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Sinusoidal Waveguides as True Time Delay Elements

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Abstract: Sinusoidal bends have been analyzed as elements for True-Time-Delay in Integrated Optics applications. The curve length of a sinusoidal bend as a function of its amplitude and wavelength has been calculated using a piecewise-linear model. The length calculated by this approach has been compared with the theoretically calculated length. The curve length of a sinusoidal bend can be used to determine the phase-shift introduced by it in an Integrated Optic environment.

I. INTRODUCTION

In Integrated Optic (IO) components, light is guided by films deposited on wafer-like substrates. IO technology has gained popularity in the recent times due to advantages like large bandwidth, low power requirements, small dimensions and immunity against Electro-Magnetic-Interference.

An IO waveguide consists of a channel having higher refractive index compared to the surrounding material as in the case of an optical fiber. The refractive index profile is primarily determined by the fabrication process.

A sinusoidal bend (S-bend) waveguide in an integrated optic environment lends itself to several applications. S-bends have been used routinely to launch light into and receive light from Directional Coupler structures. In this paper we propose the possibility of using S-bends as True-Time-Delay (TTD) elements for introducing phase shifts. S-bend structures have been analyzed for their phase shifts in this paper.

The notations for describing the sinusoidal bend have been defined in Section 2. The curve length of an S-bend has been calculated using a piecewise-linear model in Section 3. The theoretical length of an S-bend has been derived in Section 4. Section 5 compares the results obtained in Sections 3 and 4. Conclusions are summarized in Section 6.

II. NOTATIONS FOR DESCRIBING A SINUSOIDAL BEND

As in any TTD element, the length of an S-bend determines the phase shift introduced by it. The "Curve Length (C)" of the S-bend, which is a function of its "Length (L)" and "Amplitude (A)", determines the phase shift introduced in the light wave traveling through it. The parameters "Length (L)" and Amplitude (A)" of the S-bend are defined as shown in the Fig. 1.

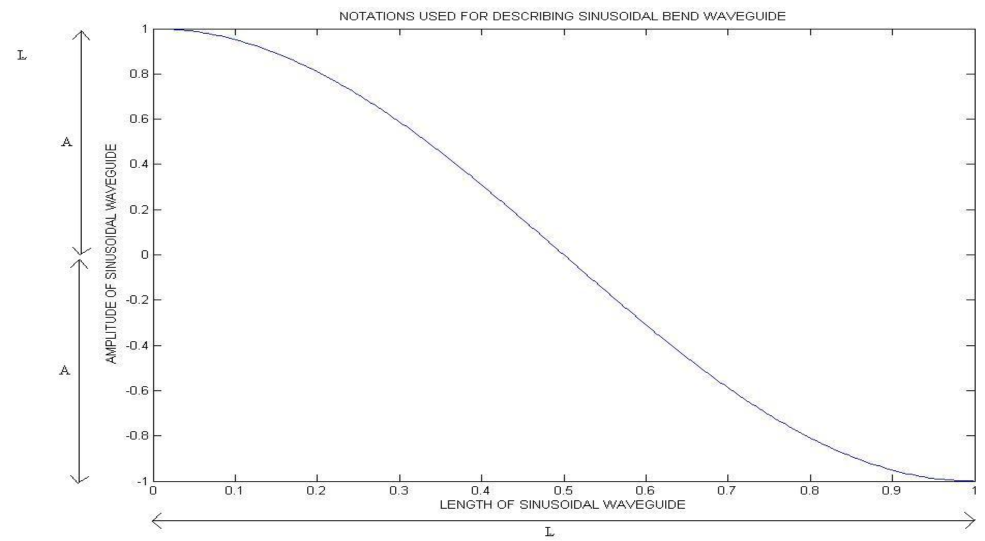

Fig. 1 Notations used for representing a Sinusoidal Bend waveguide

The S-bend shown in Fig. 1 can be mathematically represented by the expression:

$$S(x) = A \cos(\pi x / L) \tag{1}$$

III. Piecewise Linear Approach For Calculating The Curve Length Of Sinusoidal Bend

In this section, we propose an approach for calculating the curve length of an S-bend. In this approach, the S-bend is assumed to be comprised of piecewise linear elements. For illustration purposes, it is assumed that the S-bend is comprised of four linear segments s1, s2, s3 and s4 as shown in Fig. 2.

