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Alternating direction explicit and implicit methods for Schnackenberg model

Fadhil H. Easif
Department of Mathematics, Faculty of Science, University of Zakho, Duhok, Kurdistan Region, Iraq

Abstract:- alternating direction explicit and alternating direction implicit methods (ADE and ADI) were used to solve Schnackenberg model, we were found that alternating direction implicit method is much more accurate and faster than alternating direction explicit in this kind of models.

Keywords:- Schnackenberg model, ADE method, ADI method.

I. INTRODUCTION

Reaction-diffusion (RD) systems arise frequently in the study of chemical and biological phenomena and are naturally modeled by parabolic partial differential equations (PDEs). The dynamics of RD systems has been the subject of intense research activity over the past decades. The reason is that RD system exhibit very rich dynamic behavior including periodic and quasi-periodic solutions [6].

Various orders are self-organized far from the chemical equilibrium. The theoretical procedures and notions to describe the dynamics of patterns formation have been developed for the last three decades [4]. Attempts have also been made to understand morphological orders in biology [5]. Clarification of the mechanisms of the formation of orders and the relationship among them has been one of the fundamental problems in non-equilibrium statistical physics [3].

I.I. MATHEMATICAL MODEL

A general class of nonlinear-diffusion system is in the form

$$\left.\begin{aligned} \frac{\partial u}{\partial t} &= d_1\Delta u + a_1 u + b_1 v + f(u,v) + g_1(x) \\ \frac{\partial v}{\partial t} &= d_2\Delta u + a_2 u + b_2 v - f(u,v) + g_2(x) \end{aligned}\right\} \qquad (1)$$

With homogenous Dirchlet or Neumann boundary condition on a bounded domain Ω , n≤3, with locally Lipschitz continuous boundary. It is well known that reaction and diffusion of chemical or biochemical species can produce a variety of spatial patterns. This class of reaction diffusion systems includes some significant pattern formation equations arising from the modeling of kinetics of chemical or biochemical reactions and from the biological pattern formation theory.

Schnackenberg model:

$$a_1 = -k, b_1 = a_2 = b_2 = 0, f = u^2 v, g_1 = a, g_2 = b,$$

where *k*, *a* and *b* are positive constants.

Then one obtains the following system of two nonlinearly coupled reaction-diffusion equations (the Schnakenberg model),

$$\left.\begin{aligned} &\frac{\partial u}{\partial t} = d_1\Delta u - ku + u^2 v + a, \quad (t,x) \in (0,\infty)\times\Omega \\ &\frac{\partial v}{\partial t} = d_2\Delta v - u^2 v + b, \qquad (t,x) \in (0,\infty)\times\Omega \\ &\quad u(t,x) = v(t,x) = 0, \quad t>0, \quad x\epsilon\partial\Omega \\ &u(0,x) = u_0(x), \quad v(0,x) = v_0(x), \quad x\epsilon\Omega \end{aligned}\right\}$$

Where $d_1, d_2, a, k\ and\ b$ are positive constants [8]. Various finite difference algorithms or schemes have been presented for the solution of hyperbolic-parabolic problem or its simpler derivatives, such as the classical diffusion equation. It is well-known that many of these schemes are partially unsatisfactory due to the formation of oscillations and numerical diffusion within the solution [1, 7].

Solution by the finite difference method, although more general, will involve stability and convergence problems, may require special handling of boundary conditions, and may require large computer storage and execution time. The problem of numerical dispersion for finite difference solutions is also difficult to overcome [2].

