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Transportation Programming Under Uncertain Environment

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Abstract:- In a transportation problem the cost of transportation from source to destination depends on many factors which may not be deterministic in nature. In this work, the unit cost of transportation is considered to be uncertain and the rough cost is assigned to the model. In the solution of such a model, two solutions as pessimistic and optimistic solutions are obtained which suggest a range for the actual solution. Further, using uncertain distribution, one compromise solution process is also proposed.

Keywords:- Transportation Problem, Rough set, Rough variable, Uncertainty measure, Uncertain distribution

I. INTRODUCTION

Transportation problem is one of the important linear programming problems. Many efficient methods have been developed to solve a transportation problem where the unit cost of transportation, demand and supply are all deterministic. In many practical situations, the unit cost of transportation may not deterministic in nature. The cost depends upon many factors like road condition, traffic load causing delay in delivery, break down of vehicles, labor problem for loading & unloading, which may increase the cost. All these factors are uncertain which cannot be predicted beforehand due to insufficient information, unpredictable environment.

Transportation problem in various types of uncertain environment such as fuzzy, random are studied by many researchers. In this paper we have developed a transportation model with rough and uncertain environment.

Rough set theory was first developed by Pawlak[12] in 1982. Since then many researchers have developed the theoretical aspects and applied the concept to solve problems related to data mining, data development analysis, neural network, signal processing etc.

B.Liu [1] has proposed the uncertainty theory to deal with the problems related to uncertain environment. B Liu [2] has also proposed the concept of rough variable which is a measurable function from rough space to the set of real numbers.

Many researchers have used concept of rough variables in some optimization problems. Xu and Yao [8] studied a two-person zero-sum matrix games with payoffs as rough variables. Youness[5] introduced a rough programming problem considering the decision set as a rough set. Xu *et al* [9] proposed a rough DEA model to solve a supply chain performance evaluation problem with rough parameters. Xiao and Lai [11] have considered power-aware VLIW instruction scheduling problem with power consumption parameters as rough variables. Kundu [10] proposed some solid transportation models with crisp and rough costs. In this paper we have formulated transportation problems considering the unit cost of transportation from source to destination as rough variables where we get pessimistic and optimistic solution of the problem under certain level of trust(level of confidence). Again using the uncertain distribution, we have proposed solution method for a compromise solution.

The rest of the paper is organized as follows: In Section II, we provide some definitions and properties of rough set, rough variable and uncertain distribution. In Section III, we present our proposed transportation problems and model formulation. The solution procedure is discussed in section IV. One numerical example is given in section V and finally the conclusion and scope of further work is given in section VI.

II. PRELIMINARIES

A. **Rough set**:

Rough set theory developed by Z. Pawlak[12],[13] is a mathematical tool for dealing with uncertain and incomplete data without any prior knowledge about the data. We deal only with the available information provided by the data to generate conclusion.

Let U be a finite non empty set called the universal and let R be a binary relation defined on U. Let R be a equivalence relation and R(x) be the equivalence class of the relation which contain x. R shall be referred as indiscernibility relation.

For any $X \subseteq U$, the lower and upper approximation of X is defined by $\underline{R}(X) = \{x \in U : R(x) \subseteq X\}$, $\overline{R}(X) = \{x \in U : R(x) \cap X \neq \phi\}$.

The lower approximation $\underline{R}(X)$ is exact set contained in X so that the object in $\underline{R}(X)$ are members of x with certainty on the basis of knowledge in R, where the objects in the upper approximation $\overline{R}(X)$ can be classified as possible members of X. The difference between the upper and lower approximation of X will be called as R-boundary of X and is defined by $BN_R(X) = \overline{R}(X) - \underline{R}(X)$.

The set X is R-exact if $BN_R(X) = \phi$, otherwise the set is R- rough set.

B. Rough variable

The concept of rough variable is introduced by Liu[2]as uncertain variable. The following definitions are based on Liu[2].

Definition 1 **:** Let Λ be a non empty set, A be an σ - algebra of subsets of Λ, Δ be an element in A , and π be a non negative, real- valued, additive set function on A. Then $(\Lambda, \Delta, A, \pi)$ is called a rough space.

