and LH (Low-High) subbands is the next step. The scale factor is then multiplied witheach wavelet coefficient in the tree.

V. METHODOLOGY

5.1 Encoder:

The block diagram of the Fractal based wavelet encoderis shown in Fig. 1. The image is decomposed into several subbands by applying DWT (discrete wavelet transform). Haar wavelet is applied two times to the original image. It transforms the image into wavelet coefficients. The approximation subband wavelet coefficients are taken from the transformed image.The image is divided into range blocks. The variance of range blocks are calculated using the following
equation 1.

Var{R}= 1
_B_2 Σ (_ri, j_ – μ_R_)2 0≤_i, j_≤_B_---------_(1)_

where R: Range block, B: Size of the block, $r_{i,j}$: Gray
level value at position (i, j), μR: Mean of the block. The
principle behind this method is the blocks which are having the minimum variance can be coded by mean; otherwise it is coded by contractive affine transform. The first one is representing transformation whether it is of affine or mean type and the next one is the domain block position and the isometric transformation type. If the range block is coded by mean, then that block is represented by 1 and other block by 0 if it is affine transformation. It is followed by x position and y position which is represented by a maximum of two digits each. It also contains the rotation number which describes the type of isometric transformations. There are totally eight isometric transformations. Hence it takes a value between 0 and 7. The above information is kept in a file, which is the coded data of the input image.

5.2 Decoder:

The Fractal based Wavelet Compressed (FWC) image from encoder is the input to the corresponding decoder. The coded file is read one by one. If the bit is 1, then the block is replaced by mean pixel. If it is 0, then the subsequent block position is read from coded file. Hence, the block is replaced by the corresponding domain block which is taken from domain pool using block position.

VI. RESULTS AND EVALUATION

Wavelet transform can represented in continuous as well as in discrete domain as
The practical usefulness of DWT comes from its Multi-Resolution Analysis (MRA) ability and efficient Perfect Reconstruction (PR) filterbank structures
Multiresolution analysis (or Multiscale analysis) consists of a sequence of embedded subspaces $\ldots V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \ldots$..of $L^2(\Re)$.

The MRA follows the following conditions:

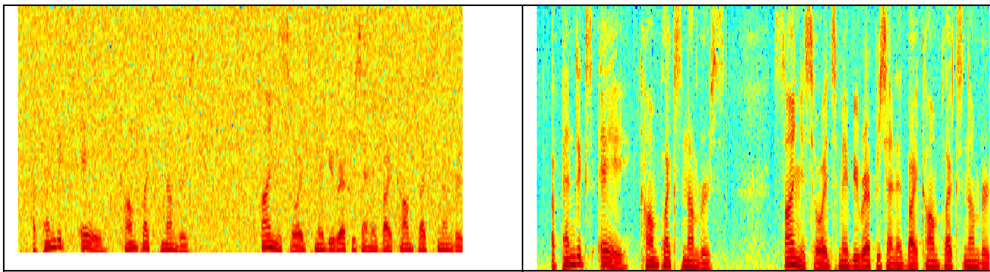
1. $V_j \subset V_{j+1}$, $j \in Z$
2. $V_{-\infty} = \{0\}$ and $V_{\infty} = L^2$
3. $f(t) \in V_j \Leftrightarrow f(2t) \in V_{j+1}$
4. V2 = V0 +W0 +W1
5. $L^2 = \ldots + W_{-2} + W_{-1} + W_0 + W_1 + W_2 + \ldots = V_0 + W_1 + W_2 + \ldots$
6. $W_{-\infty} + \ldots + W_{-2} + W_{-1} = V_0$

Standard DWT

Methods and Initial **Parameters**	Standard DWT		**DT-DWT(K)**	
	MSE	**SNR (dB)**	**MSE**	**SNR (dB)**
Thresholding Rule	MSEi= 824	SNRi= 5.00	MSEi= 824	SNRi= 5.00
Hard	343	8.81	212	10.90
Soft	237	10.40	175	11.72
Wiener	625	6.20	630	6.17
Raised-cosine	205	11.04	161	12.10

It is clear from the performance results of table that the denoising capability of redundant CWT (namely DT-DWT(K)) is superior than the standard DWT for audio speech with lower initial SNR. In both cases, hard thresholding and Wiener filtering perform poorer whereas soft thresholding and raised-cosine-law perform better.