II. MATERIALS AND METHODS

II.1. Derivation of alternating direction explicit (ADE) for schnackenberg model:
The two dimensional Schnakenberg model is given by

$$\left.\begin{aligned} \frac{\partial u}{\partial t} &= d_1\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-ku+u^2v+a, \qquad (t,x)\in(0,\infty)\times\Omega \\ \frac{\partial v}{\partial t} &= d_2\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-u^2v+b, \qquad (t,x)\in(0,\infty)\times\Omega \end{aligned}\right\} \qquad (2)$$

We consider a square region $0\le x\le 1$, $0\le y\le 1$ and u, v are known at all points within and on the boundary of the square region. We draw lines parallel to x, y, t–axis as $x=ih$, $y=jk$, and $t=nz$, $i,j=0, 1, 2,\ldots,M$ and $n=0, 1, 2,\ldots, N$, where $h=\delta x$, $k=\delta y$, $z=\delta t$.

The explicit finite difference approximation (ADE) to shnackenberg model in two-dimensions are given by:

$$\frac{U_{i,j,n+1}-U_{i,j,n}}{\delta t} = d_1\left(\frac{U_{i-1,j,n}-2U_{i,j,n}+U_{i+1,j,n}}{h^2}\right)+d_1\left(\frac{U_{i,j-1,n}-2U_{i,j,n}+U_{i,j+1,n}}{k^2}\right)-kU_{i,j,n}+\left(U_{i,j,n}\right)^2V_{i,j,n} + a$$

$$\frac{V_{i,j,n+1}-V_{i,j,n}}{\delta t} = d_2\left(\frac{V_{i-1,j,n}-2V_{i,j,n}+V_{i+1,j,n}}{h^2}\right)+d_2\left(\frac{V_{i,j-1,n}-2V_{i,j,n}+V_{i,j+1,n}}{k^2}\right)-\left(U_{i,j,n}\right)^2V_{i,j,n}+b$$

Multiplying both equations by δt and set $h=k$, then we have a rectangular region and replacing $\frac{d_1\delta t}{h^2}=r_1$ and $\frac{d_2\delta t}{h^2}=r_2$,then we get

$$U_{i,j,n+1}-U_{i,j,n} = r_1\left(U_{i-1,j,n}-2U_{i,j,n}+U_{i+1,j,n}+U_{i,j-1,n}-2U_{i,j,n}+U_{i,j+1,n}\right)-k\delta t\,U_{i,j,n}+\delta t\left(U_{i,j,n}\right)^2V_{i,j,n}+a\delta t$$

and

$$V_{i,j,n+1}-V_{i,j,n} = r_2\left(V_{i-1,j,n}-2V_{i,j,n}+V_{i+1,j,n}+V_{i,j-1,n}-2V_{i,j,n}+V_{i,j+1,n}\right)-\delta t\left(U_{i,j,n}\right)^2V_{i,j,n}+\delta t\,b$$

Then simplifying the system to obtain

$$U_{i,j,n+1} = (1-4r_1-k\delta t)U_{i,j,n}+r_1\left(U_{i-1,j,n}+U_{i+1,j,n}+U_{i,j-1,n}+U_{i,j+1,n}\right)+\delta t\left(U_{i,j,n}\right)^2V_{i,j,n}+a\delta t$$

$$V_{i,j,n+1} = (1-4r_2)V_{i,j,n}+r_2\left(V_{i-1,j,n}+V_{i+1,j,n}+V_{i,j-1,n}+V_{i,j+1,n}\right)-\delta t\left(U_{i,j,n}\right)^2V_{i,j,n}+\delta t\,b$$

This is the alternating direction explicit formula for the Schnakenberg model

II.2. Derivation of alternating direction implicit (ADI) for schnackenberg model:
In the ADI approach, the finite difference equations are written in terms of quantities at two x levels. However, two different finite difference approximations are used alternately, one to advance the calculations from the plane *n* to a plane *n+1*, and the second to advance the calculations from (*n+1*)-plane to the (*n+2*)-plane by replacing $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 v}{\partial x^2}$ by implicit finite difference approximation [5]. we get

$$\frac{U_{i,j,n+1}-U_{i,j,n}}{\delta t} = d_1\left(\frac{U_{i-1,j,n+1}-2U_{i,j,n+1}+U_{i+1,j,n+1}}{h^2}\right)+d_1\left(\frac{U_{i,j-1,n}-2U_{i,j,n}+U_{i,j+1,n}}{k^2}\right)-kU_{i,j,n} + \left(U_{i,j,n}\right)^2V_{i,j,n}+a$$