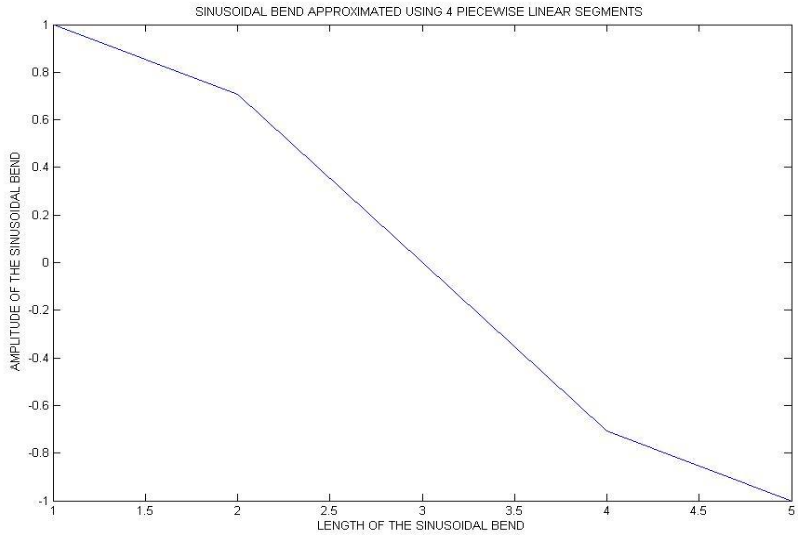

Fig. 2 Piecewise representation of S-bend using four segments

The curve length of the S-bend would then be approximated by the sum of the lengths of the individual segments. From symmetry, it can be seen that:

$C = \Delta l1 + \Delta l2 + \Delta l3 + \Delta l4 = 2(\Delta l1 + \Delta l2)$ (2)

where $\Delta l1$, $\Delta l2$, $\Delta l3$ and $\Delta l4$ are the lengths of segments s1, s2, s3 and s4 respectively.
The co-ordinates for the start and end points of the four segments are as shown in Table 1.

Segment	Co-ordinates of Start Point	Co-ordinates of End Point
s1	$(L / 4, S(z)\|_{z=0})$ $= (0, A\cos(0))$ $= (0, A)$	$(L / 4, S(z)\|_{z=L/4})$ $= (L / 4, A\cos(\pi / 4))$ $= (L / 4, 0.7071A)$
s2	$(L / 4, 0.7071A)$	$(L / 2, S(z)\|_{z=L/2})$ $= (L / 2, A\cos(\pi / 2))$ $= (L / 2, 0)$
s3	$(L / 2, 0)$	$(3L / 4, S(z)\|_{z=3L/4})$ $= (3L/4, A\cos(3\pi / 4))$ $= (3L / 4, -0.7071A)$
s4	$(3L / 4, -0.7071A)$	$(L, S(z)\|_{z=L})$ $= (L, A\cos(\pi))$ $= (L, -A)$

Table 1: Starting and end points of segments comprising the S-bend

The lengths of the individual segments can be calculated from the co-ordinates of their start and end points as follows:

$$\Delta L1 = [(L / 4 - 0)^2 + (0.7071A - A)^2]^{0.5} = [L^2 / 16 + 0.085786\,A^2]^{0.5} \quad (3)$$

$$\Delta L2 = [(L / 2 - L / 4)^2 + (0 - 0.7071A)^2]^{0.5} = [L^2 / 16 + 0.5\,A^2]^{0.5} \quad (4)$$

$$\Delta L3 = [(3L / 4 - L / 2)^2 + (-0.7071A - 0)^2]^{0.5} = [L^2 / 16 + 0.5\,A^2]^{0.5}$$

(5)

$$\Delta L4 = [(L - 3L / 4)^2 + (-A + 0.7071A)^2]^{0.5} = [L^2 / 16 + 0.085786\,A^2]^{0.5} \quad (6)$$

Hence, the approximate length of the S-bend along the curve is given by:

$C \cong \Delta L1 + \Delta L2 + \Delta L3 + \Delta L4$

$= 2\,[(L^2 / 16 + 0.085786\,A^2)^{0.5} + (L^2 / 16 + 0.5\,A^2)^{0.5}]$

$= 2L\,[(0.0625 + 0.085786\,(A / L)^2)^{0.5} + (0.0625 + 0.5\,(A / L)^2)^{0.5}]$

$= 0.5L\,[(1+1.372583\,(A / L)^2)^{0.5} + (1+8\,(A / L)^2)^{0.5}]$ (7)

If both the length 'L' and amplitude 'A' of the S-bend are normalized to a value of 1 unit, then the length of the curve as given by the above formula is 2.2702 units.