Definition 2: A rough variable ξ on the rough space $(\Lambda, \Delta, A, \pi)$ is a measurable function from Λ to the set of real numbers $\Re$ such that for every Borel set B of $\Re$, we have $\{\lambda \in \Lambda \mid \xi(\lambda) \in B\} \in A$. Then the lower and upper approximation of the rough variable ξ are defined as follows

$$\underline{\xi} = \{\xi(\lambda) \mid \lambda \in \Delta\} \ , \quad \overline{\xi} = \{\xi(\lambda) \mid \lambda \in \Lambda\}$$

Definition 3**:** Let $(\Lambda, \Delta, A, \pi)$ be a rough space. Then the upper and lower trust of event A is defined by

$$T\underline{r}(A) = \frac{\pi\{A\}}{\pi\{\Lambda\}} \quad \text{and} \quad T\overline{r}(A) = \frac{\pi\{A \cap \Delta\}}{\pi\{\Lambda\}}$$

The trust of the event A is defined as

$$Tr(A) = \frac{1}{2}\left(T\underline{r}(A) + T\overline{r}(A)\right)$$

The trust measure satisfies the followings:

$$Tr(\Lambda) = 1 \ , \ Tr(\phi) = 0$$

$$Tr(A) \leq Tr(B) \text{ where } A \subseteq B$$

$$Tr(A) + Tr(A^c) = 1$$

Definition.4**:** Let ξ_1 , ξ_2 be rough variables defined on the rough space $(\Lambda, \Delta, A, \pi)$. Then their sum and product are defined as

$$(\xi_1 + \xi_2)(\lambda) = \xi_1(\lambda) + \xi_2(\lambda)$$

$$(\xi_1 . \xi_2)(\lambda) = \xi_1(\lambda) . \xi_2(\lambda)$$

Definition 5**:** Let ξ be rough variables defined on the rough space $(\Lambda, \Delta, A, \pi)$ and $\alpha \in (0,1]$ then $\xi_{\sup}(\alpha) = \sup\{r \mid Tr\{\xi \geq r\} \geq \alpha\}$ is called α -optimistic value of ξ.

$\xi_{\inf}(\alpha) = \inf\{r \mid Tr\{\xi \leq r\} \geq \alpha\}$ is called α -pessimistic value of ξ.

Definition 6**:** Let ξ be rough variables defined on the rough space $(\Lambda, \Delta, A, \pi)$. The expected value of ξ is defined by

$$E(\xi) = \int_0^{\infty} Tr\{\xi \geq r\}\, dr - \int_{-\infty}^{0} Tr\{\xi \leq r\}\, dr$$

Definition 7**:** The trust density function $f : R \to [0,\infty)$ of a rough variable ξ is a function such that $\phi(x) = \int_{-\infty}^{x} f(y)dy$ holds for all $x \in (-\infty, \infty)$, where ϕ is trust distribution of ξ. If $\xi = ([a,b],[c,d])$ be a rough variable such that c $\leq$ a < b $\leq$ d, then the trust distribution $\phi(x) = Tr\{\xi \leq x\}$ is

$$\phi(x)=\begin{cases}0 & \text{if } x\le c\\ \dfrac{x-c}{2(d-c)} & \text{if } c\le x\le a\\ \dfrac{[(b-a)+(d-c)]x+2ac-ad-bc}{2(b-a)(d-c)} & \text{if } a\le x\le b\\ \dfrac{x+d-2c}{2(d-c)} & \text{if } b\le x\le d\\ 1 & \text{if } x\ge d\end{cases}$$

And the trust density function is defined as

$$f(x)=\begin{cases}\dfrac{1}{2(d-c)} & \text{if } c\le x\le a \text{ or } b\le x\le d\\ \dfrac{1}{2(b-c)}+\dfrac{1}{2(d-c)} & \text{if } a\le x\le b\\ 0 & \text{otherwise}\end{cases}$$

α-optimistic value to $\xi=([a,b],[c,d])$ is

$$\xi_{\sup}(\alpha)=\begin{cases}(1-2\alpha)d+2\alpha c, & \text{if } \alpha\le\dfrac{d-b}{2(d-c)};\\ 2(1-\alpha)d+(2\alpha-1)c, & \text{if } \alpha\ge\dfrac{2d-a-c}{2(d-c)};\\ \dfrac{d(b-a)+b(d-c)-2\alpha(b-a)(d-c)}{(b-a)+(d-c)}, & \text{otherwise.}\end{cases}$$