$$\frac{V_{i,j,n+1}-V_{i,j,n}}{\delta t} = d_2\left(\frac{V_{i-1,j,n+1}-2V_{i,j,n+1}+V_{i+1,j,n+1}}{h^2}\right)+d_2\left(\frac{V_{i,j-1,n}-2V_{i,j,n}+V_{i,j+1,n}}{k^2}\right)-\left(U_{i,j,n}\right)^2V_{i,j,n}+b$$

we set $h=k$, then we have a rectangular region and multiplying the equations by δt and replacing $\frac{d_1\delta t}{h^2}=r_1$ and $\frac{d_2\delta t}{h^2}=r_2$, then we get

$$U_{i,j,n+1}-U_{i,j,n} = r_1\left(U_{i-1,j,n+1}-2U_{i,j,n+1}+U_{i+1,j,n+1}+U_{i,j-1,n}-2U_{i,j,n}+U_{i,j+1,n}\right)-k\delta t\,U_{i,j,n} + \delta t\left(U_{i,j,n}\right)^2V_{i,j,n}+\delta t\,a$$

$$V_{i,j,n+1}-V_{i,j,n} = r_2\left(V_{i-1,j,n+1}-2V_{i,j,n+1}+V_{i+1,j,n+1}+V_{i,j-1,n}-2V_{i,j,n}+V_{i,j+1,n}\right)-\delta t\left(U_{i,j,n}\right)^2V_{i,j,n}+\delta t\,b$$

$$-r_2 V_{i-1,j,n+1} + (1-2r_2)V_{i,j,n+1} - r_2 V_{i+1,j+1,n+1}$$
$$= r_2 V_{i,j-1,n} + (1-2r_2)V_{i,j,n} + r_2 V_{i,j+1,n} - \delta t\,(U_{i,j,n})^2 V_{i,j,n} + \delta t\, b$$
$$-r_1 U_{i-1,j,n+1} + (1-2r_1)U_{i,j,n+1} - rU_{i+1,j,n+1} = (1-2r_1)U_{i,j,n} + r_1 U_{i,j+1,n} + \delta t\, a + \delta t\,(U_{i,j,n})^2 V_{i,j,n}$$

or

$$-r_1 U_{i-1,j,n+1} + (1-2r_1)U_{i,j,n+1} - r_1 U_{i+1,j,n+1}$$
$$= (1-2r_1)U_{i,j,n} + r_1 U_{i,j+1,n} + \delta t\,(U_{i,j,n})^2 V_{i,j,n} + \delta t\,(a - kU_{i,j,n})$$

and

$$-r_2 V_{i-1,j,n+1} + (1-2r_2)V_{i,j,n+1} - r_2 V_{i+1,j+1,n+1}$$
$$= r_2 V_{i,j-1,n} + (1-2r_2)V_{i,j,n} + r_2 V_{i,j+1,n} - \delta t\,(U_{i,j,n})^2 V_{i,j,n} + \delta t\, bt$$

Advance the solution from the (n+1)th plane to (n+2)th plane by replacing $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 v}{\partial x^2}$ with explicit finite difference approximation at (n+1)th plane then $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 v}{\partial x^2}$ by an implicit finite approximation at the (n+2)th plane

$$\frac{U_{i,j,n+2} - U_{i,j,n+1}}{\delta t} = d_1\left(\frac{U_{i-1,j,n+1} - 2U_{i,j,n+1} + U_{i+1,j,n+1}}{h^2}\right) + d_1\left(\frac{U_{i,j-1,n+2} - 2U_{i,j,n+2} + U_{i,j+1,n+2}}{k^2}\right)$$
$$-kU_{i,j,n} + (U_{i,j,n})^2 V_{i,j,n} + a$$

and

$$\frac{V_{i,j,n+2} - V_{i,j,n+1}}{\delta t} = d_2\left(\frac{V_{i-1,j,n+1} - 2V_{i,j,n+1} + V_{i+1,j,n+1}}{h^2}\right) + d_2\left(\frac{V_{i,j-1,n+2} - 2V_{i,j,n+2} + V_{i,j+1,n+2}}{k^2}\right) - (U_{i,j,n})^2 V_{i,j,n} + b$$