Once, the length of a curved waveguide is known, the phase-shift introduced by the waveguide can be determined by the equation:

$\varphi = 2\pi n_{eff} C / \lambda$ (8)

IV. THEORETICAL COMPUTATION OF THE LENGTH OF A SINUSOIDAL BEND

For a given curve, $y = f(x)$, the length of the curve can be estimated by dividing it into infinitesimally small segments of length 'dl' and summing up the lengths of all these segments along the curve.

Let 'dl' be the hypotenuse of a right angled triangle formed with 'dx' and 'dy' being the sides that are perpendicular.

Then,

$dl^2 = dx^2 + dy^2$

$dl = (dx^2 + dy^2)^{0.5}$

$= (1 + (dy / dx)^2)^{0.5}\, dx$ (9)

$l = \int (1 + (dy / dx)^2)^{0.5}\, dx,$ (10)

where the integral is over the length of the curve along the X-axis.

Theoretical Computation of the Length of S-Bend:

The mathematical representation of the S-bend under consideration is given by:

$y = f(x) = A \cos(\pi x / L)$ (11)

$dy / dx = -A\,(\pi / L) \sin(\pi x / L)$ (12)

The length of the S-bend along the curve is given by:

$C = \int (1 + (dy / dx)^2)^{0.5}\, dx$

$= \int (1 + [-A\,(\pi / L) \sin(\pi x / L)]^2)^{0.5}\, dx$

$= \int (1 + (\pi A / L)^2 \sin^2(\pi x / L))^{0.5}\, dx$ (13)

where the integral is from 0 to length 'L' along the X-axis.

The integral can be computed by using numerical methods.

Rectangular Rule for Numerical Integration:

Rectangular rule is obtained if we subdivide the interval of integration $a \le x \le b$ into n subintervals of equal length $h = (b - a) / n$ and in each subinterval approximate the function f by the constant $f(x_j^*)$, the value of f at the midpoint x_j^* of the j^{th} subinterval. Then f is approximated by a step function (piecewise constant function), the n rectangles have the areas $f(x_1^*)h, \ldots, f(x_n^*)h$, and the rectangular rule is:

$\int f(x)\, dx \cong h\,[f(x_1^*) + f(x_2^*) + \ldots + f(x_n^*)]$ (14)

where $h = (b - a) / n$ and the integral is over a to b.

Applying Rectangular Rule to evaluate the integral giving the length of the S-bend along the curve gives:

$C = \int (1 + (\pi A / L)^2 \sin^2(\pi x / L))^{0.5}\, dx$

$= [1 + (\pi A / L)^2 \sin^2(\pi / (2N))]^{0.5} (L / N) + [1 + (\pi A / L)^2 \sin^2(3\pi / (2N))]^{0.5} (L / N) + \ldots + [1 + (\pi A / L)^2 \sin^2((2N-1)\pi / (2N))]^{0.5} (L / N)$

$= (L / N) \{[1 + (\pi A / L)^2 \sin^2(\pi / (2N))]^{0.5} + [1 + (\pi A / L)^2 \sin^2(3\pi / (2N))]^{0.5} + \ldots + [1 + (\pi A / L)^2 \sin^2((2N-1)\pi / (2N))]^{0.5}\}$ (15)

For L = A = 1 and N = 2048, the above formula results in C value of 2.3049.

The above formula also suggests that for given amplitude to length ratio, A / L, the curve length C of an S-bend varies linearly with the length L.

The above formula can be can also be re-arranged in the following form:

$(C / L) = (1 / N) \{[1 + (\pi A / L)^2 \sin^2(\pi / (2N))]^{0.5} + [1 + (\pi A / L)^2 \sin^2(3\pi / (2N))]^{0.5} + \ldots + [1 + (\pi A / L)^2 \sin^2((2N-1)\pi / (2N))]^{0.5}\}$ (16)

Fig. 3 illustrates the variation of (C / L) as a function of (A / L).