α-pessimistic value to $\xi=([a,b],[c,d])$ is

$$\xi_{\inf}(\alpha)=\begin{cases}(1-2\alpha)c+2\alpha d\ , & \text{if } \alpha\le\dfrac{a-c}{2(d-c)};\\ 2(1-\alpha)c+(2\alpha-1)d\ , & \text{if } \alpha\ge\dfrac{b+d-2c}{2(d-c)};\\ \dfrac{c(b-a)+a(d-c)+2\alpha(b-a)(d-c)}{(b-a)+(d-c)}, & \text{otherwise.}\end{cases}$$

The expected value of ξ is $E(\xi)=\dfrac{1}{4}(a+b+c+d)$

C . Uncertainty Theory

B.Liu [1],[2] has developed uncertainty theory which is considered as a new approach to deal with indeterminacy factors when there is a lack of observed data. In this section, some basic concepts of uncertainty theory has been reviewed which shall be used to establish a compromise solution of transportation problem under uncertainty.

Uncertainty Measure

Let L be a δ - algebra on a nonempty set Γ. A set function M : L $\rightarrow$ [0,1] is called an uncertain measure if it satisfies the following axioms

Axiom 1: (Normality axiom) M (Γ) = 1 for the universal set Γ

***Axiom 2*:** (Duality axiom) $M(\Lambda)+M(\Lambda^c)=1$ for every event Λ

Axiom 3: (sub additive axiom) For every countable sequence of events $\Lambda_1,\Lambda_2,\ldots$ we have

$$M\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\}\le\sum_{i=1}^{\infty}M(\Lambda_i)$$

The triplet (Γ, L, M) is called an uncertain space.

Axiom 4: (Product measure) Let (Γ_k, L_k, M_k) be uncertainty spaces for k = 1, 2……The product uncertain measure M is an uncertain measure satisfying

$$M\{\prod_{k=1}^{\infty}\Lambda_k\}=\wedge_{k=1}^{\infty}M_k(\Lambda_k)$$

where, Λ_k an arbitrary chosen events for L_k for k = 1,2…… respectively.

Uncertain Variable

An uncertain variable ξ is essentially a measurable function from an uncertainty space to the set of real numbers. Let ξ be an uncertain variable. Then the uncertainty distribution of ξ is defined as $\phi(x) = M\{\xi \le x\}$ for any real number x. An uncertain variable ξ is called linear if it has linear uncertainty distribution L(a,b)

$$\phi(x) = \begin{cases} 0 & \text{if } x \le a \\ \dfrac{x-a}{b-a} & \text{if } a \le x \le b \\ 1 & \text{if } x > b \end{cases}$$

where, a and b are real numbers with a < b.

An uncertain distribution ϕ is said to be regular if its inverse function $\phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0,1)$. The linear uncertainty distribution L(a, b) is regular and its inverse uncertainty distribution is $\phi^{-1}(\alpha) = (1-\alpha)a + (\alpha)b$.

III. DESCRIPTION OF THE PROBLEM AND MODEL FORMULATION

A. Crisp Model

Suppose that there are m sources and n destinations. Let a_i be the number of supply units available at source i(i= 1,2….m) and let b_j be the number of demands units required at destination j (j= 1,2….n). Let c_{ij} represents the unit transportation cost for transportation the units from source i to destination j. The objective is to determine the number of units to be transported from source i to destination j, so that the total transportation cost is minimum. Let x_{ij} be the decision variable which denotes the number of units shipped from source i to destination j.

$$\text{Minimize } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

subject to constraint

$$\sum_{j=1}^{n} x_{ij} \le a_i \ , \ i = 1,2,\ldots\ldots,m. \qquad (1)$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j \ , \ j = 1,2,\ldots\ldots,n. \qquad (2)$$

$$\text{and } x_{ij} \ge 0 \qquad (3)$$

B. Rough Model

If the cost c_{ij} is a rough variable of the form

$$\text{Let } c_{ij} = \left(\left[c_{ij}^2, c_{ij}^3\right], \left[c_{ij}^1, c_{ij}^4\right]\right),$$
$$\text{where } c_{ij}^1 \le c_{ij}^2 < c_{ij}^3 \le c_{ij}^4$$

and the objective function becomes a rough variable of the form

$$Z = \left(\left[Z^2, Z^3\right], \left[Z^1, Z^4\right]\right)$$
$$\text{where } \quad Z^r = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^r\, x_{ij}, \quad r = 1,2,3,4$$

Then the rough model of transportation problem becomes

Minimize Z

s.t the constraints (1) – (3).