Multiplying the equations by δt and replacing $r_1 = \frac{d_1 \delta t}{h^2}$ and $r_2 = \frac{d_2 \delta t}{h^2}$ when $h = k$, then we have

$$U_{i,j,n+2} - U_{i,j,n+1} = r_1\left(U_{i-1,j,n+1} - 2U_{i,j,n+1} + U_{i+1,j,n+1}\right) + r_1\left(U_{i,j-1,n+2} - 2U_{i,j,n+2} + U_{i,j+1,n+2}\right)$$
$$- k\delta t\, U_{i,j,n} \quad + \delta t\,(U_{i,j,n})^2 V_{i,j,n} + \delta t\, a$$

and

$$V_{i,j,n+2} - V_{i,j,n+1} = r_2\left(V_{i-1,j,n+1} - 2V_{i,j,n+1} + V_{i+1,j,n+1}\right) + r_2\left(V_{i,j-1,n+2} - 2V_{i,j,n+2} + V_{i,j+1,n+2}\right)$$
$$- \delta t\,(U_{i,j,n})^2 V_{i,j,n} + \delta t\, b$$

and this implies that

$$-r_1 U_{i,j-1,n+2} + (1+2r_1)U_{i,j,n+2} - r_1 U_{i,j+1,n+2}$$
$$= r_1 U_{i-1,j,n} + (1-2r_1)U_{i,j,+1\,n} + r_1 U_{i+1,j+,n+1} + \delta t\,(a - kU_{i,j,n}) + (U_{i,j,n})^2 V_{i,j,n}$$

and

$$-r_2 V_{i,j-1,n+2} + (1-2r_2)V_{i,j,n+2} - r_2 V_{i,j+1,n+2}$$
$$= r_2 V_{i-1,j,n+1} + (1-2r_2)V_{i,j,n+1} + r_2 V_{i+1,j,n+1} - \delta t\,(U_{i,j,n})^2 V_{i,j,n} + \delta t\, b$$

Then we get two systems

$$-r_1 U_{i,j-1,n+2} + (1+2r_1)U_{i,j,n+2} - r_1 U_{i,j+1,n+2}$$
$$= r_1 U_{i-1,j,n} + (1-2r_1)U_{i,j,+1\,n} + r_1 U_{i+1,j+,n+1} + \delta t\,(a - kU_{i,j,n}) + (U_{i,j,n})^2 V_{i,j,n}$$
$$-r_2 V_{i,j-1,n+2} + (1-2r_2)V_{i,j,n+2} - r_2 V_{i,j+1,n+2}$$
$$= r_2 V_{i-1,j,n+1} + (1-2r_2)V_{i,j,n+1} + r_2 V_{i+1,j,n+1} - \delta t\,(U_{i,j,n})^2 V_{i,j,n} + \delta t\, b$$

and

$$-r_1 U_{i-1,j,n+1} + (1-2r_1)U_{i,j,n+1} - r_1 U_{i+1,j,n+1}$$
$$= (1-2r_1)U_{i,j,n} + r_1 U_{i,j+1,n} + \delta t\,(U_{i,j,n})^2 V_{i,j,n} + \delta t\,(a - kU_{i,j,n})$$
$$-r_2 V_{i-1,j,n+1} + (1-2r_2)V_{i,j,n+1} - r_2 V_{i+1,j+1,n+1}$$
$$= r_2 V_{i,j-1,n} + (1-2r_2)V_{i,j,n} + r_2 V_{i,j+1,n} - \delta t\,(U_{i,j,n})^2 V_{i,j,n} + \delta t\, b$$