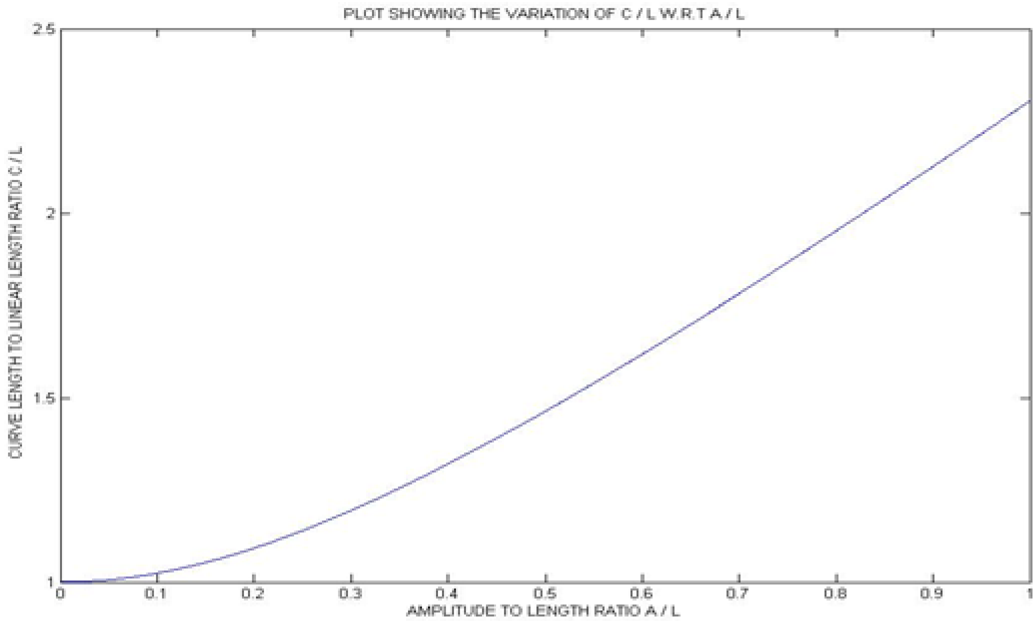

Fig. 3 Variation of C / L as a function of A / L

V. COMPARISON OF RESULTS

The curve length of an S-bend with amplitude 'A' and length 'L' being 1 unit is computed using piecewise- linear approach and rectangular rule and the results are compared.

Applying the rectangular rule with n = 2048 results in C = 2.3049 as shown in the previous section. This result is compared with the results obtained using piecewise linear approach for different number of linear segments in Table 2.

Number of linear segments	**Curve length '$C_{PW\ Lin}$' in chosen units**	**Error = C_{rect} - $C_{PW\ Lin}$ in chosen units**	**% Error = (Error / C_{rect}) *100**
4	2.2702	0.0347	1.5067
8	2.2937	0.0112	0.4867
16	2.3020	0.0029	0.1253
32	2.3042	7.2255×10^{-4}	0.0313
64	2.3047	1.8066×10^{-4}	0.0078
128	2.3048	4.5166×10^{-5}	0.0020
256	2.3049	1.1292×10^{-5}	4.8990×10^{-4}
512	2.3049	2.8229×10^{-6}	1.2247×10^{-4}
1024	2.3049	7.0573×10^{-7}	3.0619×10^{-5}
2048	2.3049	1.7643×10^{-7}	7.6547×10^{-6}

Table 2 Comparison of Sinusoidal Bend curve length using two different approaches

From the table it can be seen that the results obtained by the Piecewise Linear approach matches the theoretical results computed using the Rectangular Rule as 'n' increases.

VI. CONCLUSION

We have presented a piecewise linear approach to calculate the curve length of a sinusoidal bend. We have also derived the theoretical length of a sinusoidal bend. The results obtained by these two approaches have been compared. The approach presented here can be used to calculate the curve length of any S-bend given its amplitude and length. Desired differential phase-shifts required for any given application can be generated by appropriately choosing the lengths of the curved waveguides using the approach presented in this paper.