IV. SOLUTION PROCEDURE

A. Solution for rough model

Since the objective function of transportation problem with rough cost, so we minimize the smallest objective $\bar{Z}$ satisfying $\mathrm{Tr}(Z \le \bar{Z}) \ge \alpha$ where $\alpha \in (0,1]$ is a specified trust (confidence level) i,e. we minimize the α- pessimistic value $Z_{\inf}(\alpha)$ of Z . This implies that the optimum objective value will below the $\overline{Z}$ with a trust level at least α . So the problem becomes Min(Min $\overline{Z}$) .

From the definition of the α- pessimistic value, the above programming becomes

Min $Z^{/}$

s.t the constraints (1)-(3).

where,

$$Z^{/}=\begin{cases}(1-2\alpha)Z^1+2\alpha Z^4, & \text{if } \alpha\le\dfrac{Z^2-Z^1}{2(Z^4-Z^1)}\\ 2(1-\alpha)Z^1+(2\alpha-1)Z^4, & \text{if } \alpha\ge\dfrac{Z^3+Z^4-2Z^1}{2(Z^4-Z^1)}\\ \dfrac{Z^1(Z^3-Z^2)+Z^2(Z^4-Z^1)+2\alpha(Z^3-Z^2)(Z^4-Z^1)}{(Z^3-Z^2)+(Z^4-Z^1)}, & \text{otherwise}\end{cases}$$

Now we formulate another model to minimize the greatest objective $\underline{Z}$ satisfying $\mathrm{Tr}(Z\ge\underline{Z})\ge\alpha$ where $\alpha\in(0,1]$ is a specified trust (confidence level) i,e. we minimize the α- optimistic value $Z_{\sup}(\alpha)$ of Z . This implies that the optimum objective value will below the $\underline{Z}$ with a trust level at least α . So the model becomes Min(Max $\underline{Z}$). From the definition of the optimistic value, the above problem becomes

Min $Z^{//}$

s.t the constraints (1)- (3)

where,

$$Z^{//}=\begin{cases}(1-2\alpha)Z^4+2\alpha Z^1, & \text{if } \alpha\le\dfrac{Z^4-Z^3}{2(Z^4-Z^1)}\\ 2(1-\alpha)Z^4+(2\alpha-1)Z^1, & \text{if } \alpha\ge\dfrac{2Z^4-Z^2-Z^1}{2(Z^4-Z^1)}\\ \dfrac{Z^4(Z^3-Z^2)+Z^3(Z^4-Z^1)-2\alpha(Z^3-Z^2)(Z^4-Z^1)}{(Z^3-Z^2)+(Z^4-Z^1)}, & \text{otherwise}\end{cases}$$

Since for $0.5<\alpha\le 1$, $Z_{\inf}(\alpha)\ge Z_{\sup}(\alpha)$, so solving the problem with trust level α, we conclude that the optimum objective value lie within the range $\left[Z^{//},Z^{/}\right]$ with trust level at least α.

B. Compromise Solution

In solving the rough model of transportation problem, we find two solutions as the α- pessimistic solution and the α-optimistic solution. But in many cases the decision maker shall prefer one set of solution rather being confused with two sets of solutions. In this section, we proposed one solution procedure as a compromise solution. When the unit cost of transportation from ith source to jth destination is a rough variable as

$$c_{ij}=\left(\left[c_{ij}^2,c_{ij}^3\right],\left[c_{ij}^1,c_{ij}^4\right]\right)$$

where $c_{ij}^1\le c_{ij}^2<c_{ij}^3\le c_{ij}^4$

We find the α- pessimistic solution as p and the α-optimistic solution as q which gives two real values, where p < q and corresponding we get an interval [p,q].