The last two systems represent Alternating Direction implicit method under the conditions

$$u_{1,q,n+1} = u_{1,q+1,n+1} = 0$$
$$u_{M,q,n+1} = u_{M,q+1,n+1} = 0.$$

and

$$v_{1,q,n+1} = v_{1,q+1,n+1} = 0$$
$$v_{M,q,n+1} = v_{M,q+1,n+1} = 0.$$

The tridiagonal matrices for the system in the level n advanced to the level n+1, for both u and v can be formulated as follows $AU=B$.

$$\begin{bmatrix} (1+2r_1) & -r_1 & 0 & 0 & 0 & . & . & . & 0 \\ -r_1 & (1+2r_1) & -r_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -r_1 & (1+2r_1) & -r_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -r_1 & (1+2r_1) & -r_1 & & & & \\ . & 0 & 0 & -r_1 & . & . & & & \\ . & . & 0 & 0 & . & . & . & & \\ . & . & . & 0 & 0 & . & . & . & \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & -r_1 & (1+2r_1) & -r_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_1 & (1+2r_1) \end{bmatrix} \begin{bmatrix} u_{2,q,n+1} \\ u_{3,q,n+1} \\ u_{4,q,n+1} \\ u_{5,q+1} \\ . \\ . \\ . \\ \\ u_{n-3,q+1} \\ u_{n-2,q+1} \\ u_{M-1,q,n+1} \end{bmatrix} =$$

$$\begin{bmatrix} az + r_2(u_{2,q+1,n} + u_{2,q-1,n}) + (1-2r_2 - z)u_{2,q,n} + zu^2_{2,q,n}v_{2,q,n} \\ az + r_2(u_{3,q+1,n} + u_{3,q-1,n}) + (1-2r_2 - z)u_{3,q,n} + zu^2_{3,q,n}v_{3,q,n} \\ az + r_2(u_{4,q+1,n} + u_{4,q-1,n}) + (1-2r_2 - z)u_{4,q,n} + zu^2_{4,q,n}v_{4,q,n} \\ \\ . \\ . \\ . \\ . \\ \\ az + r_2(u_{M-1,q+1,n} + u_{M-1,q-1,n}) + (1-2r_2 - z)u_{M-1,q,n} + zu^2_{M-1,q,n}v_{M-1,q,n} \end{bmatrix}$$

$$\begin{bmatrix} (1+2r_1) & -r_1 & 0 & 0 & 0 & . & . & . & 0 \\ -r_1 & (1+2r_1) & -r_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -r_1 & (1+2r_1) & -r_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -r_1 & (1+2r_1) & -r_1 & & & & \\ . & 0 & 0 & -r_1 & . & . & & & \\ . & . & 0 & 0 & . & . & . & & \\ . & . & . & 0 & 0 & . & . & . & \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & -r_1 & (1+2r_1) & -r_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_1 & (1+2r_1) \end{bmatrix} \begin{bmatrix} v_{2,q,n+1} \\ v_{3,q,n+1} \\ v_{4,q,n+1} \\ v_{5,q,n+1} \\ . \\ . \\ . \\ \\ v_{M-3,q,n+1} \\ v_{M-2,q,n+1} \\ v_{M-1,q,n+1} \end{bmatrix} =$$

$$\begin{bmatrix} r_2(v_{2,q+1,n}+v_{2,q-1,n})+(1-2r_2-zK)v_{2,q,n}-zu^2_{2,q,n}v_{2,q,n} \\ r_2(v_{3,q+1,n}+v_{3,q-1,n})+(1-2r_2-zK)v_{3,q,n}-zu^2_{3,q,n}v_{3,q,n} \\ r_2(v_{4,q+1,n}+v_{4,q-1,n})+(1-2r_2-zK)v_{4,q,n}-zu^2_{4,q,n}v_{4,q,n} \\ . \\ . \\ . \\ . \\ r_2(v_{M-1,q+1,n}+v_{M-1,q-1,n})+(1-2r_2-zK)v_{M-1,q,n}-zu^2_{M-1,q,n}v_{M-1,q,n} \end{bmatrix}$$