Table: The MSE and SNR of denoised audio signal (y) from the observed noisy signal (x) with respect to original signal (s). The initial value of MSE and SNR for noisy signal are MSEi= 824 , SNRi= 5.00 dB . The wavelets used for standard DWT is 'bior6.8' and for DT-DWT(K) is 'near_sym_b' and 'qshift_b'. The maximum decomposition level employed is $J = \log_2(N)$, where N = 65536 = size of audio signal.



Original signal	Noisy signal

FIG: Spectrograms of audio signals with standard DWT based denoising

DWT HARD

DWT SOFT

DWT(K) HARD DWT(K)SOFT

WIENER **RASIED COSINE**

Fig:Wavelet Transform based methods for denoising of 'Lenna' image with reference to table values

Performance Measure

The quantitative measures for 2-D denoising, namely MSE (Mean Square error) and PSNR (Peak Signal to Noise Ratio) are determined as:

MSE = 1/NM (1ΣΣ=[s(n,m)-y(n,m)]2

PSNR = 10 $\log_{10}$\{255/MSE\}

where, s is an original image and y is a recovered image from a noisy image x. The qualitative performance is evaluated through human visual system by observing the recovered images with various algorithms.

The performance results of various algorithms can be evaluated for low and high noise conditions as follows:

1 Under low noise conditions (σ=10), conventional filtering methods namely Median and Weiner perform better than standard DWT. Though the performance of Mean filter is poorer than standard DWT.

2. The denoising capability (σ=10) of both DT-DWTs is better than SWT and standard DWT for all natural images as well as for synthetic 'Pattern' image. The performance of both DT-DWTs is nearly same with DT-DWT(K) slightly better then DT-DWT(S).

3. Under high noise conditions (σ=40), all conventional filtering methods give poor denoising results than even standard DWT. For all images, performance of both DT-DWTs is better than SWT and standard DWT. The performance of DT-DWT(K) is the best using both hard and soft thresholding.

4. The lowest values of optimum threshold (for both hard and soft thresholding) for DT-DWT(K) in all noise conditions (σ= 10 or 40) suggest its superior denoising capabilities with improved feature preservation (with less blurring).

5. The effect of improved directionality on denoising for redundant CWT (both DT-DWTs) compared to less directional standard DWT is quite clear for 'pattern' image for all noise conditions.

6. From human visual perspective, the performance of various algorithms for high noise conditions is quite clear. The performance of DT-DWTs is distinguishably superior to standard DWT. But under low noise conditions, minute differences are very difficult to perceive hence all wavelet based methods seem to have nearly same visual effects.

Future scope:

Future work includes other extensions of the DWT is extensively used in its non-redundant form known as standard DWT. The filterbank implementation of standard DWT for images is viewed as 2-D DWT. There are certain applications for which the optimal representation can be achieved. through more redundant extensions of standard DWT such as WP (Wavelet Packet Transform) and SWT (Stationary Wavelet Transform).

VII. CONCLUSION

DWTs are superior to standard for all 1-D and 2-D denoising application. The choice of shrinkage (thresholding) strategy, and selection of optimum threshold value are very crucial for wavelet shrinkage denoising using any form of WT. DT-DWTs are especially efficient in higher noise conditions. Entropy based WP is not suitable for generalised denoising applications in poor SNR conditions. Multiresolution fractal coders present all the advantages of conventional fractal coders and propose solutions to some of their drawbacks.

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