We define an uncertain variable ξ in the interval [p,q] whose distribution function is defined as a linear regular uncertain distribution define by Liu [1].

$$\phi(x)=\begin{cases}0 & \text{if } x\le p\\ \dfrac{x-p}{q-p} & \text{if } p\le x\le q\\ 1 & \text{if } x\ge q\end{cases}$$

The inverse uncertainty distribution will give a crisp value c_{ij} as

$$\phi^{-1}(c_{ij},\alpha)=(1-\alpha)p+\alpha q$$

In the above process we will get a set of deterministic cost c_{ij} from the set of rough costs. Then the model can be solved to give a solution which can be considered as a compromise solution.

V. NUMERICAL EXAMPLE

A. *Problems with unit transportation costs as crisp numbers*

Consider a problem with three sources (i= 1,2,3) and three destinations (j = 1,2,3).The unit transportation costs, the availability at each source, demands of each destination are given in the table:

Table I: Unit Transportation Cost c_{ij} in Crisp

j\i	1	2	3
1	8	13	11
2	6	4	7
3	5	3	10

Availability and Demands : a_1=14, a_2=12,a_3=15,b_1=21,b_2=12 and b_3=8

Then the optimum solution is given by

x_{11}=14 , x_{22}=4, x_{23}=8, x_{31}=7, x_{32}=8 and Minimum Z = 243.

B. *Problems with unit transportation costs as rough variable*

Consider a problem with three sources (i= 1,2,3) and three destinations (j = 1,2,3).The unit transportation costs are rough variables, the availability at each source, demands of each destination are crisp numbers which is given in the table:

Table II: Unit Transportation Cost c_{ij} in Rough Variable

j\i	1	2	3
1	([7,9],[6,10])	([12,14],[11,15])	([10,11],8,12])
2	([4,5],[3,6])	([3,4],[2,7])	([5,7],[4,9])
3	([5,6],[3,7])	([2,3],[1,4])	([10,11],[9,12])

Availability and Demands: a_1=14, a_2=12,a_3=15,b_1=21,b_2=12 and b_3=8

Then the optimum solution is given in rough values as Z = ([193,256],[149,297]).

If we take the trust value α = 0.8, then the α-pessimistic optimal solution will be $Z^{/}$=250.56.And the α-optimistic optimal solution will be $Z^{//}$=197.54. So the optimum objective value lie within the range $[Z^{//},Z^{/}]$ i.e, [197.56, 250.56] when the trust value (confidence level) is 0.8.

C. *Compromise Solution*

(*Problems with unit transportation costs as rough variable*)

Consider a problem with three sources (i= 1,2,3) and three destinations (j = 1,2,3).The unit transportation costs are rough variables, the availability at each source, demands of each destination are crisp number and the trust value is 0.8 which is given in the table II.

The cost value c_{ij} in terms of α-optimistic and α-pessimistic values with α=0.8 are calculated and shown in the following table

j/i	1	2	3
1	[7.2, 8.8]	[12.2,13.8]	[9.6,10.88]
2	[4.05, 4.95]	[3.17, 5]	[5.28,7]
3	[4.6, 5.88]	[2.2, 2.95]	[10.05,10.95]

Each cost c_{ij} can be considered as an uncertain variable lies in the interval [p , q], where p is the α-optimistic value and q is the α-pessimistic value of the corresponding uncertain cost. Considering the linear inverse uncertain distribution, the interval of uncertainty is transformed into a crisp value which is shown in the following table.

j/i	1	2	3
1	8.48	13.48	10.624
2	4.77	4.634	6.656
3	5.624	2.8	10.77

The corresponding optimal solution with trust value 0.8 is calculated as:

x_{11}=14, x_{21}=4, x_{23}=8, x_{31}=3, x_{32}=12 and the Minimum Z = 241.52.

The minimum value of Z under compromise solution lies within the range of objective values as obtained by rough model.

VI. CONCLUSION

In this work, the cost of transportation from the source to destination is considered to be uncertain and rough costs are assigned. The availability as well as the demand is considered to be deterministic. Using the uncertainty distribution the compromise solution is also proposed for the problem. Further investigation of the work can be made by considering the availability and demand as uncertain.

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