And the tridiagonal matrices for the system in level n+1 advanced to level n+2 for both u and v are given by AU=B.

$$\begin{bmatrix} (1+2r_2) & -r_2 & 0 & 0 & 0 & . & . & . & 0 \\ -r_2 & (1+2r_2) & -r_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -r_1 & (1+2r_1) & -r_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -r_2 & (1+2r_2) & -r_2 & & & & \\ . & 0 & 0 & -r_2 & . & . & & & \\ . & . & 0 & 0 & . & . & . & & \\ . & . & . & 0 & 0 & . & . & . & \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & -r_2 & (1+2r_2) & -r_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_2 & (1+2r_2) \end{bmatrix} \begin{bmatrix} u_{p,2,n+2} \\ u_{p,3,n+2} \\ u_{p,4,n+2} \\ u_{p,5,n+2} \\ . \\ . \\ . \\ \\ u_{p,M-3,n+2} \\ u_{p,M-2,n+2} \\ u_{p,M-1,n+2} \end{bmatrix} =$$

$$\begin{bmatrix} \rho z+r_1(u_{p+1,2,n+1}+u_{p-1,2,n+1})+(1-2r_1-z)u_{p,2,n+1}-zu_{p,2,n}+zu^2_{p,2,n}v_{p,2,n} \\ \rho z+r_1(u_{p+1,3,n+1}+u_{p-1,3,n+1})+(1-2r_1-z)u_{p,3,n+1}-zu_{p,3,n}+zu^2_{p,3,n}v_{p,3,n} \\ \rho z+r_1(u_{p+1,4,n+1}+u_{p-1,4,n+1})+(1-2r_1-z)u_{p,4,n+1}-zu_{p,4,n}+zu^2_{p,4,n}v_{p,4,n} \\ \rho z+r_1(u_{p+1,5,n+1}+u_{p-1,5,n+1})+(1-2r_1-z)u_{p,5,n+1}-zu_{p,5,n}+zu^2_{p,5,n}v_{p,5,n} \\ . \\ . \\ . \\ . \\ \rho z+r_1(u_{p+1,M-1,n+1}+u_{p-1,M-1,n+1})+(1-2r_1-z)u_{p,M-1,n+1}-zu_{pM-1,2,n}+zu^2_{p,2,n}v_{pM-1,n+1} \end{bmatrix}$$

Also for AV=B the tridiagonal is in the form

$$\begin{bmatrix} (1+2m_2) & -m_2 & 0 & 0 & 0 & . & . & . & 0 \\ -m_2 & (1+2m_2) & -m_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -m_2 & (1+2m_2) & -m_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -m_2 & (1+2m_2) & -m_2 & & & & \\ . & 0 & 0 & -m_1 & . & . & & & \\ . & . & 0 & 0 & . & . & . & & \\ . & . & . & 0 & 0 & . & . & . & \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & -m_2 & (1+2m_2) & -m_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -m_2 & (1+2m_2) \end{bmatrix} \begin{bmatrix} v_{p,2,n+2} \\ v_{p,3,n+2} \\ v_{p,4,n+2} \\ v_{p,5,n+2} \\ . \\ . \\ . \\ \\ v_{p,M-3,n+2} \\ v_{p,M-2,n+2} \\ v_{p,M-1,n+2} \end{bmatrix} =$$

$$\begin{bmatrix} m_1(v_{p+1,2,n+1}+v_{p-1,2,n+1})+(1-2m_1-zK)v_{p,2,n+1}+zu_{p,2,n}-zu^2_{p,2,n}v_{p,2,n} \\ m_1(v_{p+1,3,n+1}+v_{p-1,3,n+1})+(1-2m_1-zK)v_{p,3,n+1}+zu_{p,3,n}-zu^2_{p,3,n}v_{p,3,n} \\ m_1(v_{p+1,4,n+1}+v_{p-1,4,n+1})+(1-2m_1-zK)v_{p,4,n+1}+zu_{p,4,n}-zu^2_{p,4,n}v_{p,4,n} \\ . \\ . \\ . \\ . \\ m_1(v_{p+1,M-1,n+1}+v_{p-1,M-1,n+1})+(1-2m_1-zK)v_{p,M-1,n+1}+zu_{p,M-1,n}-zu^2_{p,M-1,n}v_{p,M-1,n} \end{bmatrix}$$

III. APPLICATION (NUMERICAL EXAMPLE)

Example: We solved the following example numerically to illustrate the efficiency of the presented methods, suppose we have the system

$$\frac{\partial u}{\partial t} = 1/4\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) - 2u + u^2 v + a,$$

$$\frac{\partial v}{\partial t} = 1/4\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) - u^2 v + u + b,$$

We the initial conditions

$u(x,y,0), v(x,y,0)) = (\exp(-x-y), \exp(x+y))$

$u(0,y,t), v(0,y,t)) = (\exp(-t/2-y), \exp(t/2+y))$

$u(x,0,t), v(x,0,t)) = (\exp(-t/2-x), \exp(t/2+x))$

$(u,v)) = \left(\exp\left(-\frac{t}{2}-x-y\right), \exp\left(\frac{t}{2}+x+y\right)\right)$ is the exact solution of the problem.

Where a=b=0.1.

Then the results in more details are shown in following table and figure:

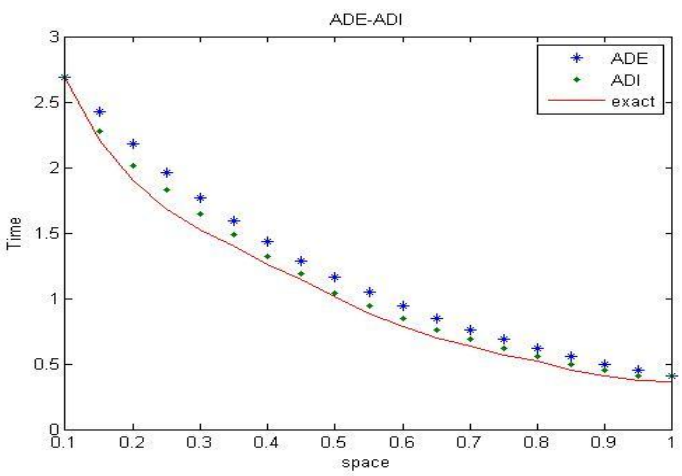

Fig. 1: Comparison of ADE, ADI with Exact solution (u).

Table 1: Comparison of ADE and ADI with Exact solution (u).

x	ADE	ADI	Exact
0.1	2.69265	2.6912344	2.6912344
0.15	2.423622	2.27384012	2.20573840
0.2	2.181472	2.01678803	1.8971263
0.25	1.963516	1.8279602	1.6823796
0.3	1.767336	1.64066	1.52614066
0.35	1.590757	1.4899208	1.401599208
0.4	1.431821	1.31680	1.254730168
0.45	1.288764	1.18920286	1.14620286
0.5	1.160001	1.0435533	1.011435533
0.55	1.044102	0.939289	0.879392894
0.6	0.939783	0.8454428	0.78454428
0.65	0.845887	0.7609726	0.697609726
0.7	0.761373	0.6849420	0.635849420
0.75	0.685302	0.6165078	0.568960165
0.8	0.616832	0.5549110	0.52549110
0.85	0.5552032	0.4994686	0.44994686
0.9	0.4997315	0.4495655	0.40495655
0.95	0.4498021	0.4046483	0.374046483
1	0.4048613	0.403642189	0.363642189

IV. CONCLUSION

We saw that alternating direction implicit is more accurate than alternating direction explicit method for solving Schnakenberg model.

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